Calculus Practice: Use Derivatives to Analyze Functions 11a

For each problem, find all points of relative minima and maxima.

1) $f(x) = \dfrac{3}{x^2 - 9}$

A) No relative minima.
Relative maximum: $\left(\frac{1}{3}, -\frac{27}{80}\right)$

B) No relative minima.
Relative maximum: $\left(4, \frac{3}{7}\right)$

C) No relative minima.
Relative maximum: $\left(0, -\frac{1}{3}\right)$

D) No relative minima.
No relative maxima.

2) $f(x) = -\frac{1}{6}(x-2)^{\frac{7}{3}} + \frac{14}{3}(x-2)^{\frac{1}{3}} + 1$

A) No relative minima.
Relative maxima: $\left(\frac{1}{3}, \frac{162 - 227\sqrt[3]{45}}{162}\right), \left(\frac{2}{3}, \frac{81 - 118\sqrt[3]{36}}{81}\right), \left(\frac{4}{3}, \frac{81 - 124\sqrt[3]{18}}{81}\right)$

B) No relative minima.
No relative maxima.

C) Relative minimum: $\left(0, 1 - 4\sqrt[3]{2}\right)$
Relative maximum: $\left(4, 1 + 4\sqrt[3]{2}\right)$

D) No relative minima.
Relative maxima: $\left(4, 1 + 4\sqrt[3]{2}\right), \left(8, \frac{3 - 4\sqrt[3]{6}}{3}\right), \left(16, 1 - 28\sqrt[3]{14}\right)$

3) $f(x) = \frac{1}{4}(x-2)^{\frac{8}{3}} - 4(x-2)^{\frac{2}{3}} + 1$

A) Relative minimum: $\left(16, 45\sqrt[3]{196} + 1\right)$
Relative maxima: $\left(4, -3\sqrt[3]{4} + 1\right), \left(8, 5\sqrt[3]{36} + 1\right)$

B) Relative minima: $\left(0, -3\sqrt[3]{4} + 1\right), \left(4, -3\sqrt[3]{4} + 1\right)$
Relative maximum: $\left(2, 1\right)$

C) Relative minima: $\left(\frac{2}{3}, \frac{-64\sqrt[3]{6} + 27}{27}\right), \left(\frac{4}{3}, \frac{-35\sqrt[3]{12} + 27}{27}\right)$
Relative maximum: $\left(\frac{1}{3}, \frac{-119\sqrt[3]{75} + 108}{108}\right)$

D) No relative minima.
No relative maxima.

4) $y = -\dfrac{x^2}{2x - 2}$

A) Relative minimum: $\left(0, 0\right)$
Relative maximum: $\left(2, -2\right)$

B) No relative minima.
No relative maxima.

C) Relative minima: $\left(4, -\frac{8}{3}\right), \left(8, -\frac{32}{7}\right)$
No relative maxima.

D) Relative minima: $\left(\frac{1}{3}, \frac{1}{12}\right), \left(\frac{2}{3}, \frac{2}{3}\right)$
No relative maxima.

5) $f(x) = 2\cos(2x)$; $[-\pi, \pi]$

A) Relative minima: $(-\pi, -2), (0, -2), (\pi, -2)$
Relative maxima: $\left(\frac{\pi}{2}, 2\right), \left(\frac{\pi}{2}, 2\right)$

B) Relative minima: $\left(-\frac{\pi}{2}, -2\right), \left(\frac{\pi}{2}, -2\right)$
Relative maxima: $(-\pi, 2), (0, 2), (\pi, 2)$

C) Relative minimum: $(0, -1)$
Relative maxima: $(-\pi, 1), (\pi, 1)$

D) Relative minima: $\left(-\frac{\pi}{4}, -2\right), \left(\frac{3\pi}{4}, -2\right), (-\pi, -2), (\pi, -2)$
Relative maxima: $\left(-\frac{3\pi}{4}, 2\right), \left(\frac{\pi}{4}, 2\right), (0, 2)$

6) $y = \csc(x)$; $[-\pi, \pi]$

A) Relative minima: $(-\pi, -1), (0, -1), (\pi, -1)$
Relative maxima: $\left(\frac{\pi}{2}, 1\right), \left(\frac{\pi}{2}, 1\right)$

B) Relative minimum: $\left(\frac{\pi}{2}, 1\right)$
Relative maximum: $\left(-\frac{\pi}{2}, -1\right)$

C) Relative minimum: $\left(\frac{\pi}{2}, -1\right)$
Relative maximum: $\left(-\frac{\pi}{2}, 1\right)$

D) No relative minima.
No relative maxima.

7) $f(x) = -2\sec(2x)$; $[-\pi, \pi]$

A) Relative minima: $\left(-\frac{3\pi}{4}, -2\right), \left(\frac{\pi}{4}, -2\right), (0, -2)$
Relative maxima: $\left(-\frac{\pi}{4}, 2\right), \left(\frac{3\pi}{4}, 2\right), (-\pi, 2), (\pi, 2)$

B) No relative minima.
No relative maxima.

C) Relative minima: $\left(-\frac{\pi}{2}, 2\right), \left(\frac{\pi}{2}, 2\right)$
Relative maxima: $(-\pi, -2), (0, -2), (\pi, -2)$

D) Relative minimum: $(0, 2)$
Relative maxima: $(-\pi, -2), (\pi, -2)$

8) $f(x) = -\sin(x)$; $[-\pi, \pi]$

A) Relative minimum: $(0, 1)$
Relative maxima: $(-\pi, -1), (\pi, -1)$

B) Relative minimum: $\left(-\frac{\pi}{2}, -2\right)$
Relative maximum: $\left(\frac{\pi}{2}, 2\right)$

C) Relative minimum: $\left(\frac{\pi}{2}, -1\right)$
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Relative maxima: $\left(-\frac{3\pi}{4}, 2\right), \left(\frac{\pi}{4}, 2\right), (0, 2)$

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*B) Relative minimum: $\left(\frac{\pi}{2}, 1\right)$
Relative maximum: $\left(-\frac{\pi}{2}, -1\right)$

C) Relative minimum: $\left(\frac{\pi}{2}, -1\right)$
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B) No relative minima.
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*C) Relative minima: $\left(-\frac{\pi}{2}, 2\right), \left(\frac{\pi}{2}, 2\right)$
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