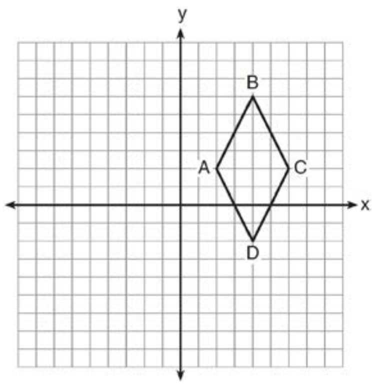
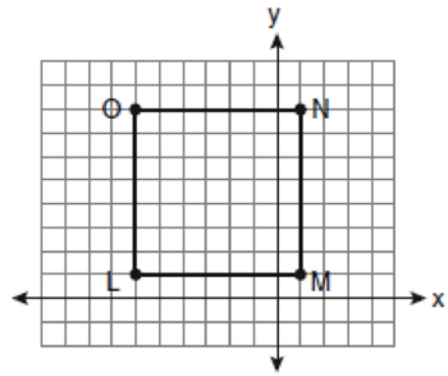
G.GPE.B.4: Quadrilaterals in the Coordinate Plane 2

1 Quadrilateral *ABCD* is graphed on the set of axes below.



Which quadrilateral best classifies *ABCD*?
1) trapezoid
2) rectangle
3) rhombus
4) square

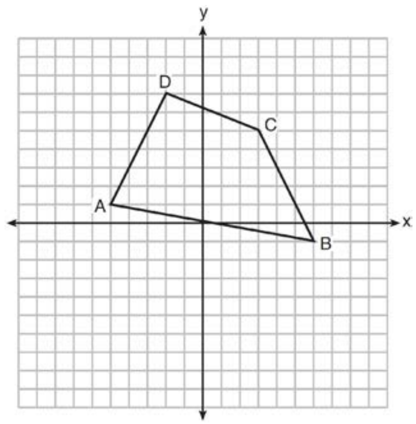
2 Square *LMNO* is shown in the diagram below.



What are the coordinates of the midpoint of diagonal $\overline{LN}$?
1) $\left(4\frac{1}{2},-2\frac{1}{2}\right)$
2) $\left(-3\frac{1}{2},3\frac{1}{2}\right)$
3) $\left(-2\frac{1}{2},3\frac{1}{2}\right)$
4) $\left(-2\frac{1}{2},4\frac{1}{2}\right)$

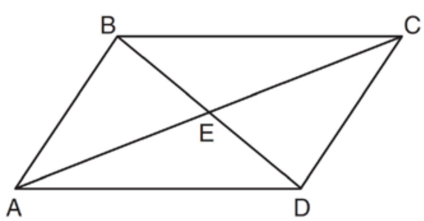
Name: ________________________

3 In the diagram below, quadrilateral $ABCD$ has vertices $A(-5,1)$, $B(6,-1)$, $C(3,5)$, and $D(-2,7)$.



What are the coordinates of the midpoint of diagonal $\overline{AC}$?
1) $(-1,3)$
2) $(1,3)$
3) $(1,4)$
4) $(2,3)$

4 In the diagram below, parallelogram $ABCD$ has vertices $A(1,3)$, $B(5,7)$, $C(10,7)$, and $D(6,3)$. Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E.



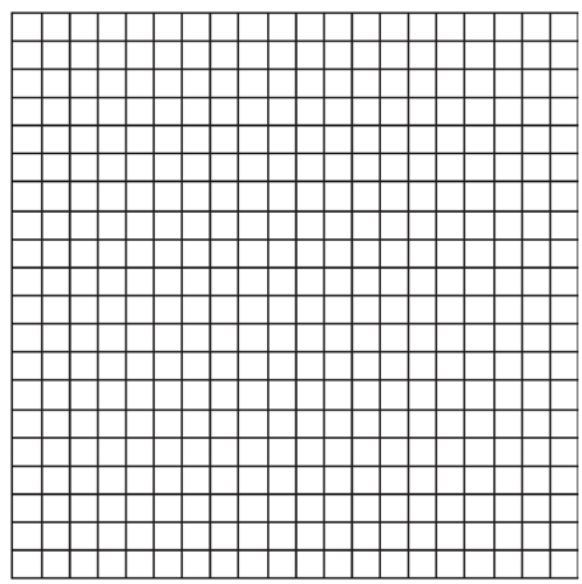
(Not drawn to scale)

What are the coordinates of point E?
1) $(0.5,2)$
2) $(4.5,2)$
3) $(5.5,5)$
4) $(7.5,7)$

5 The coordinates of the vertices of parallelogram $ABCD$ are $A(-3,2)$, $B(-2,-1)$, $C(4,1)$, and $D(3,4)$. The slopes of which line segments could be calculated to show that $ABCD$ is a rectangle?
1) $\overline{AB}$ and $\overline{DC}$
2) $\overline{AB}$ and $\overline{BC}$
3) $\overline{AD}$ and $\overline{BC}$
4) $\overline{AC}$ and $\overline{BD}$

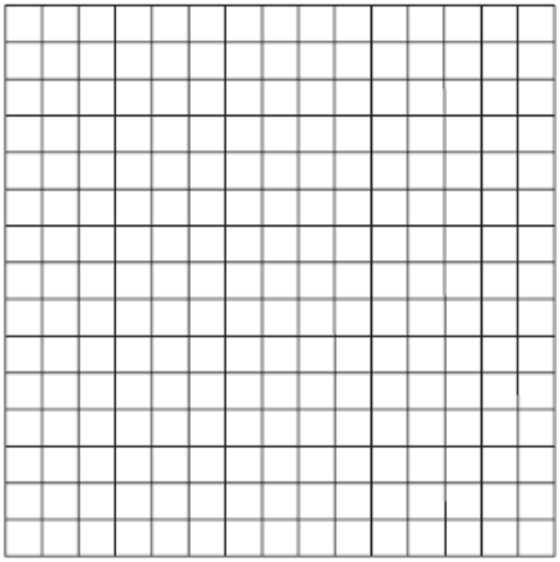
6 Parallelogram $ABCD$ has coordinates $A(1,5)$, $B(6,3)$, $C(3,-1)$, and $D(-2,1)$. What are the coordinates of E, the intersection of diagonals $\overline{AC}$ and $\overline{BD}$?
1) $(2,2)$
2) $(4.5,1)$
3) $(3.5,2)$
4) $(-1,3)$

7 The coordinates of quadrilateral $ABCD$ are $A(-1,-5)$, $B(8,2)$, $C(11,13)$, and $D(2,6)$. Using coordinate geometry, prove that quadrilateral $ABCD$ is a rhombus. [The use of the grid is optional.]

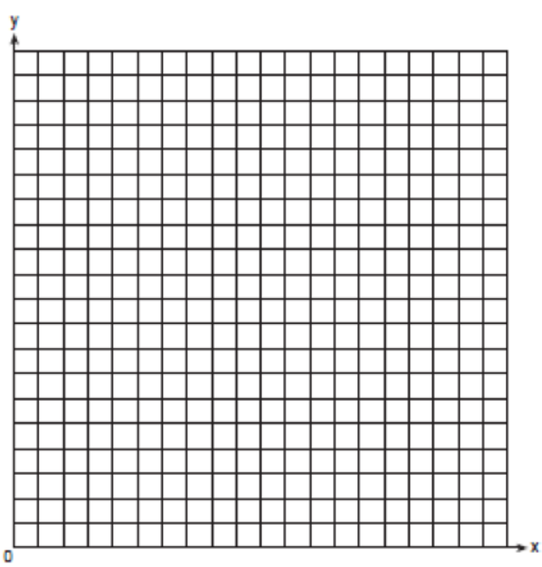


Name: ________________________

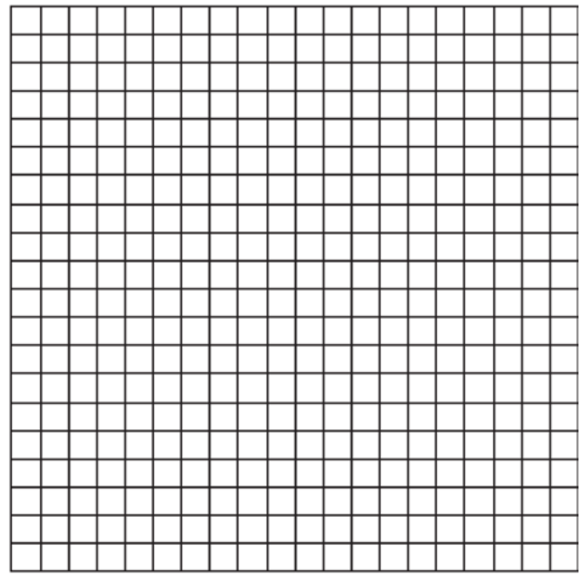
8 Given: $A(1,6), B(7,9), C(13,6)$, and $D(3,1)$
Prove: $ABCD$ is a trapezoid. [*The use of the accompanying grid is optional*.]



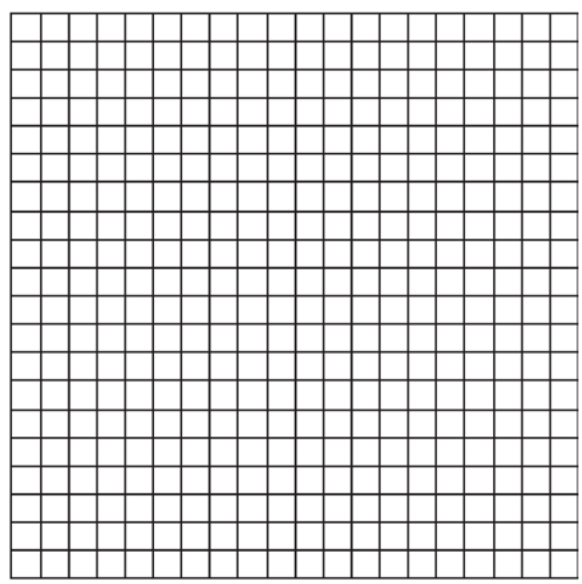
9 Ashanti is surveying for a new parking lot shaped like a parallelogram. She knows that three of the vertices of parallelogram $ABCD$ are $A(0,0)$, $B(5,2)$, and $C(6,5)$. Find the coordinates of point D and sketch parallelogram $ABCD$ on the accompanying set of axes. Justify mathematically that the figure you have drawn is a parallelogram.



10 Quadrilateral $ABCD$ has vertices $A(2,3)$, $B(7,10)$, $C(9,4)$, and $D(4,-3)$. Prove that $ABCD$ is a parallelogram but *not* a rhombus. [The use of the grid is optional.]

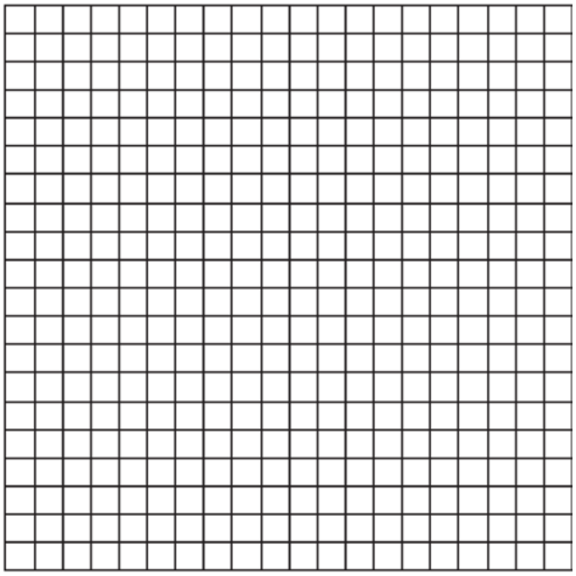


11 Given: $A(-2,2)$, $B(6,5)$, $C(4,0)$, $D(-4,-3)$
Prove: $ABCD$ is a parallelogram but not a rectangle. [The use of the grid is optional.]

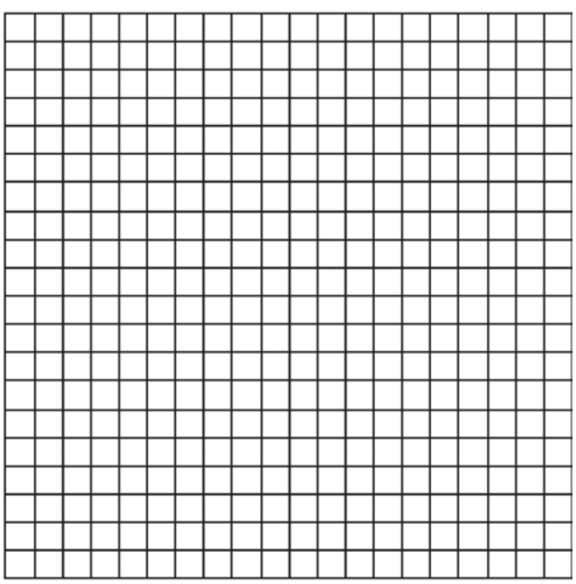


Name: ________________________

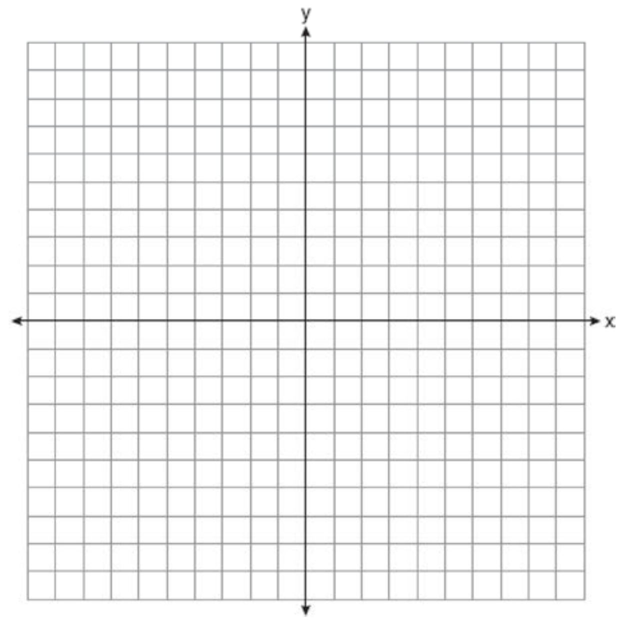
12 Quadrilateral *MATH* has coordinates $M(1,1)$, $A(-2,5)$, $T(3,5)$, and $H(6,1)$. Prove that quadrilateral *MATH* is a rhombus and prove that it is *not* a square. [The use of the grid is optional.]



13 Jim is experimenting with a new drawing program on his computer. He created quadrilateral *TEAM* with coordinates $T(-2,3)$, $E(-5,-4)$, $A(2,-1)$, and $M(5,6)$. Jim believes that he has created a rhombus but not a square. Prove that Jim is correct. [The use of the grid is optional.]



14 The vertices of quadrilateral *JKLM* have coordinates $J(-3,1)$, $K(1,-5)$, $L(7,-2)$, and $M(3,4)$. Prove that *JKLM* is a parallelogram. Prove that *JKLM* is *not* a rhombus. [The use of the set of axes below is optional.]



Name: ________________________

15 Quadrilateral $KATE$ has vertices $K(1,5)$, $A(4,7)$, $T(7,3)$, and $E(1,-1)$.
a Prove that $KATE$ is a trapezoid. [The use of the grid is optional.]
b Prove that $KATE$ is *not* an isosceles trapezoid.

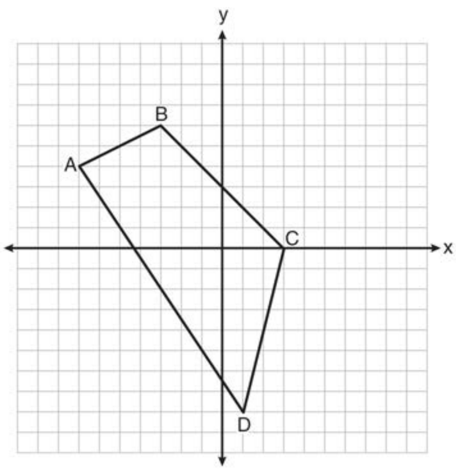
16 The coordinates of quadrilateral $JKLM$ are $J(1,-2)$, $K(13,4)$, $L(6,8)$, and $M(-2,4)$. Prove that quadrilateral $JKLM$ is a trapezoid but *not* an isosceles trapezoid. [The use of the grid is optional.]

17 Given: $T(-1,1)$, $R(3,4)$, $A(7,2)$, and $P(-1,-4)$
Prove: $TRAP$ is a trapezoid.
$TRAP$ is not an isosceles trapezoid.
[The use of the grid is optional.]

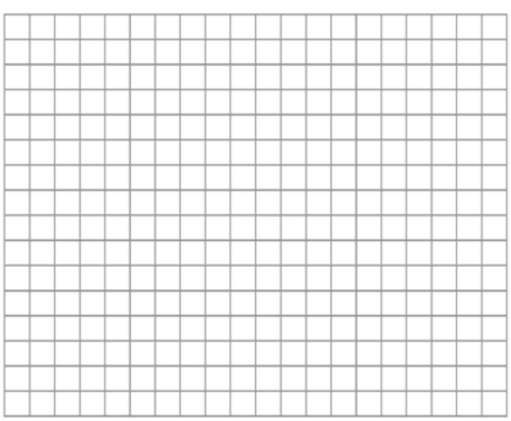
18 Given: Quadrilateral $ABCD$ has vertices $A(-5,6)$, $B(6,6)$, $C(8,-3)$, and $D(-3,-3)$.
Prove: Quadrilateral $ABCD$ is a parallelogram but is neither a rhombus nor a rectangle. [The use of the grid below is optional.]

Name: ________________________

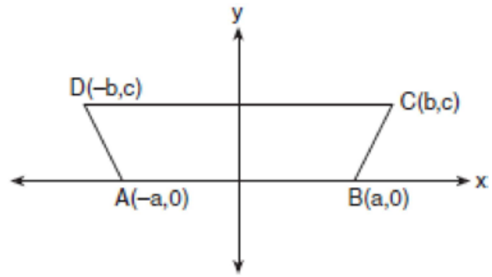
19 Quadrilateral $ABCD$ with vertices $A(-7,4)$, $B(-3,6)$, $C(3,0)$, and $D(1,-8)$ is graphed on the set of axes below. Quadrilateral $MNPQ$ is formed by joining M, N, P, and Q, the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$, respectively. Prove that quadrilateral $MNPQ$ is a parallelogram. Prove that quadrilateral $MNPQ$ is *not* a rhombus.



20 Given: $\triangle ABC$ with vertices $A(-6,-2)$, $B(2,8)$, and $C(6,-2)$. $\overline{AB}$ has midpoint D, $\overline{BC}$ has midpoint E, and $\overline{AC}$ has midpoint F.
Prove: $ADEF$ is a parallelogram
$ADEF$ is *not* a rhombus
[The use of the grid is optional.]



21 In the accompanying diagram of $ABCD$, where $a \neq b$, prove $ABCD$ is an isosceles trapezoid.



22 The coordinates of quadrilateral $PRAT$ are $P(a,b)$, $R(a,b+3)$, $A(a+3,b+4)$, and $T(a+6,b+2)$. Prove that $\overline{RA}$ is parallel to $\overline{PT}$.

23 The coordinates of two vertices of square $ABCD$ are $A(2,1)$ and $B(4,4)$. Determine the slope of side $\overline{BC}$.

24 Rectangle $KLMN$ has vertices $K(0,4)$, $L(4,2)$, $M(1,-4)$, and $N(-3,-2)$. Determine and state the coordinates of the point of intersection of the diagonals.

G.GPE.B.4: Quadrilaterals in the Coordinate Plane 2
Answer Section

1 ANS: 3
Both pairs of opposite sides are parallel, so not a trapezoid. None of the angles are right angles, so not a rectangle or square. All sides are congruent, so a rhombus.

REF: 081411ge

2 ANS: 4
$M_x = \frac{-6+1}{2} = -\frac{5}{2}$. $M_y = \frac{1+8}{2} = \frac{9}{2}$.

REF: 060919ge

3 ANS: 1
$M_x = \frac{-5+3}{2} = \frac{-2}{2} = -1$. $M_y = \frac{1+5}{2} = \frac{6}{2} = 3$.

REF: 061402ge

4 ANS: 3
$M_x = \frac{1+10}{2} = \frac{11}{2} = 5.5$ $M_y = \frac{3+7}{2} = \frac{10}{2} = 5$.

REF: 081407ge

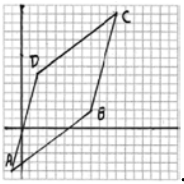
5 ANS: 2
Adjacent sides of a rectangle are perpendicular and have opposite and reciprocal slopes.

REF: 061028ge

6 ANS: 1
The diagonals of a parallelogram intersect at their midpoints. $M_{\overline{AC}}\left(\frac{1+3}{2}, \frac{5+(-1)}{2}\right) = (2,2)$

REF: 061209ge

7 ANS:



. To prove that *ABCD* is a rhombus, show that all sides are congruent using the distance formula:

$d_{\overline{AB}} = \sqrt{(8-(-1))^2 + (2-(-5))^2} = \sqrt{130}$.

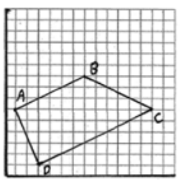
$d_{\overline{BC}} = \sqrt{(11-8)^2 + (13-2)^2} = \sqrt{130}$

$d_{\overline{CD}} = \sqrt{(11-2)^2 + (13-6)^2} = \sqrt{130}$

$d_{\overline{AD}} = \sqrt{(2-(-1))^2 + (6-(-5))^2} = \sqrt{130}$

REF: 060327b

8 ANS:

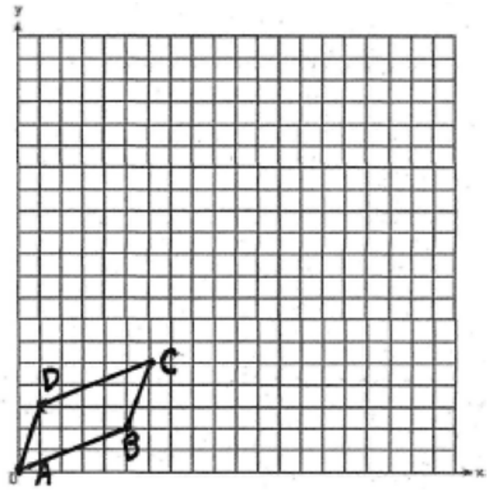


. To prove that *ABCD* is a trapezoid, show that one pair of opposite sides of the figure is parallel by showing they have the same slope and that the other pair of opposite sides is not parallel by showing they do not have the same slope: $m_{\overline{AB}} = \dfrac{9-6}{7-1} = \dfrac{3}{6} = \dfrac{1}{2}$ $m_{\overline{AD}} = \dfrac{6-1}{1-3} = -\dfrac{5}{2}$

$m_{\overline{CD}} = \dfrac{6-1}{13-3} = \dfrac{5}{10} = \dfrac{1}{2}$ $m_{\overline{BC}} = \dfrac{9-6}{7-13} = -\dfrac{3}{6} = -\dfrac{1}{2}$

REF: 080134b

9 ANS:



Both pairs of opposite sides of a parallelogram are parallel. Parallel lines have the same slope. The slope of side $\overline{BC}$ is 3. For side $\overline{AD}$ to have a slope of 3, the coordinates of point D must be (1,3). $m\overline{\text{AB}}=\frac{2-0}{5-0}=\frac{2}{5}$ $m\overline{\text{AD}}=\frac{3-0}{1-0}=3$

$m\overline{CD}=\frac{5-3}{6-1}=\frac{2}{5}$ $m\overline{BC}=\frac{5-2}{6-5}=3$

REF: 080032a

10 ANS:

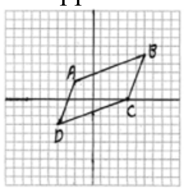
$m_{\overline{AB}}=\frac{10-3}{7-2}=\frac{7}{5}$, $m_{\overline{CD}}=\frac{4-(-3)}{9-4}=\frac{7}{5}$, $m_{\overline{AD}}=\frac{3-(-3)}{2-4}=\frac{6}{-2}=-3$, $m_{\overline{BC}}=\frac{10-4}{7-9}=\frac{6}{-2}=-3$ (Definition of slope). $\overline{AB}\|\overline{CD}$, $\overline{AD}\|\overline{BC}$ (Parallel lines have equal slope). Quadrilateral *ABCD* is a parallelogram (Definition of parallelogram). $d_{\overline{AD}}=\sqrt{(2-4)^2+(3-(-3))^2}=\sqrt{40}$, $d_{\overline{AB}}=\sqrt{(7-2)^2+(10-3)^2}=\sqrt{74}$ (Definition of distance). $\overline{AD}$ is not congruent to $\overline{AB}$ (Congruent lines have equal distance). *ABCD* is not a rhombus (A rhombus has four equal sides).

REF: 061031b

11 ANS:
To prove that *ABCD* is a parallelogram, show that both pairs of opposite sides of the parallelogram are parallel by showing the opposite sides have the same slope: $m_{\overline{AB}} = \frac{5-2}{6-(-2)} = \frac{3}{8}$ $m_{\overline{AD}} = \frac{-3-2}{-4-(-2)} = \frac{5}{2}$

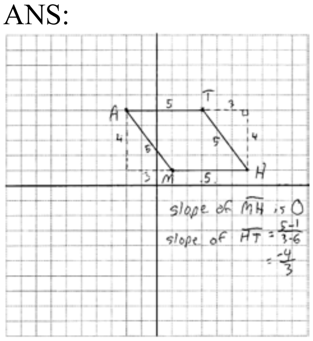
$$m_{\overline{CD}} = \frac{-3-0}{-4-4} = \frac{3}{8} \quad m_{\overline{BC}} = \frac{5-0}{6-4} = \frac{5}{2}$$

A rectangle has four right angles. If *ABCD* is a rectangle, then $\overline{AB} \perp \overline{BC}$, $\overline{BC} \perp \overline{CD}$, $\overline{CD} \perp \overline{AD}$, and $\overline{AD} \perp \overline{AB}$. Lines that are perpendicular have slopes that are the opposite and reciprocal of each other. Because $\frac{3}{8}$ and $\frac{5}{2}$ are not opposite reciprocals, the consecutive sides of *ABCD* are not perpendicular, and *ABCD* is not a rectangle.



REF: 060633b

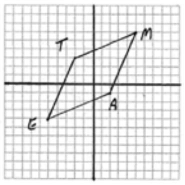
12 ANS:



The length of each side of quadrilateral is 5. Since each side is congruent, quadrilateral *MATH* is a rhombus. The slope of $\overline{MH}$ is 0 and the slope of $\overline{HT}$ is $-\frac{4}{3}$. Since the slopes are not negative reciprocals, the sides are not perpendicular and do not form rights angles. Since adjacent sides are not perpendicular, quadrilateral *MATH* is not a square.

REF: 011138ge

13 ANS:



. To prove that *TEAM* is a rhombus, show that all sides are congruent using the distance formula: $d_{\overline{ET}} = \sqrt{(-2-(-5))^2 + (3-(-4))^2} = \sqrt{58}$. A square has four right angles. If *TEAM* is a square, then $\overline{ET} \perp \overline{AE}$,

$d_{\overline{AM}} = \sqrt{(2-5)^2 + ((-1)-6)^2} = \sqrt{58}$

$d_{\overline{AE}} = \sqrt{(-5-2)^2 + (-4-(-1))^2} = \sqrt{58}$

$d_{\overline{MT}} = \sqrt{(-2-5)^2 + (3-6)^2} = \sqrt{58}$

$\overline{AE} \perp \overline{AM}$, $\overline{AM} \perp \overline{AT}$ and $\overline{MT} \perp \overline{ET}$. Lines that are perpendicular have slopes that are opposite reciprocals of each other. The slopes of sides of *TEAM* are: $m_{\overline{ET}} = \dfrac{-4-3}{-5-(-2)} = \dfrac{7}{3}$ $m_{\overline{AE}} = \dfrac{-4-(-1)}{-5-2} = \dfrac{3}{7}$ Because $\dfrac{7}{3}$ and $\dfrac{3}{7}$ are not

$m_{\overline{AM}} = \dfrac{6-(-1)}{5-2} = \dfrac{7}{3}$ $m_{\overline{MT}} = \dfrac{3-6}{-2-5} = \dfrac{3}{7}$

opposite reciprocals, consecutive sides of *TEAM* are not perpendicular, and *TEAM* is not a square.

REF: 010533b

14 ANS:

$m_{\overline{JM}} = \dfrac{1-4}{-3-3} = \dfrac{-3}{-6} = \dfrac{1}{2}$ Since both opposite sides have equal slopes and are parallel, *JKLM* is a parallelogram.

$m_{= \overline{ML}} = \dfrac{4--2}{3-7} = \dfrac{6}{-4} = -\dfrac{3}{2}$

$m_{\overline{LK}} = \dfrac{-2--5}{7-1} = \dfrac{3}{6} = \dfrac{1}{2}$

$m_{\overline{KJ}} = \dfrac{-5-1}{1--3} = \dfrac{-6}{4} = -\dfrac{3}{2}$

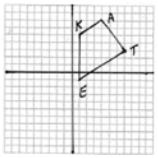
$\overline{JM} = \sqrt{(-3-3)^2 + (1-4)^2} = \sqrt{45}$. $\overline{JM}$ is not congruent to $\overline{ML}$, so *JKLM* is not a rhombus since not all sides

$\overline{ML} = \sqrt{(7-3)^2 + (-2-4)^2} = \sqrt{52}$

are congruent.

REF: 061438ge

15 ANS:



. To prove that *KATE* is a trapezoid, show that one pair of opposite sides of the figure is parallel by showing they have the same slope and that the other pair of opposite sides is not parallel by showing they do not have the same slope: $m_{\overline{AK}} = \frac{7-5}{4-1} = \frac{2}{3}$ $m_{\overline{EK}} = \frac{-1-5}{1-1} = \text{undefined}$

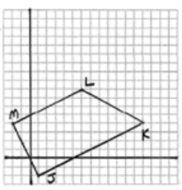
$$m_{\overline{ET}} = \frac{3-(-1)}{7-1} = \frac{4}{6} = \frac{2}{3} \quad m_{\overline{AT}} = \frac{7-3}{4-7} = -\frac{4}{3}$$

To prove that a trapezoid is not an isosceles trapezoid, show that the opposite sides that are not parallel are also not congruent using the distance formula: $d_{\overline{EK}} = \sqrt{(1-1)^2 + (5-(-1))^2} = 6$

$$d_{\overline{AT}} = \sqrt{(4-7)^2 + (7-3)^2} = 5$$

REF: 010333b

16 ANS:



. To prove that *JKLM* is a trapezoid, show that one pair of opposite sides of the figure is parallel by showing they have the same slope and that the other pair of opposite sides is not parallel by showing they do not have the same slope: $m_{\overline{JK}} = \frac{4-(-2)}{13-1} = \frac{1}{2}$ $m_{\overline{JM}} = \frac{-2-4}{1-(-2)} = -2$

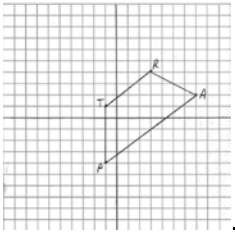
$$m_{\overline{LM}} = \frac{8-4}{6-(-2)} = \frac{1}{2} \quad m_{\overline{KL}} = \frac{4-8}{13-6} = -\frac{4}{7}$$

To prove that a trapezoid is not an isosceles trapezoid, show that the opposite sides that are not parallel are also not congruent using the distance formula: $d_{\overline{JM}} = \sqrt{(1-(-2))^2 + (-2-4)^2} = \sqrt{45}$

$$d_{\overline{KL}} = \sqrt{(13-6)^2 + (4-8)^2} = \sqrt{65}$$

REF: 080434b

17 ANS:



. To prove that *TRAP* is a trapezoid, show that one pair of opposite sides of the figure is parallel by showing they have the same slope and that the other pair of opposite sides is not parallel by showing they do not have the same slope: $m_{\overline{TR}} = \frac{1-4}{-1-3} = \frac{3}{4}$ $m_{\overline{TP}} = \frac{1-(-4)}{-1-(-1)} = \text{undefined}$

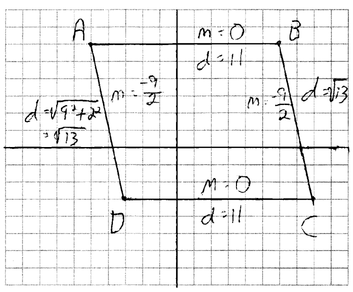
$$m_{\overline{PA}} = \frac{-4-2}{-1-7} = \frac{3}{4} \quad m_{\overline{RA}} = \frac{4-2}{3-7} = -\frac{1}{2}$$

To prove that a trapezoid is not an isosceles trapezoid, show that the opposite sides that are not parallel are also not congruent using the distance formula: $d_{\overline{TP}} = \sqrt{(-1-(-1))^2 + (1-(-4))^2} = 5$

$$d_{\overline{RA}} = \sqrt{(3-7)^2 + (4-2)^2} = \sqrt{20} = 2\sqrt{5}$$

REF: 080933b

18 ANS:



$\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{CB}$ because their slopes are equal. *ABCD* is a parallelogram because opposite side are parallel. $\overline{AB} \neq \overline{BC}$. *ABCD* is not a rhombus because all sides are not equal. $\overline{AB} \sim \perp \overline{BC}$ because their slopes are not opposite reciprocals. *ABCD* is not a rectangle because $\angle ABC$ is not a right angle.

REF: 081038ge

19 ANS:

$M\left(\frac{-7+-3}{2},\frac{4+6}{2}\right)=M(-5,5)$. $m_{\overline{MN}}=\frac{5-3}{-5-0}=\frac{2}{-5}$. Since both opposite sides have equal slopes and are

$N\left(\frac{-3+3}{2},\frac{6+0}{2}\right)=N(0,3)$ $m_{\overline{PQ}}=\frac{-4--2}{2--3}=\frac{-2}{5}$

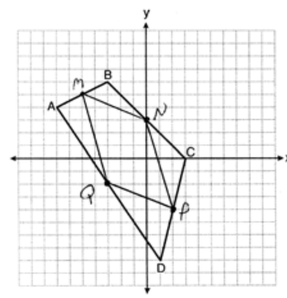
$P\left(\frac{3+1}{2},\frac{0+-8}{2}\right)=P(2,-4)$ $m_{\overline{NA}}=\frac{3--4}{0-2}=\frac{7}{-2}$

$Q\left(\frac{-7+1}{2},\frac{4+-8}{2}\right)=Q(-3,-2)$ $m_{\overline{QM}}=\frac{-2-5}{-3--5}=\frac{-7}{2}$

parallel, $MNPQ$ is a parallelogram. $\overline{MN}=\sqrt{(-5-0)^2+(5-3)^2}=\sqrt{29}$. $\overline{MN}$ is not congruent to $\overline{NP}$, so $MNPQ$

$\overline{NA}=\sqrt{(0-2)^2+(3--4)^2}=\sqrt{53}$

is not a rhombus since not all sides are congruent.



REF: 081338ge

20 ANS:

$m_{\overline{AB}}=\left(\frac{-6+2}{2},\frac{-2+8}{2}\right)=D(2,3)$ $m_{\overline{BC}}=\left(\frac{2+6}{2},\frac{8+-2}{2}\right)=E(4,3)$ $F(0,-2)$. To prove that $ADEF$ is a parallelogram, show that both pairs of opposite sides of the parallelogram are parallel by showing the opposite sides have the same slope: $\text{m}_{\overline{AD}}=\frac{3--2}{-2--6}=\frac{5}{4}$ $\overline{AF}\parallel\overline{DE}$ because all horizontal lines have the same slope. $ADEF$

$\text{m}_{FE}=\frac{3--2}{4-0}=\frac{5}{4}$

is not a rhombus because not all sides are congruent. $AD=\sqrt{5^2+4^2}=\sqrt{41}$ $AF=6$

REF: 081138ge

21 ANS:
To prove that *ABCD* is a trapezoid, show that one pair of opposite sides of the figure is parallel by showing they have the same slope and that the other pair of opposite sides is not parallel by showing they do not have the same slope: $m_{\overline{AB}} = \frac{0-0}{-a-a} = \frac{0}{-2a} = 0$ $m_{\overline{AD}} = \frac{c-0}{-b-(-a)} = \frac{c}{-b+a}$ If $\overline{AD}$ and $\overline{BC}$ are parallel, then: $\frac{c}{-b+a} = \frac{c}{b-a}$

$m_{\overline{CD}} = \frac{c-c}{-b-b} = \frac{0}{-2b} = 0$ $m_{\overline{BC}} = \frac{c-0}{b-a} = \frac{c}{b-a}$

$$c(b-a) = c(-b+a)$$
$$b-a = -b+a$$
$$2a = 2b$$
$$a = b$$

But the facts of the problem indicate $a \neq b$, so $\overline{AD}$ and $\overline{BC}$ are not parallel.
To prove that a trapezoid is an isosceles trapezoid, show that the opposite sides that are not parallel are congruent using the distance formula: $d_{\overline{BC}} = \sqrt{(b-a)^2 + (c-0)^2}$ $d_{\overline{AD}} = \sqrt{(-b-(-a))^2 + (c-0)^2}$

$$= \sqrt{b^2 - 2ab + a^2 + c^2} \qquad = \sqrt{(a-b)^2 + c^2}$$
$$= \sqrt{a^2 + b^2 - 2ab + c^2} \qquad = \sqrt{a^2 - 2ab + b^2 + c^2}$$
$$= \sqrt{a^2 + b^2 - 2ab + c^2}$$

REF: 080534b

22 ANS:
$m_{\overline{RA}} = \frac{(b+3)-(b+4)}{a-(a+3)} = \frac{-1}{-3} = \frac{1}{3}$. Because $\overline{RA}$ and $\overline{PT}$ have equal slopes, they are parallel.

$m_{\overline{PT}} = \frac{b-(b+2)}{a-(a+6)} = \frac{-2}{-6} = \frac{1}{3}$

REF: 060824b

23 ANS:
$m_{\overline{AB}} = \frac{4-1}{4-2} = \frac{3}{2}$. $m_{\overline{BC}} = -\frac{2}{3}$

REF: 061334ge

24 ANS:
$$\left(\frac{0+1}{2}, \frac{4+-4}{2}\right)$$
$$\left(\frac{1}{2}, 0\right)$$

REF: 081534ge