Calculus Practice: Linear Approximations 1b

For each problem, find a linear approximation of the given quantity.

1) $\sqrt[3]{8.2}$

2) 5.99^4

3) sin 911°

4) 2.02^4

5) sin 471°

6) sin 1491°

7) sin 1341°

8) $\sqrt{3.7}$

9) sin 891°

10) cos 1481°

11) $\cos 881°$

12) $\sin 441°$

13) 9.96^{4}

14) $\cos 591°$

15) $\sin 1481°$

16) $\sqrt[3]{27.3}$

17) 7.97^{4}

18) 6.03^{3}

19) 5.04^{4}

20) $\cos 911°$

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For each problem, find a linear approximation of the given quantity.

1) $\sqrt[3]{8.2}$

$f(x) = \sqrt[3]{x},\ f'(x) = \dfrac{1}{3x^{\frac{2}{3}}}$

$x_0 = 8,\ \Delta x = 0.2$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \dfrac{121}{60} \approx 2.0167$

2) 5.99^4

$f(x) = x^4,\ f'(x) = 4x^3$

$x_0 = 6,\ \Delta x = -0.01$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \dfrac{32184}{25} = 1287.36$

3) $\sin 911°$

$f(x) = \sin x,\ f'(x) = \cos x$

$x_0 = \dfrac{\pi}{2}$ radians, $\Delta x = \dfrac{\pi}{180}$ radians

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = 1$

4) 2.02^4

$f(x) = x^4,\ f'(x) = 4x^3$

$x_0 = 2,\ \Delta x = 0.02$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \dfrac{416}{25} = 16.64$

5) $\sin 471°$

$f(x) = \sin x,\ f'(x) = \cos x$

$x_0 = \dfrac{\pi}{4}$ radians, $\Delta x = \dfrac{\pi}{90}$ radians

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \dfrac{\sqrt{2}(90 + \pi)}{180} \approx 0.7318$

6) $\sin 1491°$

$f(x) = \sin x,\ f'(x) = \cos x$

$x_0 = \dfrac{5\pi}{6}$ radians, $\Delta x = -\dfrac{\pi}{180}$ radians

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \dfrac{180 + \pi\sqrt{3}}{360} \approx 0.5151$

7) $\sin 1341°$

$f(x) = \sin x,\ f'(x) = \cos x$

$x_0 = \dfrac{3\pi}{4}$ radians, $\Delta x = -\dfrac{\pi}{180}$ radians

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \dfrac{\sqrt{2}(180 + \pi)}{360} \approx 0.7194$

8) $\sqrt{3.7}$

$f(x) = \sqrt{x},\ f'(x) = \dfrac{1}{2x^{\frac{1}{2}}}$

$x_0 = 4,\ \Delta x = -0.3$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \dfrac{77}{40} = 1.925$

9) $\sin 891°$

$f(x) = \sin x,\ f'(x) = \cos x$

$x_0 = \dfrac{\pi}{2}$ radians, $\Delta x = -\dfrac{\pi}{180}$ radians

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = 1$

10) $\cos 1481°$

$f(x) = \cos x,\ f'(x) = -\sin x$

$x_0 = \dfrac{5\pi}{6}$ radians, $\Delta x = -\dfrac{\pi}{90}$ radians

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \dfrac{-90\sqrt{3} + \pi}{180} \approx -0.848$

11) $\cos 881°$

$f(x) = \cos x,\ f'(x) = -\sin x$

$x_0 = \frac{\pi}{2}$ radians, $\Delta x = -\frac{\pi}{90}$ radians

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \frac{\pi}{90} \approx 0.0349$

12) $\sin 441°$

$f(x) = \sin x,\ f'(x) = \cos x$

$x_0 = \frac{\pi}{4}$ radians, $\Delta x = -\frac{\pi}{180}$ radians

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \frac{\sqrt{2}(180 - \pi)}{360} \approx 0.694$

13) 9.96^4

$f(x) = x^4,\ f'(x) = 4x^3$

$x_0 = 10,\ \Delta x = -0.04$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = 9840$

14) $\cos 591°$

$f(x) = \cos x,\ f'(x) = -\sin x$

$x_0 = \frac{\pi}{3}$ radians, $\Delta x = -\frac{\pi}{180}$ radians

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \frac{180 + \pi\sqrt{3}}{360} \approx 0.5151$

15) $\sin 1481°$

$f(x) = \sin x,\ f'(x) = \cos x$

$x_0 = \frac{5\pi}{6}$ radians, $\Delta x = -\frac{\pi}{90}$ radians

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \frac{90 + \pi\sqrt{3}}{180} \approx 0.5302$

16) $\sqrt[3]{27.3}$

$f(x) = \sqrt[3]{x},\ f'(x) = \frac{1}{3x^{\frac{2}{3}}}$

$x_0 = 27,\ \Delta x = 0.3$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \frac{271}{90} \approx 3.0111$

17) 7.97^4

$f(x) = x^4,\ f'(x) = 4x^3$

$x_0 = 8,\ \Delta x = -0.03$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \frac{100864}{25} = 4034.56$

18) 6.03^3

$f(x) = x^3,\ f'(x) = 3x^2$

$x_0 = 6,\ \Delta x = 0.03$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \frac{5481}{25} = 219.24$

19) 5.04^4

$f(x) = x^4,\ f'(x) = 4x^3$

$x_0 = 5,\ \Delta x = 0.04$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = 645$

20) $\cos 911°$

$f(x) = \cos x,\ f'(x) = -\sin x$

$x_0 = \frac{\pi}{2}$ radians, $\Delta x = \frac{\pi}{180}$ radians

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = -\frac{\pi}{180} \approx -0.0175$