 Name________________________________

Calculus Practice: Optimization 1

Solve each optimization problem.

1) A cryptography expert is deciphering a computer code. To do this, the expert needs to minimize the product of a positive rational number and a negative rational number, given that the positive number is exactly 5 greater than the negative number. What final product is the expert looking for?

2) A supermarket employee wants to construct an open-top box from a 16 by 30 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?

3) A farmer wants to construct a rectangular pigpen using 200 ft of fencing. The pen will be built next to an existing stone wall, so only three sides of fencing need to be constructed to enclose the pen. What dimensions should the farmer use to construct the pen with the largest possible area?

4) A rancher wants to construct two identical rectangular corrals using 400 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?

5) A company has started selling a new type of smartphone at the price of $\$120 - 0.05x$ where x is the number of smartphones manufactured per day. The parts for each smartphone cost \$60 and the labor and overhead for running the plant cost \$6000 per day. How many smartphones should the company manufacture and sell per day to maximize profit?

6) Two vertical poles, one 8 ft high and the other 16 ft high, stand 45 feet apart on a flat field. A worker wants to support both poles by running rope from the ground to the top of each post. If the worker wants to stake both ropes in the ground at the same point, where should the stake be placed to use the least amount of rope?

7) An architect is designing a composite window by attaching a semicircular window on top of a rectangular window, so the diameter of the top window is equal to and aligned with the width of the bottom window. If the architect wants the perimeter of the composite window to be 8 ft, what dimensions should the bottom window be in order to create the composite window with the largest area?

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1) A cryptography expert is deciphering a computer code. To do this, the expert needs to minimize the product of a positive rational number and a negative rational number, given that the positive number is exactly 5 greater than the negative number. What final product is the expert looking for?

P = the product of the two numbers x = the positive number
Function to minimize: $P = x(x - 5)$ where $-\infty < x < \infty$
Smallest product of the two numbers: $-\frac{25}{4}$

2) A supermarket employee wants to construct an open-top box from a 16 by 30 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?

V = the volume of the box x = the length of the sides of the squares
Function to maximize: $V = (30 - 2x)(16 - 2x) \cdot x$ where $0 < x < 8$
Sides of the squares: $\frac{10}{3}$ in

3) A farmer wants to construct a rectangular pigpen using 200 ft of fencing. The pen will be built next to an existing stone wall, so only three sides of fencing need to be constructed to enclose the pen. What dimensions should the farmer use to construct the pen with the largest possible area?

A = the area of the pigpen x = the length of the sides perpendicular to the stone wall
Function to maximize: $A = x(200 - 2x)$ where $0 < x < 100$
Dimensions of the pigpen: 50 ft (perpendicular to wall) by 100 ft (parallel to wall)

4) A rancher wants to construct two identical rectangular corrals using 400 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?

A = the total area of the two corrals x = the length of the non-adjacent sides of each corral

Function to maximize: $A = 2x \cdot \frac{400 - 4x}{3}$ where $0 < x < 100$

Dimensions of each corall: 50 ft (non-adjacent sides) by $\frac{200}{3}$ ft (adjacent sides)

5) A company has started selling a new type of smartphone at the price of $\$120 - 0.05x$ where x is the number of smartphones manufactured per day. The parts for each smartphone cost \$60 and the labor and overhead for running the plant cost \$6000 per day. How many smartphones should the company manufacture and sell per day to maximize profit?

p = the profit per day x = the number of items manufactured per day

Function to maximize: $p = x(120 - 0.05x) - (60x + 6000)$ where $0 \le x < \infty$

Optimal number of smartphones to manufacture per day: 600

6) Two vertical poles, one 8 ft high and the other 16 ft high, stand 45 feet apart on a flat field. A worker wants to support both poles by running rope from the ground to the top of each post. If the worker wants to stake both ropes in the ground at the same point, where should the stake be placed to use the least amount of rope?

L = the total length of rope x = the horizontal distance from the short pole to the stake

Function to minimize: $L = \sqrt{x^2 + 8^2} + \sqrt{(45 - x)^2 + 16^2}$ where $0 \le x \le 45$

Stake should be placed: 15 ft from the short pole (or 30 ft from the long pole)

7) An architect is designing a composite window by attaching a semicircular window on top of a rectangular window, so the diameter of the top window is equal to and aligned with the width of the bottom window. If the architect wants the perimeter of the composite window to be 8 ft, what dimensions should the bottom window be in order to create the composite window with the largest area?

A = the area of the composite window x = the width of the bottom window = the diameter of the top window

Function to maximize: $A = x\left(\frac{8}{2} - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{1}{2}\pi \cdot \left(\frac{x}{2}\right)^2$ where $0 < x < \frac{32}{4 + \pi}$

Dimensions of the bottom window: $\frac{16}{4 + \pi}$ ft (width) by $\frac{8}{4 + \pi}$ ft (height)