Calculus Practice: Using Definite Integrals to Calculate Volume 10b

For each problem, find the volume of the specified solid.

1) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{16} + \frac{y^2}{36} = 1$. Cross-sections perpendicular to the x-axis are equilateral triangles.

2) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Cross-sections perpendicular to the x-axis are isosceles right triangles with one leg in the xy-plane.

3) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. Cross-sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse in the base.

4) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 36$. Cross-sections perpendicular to the x-axis are isosceles right triangles with one leg in the xy-plane.

5) The base of a solid is the region enclosed by the semicircle $y = \sqrt{36 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are isosceles right triangles with one leg in the xy -plane.

6) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{49} + \frac{y^2}{9} = 1$. Cross-sections perpendicular to the x-axis are isosceles right triangles with one leg in the xy-plane.

7) The base of a solid is the region enclosed by the semicircle $y = \sqrt{25 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse in the base.

8) The base of a solid is the region enclosed by the semicircle $y = \sqrt{49 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are isosceles right triangles with one leg in the xy -plane.

9) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 49$. Cross-sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse in the base.

10) The base of a solid is the region enclosed by $y = -x^2 + 4$ and $y = 0$. Cross-sections perpendicular to the x-axis are equilateral triangles.

11) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{16} + \frac{y^2}{49} = 1$. Cross-sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse in the base.

12) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 9$. Cross-sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse in the base.

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For each problem, find the volume of the specified solid.

1) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{16} + \frac{y^2}{36} = 1$. Cross-sections perpendicular to the x-axis are equilateral triangles.

$$\frac{\sqrt{3}}{4}\int_{-4}^{4}\left(\sqrt{36 - \frac{36x^2}{16}} + \sqrt{36 - \frac{36x^2}{16}}\right)^2 dx$$
$$= 192\sqrt{3} \approx 332.554$$

2) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Cross-sections perpendicular to the x-axis are isosceles right triangles with one leg in the xy-plane.

$$\frac{1}{2}\int_{-3}^{3}\left(\sqrt{4 - \frac{4x^2}{9}} + \sqrt{4 - \frac{4x^2}{9}}\right)^2 dx$$
$$= 32$$

3) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. Cross-sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse in the base.

$$\frac{1}{4}\int_{-2}^{2}\left(\sqrt{9 - \frac{9x^2}{4}} + \sqrt{9 - \frac{9x^2}{4}}\right)^2 dx$$
$$= 24$$

4) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 36$. Cross-sections perpendicular to the x-axis are isosceles right triangles with one leg in the xy-plane.

$$\frac{1}{2}\int_{-6}^{6}\left(\sqrt{36 - x^2} + \sqrt{36 - x^2}\right)^2 dx$$
$$= 576$$

5) The base of a solid is the region enclosed by the semicircle $y = \sqrt{36 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are isosceles right triangles with one leg in the xy-plane.

$$\frac{1}{2}\int_{-6}^{6}\left(\sqrt{36 - x^2}\right)^2 dx$$
$$= 144$$

6) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{49} + \frac{y^2}{9} = 1$. Cross-sections perpendicular to the x-axis are isosceles right triangles with one leg in the xy-plane.

$$\frac{1}{2}\int_{-7}^{7}\left(\sqrt{9 - \frac{9x^2}{49}} + \sqrt{9 - \frac{9x^2}{49}}\right)^2 dx$$
$$= 168$$

7) The base of a solid is the region enclosed by the semicircle $y = \sqrt{25 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse in the base.

$$\frac{1}{4}\int_{-5}^{5} \left(\sqrt{25 - x^2}\right)^2 \, dx$$
$$= \frac{125}{3} \approx 41.667$$

8) The base of a solid is the region enclosed by the semicircle $y = \sqrt{49 - x^2}$ and the x-axis. Cross-sections perpendicular to the x-axis are isosceles right triangles with one leg in the xy-plane.

$$\frac{1}{2}\int_{-7}^{7} \left(\sqrt{49 - x^2}\right)^2 \, dx$$
$$= \frac{686}{3} \approx 228.667$$

9) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 49$. Cross-sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse in the base.

$$\frac{1}{4}\int_{-7}^{7} \left(\sqrt{49 - x^2} + \sqrt{49 - x^2}\right)^2 \, dx$$
$$= \frac{1372}{3} \approx 457.333$$

10) The base of a solid is the region enclosed by $y = -x^2 + 4$ and $y = 0$. Cross-sections perpendicular to the x-axis are equilateral triangles.

$$\frac{\sqrt{3}}{4}\int_{-2}^{2} \left(-x^2 + 4\right)^2 \, dx$$
$$= \frac{128\sqrt{3}}{15} \approx 14.78$$

11) The base of a solid is the region enclosed by the ellipse $\frac{x^2}{16} + \frac{y^2}{49} = 1$. Cross-sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse in the base.

$$\frac{1}{4}\int_{-4}^{4} \left(\sqrt{49 - \frac{49x^2}{16}} + \sqrt{49 - \frac{49x^2}{16}}\right)^2 \, dx$$
$$= \frac{784}{3} \approx 261.333$$

12) The base of a solid is the region enclosed by the circle $x^2 + y^2 = 9$. Cross-sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse in the base.

$$\frac{1}{4}\int_{-3}^{3} \left(\sqrt{9 - x^2} + \sqrt{9 - x^2}\right)^2 \, dx$$
$$= 36$$