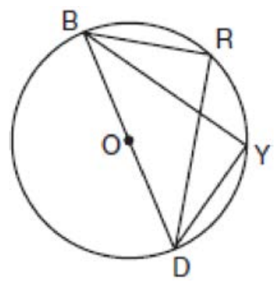
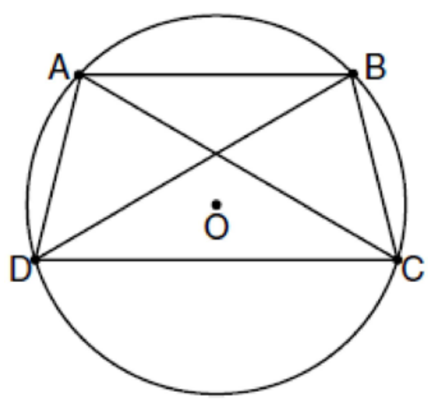
G.SRT.B.5: Circle Proofs

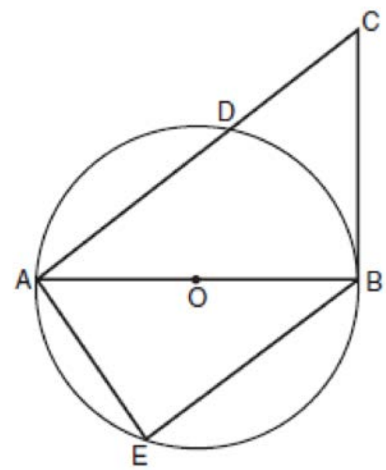
1 In the accompanying diagram, $m\overset{\frown}{BR} = 70$, $m\overset{\frown}{YD} = 70$, and $\overline{BOD}$ is the diameter of circle O. Write an explanation or a proof that shows $\triangle RBD$ and $\triangle YBD$ are congruent.



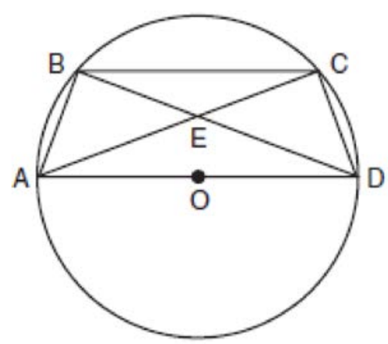
2 In the diagram below, quadrilateral $ABCD$ is inscribed in circle O, $\overline{AB} \parallel \overline{DC}$, and diagonals $\overline{AC}$ and $\overline{BD}$ are drawn. Prove that $\triangle ACD \cong \triangle BDC$.



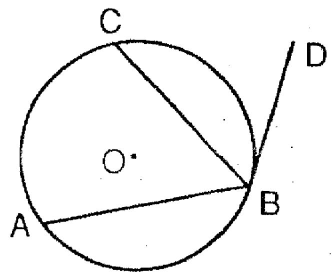
3 In the accompanying diagram of circle O, diameter $\overline{AOB}$ is drawn, tangent $\overline{CB}$ is drawn to the circle at B, E is a point on the circle, and $\overline{BE} \parallel \overline{ADC}$.
Prove: $\triangle ABE \sim \triangle CAB$



4 In the accompanying diagram of circle O, $\overline{AD}$ is a diameter with $\overline{AD}$ parallel to chord $\overline{BC}$, chords $\overline{AB}$ and $\overline{CD}$ are drawn, and chords $\overline{BD}$ and $\overline{AC}$ intersect at E.
Prove: $\overline{BE} \cong \overline{CE}$

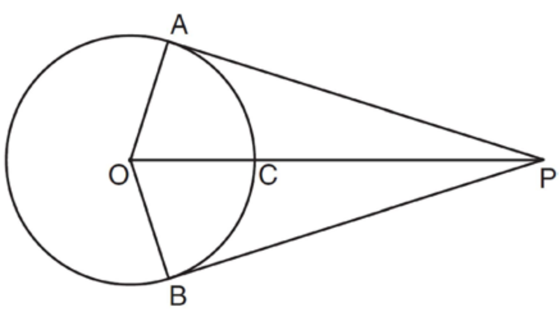


5 Given: circle O, $\overline{DB}$ is tangent to the circle at B, $\overline{BC}$ and $\overline{BA}$ are chords, and C is the midpoint of $\overset{\frown}{AB}$.

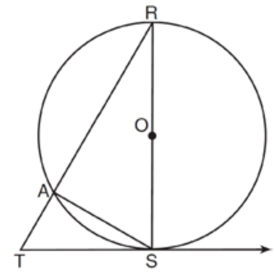


Prove: $\angle ABC \cong \angle CBD$

6 In the diagram below, $\overline{PA}$ and $\overline{PB}$ are tangent to circle O, $\overline{OA}$ and $\overline{OB}$ are radii, and $\overline{OP}$ intersects the circle at C. Prove: $\angle AOP \cong \angle BOP$



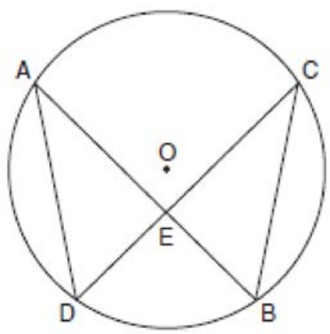
7 In the diagram of circle O below, diameter $\overline{RS}$, chord $\overline{AS}$, tangent $\overrightarrow{TS}$, and secant $\overline{TAR}$ are drawn.



Complete the following proof to show $(RS)^2 = RA \cdot RT$

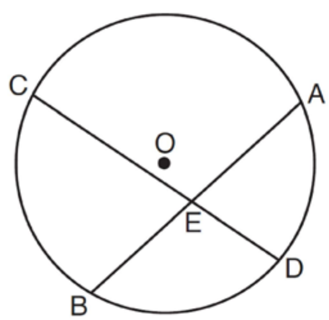
Statements	Reasons
1. circle O, diameter $\overline{RS}$, chord $\overline{AS}$, tangent $\overrightarrow{TS}$, and secant $\overline{TAR}$	1. Given
2. $\overline{RS} \perp \overrightarrow{TS}$	2. ______________________
3. $\angle RST$ is a right angle	3. $\perp$ lines form right angles
4. $\angle RAS$ is a right angle	4. ______________________
5. $\angle RST \cong \angle RAS$	5. ______________________
6. $\angle R \cong \angle R$	6. Reflexive property
7. $\triangle RST \sim \triangle RAS$	7. ______________________
8. $\frac{RS}{RA} = \frac{RT}{RS}$	8. ______________________
9. $(RS)^2 = RA \cdot RT$	9. ______________________

8 Given: chords $\overline{AB}$ and $\overline{CD}$ of circle O intersect at E, an interior point of circle O; chords $\overline{AD}$ and $\overline{CB}$ are drawn.



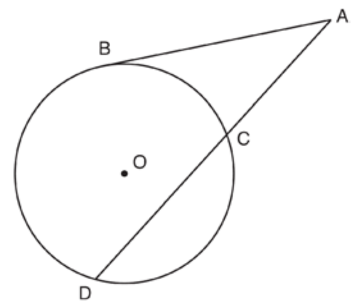
Prove: $(AE)(EB) = (CE)(ED)$

9 Given: Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E



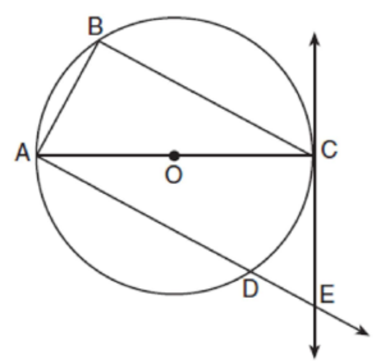
Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord. Prove this theorem by proving $AE \cdot EB = CE \cdot ED$.

10 In the diagram below, secant $\overline{ACD}$ and tangent $\overline{AB}$ are drawn from external point A to circle O.



Prove the theorem: If a secant and a tangent are drawn to a circle from an external point, the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared. ($AC \cdot AD = AB^2$)

11 In the diagram below of circle O, tangent $\overleftrightarrow{EC}$ is drawn to diameter $\overline{AC}$. Chord $\overline{BC}$ is parallel to secant $\overline{ADE}$, and chord $\overline{AB}$ is drawn.



Prove: $\dfrac{BC}{CA} = \dfrac{AB}{EC}$

G.SRT.B.5: Circle Proofs
Answer Section

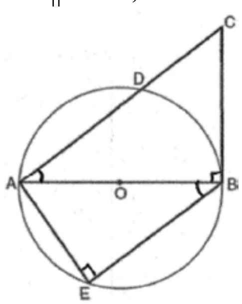
1 ANS:
The measure of an inscribed angle is half that of its intercepted arc. Therefore $m\angle RDB = m\angle YDB = 35$. Because $\angle BRD$ and $\angle DYB$ intercept a semicircle, they are both right angles. $\overline{BD} \cong \overline{DB}$ from the reflexive property. Therefore $\triangle RBD \cong \triangle YDB$ because of AAS. Alternatively, because congruent chords intersect congruent arcs, $\overline{BR} \cong \overline{YD}$ and HL applies.

REF: 010732b

2 ANS:
Because $\overline{AB} \parallel \overline{DC}$, $\overset{\frown}{AD} \cong \overset{\frown}{BC}$ since parallel chords intersect congruent arcs. $\angle BDC \cong \angle ACD$ because inscribed angles that intercept congruent arcs are congruent. $\overline{AD} \cong \overline{BC}$ since congruent chords intersect congruent arcs. $\angle DAC \cong \angle DBC$ because inscribed angles that intercept the same arc are congruent. Therefore, $\triangle ACD \cong \triangle BDC$ because of AAS.

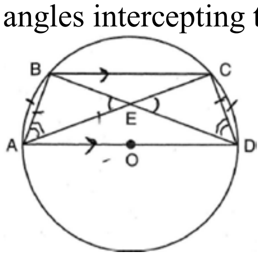
REF: fall0838ge

3 ANS:
$\angle ABC$, the angle formed by tangent $\overline{CB}$ and diameter $\overline{AOB}$ is a right angle. The measure of an inscribed angle is half that of its intercepted arc. Because $\angle BEA$ intercepts a semicircle, $\angle BEA$ is also a right angle. Since $\overline{BE} \parallel \overline{ADC}$, $\angle CAB$ and $\angle ABE$ are alternate interior angles and congruent. Therefore $\triangle ABE \sim \triangle CAB$ by AA.



REF: 080627b

4 ANS:
$\overset{\frown}{AB} \cong \overset{\frown}{DC}$ because parallel lines intercept congruent arcs. $\overline{AB} \cong \overline{DC}$ because congruent chords intercept congruent arcs. $\angle BEA$ and $\angle CED$ are congruent vertical angles. $\angle BAC$ and $\angle CDB$ are congruent inscribed angles intercepting the same arc. $\triangle BAE \cong \triangle CDE$ because of AAS. $\overline{BE} \cong \overline{CE}$ because of CPCTC.



REF: 060934b

5 ANS:
$\overset{\frown}{AC} \cong \overset{\frown}{CB}$ because of the definition of midpoint; $\angle ABC \cong \frac{1}{2}\overset{\frown}{AC}$ as the measure of an inscribed angle is one-half the measure of its intercepted arc; $\angle CBD \cong \frac{1}{2}\overset{\frown}{BC}$ as the measure of an angle formed by a tangent and a chord that intersect at the point of tangency is one-half the measure of the intercepted arc; $\frac{1}{2}\overset{\frown}{AC} \cong \frac{1}{2}\overset{\frown}{CB}$ because of the multiplication property of equalities; and $\angle ABC \cong \angle CBD$ because of substitution.

REF: 019439siii

6 ANS:
$\overline{OA} \cong \overline{OB}$ because all radii are equal. $\overline{OP} \cong \overline{OP}$ because of the reflexive property. $\overline{OA} \perp \overline{PA}$ and $\overline{OB} \perp \overline{PB}$ because tangents to a circle are perpendicular to a radius at a point on a circle. $\angle PAO$ and $\angle PBO$ are right angles because of the definition of perpendicular. $\angle PAO \cong \angle PBO$ because all right angles are congruent. $\triangle AOP \cong \triangle BOP$ because of HL. $\angle AOP \cong \angle BOP$ because of CPCTC.

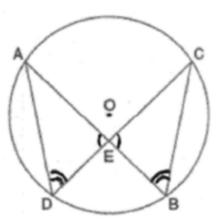
REF: 061138ge

7 ANS:
2. The diameter of a circle is $\perp$ to a tangent at the point of tangency. 4. An angle inscribed in a semicircle is a right angle. 5. All right angles are congruent. 7. AA. 8. Corresponding sides of congruent triangles are in proportion. 9. The product of the means equals the product of the extremes.

REF: 011438ge

8 ANS:
$\angle AED$ and $\angle CEB$ are congruent vertical angles. Because $\angle D$ and $\angle B$ intercept the same arc, they are congruent. $\triangle ADE \sim \triangle CBE$ by AA. Because corresponding sides of similar triangles are in proportion, $\frac{AE}{CE} = \frac{ED}{EB}$.

Cross-multiplying, $(AE)(EB) = (CE)(ED)$.



REF: 060133b

9 ANS:
Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E (Given); Chords $\overline{CB}$ and $\overline{AD}$ are drawn (auxiliary lines drawn); $\angle CEB \cong \angle AED$ (vertical angles); $\angle C \cong \angle A$ (Inscribed angles that intercept the same arc are congruent); $\triangle BCE \sim \triangle DAE$ (AA); $\frac{AE}{CE} = \frac{ED}{EB}$ (Corresponding sides of similar triangles are proportional); $AE \cdot EB = CE \cdot ED$ (The product of the means equals the product of the extremes).

REF: 081635geo

10 ANS:
Circle O, secant $\overline{ACD}$, tangent $\overline{AB}$ (Given). Chords $\overline{BC}$ and $\overline{BD}$ are drawn (Auxiliary lines). $\angle A \cong \angle A$, $\overset{\frown}{BC} \cong \overset{\frown}{BC}$ (Reflexive property). $m\angle BDC = \frac{1}{2} m\overset{\frown}{BC}$ (The measure of an inscribed angle is half the measure of the intercepted arc). $m\angle CBA = \frac{1}{2} m\overset{\frown}{BC}$ (The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc). $\angle BDC \cong \angle CBA$ (Angles equal to half of the same arc are congruent). $\triangle ABC \sim \triangle ADB$ (AA). $\frac{AB}{AC} = \frac{AD}{AB}$ (Corresponding sides of similar triangles are proportional). $AC \cdot AD = AB^2$ (In a proportion, the product of the means equals the product of the extremes).

REF: spr1413geo

11 ANS:
Circle O, tangent $\overline{EC}$ to diameter $\overline{AC}$, chord $\overline{BC} \parallel$ secant $\overline{ADE}$, and chord $\overline{AB}$ (given); $\angle B$ is a right angle (an angle inscribed in a semi-circle is a right angle); $\overleftrightarrow{EC} \perp \overline{OC}$ (a radius drawn to a point of tangency is perpendicular to the tangent); $\angle ECA$ is a right angle (perpendicular lines form right angles); $\angle B \cong \angle ECA$ (all right angles are congruent); $\angle BCA \cong \angle CAE$ (the transversal of parallel lines creates congruent alternate interior angles); $\triangle ABC \sim \triangle ECA$ (AA); $\frac{BC}{CA} = \frac{AB}{EC}$ (Corresponding sides of similar triangles are in proportion).

REF: 081733geo