Calculus Practice: Use Derivatives to Analyze Functions 5b

For each problem, find the open intervals where the function is increasing and decreasing.

1) $f(x) = \frac{1}{5}(x-4)^{\frac{5}{3}} + 2(x-4)^{\frac{2}{3}} + 2$

2) $f(x) = \frac{1}{6}(x-1)^{\frac{7}{3}} - \frac{14}{3}(x-1)^{\frac{1}{3}} + 2$

3) $f(x) = \frac{x^3}{x^2-4}$

4) $y = -\frac{1}{4}(x-1)^{\frac{8}{3}} + 4(x-1)^{\frac{2}{3}} - 2$

5) $y = \frac{x^3}{x^2-1}$

6) $f(x) = \frac{x^2}{4x+8}$

7) $f(x) = \tan(2x)$; $[-\pi, \pi]$

8) $f(x) = -2\csc(x)$; $[-\pi, \pi]$

9) $y = -2\tan(x)$; $[-\pi, \pi]$

10) $f(x) = 2\cos(x)$; $[-\pi, \pi]$

11) $y = -2\sec(2x)$; $[-\pi, \pi]$

12) $y = -2\sin(2x)$; $[-\pi, \pi]$

13) $f(x) = 2\csc(x)$; $[-\pi, \pi]$

14) $f(x) = 2\cot(x)$; $[-\pi, \pi]$

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For each problem, find the open intervals where the function is increasing and decreasing.

1) $f(x) = \frac{1}{5}(x-4)^{\frac{5}{3}} + 2(x-4)^{\frac{2}{3}} + 2$

Increasing: $(-\infty, 0), (4, \infty)$ Decreasing: $(0, 4)$

2) $f(x) = \frac{1}{6}(x-1)^{\frac{7}{3}} - \frac{14}{3}(x-1)^{\frac{1}{3}} + 2$

Increasing: $(-\infty, -1), (3, \infty)$ Decreasing: $(-1, 3)$

3) $f(x) = \frac{x^3}{x^2-4}$

Increasing: $(-\infty, -2\sqrt{3}), (2\sqrt{3}, \infty)$ Decreasing: $(-2\sqrt{3}, -2), (-2, 2), (2, 2\sqrt{3})$

4) $y = -\frac{1}{4}(x-1)^{\frac{8}{3}} + 4(x-1)^{\frac{2}{3}} - 2$

Increasing: $(-\infty, -1), (1, 3)$ Decreasing: $(-1, 1), (3, \infty)$

5) $y = \frac{x^3}{x^2-1}$

Increasing: $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$ Decreasing: $(-\sqrt{3}, -1), (-1, 1), (1, \sqrt{3})$

6) $f(x) = \frac{x^2}{4x+8}$

Increasing: $(-\infty, -4), (0, \infty)$ Decreasing: $(-4, -2), (-2, 0)$

7) $f(x) = \tan(2x);\ [-\pi, \pi]$

Increasing: $\left(-\pi, -\frac{3\pi}{4}\right), \left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right), \left(-\frac{\pi}{4}, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{3\pi}{4}\right), \left(\frac{3\pi}{4}, \pi\right)$ Decreasing: No intervals exist.

8) $f(x) = -2\csc(x);\ [-\pi, \pi]$

Increasing: $\left(-\frac{\pi}{2}, 0\right), \left(0, \frac{\pi}{2}\right)$ Decreasing: $\left(-\pi, -\frac{\pi}{2}\right), \left(\frac{\pi}{2}, \pi\right)$

9) $y = -2\tan(x);\ [-\pi, \pi]$

Increasing: No intervals exist. Decreasing: $\left(-\pi, -\frac{\pi}{2}\right), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \pi\right)$

10) $f(x) = 2\cos(x);\ [-\pi, \pi]$

Increasing: $(-\pi, 0)$ Decreasing: $(0, \pi)$

11) $y = -2\sec(2x);\ [-\pi, \pi]$

Increasing: $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right), \left(-\frac{\pi}{4}, 0\right), \left(\frac{\pi}{2}, \frac{3\pi}{4}\right), \left(\frac{3\pi}{4}, \pi\right)$ Decreasing: $\left(-\pi, -\frac{3\pi}{4}\right), \left(-\frac{3\pi}{4}, -\frac{\pi}{2}\right), \left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

12) $y = -2\sin(2x);\ [-\pi, \pi]$

Increasing: $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ Decreasing: $\left(-\pi, -\frac{3\pi}{4}\right), \left(-\frac{\pi}{4}, \frac{\pi}{4}\right), \left(\frac{3\pi}{4}, \pi\right)$

13) $f(x) = 2\csc(x);\ [-\pi, \pi]$

Increasing: $\left(-\pi, -\frac{\pi}{2}\right), \left(\frac{\pi}{2}, \pi\right)$ Decreasing: $\left(-\frac{\pi}{2}, 0\right), \left(0, \frac{\pi}{2}\right)$

14) $f(x) = 2\cot(x);\ [-\pi, \pi]$

Increasing: No intervals exist. Decreasing: $(-\pi, 0), (0, \pi)$