

Calculus Practice: Use Derivatives to Analyze Functions 8a

For each problem, find the open intervals where the function is concave up and concave down.

1) $f(x) = x^5 - 2x^3 - 2$

- A) Concave up: $\left(-\frac{\sqrt{15}}{5}, 0\right), \left(\frac{\sqrt{15}}{5}, \infty\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{15}}{5}\right), \left(0, \frac{\sqrt{15}}{5}\right)$
- B) Concave up: $\left(-\frac{4\sqrt{15}}{5}, 4\right), \left(\frac{4\sqrt{15}}{5}, \infty\right)$ Concave down: $\left(-\infty, -\frac{4\sqrt{15}}{5}\right), \left(4, \frac{4\sqrt{15}}{5}\right)$
- C) Concave up: $\left(-\infty, -\frac{\sqrt{15}}{15}\right), \left(\frac{1}{3}, \frac{\sqrt{15}}{15}\right)$ Concave down: $\left(-\frac{\sqrt{15}}{15}, \frac{1}{3}\right), \left(\frac{\sqrt{15}}{15}, \infty\right)$
- D) Concave up: $\left(-\infty, -\frac{\sqrt{15}}{5}\right), \left(0, \frac{\sqrt{15}}{5}\right)$ Concave down: $\left(-\frac{\sqrt{15}}{5}, 0\right), \left(\frac{\sqrt{15}}{5}, \infty\right)$

2) $f(x) = \frac{x^2}{2} - 2x - 1$

- A) Concave up: $(-\infty, 3)$ Concave down: $(3, \infty)$
- B) Concave up: $(-\infty, \infty)$ Concave down: No intervals exist.
- C) Concave up: $(-\infty, 2)$ Concave down: $(2, \infty)$
- D) Concave up: No intervals exist. Concave down: $(-\infty, \infty)$

3) $f(x) = \frac{x^2}{2}$

- A) Concave up: No intervals exist. Concave down: $(-\infty, \infty)$
- B) Concave up: $(-\infty, 2)$ Concave down: $(2, \infty)$
- C) Concave up: $(-\infty, \infty)$ Concave down: No intervals exist.
- D) Concave up: $(-\infty, 3)$ Concave down: $(3, \infty)$

4) $y = -x^2 - 4x - 6$

- A) Concave up: $(-\infty, \infty)$ Concave down: No intervals exist.
- B) Concave up: $(-\infty, 3)$ Concave down: $(3, \infty)$
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- D) Concave up: No intervals exist. Concave down: $(-\infty, \infty)$

5) $y = x^4 - 3x^2 + 1$

A) Concave up: $\left(-\frac{\sqrt{2}}{6}, \frac{\sqrt{2}}{6}\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{2}}{6}\right), \left(\frac{\sqrt{2}}{6}, \infty\right)$

B) Concave up: $\left(-\infty, -\frac{\sqrt{2}}{2}\right), \left(\frac{\sqrt{2}}{2}, \infty\right)$ Concave down: $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

C) Concave up: $\left(-\infty, -2\sqrt{2}\right), \left(2\sqrt{2}, \infty\right)$ Concave down: $\left(-2\sqrt{2}, 2\sqrt{2}\right)$

D) Concave up: $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{2}}{2}\right), \left(\frac{\sqrt{2}}{2}, \infty\right)$

6) $y = x^5 - 3x^3 - 3$

A) Concave up: $\left(-\frac{3\sqrt{10}}{10}, 0\right), \left(\frac{3\sqrt{10}}{10}, \infty\right)$ Concave down: $\left(-\infty, -\frac{3\sqrt{10}}{10}\right), \left(0, \frac{3\sqrt{10}}{10}\right)$

B) Concave up: $\left(-\infty, -\frac{3\sqrt{10}}{10}\right), \left(0, \frac{3\sqrt{10}}{10}\right)$ Concave down: $\left(-\frac{3\sqrt{10}}{10}, 0\right), \left(\frac{3\sqrt{10}}{10}, \infty\right)$

C) Concave up: $\left(-\frac{6\sqrt{10}}{5}, 4\right), \left(\frac{6\sqrt{10}}{5}, \infty\right)$ Concave down: $\left(-\infty, -\frac{6\sqrt{10}}{5}\right), \left(4, \frac{6\sqrt{10}}{5}\right)$

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7) $y = x^4 + 2x^3 - 2x^2 + 4$

A) Concave up: $\left(-\infty, \frac{-6 - 2\sqrt{21}}{3}\right), \left(\frac{-6 + 2\sqrt{21}}{3}, \infty\right)$ Concave down: $\left(\frac{-6 - 2\sqrt{21}}{3}, \frac{-6 + 2\sqrt{21}}{3}\right)$

B) Concave up: $\left(\frac{-3 - \sqrt{21}}{6}, \frac{-3 + \sqrt{21}}{6}\right)$ Concave down: $\left(-\infty, \frac{-3 - \sqrt{21}}{6}\right), \left(\frac{-3 + \sqrt{21}}{6}, \infty\right)$

C) Concave up: $\left(\frac{-3 - \sqrt{21}}{18}, \frac{-3 + \sqrt{21}}{18}\right)$ Concave down: $\left(-\infty, \frac{-3 - \sqrt{21}}{18}\right), \left(\frac{-3 + \sqrt{21}}{18}, \infty\right)$

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8) $f(x) = x^5 - 2x^3 - 1$

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B) Concave up: $\left(-\frac{\sqrt{15}}{5}, 0\right), \left(\frac{\sqrt{15}}{5}, \infty\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{15}}{5}\right), \left(0, \frac{\sqrt{15}}{5}\right)$

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