Calculus Practice: Using Definite Integrals to Calculate Volume 11b

For each problem, find the volume of the specified solid.

1) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 36$. Cross-sections perpendicular to the y-axis are squares.

2) The base of a solid is the region enclosed by $y = -\frac{x^2}{9} + 1$ and $y = 0$. Cross-sections perpendicular to the y-axis are semicircles.

3) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 16$. Cross-sections perpendicular to the y-axis are semicircles.

4) The base of a solid is the region enclosed by the semicircle $y = \sqrt{49 - x^2}$ and the x-axis. Cross-sections perpendicular to the y-axis are equilateral triangles.

5) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 25$. Cross-sections perpendicular to the y-axis are isosceles right triangles with one leg in the xy-plane.

6) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 49$. Cross-sections perpendicular to the y-axis are semicircles.

7) The base of a solid is the region enclosed by $y = -\frac{x^2}{4} + 1$ and $y = 0$. Cross-sections perpendicular to the y-axis are rectangles with heights twice that of the side in the xy-plane.

8) The base of a solid is the region enclosed by $y = -\frac{x^2}{4} + 1$ and $y = 0$. Cross-sections perpendicular to the y-axis are isosceles right triangles with the hypotenuse in the base.

9) The base of a solid is the region enclosed by $y = 4$ and $y = \frac{x^2}{4}$. Cross-sections perpendicular to the y-axis are isosceles right triangles with the hypotenuse in the base.

10) The base of a solid is the region enclosed by $y = 4$ and $y = x^2$. Cross-sections perpendicular to the y-axis are isosceles right triangles with one leg in the xy-plane.

11) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{16} + \frac{y^2}{36} = 1$. Cross-sections perpendicular to the y-axis are rectangles with heights twice that of the side in the xy-plane.

12) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Cross-sections perpendicular to the y-axis are isosceles right triangles with one leg in the xy-plane.

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For each problem, find the volume of the specified solid.

1) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 36$. Cross-sections perpendicular to the y-axis are squares.

$$\int_{-6}^{6} \left(\sqrt{36 - y^2} + \sqrt{36 - y^2}\right)^2 dy$$
$$= 1152$$

2) The base of a solid is the region enclosed by $y = -\frac{x^2}{9} + 1$ and $y = 0$. Cross-sections perpendicular to the y-axis are semicircles.

$$\frac{\pi}{8}\int_{0}^{1} \left(\sqrt{9 - 9y} + \sqrt{9 - 9y}\right)^2 dy$$
$$= \frac{9\pi}{4} \approx 7.069$$

3) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 16$. Cross-sections perpendicular to the y-axis are semicircles.

$$\frac{\pi}{8}\int_{-4}^{4} \left(\sqrt{16 - y^2} + \sqrt{16 - y^2}\right)^2 dy$$
$$= \frac{128\pi}{3} \approx 134.041$$

4) The base of a solid is the region enclosed by the semicircle $y = \sqrt{49 - x^2}$ and the x-axis. Cross-sections perpendicular to the y-axis are equilateral triangles.

$$\frac{\sqrt{3}}{4}\int_{0}^{7} \left(\sqrt{49 - y^2} + \sqrt{49 - y^2}\right)^2 dy$$
$$= \frac{686\sqrt{3}}{3} \approx 396.062$$

5) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 25$. Cross-sections perpendicular to the y-axis are isosceles right triangles with one leg in the xy-plane.

$$\frac{1}{2}\int_{-5}^{5} \left(\sqrt{25 - y^2} + \sqrt{25 - y^2}\right)^2 dy$$
$$= \frac{1000}{3} \approx 333.333$$

6) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 49$. Cross-sections perpendicular to the y-axis are semicircles.

$$\frac{\pi}{8}\int_{-7}^{7} \left(\sqrt{49 - y^2} + \sqrt{49 - y^2}\right)^2 dy$$
$$= \frac{686\pi}{3} \approx 718.378$$

7) The base of a solid is the region enclosed by $y = -\frac{x^2}{4} + 1$ and $y = 0$. Cross-sections perpendicular to the y-axis are rectangles with heights twice that of the side in the xy-plane.

$$2\int_0^1 \left(\sqrt{4 - 4y} + \sqrt{4 - 4y}\right)^2 dy$$
$$= 16$$

8) The base of a solid is the region enclosed by $y = -\frac{x^2}{4} + 1$ and $y = 0$. Cross-sections perpendicular to the y-axis are isosceles right triangles with the hypotenuse in the base.

$$\frac{1}{4}\int_0^1 \left(\sqrt{4 - 4y} + \sqrt{4 - 4y}\right)^2 dy$$
$$= 2$$

9) The base of a solid is the region enclosed by $y = 4$ and $y = \frac{x^2}{4}$. Cross-sections perpendicular to the y-axis are isosceles right triangles with the hypotenuse in the base.

$$\frac{1}{4}\int_0^4 \left(2\sqrt{y} + 2\sqrt{y}\right)^2 dy$$
$$= 32$$

10) The base of a solid is the region enclosed by $y = 4$ and $y = x^2$. Cross-sections perpendicular to the y-axis are isosceles right triangles with one leg in the xy-plane.

$$\frac{1}{2}\int_0^4 \left(\sqrt{y} + \sqrt{y}\right)^2 dy$$
$$= 16$$

11) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{16} + \frac{y^2}{36} = 1$. Cross-sections perpendicular to the y-axis are rectangles with heights twice that of the side in the xy-plane.

$$2\int_{-6}^{6} \left(\sqrt{16 - \frac{16y^2}{36}} + \sqrt{16 - \frac{16y^2}{36}}\right)^2 dy$$
$$= 1024$$

12) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Cross-sections perpendicular to the y-axis are isosceles right triangles with one leg in the xy-plane.

$$\frac{1}{2}\int_{-2}^{2} \left(\sqrt{9 - \frac{9y^2}{4}} + \sqrt{9 - \frac{9y^2}{4}}\right)^2 dy$$
$$= 48$$