

Calculus Practice: Differential Equations 1b

For each problem, find the particular solution of the differential equation that satisfies the initial condition.

$$1) \frac{dy}{dx} = \frac{2x}{e^y}, y(2) = 0$$

$$2) \frac{dy}{dx} = \frac{3e^x}{y^2}, y(0) = \sqrt[3]{7}$$

$$3) \frac{dy}{dx} = \frac{2x}{e^{2y}}, y(-3) = \frac{\ln 16}{2}$$

$$4) \frac{dy}{dx} = \frac{2x^2}{y^2}, y(2) = \sqrt[3]{15}$$

$$5) \frac{dy}{dx} = 2y^2, y(-3) = \frac{1}{8}$$

$$6) \frac{dy}{dx} = \frac{-3 + x^2}{y^2}, y(-2) = \sqrt[3]{10}$$

$$7) \frac{dy}{dx} = 2x\sqrt{y}, y > 0, y(3) = \frac{9}{4}$$

$$8) \frac{dy}{dx} = xe^y, y(1) = -\ln \frac{5}{2}$$

$$9) \frac{dy}{dx} = \frac{5x^2}{e^{2y}}, y(1) = \frac{\ln \frac{10}{3}}{2}$$

$$10) \frac{dy}{dx} = \frac{2y^2}{x}, y(1) = 1$$

$$11) \frac{dy}{dx} = -\frac{x}{y}, y(1) = -1$$

$$12) \frac{dy}{dx} = \frac{y+2}{x}, y(-2) = 0$$

$$13) \frac{dy}{dx} = \frac{x^2 + 3}{2y}, y(1) = -\frac{\sqrt{30}}{3}$$

$$14) \frac{dy}{dx} = -\frac{3yx}{\ln y}, y(0) = \frac{1}{e}$$

$$15) \frac{dy}{dx} = \frac{y+2}{x^2}, y(3) = \frac{-2e + 3\sqrt[3]{e^2}}{e}$$

$$16) \frac{dy}{dx} = \frac{3y}{x^2}, y(1) = \frac{3}{e^3}$$

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For each problem, find the particular solution of the differential equation that satisfies the initial condition.

1) $\frac{dy}{dx} = \frac{2x}{e^y}, y(2) = 0$

$$e^y = x^2 - 3$$

$$y = \ln(x^2 - 3), x > \sqrt{3}$$

2) $\frac{dy}{dx} = \frac{3e^x}{y^2}, y(0) = \sqrt[3]{7}$

$$\frac{y^3}{3} = 3e^x - \frac{2}{3}$$

$$y = \sqrt[3]{9e^x - 2}, x > \ln \frac{2}{9}$$

3) $\frac{dy}{dx} = \frac{2x}{e^{2y}}, y(-3) = \frac{\ln 16}{2}$

$$\frac{e^{2y}}{2} = x^2 - 1$$

$$y = \frac{\ln(2x^2 - 2)}{2}, x < -1$$

4) $\frac{dy}{dx} = \frac{2x^2}{y^2}, y(2) = \sqrt[3]{15}$

$$\frac{y^3}{3} = \frac{2x^3}{3} - \frac{1}{3}$$

$$y = \sqrt[3]{2x^3 - 1}, x > \frac{\sqrt[3]{4}}{2}$$

5) $\frac{dy}{dx} = 2y^2, y(-3) = \frac{1}{8}$

$$-\frac{1}{y} = 2x - 2$$

$$y = -\frac{1}{2x - 2}, x < 1$$

6) $\frac{dy}{dx} = \frac{-3 + x^2}{y^2}, y(-2) = \sqrt[3]{10}$

$$\frac{y^3}{3} = -3x + \frac{x^3}{3}$$

$$y = \sqrt[3]{x^3 - 9x}, -3 < x < 0$$

7) $\frac{dy}{dx} = 2x\sqrt{y}, y > 0, y(3) = \frac{9}{4}$

$$2\sqrt{y} = x^2 - 6$$

$$y = \left(\frac{x^2}{2} - 3\right)^2, x > \sqrt{6}$$

8) $\frac{dy}{dx} = xe^y, y(1) = -\ln \frac{5}{2}$

$$-e^{-y} = \frac{x^2}{2} - 3$$

$$y = -\ln\left(-\frac{x^2}{2} + 3\right), -\sqrt{6} < x < \sqrt{6}$$

$$9) \frac{dy}{dx} = \frac{5x^2}{e^{2y}}, y(1) = \frac{\ln \frac{10}{3}}{2}$$

$$\begin{aligned}\frac{e^{2y}}{2} &= \frac{5x^3}{3} \\ y &= \frac{\ln \frac{10x^3}{3}}{2}, x > 0\end{aligned}$$

$$10) \frac{dy}{dx} = \frac{2y^2}{x}, y(1) = 1$$

$$\begin{aligned}-\frac{1}{2y} &= \ln |x| - \frac{1}{2} \\ y &= -\frac{1}{2 \ln |x| - 1}, 0 < x < \sqrt{e}\end{aligned}$$

$$11) \frac{dy}{dx} = -\frac{x}{y}, y(1) = -1$$

$$\begin{aligned}\frac{y^2}{2} &= -\frac{x^2}{2} + 1 \\ y &= -\sqrt{-x^2 + 2}, -\sqrt{2} < x < \sqrt{2}\end{aligned}$$

$$12) \frac{dy}{dx} = \frac{y+2}{x}, y(-2) = 0$$

$$\begin{aligned}\ln |y+2| &= \ln |x| \\ y &= -x - 2, x < 0\end{aligned}$$

$$13) \frac{dy}{dx} = \frac{x^2 + 3}{2y}, y(1) = -\frac{\sqrt{30}}{3}$$

$$\begin{aligned}y^2 &= \frac{x^3}{3} + 3x \\ y &= -\sqrt{\frac{x^3}{3} + 3x}, x > 0\end{aligned}$$

$$14) \frac{dy}{dx} = -\frac{3yx}{\ln y}, y(0) = \frac{1}{e}$$

$$\begin{aligned}\frac{(\ln y)^2}{2} &= -\frac{3x^2}{2} + \frac{1}{2} \\ y &= e^{-\sqrt{-3x^2 + 1}}, -\frac{\sqrt{3}}{3} < x < \frac{\sqrt{3}}{3}\end{aligned}$$

$$15) \frac{dy}{dx} = \frac{y+2}{x^2}, y(3) = \frac{-2e + 3\sqrt[3]{e^2}}{e}$$

$$\ln |y+2| = -\frac{1}{x} + \frac{3 \ln 3 + 1}{3}$$

$$y = 3e^{-\frac{1}{x}} - 2, x > 0$$

$$16) \frac{dy}{dx} = \frac{3y}{x^2}, y(1) = \frac{3}{e^3}$$

$$\begin{aligned}\frac{\ln |y|}{3} &= -\frac{1}{x} + \frac{3 + \ln 3}{3} \\ y &= 3e^{-\frac{3}{x}}, x > 0\end{aligned}$$