Calculus Practice: Using Definite Integrals to Calculate Volume 9b

For each problem, find the volume of the specified solid.

1) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{16} + \frac{y^2}{36} = 1$. Cross-sections perpendicular to the x-axis are semicircles.

2) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 9$. Cross-sections perpendicular to the x-axis are rectangles with heights half that of the side in the xy-plane.

3) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Cross-sections perpendicular to the x-axis are rectangles with heights half that of the side in the xy-plane.

4) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 36$. Cross-sections perpendicular to the x-axis are semicircles.

5) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. Cross-sections perpendicular to the x-axis are squares.

6) The base of a solid is the region enclosed by the semicircle $y = \sqrt{36 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are semicircles.

7) The base of a solid is the region enclosed by the semicircle $y = \sqrt{9 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are semicircles.

8) The base of a solid is the region enclosed by the semicircle $y = \sqrt{36 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are rectangles with heights twice that of the side in the xy-plane.

9) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{49} + \frac{y^2}{36} = 1$. Cross-sections perpendicular to the x-axis are squares.

10) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 16$. Cross-sections perpendicular to the x-axis are squares.

11) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{36} + \frac{y^2}{49} = 1$. Cross-sections perpendicular to the x-axis are rectangles with heights twice that of the side in the xy-plane.

12) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{36} + \frac{y^2}{9} = 1$. Cross-sections perpendicular to the x-axis are squares.

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For each problem, find the volume of the specified solid.

1) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{16} + \frac{y^2}{36} = 1$. Cross-sections perpendicular to the x-axis are semicircles.

$$\frac{\pi}{8}\int_{-4}^{4}\left(\sqrt{36 - \frac{36x^2}{16}} + \sqrt{36 - \frac{36x^2}{16}}\right)^2 dx$$
$= 96\pi \approx 301.593$

2) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 9$. Cross-sections perpendicular to the x-axis are rectangles with heights half that of the side in the xy-plane.

$$\frac{1}{2}\int_{-3}^{3}\left(\sqrt{9 - x^2} + \sqrt{9 - x^2}\right)^2 dx$$
$= 72$

3) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Cross-sections perpendicular to the x-axis are rectangles with heights half that of the side in the xy-plane.

$$\frac{1}{2}\int_{-3}^{3}\left(\sqrt{4 - \frac{4x^2}{9}} + \sqrt{4 - \frac{4x^2}{9}}\right)^2 dx$$
$= 32$

4) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 36$. Cross-sections perpendicular to the x-axis are semicircles.

$$\frac{\pi}{8}\int_{-6}^{6}\left(\sqrt{36 - x^2} + \sqrt{36 - x^2}\right)^2 dx$$
$= 144\pi \approx 452.389$

5) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. Cross-sections perpendicular to the x-axis are squares.

$$\int_{-2}^{2}\left(\sqrt{9 - \frac{9x^2}{4}} + \sqrt{9 - \frac{9x^2}{4}}\right)^2 dx$$
$= 96$

6) The base of a solid is the region enclosed by the semicircle $y = \sqrt{36 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are semicircles.

$$\frac{\pi}{8}\int_{-6}^{6}\left(\sqrt{36 - x^2}\right)^2 dx$$
$= 36\pi \approx 113.097$

7) The base of a solid is the region enclosed by the semicircle $y = \sqrt{9 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are semicircles.

$$\frac{\pi}{8}\int_{-3}^{3}\left(\sqrt{9 - x^2}\right)^2\,dx$$
$$= \frac{9\pi}{2} \approx 14.137$$

8) The base of a solid is the region enclosed by the semicircle $y = \sqrt{36 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are rectangles with heights twice that of the side in the xy-plane.

$$2\int_{-6}^{6}\left(\sqrt{36 - x^2}\right)^2\,dx$$
$$= 576$$

9) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{49} + \frac{y^2}{36} = 1$. Cross-sections perpendicular to the x-axis are squares.

$$\int_{-7}^{7}\left(\sqrt{36 - \frac{36x^2}{49}} + \sqrt{36 - \frac{36x^2}{49}}\right)^2 dx$$
$$= 1344$$

10) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 16$. Cross-sections perpendicular to the x-axis are squares.

$$\int_{-4}^{4}\left(\sqrt{16 - x^2} + \sqrt{16 - x^2}\right)^2 dx$$
$$= \frac{1024}{3} \approx 341.333$$

11) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{36} + \frac{y^2}{49} = 1$. Cross-sections perpendicular to the x-axis are rectangles with heights twice that of the side in the xy-plane.

$$2\int_{-6}^{6}\left(\sqrt{49 - \frac{49x^2}{36}} + \sqrt{49 - \frac{49x^2}{36}}\right)^2 dx$$
$$= 3136$$

12) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{36} + \frac{y^2}{9} = 1$. Cross-sections perpendicular to the x-axis are squares.

$$\int_{-6}^{6}\left(\sqrt{9 - \frac{9x^2}{36}} + \sqrt{9 - \frac{9x^2}{36}}\right)^2 dx$$
$$= 288$$