F.TF.B.7: Trigonometric Equations 7

1 Find all values of x in the interval $0° < x < 360°$ that satisfy the equation $3\cos x + \sin 2x = 0$.

2 Find all values of θ in the interval $0° \le \theta < 360°$ that satisfy the equation $\cos\theta - \sin 2\theta = 0$.

3 Find all values of θ in the interval $0° \le \theta < 360°$ that satisfy the equation $\sin 2\theta + \cos\theta = 0$.

4 Find all values of θ in the interval $0° \le \theta < 360°$ that satisfy the equation $\sin 2\theta = \sin\theta$.

5 Solve the equation $\cos 2x = \cos x$ algebraically for all values of x in the interval $0° \le x < 360°$.

6 Find all values of θ in the interval $0° \le \theta \le 360°$ which satisfy the equation $\sin\theta - \cos 2\theta = 0$.

7 Solve for θ in the interval $0° \le \theta \le 360°$: $\cos 2\theta = -\sin\theta$.

8 Find all values of θ in the interval $0° \le \theta < 360°$ that satisfy the equation $\cos 2\theta + 2 = \sin\theta$.

9 In the interval $0° \le A \le 360°$, solve for all values of A in the equation $\cos 2A = -3\sin A - 1$.

10 A solution of the equation $\cos 2\theta + \sin 2\theta = -1$ is
1) 240º 2) 135º 3) 45º 4) –30°

11 Find, to the *nearest ten minutes* or *nearest tenth of a degree*, all values of x in the interval $0° \le x < 360°$ that satisfy the equation $2\sin 2x + \cos x = 0$.

12 Find, to the *nearest degree*, all values of θ in the interval $0° < \theta < 360°$ that satisfy the equation $3\cos 2\theta + \sin\theta - 1 = 0$.

13 Find all values of θ in the interval $0° \le \theta \le 360°$ that satisfy the equation $3\cos 2\theta + 2\sin\theta + 1 = 0$, and round all answers to the *nearest hundredth of a degree*. [Only an algebraic solution can receive full credit.]

14 Find, to the *nearest degree*, all values of θ in the interval $0° \le \theta \le 360°$ which satisfy the equation $3\cos 2\theta + \sin\theta - 2 = 0$.

15 Find all values of θ in the interval $0° \le \theta < 360°$ that satisfy the equation $5\sin\theta + 2\cos 2\theta - 3 = 0$. Express your answer to the *nearest ten minutes* or *nearest tenth of a degree*.

16 Find, to the *nearest degree*, all values of x between 0° and 360° that satisfy the equation $2\sin x + 4\cos 2x = 3$.

17 Find all positive values of θ less than 360° that satisfy the equation $2\cos 2\theta - 3\sin\theta = 1$. Express your answers to the *nearest ten minutes* or *nearest tenth of a degree.*

18 Find all values of θ in the interval $0° \le \theta \le 360°$ that satisfy the equation $\sin\theta = 2 + 3\cos 2\theta$. Express your answer to the *nearest ten minutes* or *nearest tenth of a degree.*

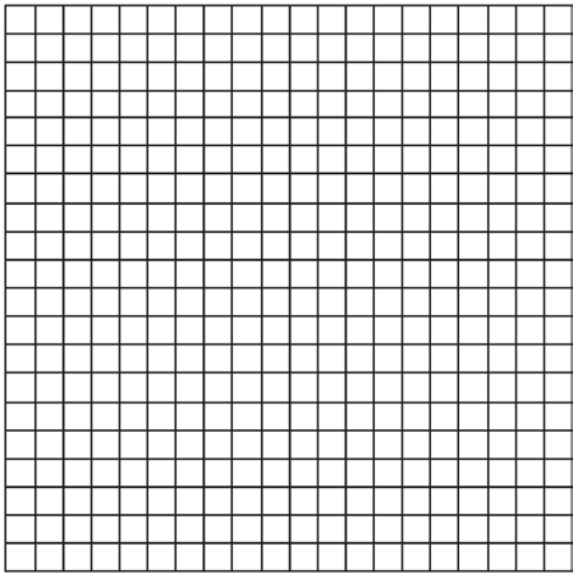
19 Solve the equation $\cos\theta = 2 + 3\cos 2\theta$ for all values of θ, to the *nearest tenth of a degree*, in the interval $0° \le \theta < 360°$.

20 Find, to the *nearest degree*, all values of x in the interval $0° \le x < 360°$ that satisfy the equation $3\cos 2x + \cos x + 2 = 0$

21 Find, to the *nearest ten minutes* or *nearest tenth of a degree*, all values of x in the interval $0° \le x < 360°$ that satisfy the equation $4\cos 2x - 2\cos x + 3 = 0$.

22 Find all values of θ in the interval $0° \le \theta < 360°$ that satisfy the equation $3\cos 2\theta = 7\cos\theta$. Express your answer to the *nearest tenth of a degree or nearest ten minutes*.

23 Find all values of x in the interval $0° \le x < 360°$ that satisfy the equation $3\cos 2x = \cos x + 2$. Express your answers to the *nearest degree*. [The use of the grid is optional.]



F.TF.B.7: Trigonometric Equations 7
Answer Section

1 ANS:

90°, 270°.

$$3\cos x + \sin 2x = 0$$
$$3\cos x + 2\sin x \cos x = 0$$
$$\cos x(3 + 2\sin x) = 0$$
$$\cos x = 0 \quad \text{or} \quad 3 + 2\sin x = 0$$
$$x = \cos^{-1} 0 \quad \text{or} \quad \sin x = -\frac{3}{2}$$
$$x = 90°, 270°$$

REF: 010829b

2 ANS:
30º, 90º, 150º, 270º

REF: 089341siii

3 ANS:
90º, 210º, 270º, 330º

REF: 019137siii

4 ANS:
0, 60, 180, 300.

$$\sin 2\theta = \sin \theta$$
$$\sin 2\theta - \sin \theta = 0$$
$$2\sin \theta \cos \theta - \sin \theta = 0$$
$$\sin \theta(2\cos \theta - 1) = 0$$
$$\sin \theta = 0 \quad 2\cos \theta - 1 = 0$$
$$\theta = 0, 180 \quad \cos \theta = \frac{1}{2}$$
$$\theta = 60, 300$$

REF: 061037a2

5 ANS:

$$2\cos^2 x - 1 = \cos x$$
$$2\cos^2 x - \cos x - 1 = 0$$
$$(2\cos x + 1)(\cos x - 1) = 0$$
$$\cos x = -\frac{1}{2}, 1$$
$$x = 0, 120, 240$$

REF: 011638a2

6 ANS:
30º, 150º, 270º

REF: 068541siii

7 ANS:
90º, 210º, 330º

REF: 018140siii

8 ANS:
90º

REF: 088737siii

9 ANS:

210°, 330°. $1-2\sin^2 A = -3\sin A - 1$. $-2\sin^2 A + 3\sin A + 2 = 0$

$-2x^2 + 3x + 2 = 0$

$2x^2 - 3x - 2 = 0$

$(2x+1)(x-2) = 0$

$2x+1=0 \quad x-2=0$

$x = -\frac{1}{2} \quad x = 2$.

$\sin A = -\frac{1}{2} \quad \sin A = 2$

$\sin^{-1}(-\frac{1}{2}) = -30°$, or $330°$, and $210°$.

$\sin^{-1}(2)$ has no solution

REF: 060131b

10 ANS: 2 REF: 060024siii

11 ANS:
90º, 194.5º, 270º, 345.5º or 90º, 194º30', 270º, 345º30'

REF: 069738siii

12 ANS:

$3\cos 2\theta + \sin\theta - 1 = 0$

$3(1 - 2\sin^2\theta) + \sin\theta - 1 = 0$

$3 - 6\sin^2\theta + \sin\theta - 1 = 0$.

$-6\sin^2\theta + \sin\theta + 2 = 0$

$6\sin^2\theta - \sin\theta - 2 = 0$

$6x^2 - x - 2 = 0$

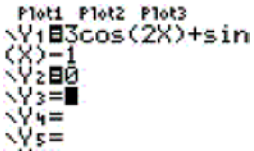
$(3x - 2)(2x + 1) = 0$

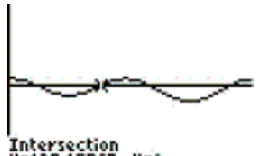
$3x - 2 = 0 \quad 2x + 1 = 0$.

$x = \frac{2}{3} \quad x = -\frac{1}{2}$

42, 138, 210, 330.

$\sin\theta = \frac{2}{3} \quad \sin\theta = -\frac{1}{2}$





$\theta = \sin^{-1}(\frac{2}{3}) \quad \theta = \sin^{-1}(-\frac{1}{2})$.

$\theta \approx 42°, 138° \quad \theta \approx 210°, 330°$

REF: 060530b

13 ANS:

$3\cos 2\theta + 2\sin\theta + 1 = 0$

$3(1 - 2\sin^2\theta) + 2\sin\theta + 1 = 0$

$3 - 6\sin^2\theta + 2\sin\theta + 1 = 0$

$-6\sin^2\theta + 2\sin\theta + 4 = 0$

$3\sin^2\theta - \sin\theta - 2 = 0$

let $\sin\theta = x$

$3x^2 - x - 2 = 0$

$(3x + 2)(x - 1) = 0$

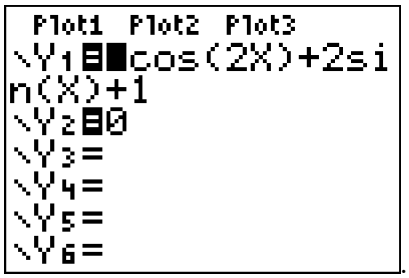
$x = -\frac{2}{3}$ or $x = 1$

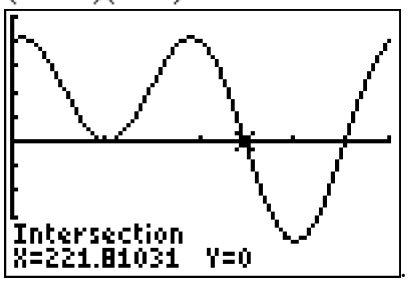
$\sin\theta = -\frac{2}{3}$ or $\sin\theta = 1$

$\theta = \sin^{-1}(-\frac{2}{3})$ or $\theta = \sin^{-1} 1$

$\theta \approx 318.19$ or 221.81 or $\theta = 90$

90°, 221.81°, 318.19°.





REF: 060829b

14 ANS:

30º, 150º, 199º, 341º

REF: 068137siii

15 ANS:

14º30', 90º, 165º30' or 14.5º, 90º, 165.5º

REF: 010437siii

16 ANS:
30, 150, 194, 346

REF: 089539siii

17 ANS:
14.5, 165.5, 270 or 14º30', 165º30', 270º

REF: 019837siii

18 ANS:
56.4º, 123.6º, 270º or 56º30', 123º30', 270º

REF: 060139siii

19 ANS:

$\cos\theta = 2 + 3\cos 2\theta$

$\cos\theta = 2 + 3(2\cos^2\theta - 1)$

$\cos\theta = 2 + 6\cos^2\theta - 3$

$0 = 6\cos^2\theta - \cos\theta - 1$

60, 109.5, 250.5, 300. $0 = 6x^2 - x - 1$

$0 = (3x + 1)(2x - 1)$

$x = -\frac{1}{3} \quad x = \frac{1}{2}$

$\cos\theta = -\frac{1}{3} \quad \cos\theta = \frac{1}{2}$

$\theta \approx 109.5^\circ, 250.5^\circ \quad \theta = 60^\circ, 300^\circ$

REF: 060932b

20 ANS:
71, 120, 240, 289

REF: 069638siii

21 ANS:
60º, 104º30’, 255º30' and 300º or 60º, 104.5º, 255.5º and 300º

REF: 060337siii

22 ANS:
109º30’, 250º30' or 109.5º, 250.5º

REF: 080340siii

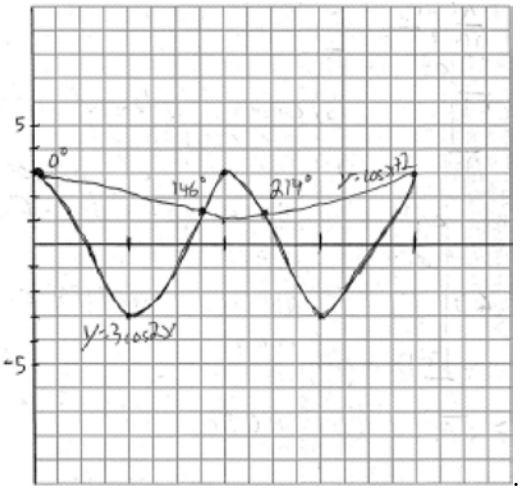
23 ANS:

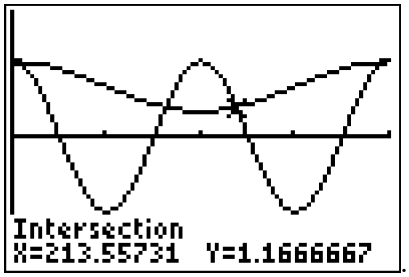
$$3\cos 2x = \cos x + 2$$
$$3(2\cos^2 x - 1) = \cos x + 2$$
$$6\cos^2 x - 3 = \cos x + 2$$
$$6\cos^2 x - \cos x - 5 = 0$$
$$6x^2 - x - 5 = 0$$
$$(6x+5)(x-1) = 0$$
$$x = -\frac{5}{6} \quad x = 1$$

$\cos x = -\frac{5}{6}$. $x = \cos^{-1} -\frac{5}{6}$. $x \approx 146°, 214°$

$\cos x = 1$. $x = \cos^{-1} 1$. $x = 0°$

0°, 146°, 214°.





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