F.LE.A.4: Exponential Decay

1 The Fahrenheit temperature, $F(t)$, of a heated object at time t, in minutes, can be modeled by the function below. F_s is the surrounding temperature, F_0 is the initial temperature of the object, and k is a constant.

$$F(t) = F_s + (F_0 - F_s)e^{-kt}$$

Coffee at a temperature of 195°F is poured into a container. The room temperature is kept at a constant 68°F and $k = 0.05$. Coffee is safe to drink when its temperature is, at most, 120°F. To the *nearest minute*, how long will it take until the coffee is safe to drink?
1) 7
2) 10
3) 11
4) 18

2 The half-life of iodine-131 is 8 days. The percent of the isotope left in the body d days after being introduced is $I = 100\left(\frac{1}{2}\right)^{\frac{d}{8}}$. When this equation is written in terms of the number e, the base of the natural logarithm, it is equivalent to $I = 100e^{kd}$. What is the approximate value of the constant, k?
1) –0.087
2) 0.087
3) –11.542
4) 11.542

3 The equation for radioactive decay is $p = (0.5)^{\frac{t}{H}}$, where p is the part of a substance with half-life H remaining radioactive after a period of time, t. A given substance has a half-life of 6,000 years. After t years, one-fifth of the original sample remains radioactive. Find t, to the *nearest thousand years*.

4 One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days. A patient is injected with 20 milligrams of I–131. Determine, to the *nearest day*, the amount of time needed before the amount of I–131 in the patient's body is approximately 7 milligrams.

5 The half-life of a radioactive substance is 15 years. Write an equation that can be used to determine the amount, $s(t)$, of 200 grams of this substance that remains after t years. Determine algebraically, to the *nearest year*, how long it will take for $\frac{1}{10}$ of this substance to remain.

6 An archaeologist can determine the approximate age of certain ancient specimens by measuring the amount of carbon-14, a radioactive substance, contained in the specimen. The formula used to determine the age of a specimen is $A = A_0 2^{\frac{-t}{5760}}$, where A is the amount of carbon-14 that a specimen contains, A_0 is the original amount of carbon-14, t is time, in years, and 5760 is the half-life of carbon-14. A specimen that originally contained 120 milligrams of carbon-14 now contains 100 milligrams of this substance. What is the age of the specimen, to the *nearest hundred years*?

7 A radioactive substance has a mass of 140 g at 3 p.m. and 100 g at 8 p.m. Write an equation in the form $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$ that models this situation, where h is the constant representing the number of hours in the half-life, A_0 is the initial mass, and A is the mass t hours after 3 p.m. Using this equation, solve for h, to the *nearest ten thousandth*. Determine when the mass of the radioactive substance will be 40 g. Round your answer to the *nearest tenth of an hour*.

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Answer Section

1 ANS: 4

$$120 = 68 + (195 - 68)e^{-0.05t}$$
$$52 = 127e^{-0.05t}$$
$$\ln\frac{52}{127} = \ln e^{-0.05t}$$
$$\ln\frac{52}{127} = -0.05t$$
$$\frac{\ln\frac{52}{127}}{-0.05} = t$$
$$18 \approx t$$

REF: 081918aii

2 ANS: 1

$$100\left(\frac{1}{2}\right)^{\frac{d}{8}} = 100e^{kd}$$
$$\left(\frac{1}{2}\right)^{\frac{1}{8}} = e^{k}$$
$$k \approx -0.087$$

REF: 061818aii

3 ANS:

$$p = (.5)^{\frac{t}{H}}$$
$$.2 = (.5)^{\frac{t}{6000}}$$
$$\log .2 = \log .5^{\frac{t}{6000}}$$
$$\log .2 = \frac{t}{6000}\log .5$$
$$\frac{\log .2}{\log .5} = \frac{t}{6000}$$
$$t \approx 14000$$

REF: 010429b

4 ANS:

$$7 = 20(0.5)^{\frac{t}{8.02}}$$

$$\log 0.35 = \log 0.5^{\frac{t}{8.02}}$$

$$\log 0.35 = \frac{t \log 0.5}{8.02}$$

$$\frac{8.02 \log 0.35}{\log 0.5} = t$$

$$t \approx 12$$

REF: 081634aii

5 ANS:

$$s(t) = 200(0.5)^{\frac{t}{15}} \qquad \frac{1}{10} = (0.5)^{\frac{t}{15}}$$

$$\log \frac{1}{10} = \log(0.5)^{\frac{t}{15}}$$

$$-1 = \frac{t \cdot \log(0.5)}{15}$$

$$t = \frac{-15}{\log(0.5)} \approx 50$$

REF: 061934aii

6 ANS:

$$100 = 120(2)^{\frac{-t}{5760}}$$

$$\frac{5}{6} = (2)^{\frac{-t}{5760}}$$

$$\log \frac{5}{6} = \log 2^{\frac{-t}{5760}}$$

$$\log \frac{5}{6} = \frac{-t}{5760} \log 2$$

$$\frac{\log \frac{5}{6}}{\log 2} = \frac{-t}{5760}$$

$$t \approx 1500$$

REF: 060431b

7 ANS:

$$100 = 140\left(\frac{1}{2}\right)^{\frac{5}{h}} \quad \log\frac{100}{140} = \log\left(\frac{1}{2}\right)^{\frac{5}{h}} \qquad 40 = 140\left(\frac{1}{2}\right)^{\frac{t}{10.3002}}$$

$$\log\frac{5}{7} = \frac{5}{h}\log\frac{1}{2} \qquad \log\frac{2}{7} = \log\left(\frac{1}{2}\right)^{\frac{t}{10.3002}}$$

$$h = \frac{5\log\frac{1}{2}}{\log\frac{5}{7}} \approx 10.3002 \qquad \log\frac{2}{7} = \frac{t\log\left(\frac{1}{2}\right)}{10.3002}$$

$$t = \frac{10.3002\log\frac{2}{7}}{\log\frac{1}{2}} \approx 18.6$$

REF: 061737aii