G.GPE.A.1: Equations of Circles 2b

1 Kevin’s work for deriving the equation of a circle is shown below.
   \[ x^2 + 4x = -(y^2 - 20) \]
   STEP 1 \[ x^2 + 4x = -y^2 + 20 \]
   STEP 2 \[ x^2 + 4x + 4 = -y^2 + 20 - 4 \]
   STEP 3 \[ (x + 2)^2 = -y^2 + 20 - 4 \]
   STEP 4 \[ (x + 2)^2 + y^2 = 16 \]
   In which step did he make an error in his work?

2 The equation \( x^2 + y^2 - 2x + 6y + 3 = 0 \) is equivalent to
   1) \( (x - 1)^2 + (y + 3)^2 = -3 \)
   2) \( (x - 1)^2 + (y + 3)^2 = 7 \)
   3) \( (x + 1)^2 + (y + 3)^2 = 7 \)
   4) \( (x + 1)^2 + (y + 3)^2 = 10 \)

3 The equation \( 4x^2 - 24x + 4y^2 + 72y = 76 \) is equivalent to
   1) \( 4(x - 3)^2 + 4(y + 9)^2 = 76 \)
   2) \( 4(x - 3)^2 + 4(y + 9)^2 = 121 \)
   3) \( 4(x - 3)^2 + 4(y + 9)^2 = 166 \)
   4) \( 4(x - 3)^2 + 4(y + 9)^2 = 436 \)

4 What are the coordinates of the center of a circle whose equation is \( x^2 + y^2 - 16x + 6y + 53 = 0 \)?

5 The equation of a circle is \( x^2 + y^2 + 6y = 7 \). What are the coordinates of the center and the length of the radius of the circle?

6 What are the center and radius of the circle whose equation is \( x^2 + y^2 + 4x = 5 \)?

7 The equation of a circle is \( x^2 + y^2 - 12y + 20 = 0 \). What are the coordinates of the center and the length of the radius of the circle?

8 What are the coordinates of the center and the length of the radius of the circle whose equation is \( x^2 + y^2 - 12y - 20.25 = 0 \)?

9 The equation of a circle is \( x^2 + y^2 - 6y + 1 = 0 \). What are the coordinates of the center and the length of the radius of this circle?

10 The equation of a circle is \( x^2 + 8x + y^2 - 12y = 144 \). What are the coordinates of the center and the length of the radius of the circle?

11 What are the coordinates of the center and length of the radius of the circle whose equation is \( x^2 + 6x + y^2 - 4y = 23 \)?

12 What are the coordinates of the center and the length of the radius of the circle represented by the equation \( x^2 + y^2 - 4x + 8y + 11 = 0 \)?

13 The equation of a circle is \( x^2 + y^2 - 6x + 2y = 6 \). What are the coordinates of the center and the length of the radius of the circle?

14 What are the coordinates of the center and the length of the radius of the circle whose equation is \( x^2 + y^2 = 8x - 6y + 39 \)?

15 If \( x^2 + 4x + y^2 - 6y - 12 = 0 \) is the equation of a circle, the length of the radius is

16 An equation of circle O is \( x^2 + y^2 + 4x - 8y = -16 \). The statement that best describes circle O is the
   1) center is (2, -4) and is tangent to the x-axis
   2) center is (2, -4) and is tangent to the y-axis
   3) center is (-2, 4) and is tangent to the x-axis
   4) center is (-2, 4) and is tangent to the y-axis

17 Determine and state the coordinates of the center and the length of the radius of a circle whose equation is \( x^2 + y^2 - 6x = 56 - 8y \).

18 Determine and state the coordinates of the center and the length of the radius of the circle whose equation is \( x^2 + y^2 + 6x = 6y + 63 \).
G.GPE.A.1: Equations of Circles 2b
Answer Section

1 ANS:
   Step 2
   REF: 061603geo

2 ANS: 2
   \[ x^2 - 2x + y^2 + 6y = -3 \]
   \[ x^2 - 2x + 1 + y^2 + 6y + 9 = -3 + 1 + 9 \]
   \[ (x - 1)^2 + (y + 3)^2 = 7 \]
   REF: 061016a2

3 ANS: 4
   \[ 4(x^2 - 6x + 9) + 4(y^2 + 18y + 81) = 76 + 36 + 324 \]
   \[ 4(x - 3)^2 + 4(y + 9)^2 = 436 \]
   REF: 061619a1i

4 ANS:
   \( (8, -3) \)
   \[ x^2 + y^2 - 16x + 6y + 53 = 0 \]
   \[ x^2 - 16x + 64 + y^2 + 6y + 9 = -53 + 64 + 9 \]
   \[ (x - 8)^2 + (y + 3)^2 = 20 \]
   REF: 011415a2

5 ANS:
   center \((0, -3)\) and radius 4
   \[ x^2 + y^2 + 6y + 9 = 7 + 9 \]
   \[ x^2 + (y + 3)^2 = 16 \]
   REF: 061514geo

6 ANS:
   \( (-2, 0) \) and 3
   \[ x^2 + y^2 + 4x = 5 \]
   \[ x^2 + 4x + 4 + y^2 = 5 + 4 \]
   \[ (x + 2)^2 + y^2 = 9 \]
   REF: 081626a2
7 ANS:
center (0, 6) and radius 4
\[ x^2 + y^2 - 12y + 36 = -20 + 36 \]
\[ x^2 + (y - 6)^2 = 16 \]
REF: 061712geo

8 ANS:
center (0, 6) and radius 7.5
\[ x^2 + y^2 - 12y + 36 = 20.25 + 36 \sqrt{56.25} = 7.5 \]
\[ x^2 + (y - 6)^2 = 56.25 \]
REF: 082219geo

9 ANS:
center (0, 3) and radius \( 2\sqrt{2} \)
\[ x^2 + y^2 - 6y + 9 = -1 + 9 \]
\[ x^2 + (y - 3)^2 = 8 \]
REF: 011718geo

10 ANS:
center (−4, 6) and radius 14
\[ x^2 + 8x + 16 + y^2 - 12y + 36 = 144 + 16 + 36 \]
\[ (x + 4)^2 + (y - 6)^2 = 196 \]
REF: 061920geo

11 ANS:
(−3, 2) and 6
\[ x^2 + 6x + 9 + y^2 - 4y + 4 = 23 + 9 + 4 \]
\[ (x + 3)^2 + (y - 2)^2 = 36 \]
REF: 011617geo

12 ANS:
center (2, −4) and radius 3
\[ x^2 - 4x + 4 + y^2 + 8y + 16 = -11 + 4 + 16 \]
\[ (x - 2)^2 + (y + 4)^2 = 9 \]
REF: 081616geo
13 ANS:  
   center (3, -1) and radius 4  
   \[ x^2 + y^2 - 6x + 2y = 6 \]  
   \[ x^2 - 6x + 9 + y^2 + 2y + 1 = 6 + 9 + 1 \]  
   \[ (x - 3)^2 + (y + 1)^2 = 16 \]  

REF: 011812geo

14 ANS:  
   center (4, -3) and radius 8  
   \[ x^2 - 8x + y^2 + 6y = 39 \]  
   \[ x^2 - 8x + 16 + y^2 + 6y + 9 = 39 + 16 + 9 \]  
   \[ (x - 4)^2 + (y + 3)^2 = 64 \]  

REF: 081906geo

15 ANS:  
   5  
   \[ x^2 + 4x + 4 + y^2 - 6y + 9 = 12 + 4 + 9 \]  
   \[ (x + 2)^2 + (y - 3)^2 = 25 \]  

REF: 081509geo

16 ANS:  
   4  
   \[ x^2 + 4x + 4 + y^2 - 8y + 16 = -16 + 4 + 16 \]  
   \[ (x + 2)^2 + (y - 4)^2 = 4 \]  

REF: 081821geo

17 ANS:  
   \[ x^2 - 6x + 9 + y^2 + 8y + 16 = 56 + 9 + 16 \]  
   \[ (3, -4); \quad r = 9 \]  
   \[ (x - 3)^2 + (y + 4)^2 = 81 \]  

REF: 081731geo

18 ANS:  
   \[ x^2 + 6x + 9 + y^2 - 6y + 9 = 63 + 9 + 9 \]  
   \[ (-3, 3); \quad r = 9 \]  
   \[ (x + 3)^2 + (y - 3)^2 = 81 \]  

REF: 062230geo