Calculus Practice: Related Rates 1

Solve each related rate problem.

1) A hypothetical square grows so that the length of its diagonals are increasing at a rate of 5 m/min. How fast is the area of the square increasing when the diagonals are 12 m each?

2) A 17 ft ladder is leaning against a wall and sliding towards the floor. The top of the ladder is sliding down the wall at a rate of 5 ft/sec. How fast is the base of the ladder sliding away from the wall when the base of the ladder is 15 ft from the wall?

3) Water slowly evaporates from a circular shaped puddle. The radius of the puddle decreases at a rate of 9 in/hr. Assuming the puddle retains its circular shape, at what rate is the area of the puddle changing when the radius is 6 in?

4) A hypothetical cube grows so that the length of its sides are increasing at a rate of 3 m/min. How fast is the volume of the cube increasing when the sides are 2 m each?

5) A spherical snowball is rolled in fresh snow, causing it to grow so that its radius increases at a rate of 2 in/sec. How fast is the volume of the snowball increasing when the radius is 8 in?

6) A conical paper cup is 10 cm tall with a radius of 20 cm. The cup is being filled with water so that the water level rises at a rate of 4 cm/sec. At what rate is water being poured into the cup when the water level is 7 cm?

Name________________________________

Calculus Practice: Related Rates 1

Solve each related rate problem.

1) A hypothetical square grows so that the length of its diagonals are increasing at a rate of 5 m/min. How fast is the area of the square increasing when the diagonals are 12 m each?

A = area of square x = length of diagonals t = time

Equation: $A = \dfrac{x^2}{2}$ Given rate: $\dfrac{dx}{dt} = 5$ Find: $\left.\dfrac{dA}{dt}\right|_{x=12}$

$$\left.\frac{dA}{dt}\right|_{x=12} = x \cdot \frac{dx}{dt} = 60 \text{ m}^2/\text{min}$$

2) A 17 ft ladder is leaning against a wall and sliding towards the floor. The top of the ladder is sliding down the wall at a rate of 5 ft/sec. How fast is the base of the ladder sliding away from the wall when the base of the ladder is 15 ft from the wall?

x = horizontal distance from base of ladder to wall y = vertical distance from top of ladder to floor t = time

Equation: $x^2 + y^2 = 17^2$ Given rate: $\dfrac{dy}{dt} = -5$ Find: $\left.\dfrac{dx}{dt}\right|_{x=15}$

$$\left.\frac{dx}{dt}\right|_{x=15} = -\frac{y}{x} \cdot \frac{dy}{dt} = \frac{8}{3} \text{ ft/sec}$$

3) Water slowly evaporates from a circular shaped puddle. The radius of the puddle decreases at a rate of 9 in/hr. Assuming the puddle retains its circular shape, at what rate is the area of the puddle changing when the radius is 6 in?

A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\dfrac{dr}{dt} = -9$ Find: $\left.\dfrac{dA}{dt}\right|_{r=6}$

$$\left.\frac{dA}{dt}\right|_{r=6} = 2\pi r \cdot \frac{dr}{dt} = -108\pi \text{ in}^2/\text{hr}$$

4) A hypothetical cube grows so that the length of its sides are increasing at a rate of 3 m/min. How fast is the volume of the cube increasing when the sides are 2 m each?

V = volume of cube $\quad s$ = length of sides $\quad t$ = time

Equation: $V = s^3$ $\quad$ Given rate: $\frac{ds}{dt} = 3$ $\quad$ Find: $\left.\frac{dV}{dt}\right|_{s=2}$

$$\left.\frac{dV}{dt}\right|_{s=2} = 3s^2 \cdot \frac{ds}{dt} = 36 \text{ m}^3\text{/min}$$

5) A spherical snowball is rolled in fresh snow, causing it to grow so that its radius increases at a rate of 2 in/sec. How fast is the volume of the snowball increasing when the radius is 8 in?

V = volume of sphere $\quad r$ = radius $\quad t$ = time

Equation: $V = \frac{4}{3}\pi r^3$ $\quad$ Given rate: $\frac{dr}{dt} = 2$ $\quad$ Find: $\left.\frac{dV}{dt}\right|_{r=8}$

$$\left.\frac{dV}{dt}\right|_{r=8} = 4\pi r^2 \cdot \frac{dr}{dt} = 512\pi \text{ in}^3\text{/sec}$$

6) A conical paper cup is 10 cm tall with a radius of 20 cm. The cup is being filled with water so that the water level rises at a rate of 4 cm/sec. At what rate is water being poured into the cup when the water level is 7 cm?

V = volume of material in cone $\quad h$ = height $\quad t$ = time

Equation: $V = \frac{4\pi h^3}{3}$ $\quad$ Given rate: $\frac{dh}{dt} = 4$ $\quad$ Find: $\left.\frac{dV}{dt}\right|_{h=7}$

$$\left.\frac{dV}{dt}\right|_{h=7} = 4\pi h^2 \cdot \frac{dh}{dt} = 784\pi \text{ cm}^3\text{/sec}$$