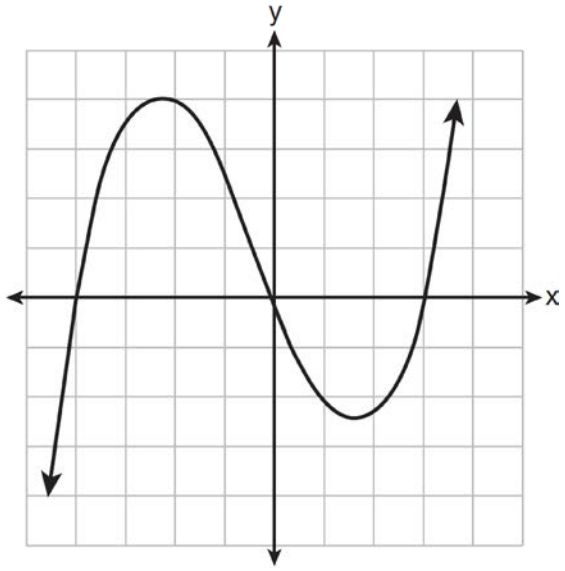
A.APR.B.2: Remainder and Factor Theorems

1 The graph of $p(x)$ is shown below.



What is the remainder when $p(x)$ is divided by $x+4$?
1) $x-4$
2) -4
3) 0
4) 4

2 Which binomial is *not* a factor of the expression $x^3-11x^2+16x+84$?
1) $x+2$
2) $x+4$
3) $x-6$
4) $x-7$

3 Which expression is a factor of $x^4-x^3-11x^2+5x+30$?
1) $x+2$
2) $x-2$
3) $x+5$
4) $x-5$

4 What is the remainder when $4x^3-3x+3$ is divided by $x-2$?
1) -23
2) -7
3) 13
4) 29

5 If $p(x)=2x^3-3x+5$, what is the remainder of $p(x)\div(x-5)$?
1) -230
2) 0
3) 40
4) 240

6 If $x-1$ is a factor of x^3-kx^2+2x, what is the value of k?
1) 0
2) 2
3) 3
4) -3

7 Which binomial is a factor of x^4-4x^2-4x+8?
1) $x-2$
2) $x+2$
3) $x-4$
4) $x+4$

8 Given $P(x)=x^3-3x^2-2x+4$, which statement is true?
1) $(x-1)$ is a factor because $P(-1)=2$.
2) $(x+1)$ is a factor because $P(-1)=2$.
3) $(x+1)$ is a factor because $P(1)=0$.
4) $(x-1)$ is a factor because $P(1)=0$.

9 Consider the function $f(x)=2x^3+x^2-18x-9$. Which statement is true?
1) $2x-1$ is a factor of $f(x)$.
2) $x-3$ is a factor of $f(x)$.
3) $f(3)\neq f\left(-\frac{1}{2}\right)$
4) $f\left(\frac{1}{2}\right)=0$

10 For the polynomial $p(x)$, if $p(3) = 0$, it can be concluded that
1) $x + 3$ is a factor of $p(x)$
2) $x - 3$ is a factor of $p(x)$
3) when $p(x)$ is divided by 3, the remainder is zero
4) when $p(x)$ is divided by -3, the remainder is zero

11 When $g(x)$ is divided by $x + 4$, the remainder is 0. Given $g(x) = x^4 + 3x^3 - 6x^2 - 6x + 8$, which conclusion about $g(x)$ is true?
1) $g(4) = 0$
2) $g(-4) = 0$
3) $x - 4$ is a factor of $g(x)$.
4) No conclusion can be made regarding $g(x)$.

12 If $f(x) = 2x^4 - x^3 - 16x + 8$, then $f\left(\frac{1}{2}\right)$
1) equals 0 and $2x + 1$ is a factor of $f(x)$
2) equals 0 and $2x - 1$ is a factor of $f(x)$
3) does not equal 0 and $2x + 1$ is not a factor of $f(x)$
4) does not equal 0 and $2x - 1$ is a factor of $f(x)$

13 Which statements must be true about the polynomial function $k(x) = -2x^3 - 11x^2 - 12x + 9$?
I. $(x - 3)$ is a factor of $k(x)$
II. $k(0) = 9$
III. $\frac{k(x)}{x+2}$ has a remainder of 5
1) II, only
2) I and II
3) II and III
4) I, II, and III

14 Show why $x - 3$ is a factor of $m(x) = x^3 - x^2 - 5x - 3$. Justify your answer.

15 Use an appropriate procedure to show that $x - 4$ is a factor of the function $f(x) = 2x^3 - 5x^2 - 11x - 4$. Explain your answer.

16 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

17 Determine if $x + 4$ is a factor of $2x^3 + 10x^2 + 4x - 16$. Explain your answer.

18 Is $x + 3$ a factor of $7x^3 + 27x^2 + 9x - 27$? Justify your answer.

19 Given $r(x) = x^3 - 4x^2 + 4x - 6$, find the value of $r(2)$. What does your answer tell you about $x - 2$ as a factor of $r(x)$? Explain.

20 Determine for which polynomial(s) $(x + 2)$ is a factor. Explain your answer.

$$P(x) = x^4 - 3x^3 - 16x - 12$$

$$Q(x) = x^3 - 3x^2 - 16x - 12$$

21 The polynomial function $g(x) = x^3 + ax^2 - 5x + 6$ has a factor of $(x - 3)$. Determine the value of a.

22 Evaluate $j(-1)$ given $j(x) = 2x^4 - x^3 - 35x^2 + 16x + 48$. Explain what your answer tells you about $x + 1$ as a factor. Algebraically find the remaining zeros of $j(x)$.

A.APR.B.2: Remainder and Factor Theorems
Answer Section

1 ANS: 3
Since $x + 4$ is a factor of $p(x)$, there is no remainder.

REF: 081621aii

2 ANS: 2

-4	1	-11	16	84
		-4	60	-304
	1	-15	76	

Since there is a remainder when the cubic is divided by $x + 4$, this binomial is not a factor.

REF: 081720aii

3 ANS: 1

-2	1	-1	-11	5	30
		-2	6	10	-30
	1	-3	-5	15	0

Since there is no remainder when the quartic is divided by $x + 2$, this binomial is a factor.

REF: 082320aii

4 ANS: 4
$p(2) = 4(2)^3 - 3(2) + 3 = 29$

REF: 062422aii

5 ANS: 4
$p(5) = 2(5)^3 - 3(5) + 5 = 240$

REF: 011819aii

6 ANS: 3
$1^3 - k(1)^2 + 2(1) = 0$

$k = 3$

REF: 061812aii

7 ANS: 1

2	1	0	-4	-4	8
		2	4	0	-8
	1	2	0	-4	0

Since there is no remainder when the quartic is divided by $x - 2$, this binomial is a factor.

REF: 061711aii

8 ANS: 4 REF: 061907aii

9 ANS: 2

$2x^3 + x^2 - 18x - 9$

$x^2(2x+1) - 9(2x+1)$

$(x^2 - 9)(2x+1)$

$(x+3)(x-3)(2x+1)$

REF: 082206aii

10 ANS: 2 REF: 062206aii

11 ANS: 2 REF: 011720aii

12 ANS: 2

$2x^4 - x^3 - 16x + 8 = 0$

$x^3(2x-1) - 8(2x-1) = 0$

$(x^3 - 8)(2x-1) = 0$

$x = 2, \frac{1}{2}$

REF: 012307aii

13 ANS: 3

$3|-2 \quad -11 \quad -12 \quad 9$ $x-3$ is not a factor since there is a remainder. $-2|-2 \quad -11 \quad -12 \quad 9$

$\quad | \quad -6 \quad -51 \quad -189$ $\quad | \quad 4 \quad 14 \quad -4$

$-2 \quad -17 \quad -63 \quad -180$ $-2 \quad -7 \quad 2 \quad 5$

REF: 062414aii

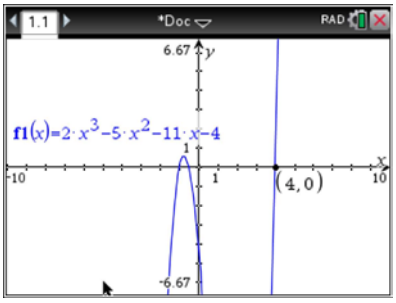
14 ANS:

$m(3) = 3^3 - 3^2 - 5(3) - 3 = 27 - 9 - 15 - 3 = 0$ Since $m(3) = 0$, there is no remainder when $m(x)$ is divided by $x-3$, and so $x-3$ is a factor.

REF: 012026aii

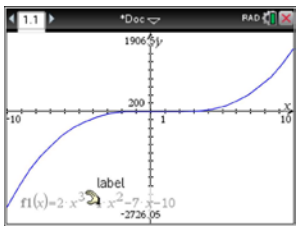
15 ANS:

$f(4) = 2(4)^3 - 5(4)^2 - 11(4) - 4 = 128 - 80 - 44 - 4 = 0$ Any method that demonstrates 4 is a zero of $f(x)$ confirms that $x-4$ is a factor, as suggested by the Remainder Theorem.



REF: spr1507aii

16 ANS:



$$x-5\overline{)\,2x^3-4x^2-7x-10}\qquad 2x^2+6x+23$$

$$\underline{2x^3-10x^2}$$

$$6x^2-7x$$

$$\underline{6x^2-30x}$$

$$23x-10$$

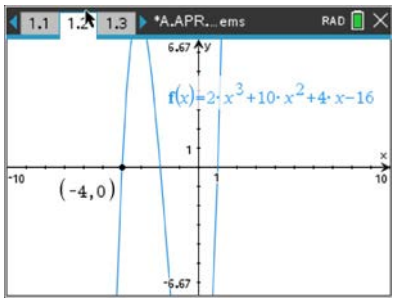
$$\underline{23x-115}$$

$$105$$

Since there is a remainder, $x-5$ is not a factor.

REF: 061627aii

17 ANS:



Since -4 is a zero, $x+4$ is a factor.

REF: 012426aii

18 ANS:

Since there is no remainder when the cubic is divided by $x+3$, this binomial is a factor.

-3	7	27	9	-27
		-21	-18	27
	7	6	-9	0

REF: 082426aii

19 ANS:

$r(2) = -6$. Since there is a remainder when the cubic is divided by $x-2$, this binomial is not a factor.

2	1	-4	4	6
		2	-4	0
	1	-2	0	-6

REF: 061725aii

20 ANS:
$P(-2) = 60$ $Q(-2) = 0$ $(x + 2)$ is a factor of $Q(x)$ since $Q(-2) = 0$.

REF: 081929aii

21 ANS:
$g(3) = 0; \quad 0 = 3^3 + a(3)^2 - 5(3) + 6$

$0 = 27 + 9a - 15 + 6$

$-18 = 9a$

$a = -2$

REF: 062328aii

22 ANS:
$j(-1) = 2(-1)^4 - (-1)^3 - 35(-1)^2 + 16(-1) + 48 = 2 + 1 - 35 - 16 + 48 = 0;$ $x + 1$ is a factor of $j(x)$;
$2x^3 - 3x^2 - 32x + 48 = 0$

$x^2(2x - 3) - 16(2x - 3) = 0$

$\left(x^2 - 16\right)(2x - 3) = 0$

$x = \pm 4, \frac{3}{2}$

REF: 081834aii