F.TF.C.9: Double Angle Identities 2

1 Is $\frac{1}{2}\sin 2x$ the same expression as $\sin x$? Justify your answer.

2 For all values of θ for which the expressions are defined, prove that the following is an identity:
$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

3 For all values of θ for which the expressions are defined, prove the following is an identity:
$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

4 For all value of x for which the expressions are defined, prove the following is an identity:
$\sec x \sin 2x = \frac{2-2\cos^2 x}{\sin x}$

5 For all values of x for which the expressions are defined, show that the following equation is an identity: $\sin 2x = \frac{2\tan x}{1+\tan^2 x}$

6 For all values of θ for which the expressions are defined, prove that the following is an identity:
$1+\tan^2\theta = \frac{2\sin\theta}{\cos\theta \sin 2\theta}$

7 Prove the identity: $\sin 2x = \tan x(2-2\sin^2 x)$

8 For all values of A for which the expressions are defined, show that the following is an identity:
$\frac{2\sin^2 A}{\sin 2A} + \cot A = \sec A \csc A$

9 For all values of θ for which the expressions are defined, prove the following is an identity:
$\frac{\cos(90-\theta)}{\sin 2\theta} = \frac{\sec\theta}{2}$

10 For all value of x for which the expressions are defined, prove the following is an identity:
$2\sin x \cos x = \tan 2x \cos 2x$

11 For all values of A for which the expressions are defined, prove the following is an identity:
$\cot A = \frac{1+\cos A+\cos 2A}{\sin A+\sin 2A}$

12 For all values of θ for which the expressions are defined, prove the following is an identity:
$\frac{\sin 2\theta+\sin\theta}{\cos 2\theta+\cos\theta+1} = \tan\theta$

13 For all values of x for which the expressions are defined, prove that the following is an identity:
$\tan x + \cot x = 2\csc 2x$

14 For all value of x for which the expression is defined, prove that the following is an identity:
$\cot x = \frac{\sin 2x}{1-\cos 2x}$

15 For all values of x for which the expressions are defined, prove the following is an identity:
$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \csc x$

16 Express $\frac{\cos 2A+\sin^2 A}{\cos A}$ as a single trigonometric function for all values of A for which the fraction is defined.

17 For all value of x for which the expressions are defined, prove the following is an identity:
$\frac{\cos 2x}{\sin x} + \sin x = \csc x - \sin x$

18 Prove the following identity:
$\frac{\sin\theta}{\sin^2\theta+\cos 2\theta} = \frac{\sec\theta}{\cot\theta}$

19 Prove the following identity:
$\frac{(\sin x+\cos x)^2-1}{\cos x} = (\sin 2x)(\tan x)(\csc x)$

F.TF.C.9: Double Angle Identities 2
Answer Section

1 ANS:

$$\frac{1}{2}\sin 2x = \sin x$$

$$\frac{1}{2}(2\sin x\cos x) = \sin x$$

$$\sin x\cos x = \sin x$$

$$\sin x\cos x - \sin x = 0$$

Not an identity, so the expressions are not equal.

$$\sin x(\cos x - 1) = 0$$

$$\sin x = 0 \text{ or } \cos x - 1 = 0$$

$$x = 0° \text{ or } 180°$$

REF: 060222b

2 ANS:

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$2\sin\theta\cos\theta = \frac{\frac{2\sin\theta}{\cos\theta}}{\sec^2\theta}$$

$$\cos\theta = \frac{\frac{1}{\cos\theta}}{\frac{1}{\cos^2\theta}}$$

$$\cos\theta = \cos\theta$$

REF: 088442siii

3 ANS:

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\cos 2\theta = \frac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{\sec^2\theta}$$

$$\cos 2\theta = \frac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{\dfrac{1}{\cos^2\theta}}$$

$$\cos^2\theta - \sin^2\theta = \left(1-\frac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta$$

$$\cos^2\theta - \sin^2\theta = \cos^2\theta - \sin^2\theta$$

REF: 018542siii

4 ANS:

$$\sec x \sin 2x = \frac{2-2\cos^2 x}{\sin x}$$

$$\frac{1}{\cos x}\cdot 2\sin x\cos x = \frac{2(1-\cos^2 x)}{\sin x}$$

$$\sin x = \frac{\sin^2 x}{\sin x}$$

$$\sin x = \sin x$$

REF: 068541siii

5 ANS:

$$\sin 2x = \frac{2\tan x}{1+\tan^2 x}$$

$$2\sin x\cos x = \frac{\dfrac{2\sin x}{\cos x}}{\sec^2 x}$$

$$2\sin x\cos x = \frac{\dfrac{2\sin x}{\cos x}}{\dfrac{1}{\cos^2 x}}$$

$$\frac{2\sin x}{\cos x} = \frac{2\sin x}{\cos x}$$

REF: 068738siii

6 ANS:

$$\sec^2\theta = \frac{2\sin\theta}{\cos\theta \cdot 2\sin\theta \cdot \cos\theta}$$

$$\frac{1}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

REF: 088737siii

7 ANS:

$$\sin 2x = \tan x(2 - 2\sin^2 x)$$

$$2\sin x\cos x = \frac{\sin x}{\cos x}(2(1 - \sin^2 x))$$

$$2\cos x = \frac{(2(\cos^2 x))}{\cos x}$$

$$\cos x = \cos x$$

REF: 018941siii

8 ANS:

$$\frac{2\sin^2 A}{\sin 2A} + \cot A = \sec A \csc A$$

$$\frac{2\sin^2 A}{2\sin A\cos A} + \frac{\cos A}{\sin A} = \frac{1}{\cos A} \cdot \frac{1}{\sin A}$$

$$\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} = \frac{1}{\cos A} \cdot \frac{1}{\sin A}$$

$$\frac{\sin^2 A + \cos^2 A}{\cos A \cdot \sin A} = \frac{1}{\cos A \cdot \sin A}$$

$$\frac{1}{\cos A \cdot \sin A} = \frac{1}{\cos A \cdot \sin A}$$

REF: 068942siii

9 ANS:

$$\frac{\cos(90 - \theta)}{\sin 2\theta} = \frac{\sec\theta}{2}$$

$$\frac{\cos 90\cos\theta + \sin 90\sin\theta}{2\sin\theta\cos\theta} = \frac{\sec\theta}{2}$$

$$\frac{\sin\theta}{2\sin\theta\cos\theta} = \frac{\sec\theta}{2}$$

$$\frac{\sec\theta}{2} = \frac{\sec\theta}{2}$$

REF: 069040siii

10 ANS:

$2\sin x\cos x = \tan 2x\cos 2x$

$$\sin 2x = \frac{\sin 2x}{\cos 2x}\cos 2x$$

$$\sin 2x = \sin 2x$$

REF: 069442siii

11 ANS:

$$\cot A = \frac{1+\cos A+\cos 2A}{\sin A+\sin 2A}$$

$$\frac{\cos A}{\sin A} = \frac{1+\cos A+2\cos^2 A-1}{\sin A+2\sin A\cos A}$$

$$\frac{\cos A}{\sin A} = \frac{\cos A(1+2\cos A)}{\sin A(1+2\cos A)}$$

$$\frac{\cos A}{\sin A} = \frac{\cos A}{\sin A}$$

REF: 089442siii

12 ANS:

$$\frac{\sin 2\theta+\sin\theta}{\cos 2\theta+\cos\theta+1} = \tan\theta$$

$$\frac{2\sin\theta\cos\theta+\sin\theta}{2\cos^2\theta-1+\cos\theta+1} = \tan\theta$$

$$\frac{\sin\theta(2\cos\theta+1)}{\cos\theta(2\cos\theta+1)} = \tan\theta$$

$$\tan\theta = \tan\theta$$

REF: 019637siii

13 ANS:

$\tan x+\cot x = 2\csc 2x$

$$\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x} = \frac{2}{\sin 2x}$$

$$\frac{\sin^2 x+\cos^2 x}{\sin x\cos x} = \frac{2}{2\sin x\cos x}$$

$$\frac{1}{\sin x\cos x} = \frac{1}{\sin x\cos x}$$

REF: 069640siii

14 ANS:

$$\cot x = \frac{\sin 2x}{1-\cos 2x}$$

$$\frac{\cos x}{\sin x} = \frac{2\sin x\cos x}{1-(1-2\sin^2 x)}$$

$$\frac{\cos x}{\sin x} = \frac{2\sin x\cos x}{2\sin^2 x}$$

$$\frac{\cos x}{\sin x} = \frac{\cos x}{\sin x}$$

REF: 089638siii

15 ANS:

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \csc x$$

$$\frac{1-2\sin^2 x}{\sin x} + \frac{2\sin x\cos x}{\cos x} = \frac{1}{\sin x}$$

$$\frac{1-2\sin^2 x}{\sin x} + 2\sin x = \frac{1}{\sin x}$$

$$\frac{1-2\sin^2 x+2\sin^2 x}{\sin x} = \frac{1}{\sin x}$$

$$\frac{1}{\sin x} = \frac{1}{\sin x}$$

REF: 019740siii

16 ANS:

$\cos A$

REF: 010014siii

17 ANS:

$$\frac{\cos 2x}{\sin x} + \sin x = \csc x - \sin x$$

$$\frac{\cos 2x}{\sin x} + \sin x = \frac{1}{\sin x} - \sin x$$

$$\cos 2x + \sin^2 x = 1 - \sin^2 x$$

$$1-2\sin^2 x+\sin^2 x = 1-\sin^2 x$$

$$1-\sin^2 x = 1-\sin^2 x$$

REF: 060141siii

18 ANS:

$$\frac{\sin\theta}{\sin^2\theta+\cos 2\theta}=\frac{\sec\theta}{\cot\theta}$$

$$\frac{\sin\theta}{\sin^2\theta+\cos^2\theta-\sin^2\theta}=\frac{\frac{1}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}}$$

$$\frac{\sin\theta}{\cos^2\theta}=\frac{1}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta}$$

$$\frac{\sin\theta}{\cos^2\theta}=\frac{\sin\theta}{\cos^2\theta}$$

REF: 010242siii

19 ANS:

$$\frac{(\sin x+\cos x)^2-1}{\cos x}=(\sin 2x)(\tan x)(\csc x)$$

$$\frac{\sin^2 x+2\sin x\cos x+\cos^2 x-1}{\cos x}=2\sin x\cos x\left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x}\right)$$

$$\frac{\sin^2 x+2\sin x\cos x-\sin^2 x}{\cos x}=2\sin x$$

$$\frac{2\sin x\cos x}{\cos x}=2\sin x$$

$$2\sin x=2\sin x$$

REF: 080239siii