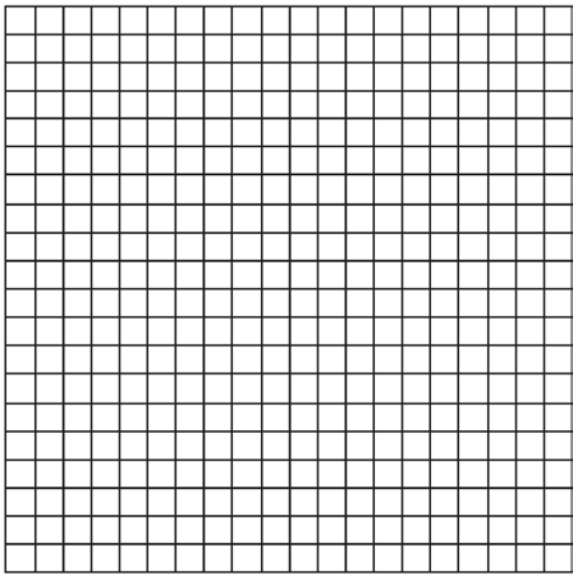
A.CED.A.1: Exponential Decay

1 A retailer advertises that items will be discounted by 10% every Monday until they are sold. In how many weeks will an item costing $50 first be sold for under half price?
1) 7
2) 6
3) 5
4) 4

2 The amount *A,* in milligrams, of a 10-milligram dose of a drug remaining in the body after *t* hours is given by the formula $A = 10(0.8)^t$. Find, to the *nearest tenth of an hour*, how long it takes for half of the drug dose to be left in the body.

3 Depreciation (the decline in cash value) on a car can be determined by the formula $V = C(1 - r)^t$, where V is the value of the car after t years, C is the original cost, and r is the rate of depreciation. If a car's cost, when new, is $15,000, the rate of depreciation is 30%, and the value of the car now is $3,000, how old is the car to the *nearest tenth of a year*?

4 The current population of Little Pond, New York, is 20,000. The population is *decreasing*, as represented by the formula $P = A(1.3)^{-0.234t}$, where P = final population, t = time, in years, and A = initial population. What will the population be 3 years from now? Round your answer to the *nearest hundred people*. To the *nearest tenth of a year*, how many years will it take for the population to reach half the present population? [The use of the grid is optional.]



5 Megan is performing an experiment in a lab where the air temperature is a constant 73ºF and the liquid is 237ºF. One and a half hours later, the temperature of the liquid is 112ºF. Newton's law of cooling states $T(t) = T_a + (T_0 - T_a)e^{-kt}$ where:

$T(t)$: temperature, °F, of the liquid at t hours

T_a: air temperature

T_0: initial temperature of the liquid

k: constant

Determine the value of k, to the *nearest thousandth*, for this liquid. Determine the temperature of the liquid using your value for k, to the *nearest degree*, after two and a half hours. Megan needs the temperature of the liquid to be 80ºF to perform the next step in her experiment. Use your value for k to determine, to the *nearest tenth of an hour*, how much time she must wait since she first began the experiment.

6 Objects cool at different rates based on the formula below.

$$T = (T_0 - T_R)e^{-rt} + T_R$$

T_0: initial temperature

T_R: room temperature

r: rate of cooling of the object

t: time in minutes that the object cools to a temperature, T

Mark makes T-shirts using a hot press to transfer designs to the shirts. He removes a shirt from a press that heats the shirt to 400°F. The rate of cooling for the shirt is 0.0735 and the room temperature is 75°F. Using this information, write an equation for the temperature of the shirt, T, after t minutes. Use the equation to find the temperature of the shirt, to the *nearest degree*, after five minutes. At the same time, Mark's friend Jeanine removes a hoodie from a press that heats the hoodie to 450°F. After eight minutes, the hoodie measured 270°F. The room temperature is still 75°F. Determine the rate of cooling of the hoodie, to the *nearest ten thousandth*. The T-shirt and hoodie were removed at the same time. Determine when the temperature will be the same, to the *nearest minute*.

A.CED.A.1: Exponential Decay
Answer Section

1 ANS: 1

$50(.9)^t = 25$

$t \approx 6.57$

REF: 082317aii

2 ANS:

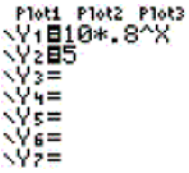
$5 = 10(0.8)^t$

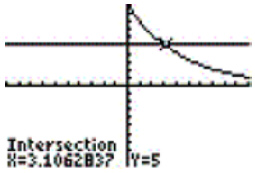
$\frac{5}{10} = 0.8^t$

3.1. Half the dose is left, $A = 5$: $\log\frac{5}{10} = \log 0.8^t$

$t\log 0.8 = \log 0.5$

$t = \frac{\log 0.5}{\log 0.8} \approx 3.1$





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3 ANS:

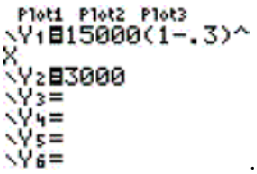
$V = C(1-r)^t$

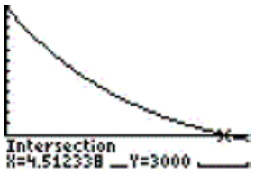
$3000 = 15000(1-.3)^t$

4.5. $.2 = .7^t$

$\log.2 = \log.7^t$

$t = \frac{\log 0.2}{\log 0.7} \approx 4.5$





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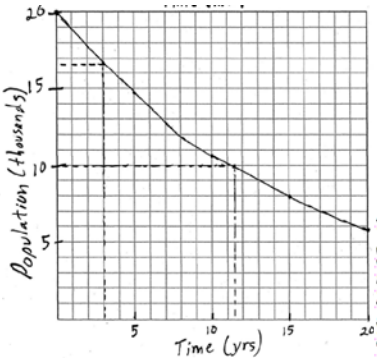
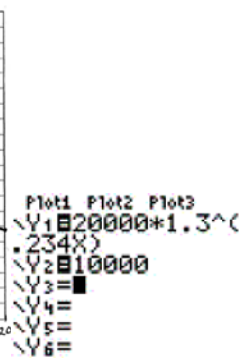
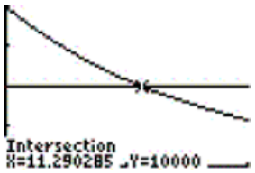
4 ANS:

16,600, 11.3. $P = 20000(1.3)^{-0.234 \cdot 3} \approx 16600$. Half of Little Pond's present population is 10,000.







$$10000 = 20000(1.3)^{-0.234t}$$

$$.5 = (1.3)^{-0.234t}$$

$$\log .5 = \log 1.3^{-0.234t}$$

$$\log .5 = -0.234t \cdot \log 1.3$$

$$\frac{\log .5}{\log 1.3} = -0.234t$$

$$t \approx 11.3$$

REF: 010632b

5 ANS:

$112 = 73 + (237 - 73)e^{-1.5k}$ $T(2.5) = 73 + (237 - 73)e^{(-.958)(2.5)} \approx 88$ $80 = 73 + (237 - 73)e^{-.958t}$

$k \approx .958$ $t \approx 3.3$

REF: 062437aii

6 ANS:

$T = (400 - 75)e^{-0.0735t} + 75$, $325e^{-0.0735(5)} + 75 \approx 300$, $270 = (450 - 75)e^{-8r} + 75$, $325e^{-0.0735t} + 75 = 375e^{-0.0817t} + 75$

$r \approx 0.0817$ $t \approx 17$

REF: 012337aii