Calculus Practice: Differentiating Products and Quotients of Functions 4a

Differentiate each function with respect to *x*.

1) $f(x) = \dfrac{3x^2}{4x^{\frac{2}{3}} - 5}$

A) $f'(x) = 3x^2 \cdot \dfrac{8}{3}x^{-\frac{1}{3}}$

$= 8x^{\frac{5}{3}}$

B) $f'(x) = \dfrac{\left(4x^{\frac{2}{3}} - 5\right) \cdot 6x - 3x^2 \cdot \dfrac{8}{3}x^{-\frac{1}{3}}}{(3x^2)^2}$

$= \dfrac{16x^{\frac{2}{3}} - 30}{9x^3}$

C) $f'(x) = \dfrac{\left(4x^{\frac{2}{3}} - 5\right) \cdot 6x - 3x^2 \cdot \dfrac{8}{3}x^{-\frac{1}{3}}}{\left(4x^{\frac{2}{3}} - 5\right)^2}$

$= \dfrac{16x^{\frac{5}{3}} - 30x}{16x^{\frac{4}{3}} - 40x^{\frac{2}{3}} + 25}$

D) $f'(x) = \dfrac{\left(4x^{\frac{2}{3}} - 5\right) \cdot 6x - 3x^2 \cdot \dfrac{8}{3}x^{-\frac{1}{3}}}{4x^{\frac{2}{3}} - 5}$

$= \dfrac{16x^{\frac{5}{3}} - 30x}{4x^{\frac{2}{3}} - 5}$

2) $f(x) = \dfrac{5}{\sqrt[3]{x} + 2}$

A) $f'(x) = -\dfrac{5 \cdot \dfrac{1}{3}x^{-\frac{2}{3}}}{5^2}$

$= -\dfrac{1}{15x^{\frac{2}{3}}}$

B) $f'(x) = -\dfrac{\dfrac{1}{3}x^{-\frac{2}{3}}}{\left(x^{\frac{1}{3}} + 2\right)^2}$

$= -\dfrac{1}{3x^{\frac{4}{3}} + 12x + 12x^{\frac{2}{3}}}$

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D) $f'(x) = 5 \cdot \dfrac{1}{3}x^{-\frac{2}{3}}$

$= \dfrac{5}{3x^{\frac{2}{3}}}$

3) $y = \dfrac{x^2 - 4}{3 + \dfrac{1}{x^4}}$

A) $\dfrac{dy}{dx} = \dfrac{\left(3 + x^{-4}\right) \cdot 2x - \left(x^2 - 4\right) \cdot -4x^{-5}}{\left(x^2 - 4\right)^2}$

$= \dfrac{6x^6 + 6x^2 - 16}{x^9 - 8x^7 + 16x^5}$

B) $\dfrac{dy}{dx} = \dfrac{\left(3 + x^{-4}\right) \cdot 2x - \left(x^2 - 4\right) \cdot -4x^{-5}}{\left(3 + x^{-4}\right)^2}$

$= \dfrac{6x^9 + 6x^5 - 16x^3}{9x^8 + 6x^4 + 1}$

C) $\dfrac{dy}{dx} = \left(x^2 - 4\right) \cdot -4x^{-5}$

$= -\dfrac{4}{x^3} + \dfrac{16}{x^5}$

D) $\dfrac{dy}{dx} = \dfrac{2x - -4x^{-5}}{\left(3 + x^{-4}\right)^2}$

$= \dfrac{2x^9 + 4x^3}{9x^8 + 6x^4 + 1}$

4) $y = \dfrac{4x^4 - 4x^3 + 2x^2}{3 + 2x^{-3}}$

A) $\dfrac{dy}{dx} = \dfrac{\left(3 + 2x^{-3}\right)\left(16x^3 - 12x^2 + 4x\right) - \left(4x^4 - 4x^3 + 2x^2\right) \cdot -6x^{-4}}{\left(4x^4 - 4x^3 + 2x^2\right)^2}$

$= \dfrac{12x^5 - 9x^4 + 3x^3 + 14x^2 - 12x + 5}{4x^{10} - 8x^9 + 8x^8 - 4x^7 + x^6}$

B) $\dfrac{dy}{dx} = \dfrac{\left(3 + 2x^{-3}\right)\left(16x^3 - 12x^2 + 4x\right) - \left(4x^4 - 4x^3 + 2x^2\right) \cdot -6x^{-4}}{3 + 2x^{-3}}$

$= \dfrac{48x^6 - 36x^5 + 12x^4 + 56x^3 - 48x^2 + 20x}{3x^3 + 2}$

C) $\dfrac{dy}{dx} = \dfrac{\left(3 + 2x^{-3}\right)\left(16x^3 - 12x^2 + 4x\right) - \left(4x^4 - 4x^3 + 2x^2\right) \cdot -6x^{-4}}{\left(3 + 2x^{-3}\right)^2}$

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D) $\dfrac{dy}{dx} = \left(4x^4 - 4x^3 + 2x^2\right) \cdot -6x^{-4}$

$= -24 + \dfrac{24}{x} - \dfrac{12}{x^2}$

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C) $\dfrac{dy}{dx} = \left(x^2 - 4\right) \cdot -4x^{-5}$

$= -\dfrac{4}{x^3} + \dfrac{16}{x^5}$

D) $\dfrac{dy}{dx} = \dfrac{2x - -4x^{-5}}{\left(3 + x^{-4}\right)^2}$

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B) $\dfrac{dy}{dx} = \dfrac{\left(3 + 2x^{-3}\right)\left(16x^3 - 12x^2 + 4x\right) - \left(4x^4 - 4x^3 + 2x^2\right) \cdot -6x^{-4}}{3 + 2x^{-3}}$

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