

Calculus Practice: Differentiating Products and Quotients of Functions 4b**Differentiate each function with respect to x .**

1) $y = \frac{4x^5}{3 + 2x^{-3}}$

2) $y = \frac{4x^5}{3 + \frac{5}{x^5}}$

3) $f(x) = \frac{5x^5}{2 + \frac{4}{x^5}}$

4) $y = \frac{3x^5 + 2x^2}{2\sqrt[5]{x+4}}$

5) $f(x) = \frac{4x^5 + 4x^3}{5 + \frac{3}{x^5}}$

6) $f(x) = \frac{x^2 - 2}{2 + 5x^{-2}}$

$$7) y = \frac{5x^4 + 3x^3 + 3x^2}{2 - 3x^{-3}}$$

$$8) f(x) = \frac{5x^5 + 2x^2 - 4}{2 + \frac{2}{x^2}}$$

$$9) y = \frac{2x^5 + 5x^2 - 3}{5 + 2x^{-2}}$$

$$10) f(x) = \frac{5x^5 + 5x^3 + 3}{4x^{\frac{3}{5}} - 3}$$

Calculus Practice: Differentiating Products and Quotients of Functions 4b

Differentiate each function with respect to x .

1) $y = \frac{4x^5}{3 + 2x^{-3}}$

$$\frac{dy}{dx} = \frac{(3 + 2x^{-3}) \cdot 20x^4 - 4x^5 \cdot -6x^{-4}}{(3 + 2x^{-3})^2}$$

$$= \frac{60x^{10} + 64x^7}{9x^6 + 12x^3 + 4}$$

2) $y = \frac{4x^5}{3 + \frac{5}{x^5}}$

$$\frac{dy}{dx} = \frac{(3 + 5x^{-5}) \cdot 20x^4 - 4x^5 \cdot -25x^{-6}}{(3 + 5x^{-5})^2}$$

$$= \frac{60x^{14} + 200x^9}{9x^{10} + 30x^5 + 25}$$

3) $f(x) = \frac{5x^5}{2 + \frac{4}{x^5}}$

$$f'(x) = \frac{(2 + 4x^{-5}) \cdot 25x^4 - 5x^5 \cdot -20x^{-6}}{(2 + 4x^{-5})^2}$$

$$= \frac{25x^{14} + 100x^9}{2x^{10} + 8x^5 + 8}$$

4) $y = \frac{3x^5 + 2x^2}{2\sqrt[5]{x+4}}$

$$\frac{dy}{dx} = \frac{\left(2x^{\frac{1}{5}} + 4\right)(15x^4 + 4x) - (3x^5 + 2x^2) \cdot \frac{2}{5}x^{-\frac{4}{5}}}{\left(2x^{\frac{1}{5}} + 4\right)^2}$$

$$= \frac{36x^{\frac{21}{5}} + 75x^4 + 9x^{\frac{6}{5}} + 20x}{5x^{\frac{2}{5}} + 20x^{\frac{1}{5}} + 20}$$

5) $f(x) = \frac{4x^5 + 4x^3}{5 + \frac{3}{x^5}}$

$$f'(x) = \frac{(5 + 3x^{-5})(20x^4 + 12x^2) - (4x^5 + 4x^3) \cdot -15x^{-6}}{(5 + 3x^{-5})^2}$$

$$= \frac{100x^{14} + 60x^{12} + 120x^9 + 96x^7}{25x^{10} + 30x^5 + 9}$$

6) $f(x) = \frac{x^2 - 2}{2 + 5x^{-2}}$

$$f'(x) = \frac{(2 + 5x^{-2}) \cdot 2x - (x^2 - 2) \cdot -10x^{-3}}{(2 + 5x^{-2})^2}$$

$$= \frac{4x^5 + 20x^3 - 20x}{4x^4 + 20x^2 + 25}$$

$$7) y = \frac{5x^4 + 3x^3 + 3x^2}{2 - 3x^{-3}}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(2 - 3x^{-3})(20x^3 + 9x^2 + 6x) - (5x^4 + 3x^3 + 3x^2) \cdot 9x^{-4}}{(2 - 3x^{-3})^2} \\ &= \frac{40x^9 + 18x^8 + 12x^7 - 105x^6 - 54x^5 - 45x^4}{4x^6 - 12x^3 + 9} \end{aligned}$$

$$8) f(x) = \frac{5x^5 + 2x^2 - 4}{2 + \frac{2}{x^2}}$$

$$\begin{aligned} f'(x) &= \frac{(2 + 2x^{-2})(25x^4 + 4x) - (5x^5 + 2x^2 - 4) \cdot -4x^{-3}}{(2 + 2x^{-2})^2} \\ &= \frac{25x^8 + 35x^6 + 4x^5 + 8x^3 - 8x}{2x^4 + 4x^2 + 2} \end{aligned}$$

$$9) y = \frac{2x^5 + 5x^2 - 3}{5 + 2x^{-2}}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(5 + 2x^{-2})(10x^4 + 10x) - (2x^5 + 5x^2 - 3) \cdot -4x^{-3}}{(5 + 2x^{-2})^2} \\ &= \frac{50x^8 + 28x^6 + 50x^5 + 40x^3 - 12x}{25x^4 + 20x^2 + 4} \end{aligned}$$

$$10) f(x) = \frac{5x^5 + 5x^3 + 3}{4x^{\frac{3}{5}} - 3}$$

$$\begin{aligned} f'(x) &= \frac{\left(4x^{\frac{3}{5}} - 3\right)(25x^4 + 15x^2) - (5x^5 + 5x^3 + 3) \cdot \frac{12}{5}x^{-\frac{2}{5}}}{\left(4x^{\frac{3}{5}} - 3\right)^2} \\ &= \frac{440x^5 - 375x^{\frac{22}{5}} + 240x^3 - 225x^{\frac{12}{5}} - 36}{80x^{\frac{8}{5}} - 120x + 45x^{\frac{2}{5}}} \end{aligned}$$