Name_____

Calculus Practice: Use Derivatives to Analyze Functions 13a

For each problem, find all points of absolute minima and maxima on the given interval.

1) $y = \frac{1}{4}(x+3)^{\frac{8}{3}} - 4(x+3)^{\frac{2}{3}} + 2$; $(-7, -3)$

  A) No absolute minima.
     No absolute maxima.

  B) Absolute minimum: $\left(-1, -3\sqrt[3]{4} + 2\right)$
     Absolute maximum: $\left(-5, -3\sqrt[3]{4} + 2\right)$

  C) Absolute minimum: $(-3, 2)$
     Absolute maximum: $\left(-1, -3\sqrt[3]{4} + 2\right)$

  D) Absolute minimum: $\left(-5, -3\sqrt[3]{4} + 2\right)$
     No absolute maxima.

2) $y = \frac{x^2}{4x-4}$; $(-2, 2]$

  A) No absolute minima.
     No absolute maxima.

  B) Absolute minimum: $(0, 0)$
     Absolute maximum: $(2, 1)$

  C) Absolute minimum: $(2, 1)$
     Absolute maximum: $(0, 0)$

  D) Absolute minimum: $(2, 1)$
     No absolute maxima.

3) $y = \frac{3}{16}(x+1)^{\frac{4}{3}} - \frac{3}{2}(x+1)^{\frac{1}{3}} - 2$; $[-2, \infty)$

  A) Absolute minimum: $\left(1, \frac{-16 - 9\sqrt[3]{2}}{8}\right)$

     No absolute maxima.

  B) Absolute minimum: $\left(1, \frac{-16 - 9\sqrt[3]{2}}{8}\right)$

     Absolute maximum: $(-1, -2)$

  C) Absolute minimum: $\left(1, \frac{-16 - 9\sqrt[3]{2}}{8}\right)$

     Absolute maximum: $\left(-2, -\frac{5}{16}\right)$

  D) No absolute minima.
     No absolute maxima.

4) $y = \frac{1}{5}(x+3)^{\frac{5}{3}} - 2(x+3)^{\frac{2}{3}}$; $(-4, 2)$

  A) Absolute minimum: $\left(1, -\frac{12\sqrt[3]{2}}{5}\right)$

     No absolute maxima.

  B) No absolute minima.
     No absolute maxima.

  C) Absolute minimum: $\left(1, -\frac{12\sqrt[3]{2}}{5}\right)$

     Absolute maximum: $(-3, 0)$

  D) Absolute minimum: $\left(1, -\frac{12\sqrt[3]{2}}{5}\right)$

     Absolute maximum: $\left(2, -\sqrt[3]{25}\right)$

5) $f(x) = -\frac{15}{x^2 + 5}$; $[-1, \infty)$

  A) No absolute minima.
     Absolute maximum: $(0, -3)$

  B) No absolute minima.
     No absolute maxima.

  C) Absolute minimum: $(0, -3)$
     Absolute maximum: $\left(2, -\frac{5}{3}\right)$

  D) Absolute minimum: $(0, -3)$
     No absolute maxima.

6) $y = \frac{9x}{x^2 + 9}$; $[2, 6]$

  A) Absolute minimum: $\left(6, \frac{6}{5}\right)$

     Absolute maximum: $\left(3, \frac{3}{2}\right)$

  B) Absolute minimum: $\left(2, \frac{18}{13}\right)$

     Absolute maximum: $\left(3, \frac{3}{2}\right)$

  C) No absolute minima.
     No absolute maxima.

  D) Absolute minimum: $\left(-3, -\frac{3}{2}\right)$

     Absolute maximum: $\left(3, \frac{3}{2}\right)$

7) $f(x) = -\cot(2x)$; $[\frac{\pi}{4}, \frac{\pi}{3}]$

   A) Absolute minimum: $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{3}\right)$

      Absolute maximum: $\left(\frac{\pi}{4}, 0\right)$

   B) Absolute minimum: $\left(\frac{\pi}{4}, 0\right)$

      No absolute maxima.

   C) Absolute minimum: $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{3}\right)$

      No absolute maxima.

   D) Absolute minimum: $\left(\frac{\pi}{4}, 0\right)$

      Absolute maximum: $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{3}\right)$

8) $y = 2\csc(2x)$; $[\frac{\pi}{6}, \frac{\pi}{4}]$

   A) No absolute minima.
      No absolute maxima.

   B) Absolute minimum: $\left(\frac{3\pi}{4}, -2\right)$

      Absolute maximum: $\left(-\frac{3\pi}{4}, 2\right)$

   C) Absolute minimum: $\left(\frac{\pi}{4}, 2\right)$

      Absolute maximum: $\left(\frac{\pi}{6}, \frac{4\sqrt{3}}{3}\right)$

   D) Absolute minimum: $\left(\frac{3\pi}{4}, -2\right)$

      Absolute maximum: $\left(\frac{\pi}{4}, 2\right)$

9) $f(x) = -\sin(x)$; $[\frac{\pi}{6}, \frac{\pi}{4})$

   A) No absolute minima.

      Absolute maximum: $\left(\frac{\pi}{6}, -\frac{1}{2}\right)$

   B) No absolute minima.
      No absolute maxima.

   C) Absolute minimum: $\left(\frac{\pi}{6}, -\frac{1}{2}\right)$

      Absolute maximum: $\left(\frac{\pi}{2}, -1\right)$

   D) Absolute minimum: $\left(-\frac{\pi}{2}, 1\right)$

      Absolute maximum: $\left(\frac{\pi}{2}, -1\right)$

10) $f(x) = \tan(2x)$; $(-\frac{\pi}{2}, -\frac{\pi}{3}]$

   A) Absolute minimum: $\left(-\frac{\pi}{2}, 0\right)$

      Absolute maximum: $\left(-\frac{\pi}{3}, \sqrt{3}\right)$

   B) No absolute minima.

      Absolute maximum: $\left(-\frac{\pi}{3}, \sqrt{3}\right)$

   C) Absolute minimum: $\left(-\frac{\pi}{3}, \sqrt{3}\right)$

      Absolute maximum: $\left(-\frac{\pi}{2}, 0\right)$

   D) No absolute minima.

      Absolute maximum: $\left(-\frac{\pi}{2}, 0\right)$

11) $f(x) = -2\sec(2x)$; $[-\frac{\pi}{2}, \frac{\pi}{2}]$

   A) Absolute minimum: $(-\pi, -2)$
      Absolute maximum: $(\pi, -2)$
   B) Absolute minimum: $(\pi, -2)$
      Absolute maximum: $(-\pi, -2)$
   C) Absolute minimum: $(\pi, -2)$

      Absolute maximum: $\left(\frac{\pi}{2}, 2\right)$

   D) No absolute minima.
      No absolute maxima.

12) $f(x) = 2\cos(x)$; $[-\frac{\pi}{6}, \frac{3\pi}{4}]$

   A) Absolute minimum: $(0, 2)$
      Absolute maximum: $(\pi, -2)$
   B) Absolute minimum: $(\pi, -2)$
      Absolute maximum: $(0, 2)$
   C) No absolute minima.
      No absolute maxima.

   D) Absolute minimum: $\left(\frac{3\pi}{4}, -\sqrt{2}\right)$

      Absolute maximum: $(0, 2)$

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B) Absolute minimum: $\left(1, \dfrac{-16 - 9\sqrt[3]{2}}{8}\right)$
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*C) Absolute minimum: $\left(1, \dfrac{-16 - 9\sqrt[3]{2}}{8}\right)$
   Absolute maximum: $\left(-2, -\dfrac{5}{16}\right)$
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4) $y = \dfrac{1}{5}(x+3)^{\frac{5}{3}} - 2(x+3)^{\frac{2}{3}}; \quad (-4, 2)$

A) Absolute minimum: $\left(1, -\dfrac{12\sqrt[3]{2}}{5}\right)$
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D) Absolute minimum: $\left(1, -\dfrac{12\sqrt[3]{2}}{5}\right)$
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  C) Absolute minimum: $\left(-\frac{\pi}{3}, \sqrt{3}\right)$

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