

Calculus Practice: Use Derivatives to Analyze Functions 9a

For each problem, find the open intervals where the function is concave up and concave down.

1) $f(x) = (7x - 21)^{\frac{2}{3}}$

- A) Concave up: $(-\infty, 1), (1, \infty)$ Concave down: No intervals exist.
 B) Concave up: No intervals exist. Concave down: $(-\infty, 3), (3, \infty)$
 C) Concave up: No intervals exist. Concave down: $(-\infty, 12), (12, \infty)$
 D) Concave up: $(-\infty, 3), (3, \infty)$ Concave down: No intervals exist.

2) $f(x) = -(-6x + 24)^{\frac{1}{2}}$

- A) Concave up: $(-\infty, 2)$ Concave down: $(2, \infty)$
 B) Concave up: No intervals exist. Concave down: $(-\infty, 4)$
 C) Concave up: $(-\infty, 3)$ Concave down: $(3, \infty)$
 D) Concave up: $(-\infty, 4)$ Concave down: No intervals exist.

3) $y = \frac{1}{6}(x - 1)^{\frac{7}{3}} - \frac{14}{3}(x - 1)^{\frac{1}{3}} + 1$

- A) Concave up: $(1, \infty)$ Concave down: $(-\infty, 1)$
 B) Concave up: $(-\infty, 1)$ Concave down: $(1, \infty)$
 C) Concave up: $(4, \infty)$ Concave down: $(-\infty, 4)$
 D) Concave up: $(-\infty, \frac{1}{3})$ Concave down: $(\frac{1}{3}, \infty)$

4) $y = -\frac{1}{5}(x + 4)^{\frac{5}{3}} + 2(x + 4)^{\frac{2}{3}} + 1$

- A) Concave up: $(-\infty, -24)$ Concave down: $(-24, -16), (-16, \infty)$
 B) Concave up: $(-2, -\frac{4}{3}), (-\frac{4}{3}, \infty)$ Concave down: $(-\infty, -2)$
 C) Concave up: $(-6, -4), (-4, \infty)$ Concave down: $(-\infty, -6)$
 D) Concave up: $(-\infty, -6)$ Concave down: $(-6, -4), (-4, \infty)$

5) $f(x) = -\sec(2x); [-\pi, \pi]$

- A) Concave up: $(-\frac{\pi}{2}, 0), (\frac{\pi}{2}, \pi)$ Concave down: $(-\pi, -\frac{\pi}{2}), (0, \frac{\pi}{2})$
 B) Concave up: $(-\frac{\pi}{2}, \frac{\pi}{2})$ Concave down: $(-\pi, -\frac{\pi}{2}), (\frac{\pi}{2}, \pi)$
 C) Concave up: $(-\frac{3\pi}{4}, -\frac{\pi}{4}), (\frac{\pi}{4}, \frac{3\pi}{4})$ Concave down: $(-\pi, -\frac{3\pi}{4}), (-\frac{\pi}{4}, \frac{\pi}{4}), (\frac{3\pi}{4}, \pi)$
 D) Concave up: $(-\frac{\pi}{2}, 0), (\frac{\pi}{2}, \pi), (-\pi, -\frac{\pi}{2}), (\frac{\pi}{2}, \pi)$ Concave down: $(-\pi, -\frac{\pi}{2}), (0, \frac{\pi}{2}), (-\frac{\pi}{2}, \frac{\pi}{2})$

6) $f(x) = -2\sin(2x)$; $[-\pi, \pi]$

A) Concave up: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ Concave down: $\left(-\pi, -\frac{\pi}{2}\right), \left(\frac{\pi}{2}, \pi\right)$

B) Concave up: $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, \pi\right)$ Concave down: $\left(-\pi, -\frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$

C) Concave up: $\left(-\pi, -\frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$ Concave down: $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, \pi\right)$

D) Concave up: $\left(-\pi, -\frac{3\pi}{4}\right), \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right), \left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$ Concave down: $\left(-\frac{3\pi}{4}, -\frac{\pi}{2}\right), \left(-\frac{\pi}{4}, 0\right), \left(\frac{\pi}{4}, \frac{\pi}{2}\right), \left(\frac{3\pi}{4}, \pi\right)$

7) $f(x) = 2\tan(2x)$; $[-\pi, \pi]$

A) Concave up: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ Concave down: $\left(-\pi, -\frac{\pi}{2}\right), \left(\frac{\pi}{2}, \pi\right)$

B) Concave up: $\left(-\frac{3\pi}{4}, -\frac{\pi}{2}\right), \left(-\frac{\pi}{4}, 0\right), \left(\frac{\pi}{4}, \frac{\pi}{2}\right), \left(\frac{3\pi}{4}, \pi\right)$ Concave down: $\left(-\pi, -\frac{3\pi}{4}\right), \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right), \left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

C) Concave up: $\left(-\pi, -\frac{3\pi}{4}\right), \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right), \left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$ Concave down: $\left(-\frac{3\pi}{4}, -\frac{\pi}{2}\right), \left(-\frac{\pi}{4}, 0\right), \left(\frac{\pi}{4}, \frac{\pi}{2}\right), \left(\frac{3\pi}{4}, \pi\right)$

D) Concave up: $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, \pi\right)$ Concave down: $\left(-\pi, -\frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$

8) $f(x) = -\cos(2x)$; $[-\pi, \pi]$

A) Concave up: $\left(-\pi, -\frac{3\pi}{4}\right), \left(-\frac{\pi}{4}, \frac{\pi}{4}\right), \left(\frac{3\pi}{4}, \pi\right)$ Concave down: $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

B) Concave up: $(-\pi, 0), \left(-\pi, -\frac{\pi}{2}\right), \left(\frac{\pi}{2}, \pi\right)$ Concave down: $(0, \pi), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

C) Concave up: $\left(-\pi, -\frac{\pi}{2}\right), \left(\frac{\pi}{2}, \pi\right)$ Concave down: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

D) Concave up: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ Concave down: $\left(-\pi, -\frac{\pi}{2}\right), \left(\frac{\pi}{2}, \pi\right)$

9) $y = \cot(x)$; $[-\pi, \pi]$

A) Concave up: $\left(-\pi, -\frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$ Concave down: $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, \pi\right)$

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10) $f(x) = -2\csc(x)$; $[-\pi, \pi]$

A) Concave up: $(-\pi, 0)$ Concave down: $(0, \pi)$

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