F.IF.C.9: Comparing Quadratic Functions

1 Which quadratic function has the largest maximum over the set of real numbers?

1) $f(x) = -x^2 + 2x + 4$

2)

x	k(x)
−1	−1
0	3
1	5
2	5
3	3
4	−1

3) $g(x) = -(x-5)^2 + 5$

4)

x	h(x)
−2	−9
−1	−3
0	1
1	3
2	3
3	1

2 Which quadratic function has the largest maximum?

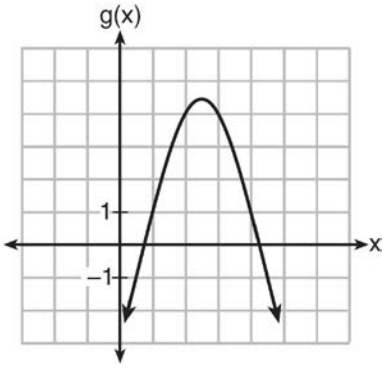
1) $h(x) = (3-x)(2+x)$

2)

x	f(x)
−1	−3
0	5
1	9
2	9
3	5
4	−3

3) $k(x) = -5x^2 - 12x + 4$

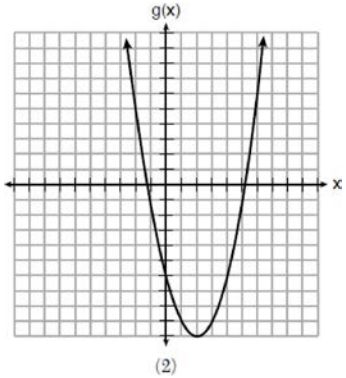
4)



3 Which of the quadratic functions below has the *smallest* minimum value?

1) $h(x) = x^2 + 2x - 6$

2)



3) $k(x) = (x+5)(x+2)$

4)

x	f(x)
−1	−2
0	−5
1	−6
2	−5
3	−2

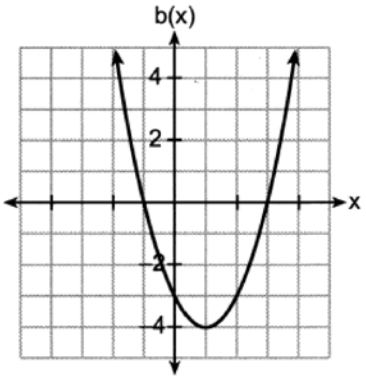
4 Four quadratic functions are represented below.

$a(x) = (x - 3)^2 - 7$

I

$c(x) = x^2 + 6x + 3$

III



II

x	d(x)
–4	–1
–3	–4
–2	–5
–1	–4
0	–1

IV

Which function has the *smallest* minimum value?

1) I
2) II
3) III
4) IV

5 Which quadratic function has the *smallest* minimum value?

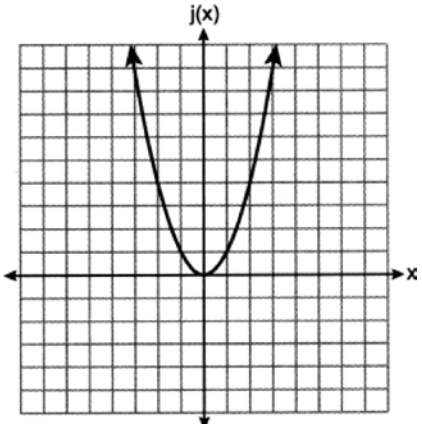
1) $f(x) = 6x^2 + 5x - 2$

2)

x	h(x)
0	6
1	2
2	0
3	0
4	2
5	6

3) $g(x) = 6(x - 2)^2 - 2$

4)



6 The quadratic functions $r(x)$ and $q(x)$ are given below.

x	$r(x)$
–4	–12
–3	–15
–2	–16
–1	–15
0	–12
1	7

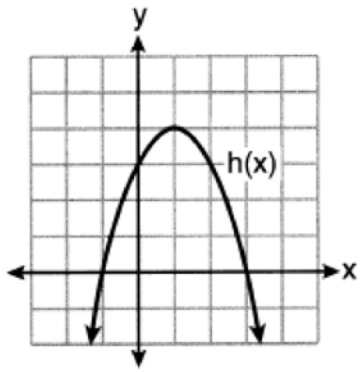
$$q(x) = x^2 + 2x - 8$$

The function with the *smaller* minimum value is

1) $q(x)$, and the value is –9
2) $q(x)$, and the value is –1
3) $r(x)$, and the value is –16
4) $r(x)$, and the value is –2

7 Four quadratic functions are shown below.

x	f(x)
−4	−4
−2	4
−1	5
0	4
2	−4



$$g(x) = -(x-4)^2 + 5$$

$$j(x) = -\frac{1}{2}x^2 + x + 4$$

Which statement is true?

1) The maximum of $f(x)$ is less than the maximum of $j(x)$.
2) The maximum of $g(x)$ is less than the maximum of $h(x)$.
3) The maximum of $f(x)$ equals the maximum of $g(x)$.
4) The maximum of $h(x)$ equals the maximum of $j(x)$.

8 Three quadratic functions are given below.

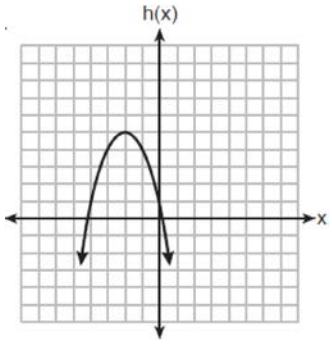
I.

$$f(x) = (x+2)^2 + 5$$

II.

x	–4	–3	–2	–1	0	1
g(x)	–3	2	5	5	2	–3

III.



Which of these functions have the same vertex?

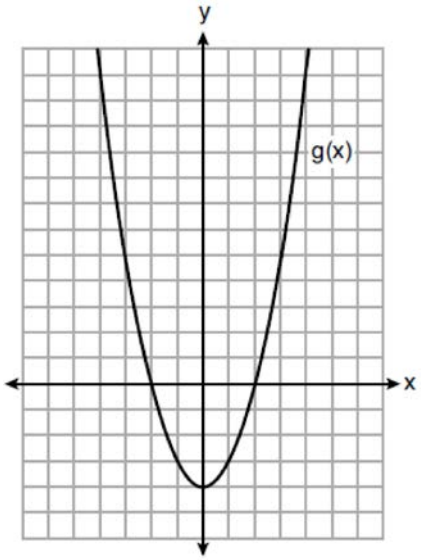
1) I and II, only
2) II and III, only
3) I and III, only
4) I, II, and III

9 Which statement is true about the quadratic functions $g(x)$, shown in the table below, and $f(x) = (x-3)^2 + 2$?

x	g(x)
0	4
1	–1
2	–4
3	–5
4	–4
5	–1
6	4

1) They have the same vertex.
2) They have the same zeros.
3) They have the same axis of symmetry.
4) They intersect at two points.

10 Which statement is true about the functions $f(x)$ and $g(x)$, given below?



$$f(x) = -x^2 - 4x - 4$$

1) The minimum value of $g(x)$ is greater than the maximum value of $f(x)$.
2) $f(x)$ and $g(x)$ have the same y-intercept.
3) $f(x)$ and $g(x)$ have the same roots.
4) $f(x) = g(x)$ when $x = -4$.

11 Given the following quadratic functions:

$$g(x) = -x^2 - x + 6$$

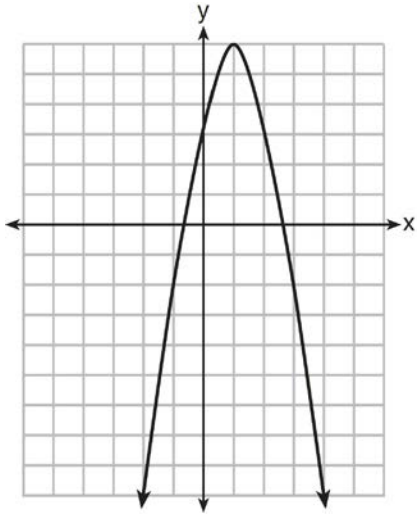
and

x	–3	–2	–1	0	1	2	3	4	5
n(x)	–7	0	5	8	9	8	5	0	–7

Which statement about these functions is true?

1) Over the interval $-1 \le x \le 1$, the average rate of change for $n(x)$ is less than that for $g(x)$.
2) The y-intercept of $g(x)$ is greater than the y-intercept for $n(x)$.
3) The function $g(x)$ has a greater maximum value than $n(x)$.
4) The sum of the roots of $n(x) = 0$ is greater than the sum of the roots of $g(x) = 0$.

12 Let f be the function represented by the graph below.



Let g be a function such that $g(x) = -\frac{1}{2}x^2 + 4x + 3$. Determine which function has the larger maximum value. Justify your answer.

F.IF.C.9: Comparing Quadratic Functions
Answer Section

1 ANS: 2

1) $x = \frac{-2}{2(-1)} = 1$; 2) $h = \frac{3}{2}$ Using (0,3), $3 = a\left(0 - \frac{3}{2}\right)^2 + k$; Using (1,5), $5 = a\left(1 - \frac{3}{2}\right)^2 + k$

$y = -1^2 + 2(1) + 4 = 5$ $3 = \frac{9}{4}a + k$ $5 = \frac{1}{4}a + k$

vertex (1,5)

$k = 3 - \frac{9}{4}a$ $k = 5 - \frac{1}{4}a$

$5 - \frac{1}{4}a = 3 - \frac{9}{4}a$ $k = 5 - \frac{1}{4}(-1) = \frac{21}{4}$; 3) vertex (5,5); 4) Using $c = 1$ $-9 = (-2)^2 a + (-2)b + 1$

$20 - a = 12 - 9a$ vertex $\left(\frac{3}{2}, \frac{21}{4}\right)$ $-10 = 4a - 2b$

$8a = -8$ $b = 2a + 5$

$a = -1$

$-3 = (-1)^2 a + (-1)b + 1$ $2a + 5 = a + 4$ $x = \frac{-3}{2(-1)} = \frac{3}{2}$ vertex $\left(\frac{3}{2}, \frac{13}{4}\right)$

$-3 = a - b + 1$ $a = -1$

$b = a + 4$ $b = -1 + 4 = 3$ $y = -\left(\frac{3}{2}\right)^2 + 3\left(\frac{3}{2}\right) + 1 = -\frac{9}{4} + \frac{18}{4} + \frac{4}{4} = \frac{13}{4}$

REF: 011823ai

2 ANS: 3

$h(x) = -x^2 + x + 6$ Maximum of $f(x) = 9$ $k(x) = -5x^2 - 12x + 4$ Maximum of $g(x) < 5$

$x = \frac{-1}{2(-1)} = \frac{1}{2}$ $x = \frac{12}{2(-5)} = -\frac{6}{5}$

$y = -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 6$ $y = -5\left(-\frac{6}{5}\right)^2 - 12\left(-\frac{6}{5}\right) + 4$

$= -\frac{1}{4} + \frac{2}{4} + 6$ $= -\frac{36}{5} + \frac{72}{5} + \frac{20}{5}$

$= 6\frac{1}{4}$ $= \frac{56}{5}$

$= 11\frac{1}{5}$

REF: 061514ai

3 ANS: 2

1) $x = \frac{-2}{2(1)} = -1$, $h(-1) = (-1)^2 + 2(-1) - 6 = -7$; 2) $y = -10$; 3) $k\left(\frac{-5+-2}{2}\right) = (-3.5+5)(-3.5+2) = -2.25$; 4) $y = -6$

REF: 061813ai

4 ANS: 1

1) -7; 2) -4; 3) $x = \frac{-6}{2(1)} = -3$, $c(-3) = (-3)^2 + 6(-3) + 3 = -6$; 4) -5

REF: 062414ai

5 ANS: 1

1) $f\left(\frac{-5}{2(6)}\right) \approx -3.04$; 2) $h(2.5) = (2.5-2)(2.5-3) = -0.25$; 3) $g(2) = -2$; 4) 0

REF: 012320ai

6 ANS: 3

The minimum of $r(x)$ is -16. The minimum of $q(x)$ is -9 $\left(x = \frac{-2}{2(1)} = -1, q(-1) = -9\right)$.

REF: 081917ai

7 ANS: 3

Maximum of $f(x) = 5$ Maximum of $h(x) = 4$ Maximum of $g(x) = 5$ $j(x) = -\frac{1}{2}x^2 + x + 4$

$$x = \frac{-1}{2\left(-\frac{1}{2}\right)} = 1$$

$$j(1) = -\frac{1}{2}(1)^2 + 1 + 4 = 4\frac{1}{2}$$

REF: 062219ai

8 ANS: 3

f and h's vertex is $(-2,5)$. g's axis of symmetry is $x = -1.5$.

REF: 062319ai

9 ANS: 3

$x = 3$

REF: 061717ai

10 ANS: 2

The y-intercept of both $f(x)$ and $g(x)$ is -4.

REF: 012013ai

11 ANS: 4

1) $\dfrac{g(1)-g(-1)}{1--1}=\dfrac{4-6}{2}=\dfrac{-2}{2}=-1$ 2) $g(0)=6$ 3) $x=\dfrac{-(-1)}{2(-1)}=-\dfrac{1}{2};\ g\left(-\dfrac{1}{2}\right)=-\left(-\dfrac{1}{2}\right)^2+\dfrac{1}{2}+6=6\dfrac{1}{4}$

$\dfrac{n(1)-n(-1)}{1--1}=\dfrac{9-5}{2}=\dfrac{4}{2}=2$ $n(0)=8$ $x=1;\ n(1)=9$

4) $g{:}S=\dfrac{-(-1)}{-1}=-1$

$n{:}\ S=-2+4=2$

REF: 081521ai

12 ANS:

g. The maximum of f is 6. For g, the maximum is 11. $x=\dfrac{-b}{2a}=\dfrac{-4}{2\left(-\dfrac{1}{2}\right)}=\dfrac{-4}{-1}=4$

$$y=-\frac{1}{2}(4)^2+4(4)+3=-8+16+3=11$$

REF: 081429ai