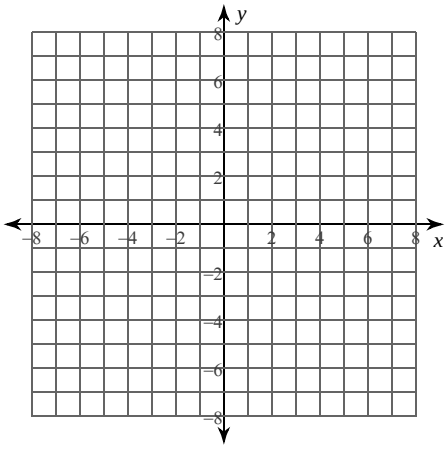
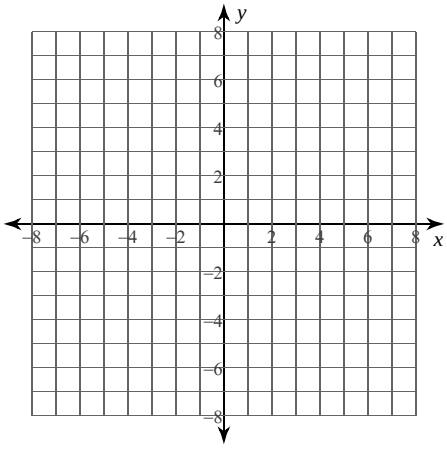
Calculus Practice: Curve Sketching 4

For each problem, find the: x and y intercepts, asymptotes, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

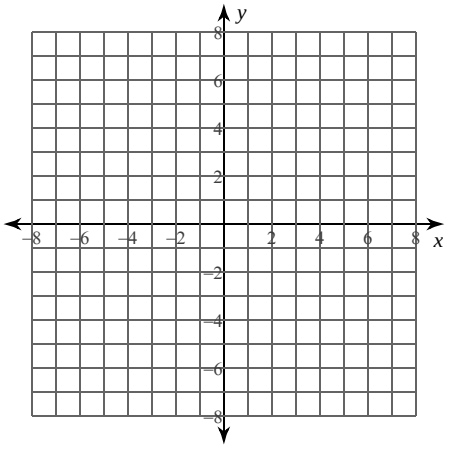
1) $y = -\dfrac{x^2}{4x - 8}$



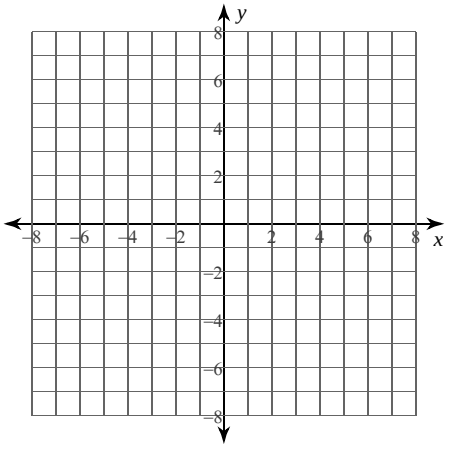
2) $y = -\dfrac{3x}{x - 3}$



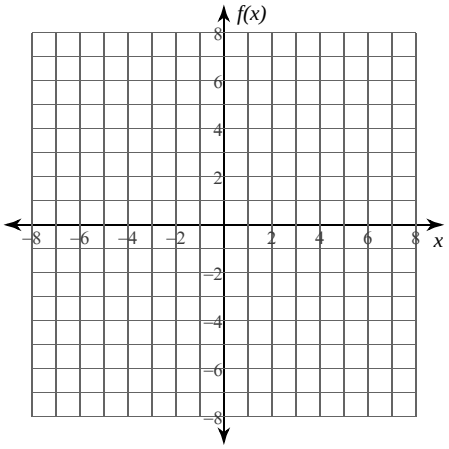
3) $y = \dfrac{2x^2 - 2}{x^3}$



4) $y = -\dfrac{6}{x^2 + 3}$



5) $f(x) = \dfrac{x^3}{x^2 - 1}$



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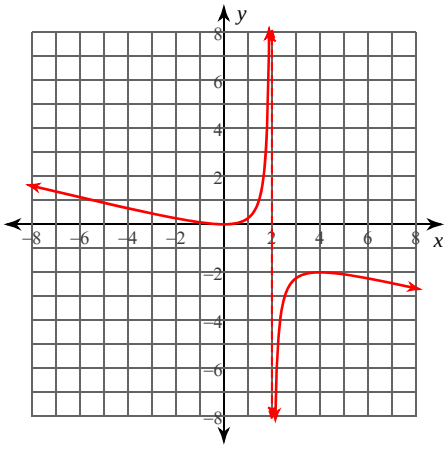
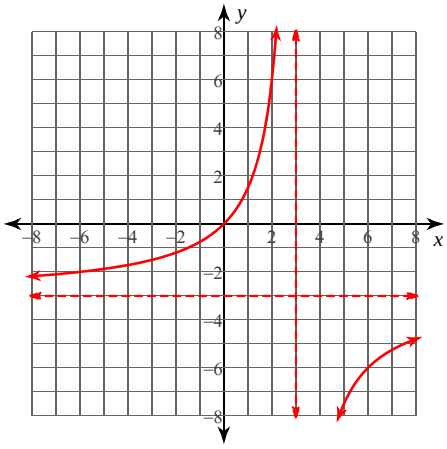


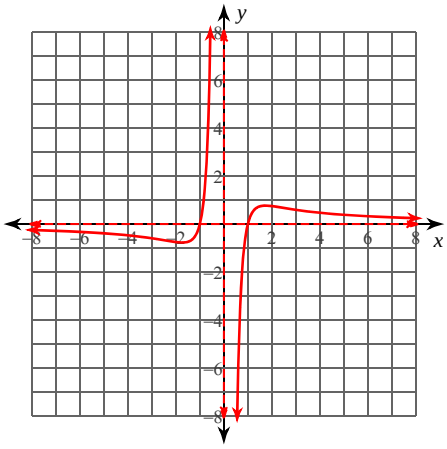
x-intercept at $x = 0$ y-intercept at $y = 0$
Vertical asymptote at: $x = 2$
No horizontal asymptotes exist.
Slant asymptote: $y = -\dfrac{x}{4} - \dfrac{1}{2}$
Critical points at: $x = 0, 4$
Increasing: $(0, 2), (2, 4)$ Decreasing: $(-\infty, 0), (4, \infty)$
No inflection points exist.
Concave up: $(-\infty, 2)$ Concave down: $(2, \infty)$
Relative minimum: $(0, 0)$ Relative maximum: $(4, -2)$

2) $y = -\dfrac{3x}{x - 3}$



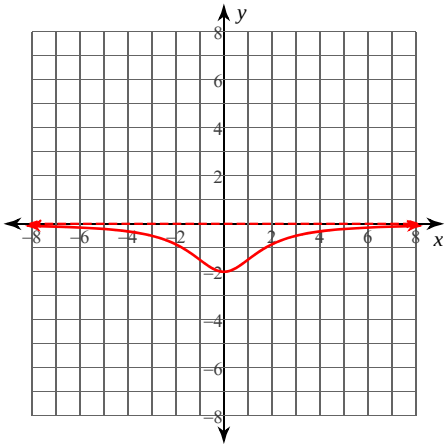
x-intercept at $x = 0$ y-intercept at $y = 0$
Vertical asymptote at: $x = 3$
Horizontal asymptote at: $y = -3$
No critical points exist.
Increasing: $(-\infty, 3), (3, \infty)$ Decreasing: No intervals exist.
No inflection points exist.
Concave up: $(-\infty, 3)$ Concave down: $(3, \infty)$
No relative minima. No relative maxima.

3) $y = \dfrac{2x^2 - 2}{x^3}$



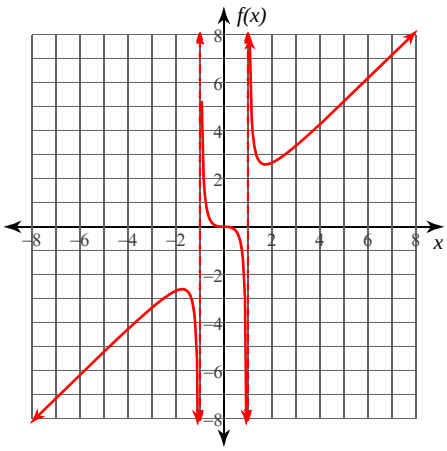
x-intercepts at $x = -1, 1$ No y-intercepts.
Vertical asymptote at: $x = 0$
Horizontal asymptote at: $y = 0$
Critical points at: $x = -\sqrt{3}, \sqrt{3}$
Increasing: $(-\sqrt{3}, 0), (0, \sqrt{3})$ Decreasing: $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$
Inflection points at: $x = -\sqrt{6}, \sqrt{6}$
Concave up: $(-\sqrt{6}, 0), (\sqrt{6}, \infty)$ Concave down: $(-\infty, -\sqrt{6}), (0, \sqrt{6})$
Relative minimum: $\left(-\sqrt{3}, -\dfrac{4\sqrt{3}}{9}\right)$ Relative maximum: $\left(\sqrt{3}, \dfrac{4\sqrt{3}}{9}\right)$

4) $y = -\dfrac{6}{x^2 + 3}$



No x-intercepts. y-intercept at $y = -2$
No vertical asymptotes exist.
Horizontal asymptote at: $y = 0$
Critical point at: $x = 0$
Increasing: $(0, \infty)$ Decreasing: $(-\infty, 0)$
Inflection points at: $x = -1, 1$
Concave up: $(-1, 1)$ Concave down: $(-\infty, -1), (1, \infty)$
Relative minimum: $(0, -2)$ No relative maxima.

5) $f(x) = \dfrac{x^3}{x^2 - 1}$



x-intercept at $x = 0$ y-intercept at $y = 0$
Vertical asymptotes at: $x = -1, 1$
No horizontal asymptotes exist.
Slant asymptote: $y = x$
Critical points at: $x = -\sqrt{3}, 0, \sqrt{3}$
Increasing: $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$ Decreasing: $(-\sqrt{3}, -1), (-1, 1), (1, \sqrt{3})$
Inflection point at: $x = 0$
Concave up: $(-1, 0), (1, \infty)$ Concave down: $(-\infty, -1), (0, 1)$
Relative minimum: $\left(\sqrt{3}, \dfrac{3\sqrt{3}}{2}\right)$ Relative maximum: $\left(-\sqrt{3}, -\dfrac{3\sqrt{3}}{2}\right)$