

Calculus Practice: Differentiating Products and Quotients of Functions 3a

Differentiate each function with respect to x .

1) $f(x) = \frac{4}{3x^2 + 5}$

$$\begin{aligned} A) f'(x) &= -\frac{4 \cdot 6x}{(3x^2 + 5)^2} \\ &= -\frac{24x}{9x^4 + 30x^2 + 25} \end{aligned}$$

$$\begin{aligned} B) f'(x) &= -\frac{4 \cdot 6x}{4^2} \\ &= -\frac{3x}{2} \end{aligned}$$

$$\begin{aligned} C) f'(x) &= -\frac{6x}{(3x^2 + 5)^2} \\ &= -\frac{6x}{9x^4 + 30x^2 + 25} \end{aligned}$$

$$\begin{aligned} D) f'(x) &= -\frac{4 \cdot 6x}{3x^2 + 5} \\ &= -\frac{24x}{3x^2 + 5} \end{aligned}$$

3) $y = \frac{4x^4}{x^2 - 5}$

$$\begin{aligned} A) \frac{dy}{dx} &= (x^2 - 5) \cdot 16x^3 - 4x^4 \cdot 2x \\ &= 8x^5 - 80x^3 \end{aligned}$$

$$\begin{aligned} B) \frac{dy}{dx} &= \frac{(x^2 - 5) \cdot 16x^3 - 4x^4 \cdot 2x}{(4x^4)^2} \\ &= \frac{x^2 - 10}{2x^5} \end{aligned}$$

$$\begin{aligned} C) \frac{dy}{dx} &= \frac{(x^2 - 5) \cdot 16x^3 - 4x^4 \cdot 2x}{(x^2 - 5)^2} \\ &= \frac{8x^5 - 80x^3}{x^4 - 10x^2 + 25} \end{aligned}$$

$$\begin{aligned} D) \frac{dy}{dx} &= \frac{16x^3 - 2x}{(x^2 - 5)^2} \\ &= \frac{16x^3 - 2x}{x^4 - 10x^2 + 25} \end{aligned}$$

2) $f(x) = \frac{4}{4x^5 + 5}$

$$\begin{aligned} A) f'(x) &= -\frac{4 \cdot 20x^4}{(4x^5 + 5)^2} \\ &= -\frac{80x^4}{16x^{10} + 40x^5 + 25} \end{aligned}$$

$$\begin{aligned} B) f'(x) &= -\frac{4 \cdot 20x^4}{4^2} \\ &= -\frac{80x^4}{16} \end{aligned}$$

$$\begin{aligned} C) f'(x) &= 4 \cdot 20x^4 \\ &= 80x^4 \end{aligned}$$

$$\begin{aligned} D) f'(x) &= -\frac{4 \cdot 20x^4}{4x^5 + 5} \\ &= -\frac{80x^4}{4x^5 + 5} \end{aligned}$$

4) $f(x) = \frac{5x^5 + 2}{4x^3 + 2}$

$$\begin{aligned} A) f'(x) &= \frac{(4x^3 + 2) \cdot 25x^4 - (5x^5 + 2) \cdot 12x^2}{(4x^3 + 2)^2} \\ &= \frac{20x^7 + 25x^4 - 12x^2}{8x^6 + 8x^3 + 2} \end{aligned}$$

$$\begin{aligned} B) f'(x) &= \frac{(4x^3 + 2) \cdot 25x^4 - (5x^5 + 2) \cdot 12x^2}{(5x^5 + 2)^2} \\ &= \frac{40x^7 + 50x^4 - 24x^2}{25x^{10} + 20x^5 + 4} \end{aligned}$$

$$\begin{aligned} C) f'(x) &= (4x^3 + 2) \cdot 25x^4 - (5x^5 + 2) \cdot 12x^2 \\ &= 40x^7 + 50x^4 - 24x^2 \end{aligned}$$

$$\begin{aligned} D) f'(x) &= \frac{(4x^3 + 2) \cdot 25x^4 - (5x^5 + 2) \cdot 12x^2}{4x^3 + 2} \\ &= \frac{20x^7 + 25x^4 - 12x^2}{2x^3 + 1} \end{aligned}$$

5) $y = \frac{2x^4 + 1}{5x^3 + 5}$

A) $\frac{dy}{dx} = (5x^3 + 5) \cdot 8x^3 - (2x^4 + 1) \cdot 15x^2$
 $= 10x^6 + 40x^3 - 15x^2$

B) $\frac{dy}{dx} = \frac{(5x^3 + 5) \cdot 8x^3 - (2x^4 + 1) \cdot 15x^2}{5x^3 + 5}$
 $= \frac{2x^6 + 8x^3 - 3x^2}{x^3 + 1}$

C) $\frac{dy}{dx} = \frac{(5x^3 + 5) \cdot 8x^3 - (2x^4 + 1) \cdot 15x^2}{(2x^4 + 1)^2}$
 $= \frac{10x^6 + 40x^3 - 15x^2}{4x^8 + 4x^4 + 1}$

D) $\frac{dy}{dx} = \frac{(5x^3 + 5) \cdot 8x^3 - (2x^4 + 1) \cdot 15x^2}{(5x^3 + 5)^2}$
 $= \frac{2x^6 + 8x^3 - 3x^2}{5x^6 + 10x^3 + 5}$

6) $y = \frac{5x^4 - 5}{4x^5 + 2}$

A) $\frac{dy}{dx} = \frac{20x^3 - 20x^4}{(4x^5 + 2)^2}$
 $= \frac{-5x^4 + 5x^3}{4x^{10} + 4x^5 + 1}$

B) $\frac{dy}{dx} = \frac{(4x^5 + 2) \cdot 20x^3 - (5x^4 - 5) \cdot 20x^4}{4x^5 + 2}$
 $= \frac{-10x^8 + 50x^4 + 20x^3}{2x^5 + 1}$

C) $\frac{dy}{dx} = \frac{(4x^5 + 2) \cdot 20x^3 - (5x^4 - 5) \cdot 20x^4}{(4x^5 + 2)^2}$
 $= \frac{-5x^8 + 25x^4 + 10x^3}{4x^{10} + 4x^5 + 1}$

D) $\frac{dy}{dx} = \frac{(4x^5 + 2) \cdot 20x^3 - (5x^4 - 5) \cdot 20x^4}{(5x^4 - 5)^2}$
 $= \frac{-4x^8 + 20x^4 + 8x^3}{5x^8 - 10x^4 + 5}$

7) $y = \frac{4x^5 - 5x^4 - 4x^3}{2x^2 + 2}$

A) $\frac{dy}{dx} = \frac{(2x^2 + 2)(20x^4 - 20x^3 - 12x^2) - (4x^5 - 5x^4 - 4x^3) \cdot 4x}{(4x^5 - 5x^4 - 4x^3)^2}$
 $= \frac{24x^4 - 20x^3 + 32x^2 - 40x - 24}{16x^8 - 40x^7 - 7x^6 + 40x^5 + 16x^4}$

B) $\frac{dy}{dx} = \frac{(2x^2 + 2)(20x^4 - 20x^3 - 12x^2) - (4x^5 - 5x^4 - 4x^3) \cdot 4x}{(2x^2 + 2)^2}$
 $= \frac{6x^6 - 5x^5 + 8x^4 - 10x^3 - 6x^2}{x^4 + 2x^2 + 1}$

C) $\frac{dy}{dx} = \frac{20x^4 - 20x^3 - 12x^2 - 4x}{(2x^2 + 2)^2}$
 $= \frac{5x^4 - 5x^3 - 3x^2 - x}{x^4 + 2x^2 + 1}$

D) $\frac{dy}{dx} = \frac{(2x^2 + 2)(20x^4 - 20x^3 - 12x^2) - (4x^5 - 5x^4 - 4x^3) \cdot 4x}{2x^2 + 2}$
 $= \frac{12x^6 - 10x^5 + 16x^4 - 20x^3 - 12x^2}{x^2 + 1}$

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A) $\frac{dy}{dx} = (5x^3 + 5) \cdot 8x^3 - (2x^4 + 1) \cdot 15x^2$
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B) $\frac{dy}{dx} = \frac{(5x^3 + 5) \cdot 8x^3 - (2x^4 + 1) \cdot 15x^2}{5x^3 + 5}$
 $= \frac{2x^6 + 8x^3 - 3x^2}{x^3 + 1}$

C) $\frac{dy}{dx} = \frac{(5x^3 + 5) \cdot 8x^3 - (2x^4 + 1) \cdot 15x^2}{(2x^4 + 1)^2}$
 $= \frac{10x^6 + 40x^3 - 15x^2}{4x^8 + 4x^4 + 1}$

*D) $\frac{dy}{dx} = \frac{(5x^3 + 5) \cdot 8x^3 - (2x^4 + 1) \cdot 15x^2}{(5x^3 + 5)^2}$
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A) $\frac{dy}{dx} = \frac{20x^3 - 20x^4}{(4x^5 + 2)^2}$
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A) $\frac{dy}{dx} = \frac{(2x^2 + 2)(20x^4 - 20x^3 - 12x^2) - (4x^5 - 5x^4 - 4x^3) \cdot 4x}{(4x^5 - 5x^4 - 4x^3)^2}$
 $= \frac{24x^4 - 20x^3 + 32x^2 - 40x - 24}{16x^8 - 40x^7 - 7x^6 + 40x^5 + 16x^4}$

*B) $\frac{dy}{dx} = \frac{(2x^2 + 2)(20x^4 - 20x^3 - 12x^2) - (4x^5 - 5x^4 - 4x^3) \cdot 4x}{(2x^2 + 2)^2}$
 $= \frac{6x^6 - 5x^5 + 8x^4 - 10x^3 - 6x^2}{x^4 + 2x^2 + 1}$

C) $\frac{dy}{dx} = \frac{20x^4 - 20x^3 - 12x^2 - 4x}{(2x^2 + 2)^2}$
 $= \frac{5x^4 - 5x^3 - 3x^2 - x}{x^4 + 2x^2 + 1}$

D) $\frac{dy}{dx} = \frac{(2x^2 + 2)(20x^4 - 20x^3 - 12x^2) - (4x^5 - 5x^4 - 4x^3) \cdot 4x}{2x^2 + 2}$
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