Calculus Practice: Chain Rule 4a

Differentiate each function with respect to *x*.

1) $f(x) = \cot 5x^5$

A) $f'(x) = -\cos^2 5x^5 \cdot 25x^4$
$= -25x^4\cos^2 5x^5$

B) $f'(x) = -\sin^2 5x^5 \cdot 25x^4$
$= -25x^4\sin^2 5x^5$

C) $f'(x) = \csc^2 5x^5 \cdot 25x^4$
$= 25x^4\csc^2 5x^5$

D) $f'(x) = -\csc^2 5x^5 \cdot 25x^4$
$= -25x^4\csc^2 5x^5$

2) $y = \csc x^2$

A) $\frac{dy}{dx} = -\csc x^2\tan x^2 \cdot 2x$
$= -2x\csc x^2\tan x^2$

B) $\frac{dy}{dx} = -\csc x^2\cot x^2 \cdot 2x$
$= -2x\csc x^2\cot x^2$

C) $\frac{dy}{dx} = \csc x^2\cot x^2 \cdot 2x$
$= 2x\csc x^2\cot x^2$

D) $\frac{dy}{dx} = -\csc x^2\csc x^2 \cdot 2x$
$= -2x\csc^2 x^2$

3) $f(x) = \sec(\cot 2x^2)$

A) $f'(x) = \sec(\cot 2x^2)\tan(\cot 2x^2) \cdot -\csc^2 2x^2 \cdot 4x$
$= -4x\sec(\cot 2x^2)\tan(\cot 2x^2)\csc^2 2x^2$

B) $f'(x) = \sec(\cot 2x^2)\sec(\cot 2x^2) \cdot -\csc^2 2x^2 \cdot 4x$
$= -4x\sec^2(\cot 2x^2)\csc^2 2x^2$

C) $f'(x) = \sec(\cot 2x^2)\cot(\cot 2x^2) \cdot -\csc^2 2x^2 \cdot 4x$
$= -4x\sec(\cot 2x^2)\cot(\cot 2x^2)\csc^2 2x^2$

D) $f'(x) = -\sec(\cot 2x^2)\tan(\cot 2x^2) \cdot -\csc^2 2x^2 \cdot 4x$
$= 4x\sec(\cot 2x^2)\tan(\cot 2x^2)\csc^2 2x^2$

4) $f(x) = \csc(\sec 5x^3)$

A) $f'(x) = \csc(\sec 5x^3)\cot(\sec 5x^3) \cdot \sec 5x^3\tan 5x^3 \cdot 15x^2$
$= 15x^2\csc(\sec 5x^3)\cot(\sec 5x^3)\sec 5x^3\tan 5x^3$

B) $f'(x) = -\csc(\sec 5x^3)\cot(\sec 5x^3) \cdot \sec 5x^3\tan 5x^3 \cdot 15x^2$
$= -15x^2\csc(\sec 5x^3)\cot(\sec 5x^3)\sec 5x^3\tan 5x^3$

C) $f'(x) = -\csc(\sec 5x^3)\csc(\sec 5x^3) \cdot \sec 5x^3\tan 5x^3 \cdot 15x^2$
$= -15x^2\csc^2(\sec 5x^3)\sec 5x^3\tan 5x^3$

D) $f'(x) = -\csc(\sec 5x^3)\tan(\sec 5x^3) \cdot \sec 5x^3\tan 5x^3 \cdot 15x^2$
$= -15x^2\csc(\sec 5x^3)\tan(\sec 5x^3)\sec 5x^3\tan 5x^3$

5) $y = \tan 3x^2 \cdot \left(2x^3 - 3\right)$

A) $\dfrac{dy}{dx} = \sec^2 3x^2 \cdot 6x \cdot 6x^2 + \sec^2 3x^2 \cdot 6x \cdot 6x^2$

$= 72x^3 \sec^2 3x^2$

B) $\dfrac{dy}{dx} = \tan 3x^2 \cdot 6x^2$

$= 6x^2 \tan 3x^2$

C) $\dfrac{dy}{dx} = \tan 3x^2 \cdot 6x^2 + \left(2x^3 - 3\right) \cdot \sec^2 3x^2 \cdot 6x$

$= 6x\left(x\tan 3x^2 + 2x^3 \sec^2 3x^2 - 3\sec^2 3x^2\right)$

D) $\dfrac{dy}{dx} = \sec^2 3x^2 \cdot 6x + 6x^2$

$= 6x\left(\sec^2 3x^2 + x\right)$

6) $f\left(x\right) = \left(x^5 + 5\right)\csc 2x^3$

A) $f'\left(x\right) = 5x^4 \cdot -\csc 2x^3 \cot 2x^3 \cdot 6x^2 + 5x^4 \cdot -\csc 2x^3 \cot 2x^3 \cdot 6x^2$

$= -60x^6 \csc 2x^3 \cot 2x^3$

B) $f'\left(x\right) = 5x^4 - \csc 2x^3 \cot 2x^3 \cdot 6x^2$

$= x^2\left(5x^2 - 6\csc 2x^3 \cot 2x^3\right)$

C) $f'\left(x\right) = \left(x^5 + 5\right) \cdot -\csc 2x^3 \cot 2x^3 \cdot 6x^2 + \csc 2x^3 \cdot 5x^4$

$= x^2 \csc 2x^3 \cdot \left(-6x^5 \cot 2x^3 - 30\cot 2x^3 + 5x^2\right)$

D) $f'\left(x\right) = \left(x^5 + 5\right) \cdot -\csc 2x^3 \cot 2x^3 \cdot 6x^2$

$= -6x^2 \csc 2x^3 \cot 2x^3 \cdot \left(x^5 + 5\right)$

7) $y = \dfrac{\tan 3x^2}{4x^4 + 5}$

A) $\dfrac{dy}{dx} = \dfrac{\left(4x^4 + 5\right) \cdot \sec^2 3x^2 \cdot 6x - \tan 3x^2 \cdot 16x^3}{4x^4 + 5}$

$= \dfrac{2x\left(12x^4 \sec^2 3x^2 + 15\sec^2 3x^2 - 8x^2 \tan 3x^2\right)}{4x^4 + 5}$

B) $\dfrac{dy}{dx} = \dfrac{\left(4x^4 + 5\right) \cdot \sec^2 3x^2 \cdot 6x - \tan 3x^2 \cdot 16x^3}{\tan^2 3x^2}$

$= \dfrac{2x\left(12x^4 \sec^2 3x^2 + 15\sec^2 3x^2 - 8x^2 \tan 3x^2\right)}{\tan^2 3x^2}$

C) $\dfrac{dy}{dx} = \dfrac{\left(4x^4 + 5\right) \cdot \sec^2 3x^2 \cdot 6x - \tan 3x^2 \cdot 16x^3}{\left(4x^4 + 5\right)^2}$

$= \dfrac{2x\left(12x^4 \sec^2 3x^2 + 15\sec^2 3x^2 - 8x^2 \tan 3x^2\right)}{\left(4x^4 + 5\right)^2}$

D) $\dfrac{dy}{dx} = \left(4x^4 + 5\right) \cdot \sec^2 3x^2 \cdot 6x - \tan 3x^2 \cdot 16x^3$

$= 2x\left(12x^4 \sec^2 3x^2 + 15\sec^2 3x^2 - 8x^2 \tan 3x^2\right)$

Calculus Practice: Chain Rule 4a

Differentiate each function with respect to *x*.

1) $f(x) = \cot 5x^5$

A) $f'(x) = -\cos^2 5x^5 \cdot 25x^4$
$= -25x^4\cos^2 5x^5$

B) $f'(x) = -\sin^2 5x^5 \cdot 25x^4$
$= -25x^4\sin^2 5x^5$

C) $f'(x) = \csc^2 5x^5 \cdot 25x^4$
$= 25x^4\csc^2 5x^5$

*D) $f'(x) = -\csc^2 5x^5 \cdot 25x^4$
$= -25x^4\csc^2 5x^5$

2) $y = \csc x^2$

A) $\frac{dy}{dx} = -\csc x^2\tan x^2 \cdot 2x$
$= -2x\csc x^2\tan x^2$

*B) $\frac{dy}{dx} = -\csc x^2\cot x^2 \cdot 2x$
$= -2x\csc x^2\cot x^2$

C) $\frac{dy}{dx} = \csc x^2\cot x^2 \cdot 2x$
$= 2x\csc x^2\cot x^2$

D) $\frac{dy}{dx} = -\csc x^2\csc x^2 \cdot 2x$
$= -2x\csc^2 x^2$

3) $f(x) = \sec(\cot 2x^2)$

*A) $f'(x) = \sec(\cot 2x^2)\tan(\cot 2x^2) \cdot -\csc^2 2x^2 \cdot 4x$
$= -4x\sec(\cot 2x^2)\tan(\cot 2x^2)\csc^2 2x^2$

B) $f'(x) = \sec(\cot 2x^2)\sec(\cot 2x^2) \cdot -\csc^2 2x^2 \cdot 4x$
$= -4x\sec^2(\cot 2x^2)\csc^2 2x^2$

C) $f'(x) = \sec(\cot 2x^2)\cot(\cot 2x^2) \cdot -\csc^2 2x^2 \cdot 4x$
$= -4x\sec(\cot 2x^2)\cot(\cot 2x^2)\csc^2 2x^2$

D) $f'(x) = -\sec(\cot 2x^2)\tan(\cot 2x^2) \cdot -\csc^2 2x^2 \cdot 4x$
$= 4x\sec(\cot 2x^2)\tan(\cot 2x^2)\csc^2 2x^2$

4) $f(x) = \csc(\sec 5x^3)$

A) $f'(x) = \csc(\sec 5x^3)\cot(\sec 5x^3) \cdot \sec 5x^3\tan 5x^3 \cdot 15x^2$
$= 15x^2\csc(\sec 5x^3)\cot(\sec 5x^3)\sec 5x^3\tan 5x^3$

*B) $f'(x) = -\csc(\sec 5x^3)\cot(\sec 5x^3) \cdot \sec 5x^3\tan 5x^3 \cdot 15x^2$
$= -15x^2\csc(\sec 5x^3)\cot(\sec 5x^3)\sec 5x^3\tan 5x^3$

C) $f'(x) = -\csc(\sec 5x^3)\csc(\sec 5x^3) \cdot \sec 5x^3\tan 5x^3 \cdot 15x^2$
$= -15x^2\csc^2(\sec 5x^3)\sec 5x^3\tan 5x^3$

D) $f'(x) = -\csc(\sec 5x^3)\tan(\sec 5x^3) \cdot \sec 5x^3\tan 5x^3 \cdot 15x^2$
$= -15x^2\csc(\sec 5x^3)\tan(\sec 5x^3)\sec 5x^3\tan 5x^3$

5) $y = \tan 3x^2 \cdot (2x^3 - 3)$

A) $\frac{dy}{dx} = \sec^2 3x^2 \cdot 6x \cdot 6x^2 + \sec^2 3x^2 \cdot 6x \cdot 6x^2$

$= 72x^3 \sec^2 3x^2$

B) $\frac{dy}{dx} = \tan 3x^2 \cdot 6x^2$

$= 6x^2 \tan 3x^2$

*C) $\frac{dy}{dx} = \tan 3x^2 \cdot 6x^2 + (2x^3 - 3) \cdot \sec^2 3x^2 \cdot 6x$

$= 6x(x\tan 3x^2 + 2x^3 \sec^2 3x^2 - 3\sec^2 3x^2)$

D) $\frac{dy}{dx} = \sec^2 3x^2 \cdot 6x + 6x^2$

$= 6x(\sec^2 3x^2 + x)$

6) $f(x) = (x^5 + 5)\csc 2x^3$

A) $f'(x) = 5x^4 \cdot -\csc 2x^3 \cot 2x^3 \cdot 6x^2 + 5x^4 \cdot -\csc 2x^3 \cot 2x^3 \cdot 6x^2$

$= -60x^6 \csc 2x^3 \cot 2x^3$

B) $f'(x) = 5x^4 - \csc 2x^3 \cot 2x^3 \cdot 6x^2$

$= x^2(5x^2 - 6\csc 2x^3 \cot 2x^3)$

*C) $f'(x) = (x^5 + 5) \cdot -\csc 2x^3 \cot 2x^3 \cdot 6x^2 + \csc 2x^3 \cdot 5x^4$

$= x^2 \csc 2x^3 \cdot (-6x^5 \cot 2x^3 - 30\cot 2x^3 + 5x^2)$

D) $f'(x) = (x^5 + 5) \cdot -\csc 2x^3 \cot 2x^3 \cdot 6x^2$

$= -6x^2 \csc 2x^3 \cot 2x^3 \cdot (x^5 + 5)$

7) $y = \frac{\tan 3x^2}{4x^4 + 5}$

A) $\frac{dy}{dx} = \frac{(4x^4 + 5) \cdot \sec^2 3x^2 \cdot 6x - \tan 3x^2 \cdot 16x^3}{4x^4 + 5}$

$= \frac{2x(12x^4 \sec^2 3x^2 + 15\sec^2 3x^2 - 8x^2 \tan 3x^2)}{4x^4 + 5}$

B) $\frac{dy}{dx} = \frac{(4x^4 + 5) \cdot \sec^2 3x^2 \cdot 6x - \tan 3x^2 \cdot 16x^3}{\tan^2 3x^2}$

$= \frac{2x(12x^4 \sec^2 3x^2 + 15\sec^2 3x^2 - 8x^2 \tan 3x^2)}{\tan^2 3x^2}$

*C) $\frac{dy}{dx} = \frac{(4x^4 + 5) \cdot \sec^2 3x^2 \cdot 6x - \tan 3x^2 \cdot 16x^3}{(4x^4 + 5)^2}$

$= \frac{2x(12x^4 \sec^2 3x^2 + 15\sec^2 3x^2 - 8x^2 \tan 3x^2)}{(4x^4 + 5)^2}$

D) $\frac{dy}{dx} = (4x^4 + 5) \cdot \sec^2 3x^2 \cdot 6x - \tan 3x^2 \cdot 16x^3$

$= 2x(12x^4 \sec^2 3x^2 + 15\sec^2 3x^2 - 8x^2 \tan 3x^2)$