

F.TF.A.2: Reciprocal Trigonometric Relationships 1

- 1 Which equation is *not* true?
 - 1) $\cot^2 \theta = 1 - \sec^2 \theta$
 - 2) $\sin^2 \theta = 1 - \cos^2 \theta$
 - 3) $\sec^2 \theta = \tan^2 \theta + 1$
 - 4) $\csc^2 \theta = 1 + \cot^2 \theta$

- 2 Which trigonometric expression does *not* simplify to 1?
 - 1) $\sin^2 x(1 + \cot^2 x)$
 - 2) $\sec^2 x(1 - \sin^2 x)$
 - 3) $\cos^2 x(\tan^2 x - 1)$
 - 4) $\cot^2 x(\sec^2 x - 1)$

- 3 The expression $\frac{\cot x}{\csc x}$ is equivalent to
 - 1) $\sin x$
 - 2) $\cos x$
 - 3) $\tan x$
 - 4) $\sec x$

- 4 The expression $\frac{\sin^2 \theta + \cos^2 \theta}{1 - \sin^2 \theta}$ is equivalent to
 - 1) $\cos^2 \theta$
 - 2) $\sin^2 \theta$
 - 3) $\sec^2 \theta$
 - 4) $\csc^2 \theta$

- 5 Express $\cos \theta(\sec \theta - \cos \theta)$, in terms of $\sin \theta$.

- 6 Express $\frac{\cot x \sin x}{\sec x}$ as a single trigonometric function, in simplest form, for all values of x for which it is defined.

- 7 Show that $\frac{\sec^2 x - 1}{\sec^2 x}$ is equivalent to $\sin^2 x$.

- 8 The exact value of $\csc 120^\circ$ is
 - 1) $\frac{2\sqrt{3}}{3}$
 - 2) 2
 - 3) $-\frac{2\sqrt{3}}{3}$
 - 4) -2

- 9 Which expression has a value of $\frac{\sqrt{3}}{3}$?
 - 1) $\cot 60^\circ$
 - 2) $\tan 60^\circ$
 - 3) $\csc 30^\circ$
 - 4) $\sec 30^\circ$

- 10 Express the exact value of $\csc 60^\circ$, with a rational denominator.

11 If angles A and B are complementary, then $\sec B$ equals

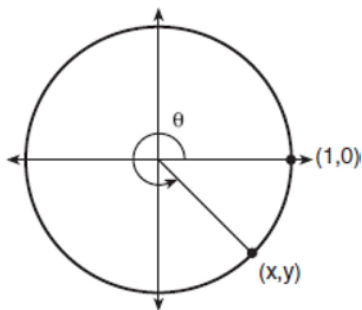
- 1) $\csc(90^\circ - B)$
- 2) $\csc(B - 90^\circ)$
- 3) $\cos(B - 90^\circ)$
- 4) $\cos(90^\circ - B)$

12 If $\angle A$ is acute and $\tan A = \frac{2}{3}$, then

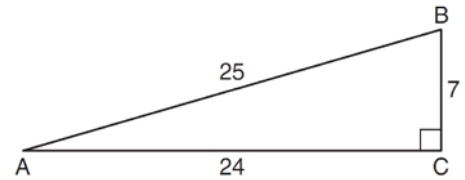
- 1) $\cot A = \frac{2}{3}$
- 2) $\cot A = \frac{1}{3}$
- 3) $\cot(90^\circ - A) = \frac{2}{3}$
- 4) $\cot(90^\circ - A) = \frac{1}{3}$

13 If $\sec(a + 15)^\circ = \csc(2a)^\circ$, find the smallest positive value of a , in degrees.

14 Using the unit circle below, explain why $\csc \theta = \frac{1}{y}$.

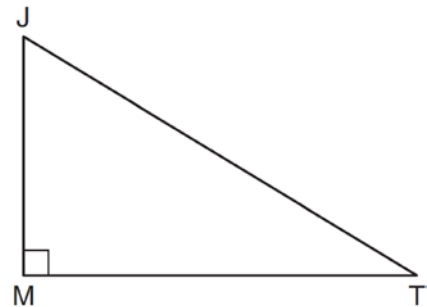


15 Which ratio represents $\csc A$ in the diagram below?



- 1) $\frac{25}{24}$
- 2) $\frac{25}{7}$
- 3) $\frac{24}{7}$
- 4) $\frac{7}{24}$

16 In the diagram below of right triangle JTM , $JT = 12$, $JM = 6$, and $m\angle JMT = 90$.



What is the value of $\cot J$?

- 1) $\frac{\sqrt{3}}{3}$
- 2) 2
- 3) $\sqrt{3}$
- 4) $\frac{2\sqrt{3}}{3}$

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Answer Section

1 ANS: 1

$$\cot^2 \theta = 1 - \sec^2 \theta$$

$$\frac{\cos^2 \theta}{\sin^2 \theta} = 1 - \frac{1}{\cos^2 \theta}$$

$$\frac{\cos^2 \theta}{\sin^2 \theta} = \frac{\cos^2 \theta - 1}{\cos^2 \theta}$$

$$\frac{\cos^2 \theta}{\sin^2 \theta} \neq \frac{-\sin^2 \theta}{\cos^2 \theta}$$

REF: 061626a2

2 ANS: 3

$$\sin^2 x \left(1 + \frac{\cos^2 x}{\sin^2 x} \right) = \sin^2 x + \cos^2 x = 1 \cdot \frac{1}{\cos^2 x} (\cos^2 x) = 1 \cdot \cos^2 x \left(\frac{\sin^2 x}{\cos^2 x} - 1 \right) = \sin^2 x - \cos^2 x \neq 1$$

$$\frac{\cos^2 x}{\sin^2 x} \left(\frac{1}{\cos^2 x} - 1 \right) = \frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x} = \csc^2 x - \cot x = 1$$

REF: 011515a2

3 ANS: 2

$$\frac{\cot x}{\csc x} = \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}} = \cos x$$

REF: 061410a2

4 ANS: 3

$$\frac{\sin^2 \theta + \cos^2 \theta}{1 - \sin^2 \theta} = \frac{1}{\cos^2 \theta} = \sec^2 \theta$$

REF: 061123a2

5 ANS:

$$\cos \theta \cdot \frac{1}{\cos \theta} - \cos^2 \theta = 1 - \cos^2 \theta = \sin^2 \theta$$

REF: 061230a2

6 ANS:

$$\frac{\cot x \sin x}{\sec x} = \frac{\frac{\cos x}{\sin x} \sin x}{\frac{1}{\cos x}} = \cos^2 x$$

REF: 061334a2

7 ANS:

$$\frac{\frac{1}{\cos^2 x} - 1}{\frac{1}{\cos^2 x}} \cdot \frac{\cos^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{1} = \sin^2 x$$

REF: 081533a2

8 ANS: 1

$$\sin 120 = \frac{\sqrt{3}}{2} \quad \csc 120 = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

REF: 081505a2

9 ANS: 1

REF: 081613a2

10 ANS:

$$\frac{2\sqrt{3}}{3}. \text{ If } \sin 60 = \frac{\sqrt{3}}{2}, \text{ then } \csc 60 = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

REF: 011235a2

11 ANS: 3

Cofunctions secant and cosecant are complementary

REF: 011625a2

12 ANS: 3

Cofunctions tangent and cotangent are complementary

REF: 061014a2

13 ANS:

$$a + 15 + 2a = 90$$

$$3a + 15 = 90$$

$$3a = 75$$

$$a = 25$$

REF: 011330a2

14 ANS:

$\csc \theta = \frac{1}{\sin \theta}$, and $\sin \theta$ on a unit circle represents the y value of a point on the unit circle. Since $y = \sin \theta$,

$$\csc \theta = \frac{1}{y}.$$

REF: 011727aii

15 ANS: 2

REF: 081010a2

16 ANS: 1

$$\sqrt{12^2 - 6^2} = \sqrt{108} = \sqrt{36} \sqrt{3} = 6\sqrt{3}. \quad \cot J = \frac{A}{O} = \frac{6}{6\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

REF: 011120a2