A.APR.C.4: Polynomial Identities

1 Emmeline is working on one side of a polynomial identity proof used to form Pythagorean triples. Her work is shown below:

$$(5x)^2 + (5x^2 - 5)^2$$

Step 1: $25x^2 + (5x^2 - 5)^2$

Step 2: $25x^2 + 25x^2 + 25$

Step 3: $50x^2 + 25$

Step 4: $75x^2$

What statement is true regarding Emmeline's work?

1) Emmeline's work is entirely correct.
2) There is a mistake in step 2, only.
3) There are mistakes in step 2 and step 4.
4) There is a mistake in step 4, only.

2 The expression $(x + a)(x + b)$ can *not* be written as

1) $a(x + b) + x(x + b)$
2) $x^2 + abx + ab$
3) $x^2 + (a + b)x + ab$
4) $x(x + a) + b(x + a)$

3 Which equation does *not* represent an identity?

1) $x^2 - y^2 = (x + y)(x - y)$
2) $(x - y)^2 = (x - y)(x - y)$
3) $(x + y)^2 = x^2 + 2xy + y^2$
4) $(x + y)^3 = x^3 + 3xy + y^3$

4 Which statement(s) are true for all real numbers?

I $(x - y)^2 = x^2 + y^2$

II $(x + y)^3 = x^3 + 3xy + y^3$

1) I, only
2) II, only
3) I and II
4) neither I nor II

5 For which equations will the value $s = 4$ make the statement an identity?

I $(2x - 3)^2 = 4x^2 - 3sx + 9$

II $(x - 2)^3 = (x - 2)(x^2 + sx + s)$

1) I, only
2) II, only
3) I and II
4) neither I nor II

6 Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?

I $(m + p)^2 = m^2 + 2mp + p^2$

II $(x + y)^3 = x^3 + 3xy + y^3$

III $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

1) I, only
2) I and II
3) II and III
4) I and III

7 Which equation represents a polynomial identity?

1) $x^3 + y^3 = (x + y)^3$
2) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
3) $x^3 + y^3 = (x + y)(x^2 - xy - y^2)$
4) $x^3 + y^3 = (x - y)(x^2 + xy + y^2)$

8 Which equation is true for all real values of x?

1) $x^4 + x = (x+1)(x^3 - x^2 + x)$
2) $x^4 + x = (x+1)(x^3 + x)$
3) $x^4 + x = (x^2 + x)^2$
4) $x^4 + x = (x-1)(x^3 + x^2 + x)$

9 How many equations below are identities?

- $x^2 + y^2 = (x^2 - y^2) + (2xy)^2$
- $x^3 + y^3 = (x - y) + (x^2 - xy + y^2)$
- $x^4 + y^4 = (x - y)(x - y)(x^2 + y^2)$

1) 1
2) 2
3) 3
4) 0

10 Given the following polynomials

$$x = (a + b + c)^2$$
$$y = a^2 + b^2 + c^2$$
$$z = ab + bc + ac$$

Which identity is true?

1) $x = y - z$
2) $x = y + z$
3) $x = y - 2z$
4) $x = y + 2z$

11 Given the polynomial identity $x^6 + y^6 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$, which equation must also be true for all values of x and y?

1) $x^6 + y^6 = x^2(x^4 - x^2y^2 + y^4) + y^2(x^4 - x^2y^2 + y^4)$
2) $x^6 + y^6 = (x^2 + y^2)(x^2 - y^2)(x^2 - y^2)$
3) $(x^3 + y^3)^2 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$
4) $(x^6 + y^6) - (x^2 + y^2) = x^4 - x^2y^2 + y^4$

12 Algebraically prove that the difference of the squares of any two consecutive integers is an odd integer.

13 Verify the following Pythagorean identity for all values of x and y:

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$$

14 Erin and Christa were working on cubing binomials for math homework. Erin believed they could save time with a shortcut. She wrote down the rule below for Christa to follow.

$$(a + b)^3 = a^3 + b^3$$

Does Erin's shortcut always work? Justify your result algebraically.

15 Algebraically determine the values of h and k to correctly complete the identity stated below.

$$2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k$$

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Answer Section

1 ANS: 3 REF: 012003aii

2 ANS: 2 REF: 011806aii

3 ANS: 4

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

REF: 012417aii

4 ANS: 4

$(x-y)^2 = x^2 - 2xy + y^2$ $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

REF: 061902aii

5 ANS: 1

$(2x-3)^2 = 4x^2 - 12x + 9$ $(x-2)^3 = (x-2)(x-2)^2 = (x-2)(x^2 - 4x + 4)$

$s = 4$ $s = -4 \text{ and } 4$

REF: 062405aii

6 ANS: 4

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \neq x^3 + 3xy + y^3$

REF: 081620aii

7 ANS: 2 REF: 012311aii

8 ANS: 1

$x^4 + x$

$x(x^3 + 1)$

$x(x+1)(x^2 - x + 1)$

$(x+1)(x^3 - x^2 + x)$

REF: 082404aii

9 ANS: 4

$(x^2 - y^2) + (2xy)^2 = x^2 + 4x^2y^2 - y^2$

$(x-y) + (x^2 - xy + y^2) = x^2 + x - y - xy + y^2$

$(x-y)(x-y)(x^2 + y^2) = (x^2 - 2xy + y^2)(x^2 + y^2) = x^4 - 2x^3y + x^2y^2 + x^2y^2 - 2xy^3 + y^4$

REF: 062322aii

10 ANS: 4

$(a+b+c)^2 = a^2 + ab + ac + ab + b^2 + bc + ac + ab + c^2$

$x = a^2 + b^2 + c^2 + 2(ab + bc + ac)$

$x = y + 2z$

REF: 061822aii

11 ANS: 1

2) $(x^4 - x^2y^2 + y^4) \neq (x^2 - y^2)(x^2 - y^2)$; 3) $x^6 + y^6 \neq (x^3 + y^3)^2$; 4) $\dfrac{x^6 + y^6}{x^2 + y^2} \neq x^6 + y^6 - (x^2 + y^2)$

REF: 082219aii

12 ANS:

Let x equal the first integer and $x + 1$ equal the next. $(x+1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1$. $2x + 1$ is an odd integer.

REF: fall1511aii

13 ANS:

$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$

$x^4 + 2x^2y^2 + y^4 = x^4 - 2x^2y^2 + y^4 + 4x^2y^2$

$x^4 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4$

REF: 081727aii

14 ANS:

$(a+b)^3 = a^3 + b^3$ No. Erin's shortcut only works if $a = 0$, $b = 0$ or $a = -b$.

$a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3$

$3ab^2 + 3a^2b = 0$

$3ab(b + a) = 0$

$a = 0, b = 0, a = -b$

REF: 011927aii

15 ANS:

$2x^3 - 10x^2 + 11x - 7 = 2x^3 + hx^2 + 3x - 8x^2 - 4hx - 12 + k$ $h = -2$

$-2x^2 + 8x + 5 = hx^2 - 4hx + k$ $k = 5$

REF: 011733aii