A.APR.B.3: Solving Polynomial Equations

1 When factoring to reveal the roots of the equation $x^3 + 2x^2 - 9x - 18 = 0$, which equations can be used?

    I. $x^2(x+2) - 9(x+2) = 0$
    II. $x(x^2 - 9) + 2(x^2 - 9) = 0$
    III. $(x-2)(x^2 - 9) = 0$

1) I and II, only
2) I and III, only
3) II and III, only
4) I, II, and III

2 How many negative solutions to the equation $2x^3 - 4x^2 + 3x - 1 = 0$ exist?
1) 1
2) 2
3) 3
4) 0

3 The zeros of the function $f(x) = x^3 - 9x^2$ are
1) 9, only
2) 0 and 9
3) 0 and 3, only
4) −3, 0, and 3

4 Which values of x are solutions of the equation $x^3 + x^2 - 2x = 0$?
1) 0,1,2
2) 0,1,−2
3) 0,−1,2
4) 0,−1,−2

5 The zeros of the function $f(x) = 2x^3 + 12x - 10x^2$ are
1) {2,3}
2) {−1,6}
3) {0,2,3}
4) {0,−1,6}

6 Given $c(m) = m^3 - 2m^2 + 4m - 8$, the solution of $c(m) = 0$ is
1) ±2
2) 2, only
3) $2i,2$
4) $\pm 2i,2$

7 Evan graphed a cubic function, $f(x) = ax^3 + bx^2 + cx + d$, and determined the roots of $f(x)$ to be ±1 and 2. What is the value of b, if $a = 1$?
1) 1
2) 2
3) −1
4) −2

8 Which statement regarding polynomials and their zeros is true?
1) $f(x) = (x^2 - 1)(x+a)$ has zeros of 1 and −a, only.
2) $f(x) = x^3 - ax^2 + 16x - 16a$ has zeros of 4 and a, only.
3) $f(x) = (x^2 + 25)(x+a)$ has zeros of ±5 and −a.
4) $f(x) = x^3 - ax^2 - 9x + 9a$ has zeros of ±3 and a.

9 What are the zeros of $P(m) = (m^2 - 4)(m^2 + 1)$?
1) 2 and − 2, only
2) 2, − 2, and − 4
3) −4, i, and − i
4) 2, − 2, i, and − i

10 Given $f(x) = x^4 + x^3 - 3x^2 + 9x - 108$ and $f(3) = 0$, which values satisfy $f(x) = 0$?
1) −4,3 only
2) −3,4 only
3) $\pm 3i,-4,3$
4) $\pm 3i,-3,4$

11 The zeros for $f(x) = x^4 - 4x^3 - 9x^2 + 36x$ are
1) $\{0, \pm 3, 4\}$
2) $\{0, 3, 4\}$
3) $\{0, \pm 3, -4\}$
4) $\{0, 3, -4\}$

12 What is the solution set of the equation
$3x^5 - 48x = 0$?
1) $\{0, \pm 2\}$
2) $\{0, \pm 2, 3\}$
3) $\{0, \pm 2, \pm 2i\}$
4) $\{\pm 2, \pm 2i\}$

13 What are the zeros of
$s(x) = x^4 - 9x^2 + 3x^3 - 27x - 10x^2 + 90$?
1) $\{-3, -2, 5\}$
2) $\{-2, 3, 5\}$
3) $\{-3, -2, 3, 5\}$
4) $\{-5, -3, 2, 3\}$

14 Given 3 is a root of $f(x) = x^4 - x^3 - 21x^2 + 45x$,
what are the other unique roots of $f(x)$?
1) -5, only
2) -5 and 0
3) -3, 1 and 5
4) -5, -3 and 0

15 Determine algebraically the zeros of
$f(x) = 3x^3 + 21x^2 + 36x$.

16 Solve the equation $2x^3 - x^2 - 8x + 4 = 0$
algebraically for all values of x.

17 Algebraically determine the zeros of the function
below.
$$r(x) = 3x^3 + 12x^2 - 3x - 12$$

18 Solve the equation $8x^3 + 4x^2 - 18x - 9 = 0$
algebraically for all values of x.

19 Solve $x^3 + 5x^2 = 4x + 20$ algebraically.

20 Solve algebraically for all values of x:
$x^4 + 4x^3 + 4x^2 = -16x$

21 Solve, giving 4 roots, $x^4 - 13x^2 + 36 = 0$

22 Solve: $x^4 - 3x^2 = 4$ (Find 4 roots.)

23 Solve: $x^4 + 4x^2 = 32$. Find 4 roots.

24 Find *four* roots of the following equation:
$2x^4 + 5x^2 = 207$

25 Given $z(x) = 6x^3 + bx^2 - 52x + 15$, $z(2) = 35$, and
$z(-5) = 0$, algebraically determine all the zeros of
$z(x)$.

26 Denise is designing a storage box in the shape of a
cube. Each side of the box has a length of 10
inches. She needs more room and decides to
construct a larger box in the shape of a cube with a
volume of 2,000 cubic inches. By how many
inches, to the *nearest tenth,* should she *increase* the
length of each side of the original box?

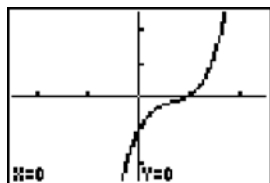## A.APR.B.3: Solving Polynomial Equations
Answer Section

1 ANS: 1

$x^3 + 2x^2 - 9x - 18 = 0$ $x^3 - 9x + 2x^2 - 18 = 0$ $x^3 - 9x + 2x^2 - 18 = 0$

$x^2(x + 2) - 9(x + 2) = 0$ $x(x^2 - 9) + 2(x^2 - 9) = 0$ $x(x^2 - 9) + 2(x^2 - 9) = 0$

$(x + 2)(x^2 - 9) = 0$

REF: 011903aii

2 ANS: 4

REF: 061222a2

3 ANS: 2

$f(x) = x^3 - 9x^2 = x^2(x - 9) = 0$

$x = 0, 9$

REF: 012009ai

4 ANS: 2

$x^3 + x^2 - 2x = 0$

$x(x^2 + x - 2) = 0$

$x(x + 2)(x - 1) = 0$

$x = 0, -2, 1$

REF: 011103a2

5 ANS: 3

$2x^3 + 12x - 10x^2 = 0$

$2x(x^2 - 5x + 6) = 0$

$2x(x - 3)(x - 2) = 0$

$x = 0, 2, 3$

REF: 081719ai

6 ANS: 4

$$m^3 - 2m^2 + 4m - 8 = 0$$

$$m^2(m-2) + 4(m-2) = 0$$

$$\left(m^2 + 4\right)(m-2) = 0$$

REF: 081821aii

7 ANS: 4

$$f(x) = (x+1)(x-1)(x-2) = (x^2-1)(x-2) = x^3 - 2x^2 - x + 2$$

REF: 081921aii

8 ANS: 4

1) -1 is also a zero. 2) $x^2(x-a) + 16(x-a) = (x^2+16)(x-a)$ a is the only zero. 3) $-a$ is the only zero. 4) $x^2(x-a) - 9(x-a) = (x^2-9)(x-a)$.

REF: 012019aii

9 ANS: 4   REF: 081708aii

10 ANS: 3

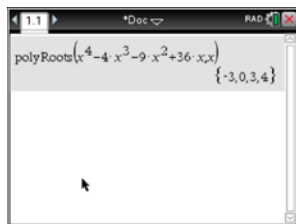$$3 \mid 1 \ 1 - 3 \quad 9 - 108 \quad x^3 + 4x^2 + 9x + 36 = 0$$

$$\mid \ \underline{3 \ 12 \ 27 \quad 108} \quad x^2(x+4) + 9(x+4) = 0$$

$$1 \ 4 \ \ 9 \ 36 \quad \ 0 \qquad (x^2+9)(x+4) = 0$$

$$x = \pm 3i, -4$$

REF: 062420aii

11 ANS: 1

$$x^4 - 4x^3 - 9x^2 + 36x = 0$$

$$x^3(x-4) - 9x(x-4) = 0$$

$$(x^3 - 9x)(x-4) = 0$$

$$x(x^2 - 9)(x-4) = 0$$

$$x(x+3)(x-3)(x-4) = 0$$

$$x = 0, \pm 3, 4$$

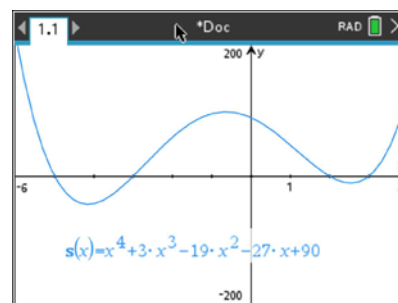REF: 061606aii

12 ANS: 3

$$3x^5 - 48x = 0$$

$$3x(x^4 - 16) = 0$$

$$3x(x^2 + 4)(x^2 - 4) = 0$$

$$3x(x^2 + 4)(x + 2)(x - 2) = 0$$

REF: 011216a2

13 ANS: 4

$$s(x) = x^4 - 9x^2 + 3x^3 - 27x - 10x^2 + 90 = x^4 + 3x^3 - 19x^2 - 27x + 90$$

REF: 062303aii

14 ANS: 2

3	1	-1	-21	45	0
		3	6	-45	0
	1	2	-15	0	0

$$x^3 + 2x^2 - 15x = 0$$

$$x(x^2 + 2x - 15) = 0$$

$$x(x + 5)(x - 3) = 0$$

$$x = 0, -5, 3$$

REF: 012403aii

15 ANS:

$$3x^3 + 21x^2 + 36x = 0$$

$$3x(x^2 + 7x + 12) = 0$$

$$3x(x + 4)(x + 3) = 0$$

$$x = 0, -4, -3$$

REF: 011930ai

16 ANS:

$$x^2(2x-1) - 4(2x-1) = 0$$

$$(x^2 - 4)(2x - 1) = 0$$

$$(x+2)(x-2)(2x-1) = 0$$

$$x = \pm 2, \ \frac{1}{2}$$

REF: 081537a2

17 ANS:

$$3(x^3 + 4x^2 - x - 4) = 0$$

$$(x^2(x+4) - (x+4)) = 0$$

$$(x^2 - 1)(x+4) = 0$$

$$x = \pm 1, -4$$

REF: 012325aii

18 ANS:

$$\pm \frac{3}{2}, \ -\frac{1}{2}. \qquad 8x^3 + 4x^2 - 18x - 9 = 0$$

$$4x^2(2x+1) - 9(2x+1) = 0$$

$$(4x^2 - 9)(2x+1) = 0$$

$$4x^2 - 9 = 0 \text{ or } 2x + 1 = 0$$

$$(2x+3)(2x-3) = 0 \quad x = -\frac{1}{2}$$

$$x = \pm \frac{3}{2}$$

REF: fall0937a2

19 ANS:

$$x^3 + 5x^2 - 4x - 20 = 0$$

$$x^2(x+5) - 4(x+5) = 0$$

$$(x^2 - 4)(x+5) = 0$$

$$(x+2)(x-2)(x+5) = 0$$

$$x = \pm 2, -5$$

REF: 061437a2

20 ANS:

$$x^4 + 4x^3 + 4x^2 + 16x = 0$$

$$x(x^3 + 4x^2 + 4x + 16) = 0$$

$$x(x^2(x+4) + 4(x+4)) = 0$$

$$x(x^2 + 4)(x + 4) = 0$$

$$x = 0, \pm 2i, -4$$

REF: 061339a2

21 ANS:
$\pm 2, \pm 3$

REF: 089604al

22 ANS:
$\pm 2, \pm i$

REF: 069407al

23 ANS:
$\pm 2, \pm 2i\sqrt{2}$

REF: 089409al

24 ANS:
$\pm 3, \pm \dfrac{i\sqrt{46}}{2}$

REF: 089812al

25 ANS:

$$0 = 6(-5)^3 + b(-5)^2 - 52(-5) + 15 \quad z(x) = 6x^3 + 19x^2 - 52x + 15$$

$$0 = -750 + 25b + 260 + 15$$

$$475 = 25b$$

$$19 = b$$

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-5 |  6    19   -52    15
         -30    55    15
      6   -11     3     0
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$$6x^2 - 11x + 3 = 0$$

$$(2x - 3)(3x - 1) = 0$$

$$x = \frac{3}{2}, \frac{1}{3}, -5$$

REF: fall1515aii

5

26 ANS:

$$V = s^3$$

2.6. $$\begin{aligned} 2000 &= (s+10)^3 \\ \sqrt[3]{2000} &= s+10 \\ s &\approx 12.6 \end{aligned}$$. Increase the length of each side of the box by 2.6 inches (12.6 – 10).

REF: 060724b