Algebra II Overview

The content standards associated with Algebra II are based on [the New York State Common Core Learning Standards for Mathematics] and the [PARCC Model Content Framework for Algebra II]. The content standards define what students should understand and be able to do at the high school level; the Model Content Framework describes which content is included and emphasized within the Algebra II course, specifically. More information about the relationship between the NYS CCLS and the PARCC Model Content Frameworks can be found in [this memo].

For high school mathematics, the standards are organized at three levels: *conceptual categories*, *domains* and *clusters*.

Algebra II is associated with high school content standards within five *conceptual categories*: **Number & Quantity, Algebra, Functions, Geometry** and **Statistics & Probability**. Each conceptual category contains *domains* of related *clusters* of standards. This chart shows the high school mathematics domains included in Algebra II, as well as the corresponding percent of credits on the Algebra II Regents Exam:

Conceptual Category	**Percent of Algebra II Regents Exam**	**High School Mathematics Domains Included in Algebra II**
Number & Quantity	5%-12%	The Real Number System (N-RN) Quantities (N-Q) The Complex Number System (N-CN)
Algebra	35%-44%	Seeing Structure in Expressions (A-SSE) Arithmetic with Polynomials and Rational Expressions (A-APR) Creating Equations (A-CED) Reasoning with Equations and Inequalities (A-REI) Expressing Geometric Properties with Equations (G-GPE) *
Functions	30% - 40%	Interpreting Functions (F-IF) Building Functions (F-BF) Linear, Quadratic, and Exponential Models (F-LE) Trigonometric Functions (F-TF)
Statistics & Probability	14% - 21%	Interpreting categorical and quantitative data (S-ID) Making Inferences and Justifying Conclusions (S-IC) Conditional Probability and the Rules of Probability (S-CP)

***Although the organization of the CCLS places one standard from the **G-GPE** domain into the Geometry Conceptual Category, the content within this domain will be **assessed as part of the Algebra Conceptual Category** for the Regents Examination in Algebra II (Common core).

The conceptual category of **Modeling** is also included in Algebra II, and is best interpreted not as a collection of isolated topics, but rather in relation to other standards:

Modeling	Specific *modeling* domains, clusters, and standards, indicated by a star symbol(★)

For more information about modeling at the high school level, please consult the High School Progression on Modeling.

Not all of the content in a given grade is emphasized equally in the standards. The list of content standards for each grade is not a flat, one-dimensional checklist; this is by design. There are sometimes strong differences of emphasis even within a single domain. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice. Without such focus, attention to the practices would be difficult and unrealistic, as would best practices like formative assessment.

The Regents Examination in Algebra II test will mirror the organization of the standards: **Major Clusters will account for a majority (51% - 65%) of the credits on the test, while Supporting Clusters (14% - 28%) and Additional Clusters (19% - 33%) will together constitute less than half the possible credits.**

The chart below shows:

- **Clusters and standards associated with Algebra II within each domain.** Clusters are identified as ■ Major Content, ◘ Supporting Content, or ○ Additional Content, to indicate emphasis within the course.

- **NYSED standards clarifications and PARCC assessment limits, in *italics* following the applicable standards.** More information about the assessment limits, which offer clarification for standards shared with other high school courses, can be found in the Model Content Framework linked above. Additionally, more information about the NYSED standards clarifications can be found here. The PARCC assessment limits are denoted by, "**PARCC**," and the NYSED standards clarifications are denoted by, "**NYSED.**"

Note: The Standards for Mathematical Practice form an important part of the Algebra II course, as well:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematics – High School Number & Quantity

The Real Number System (N-RN)

A. Extend the properties of exponents to rational exponents.

N-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

N-RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

NYSED: *Includes expressions with variable factors***,** *such as* $\sqrt[3]{27x^5y^3}$

Quantities (N-Q) ★

A. Reason quantitatively and use units to solve problems.

N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling. **(Shared with AI)**

PARCC: *This standard will be assessed in Algebra II by ensuring that some modeling tasks (involving Algebra II content or securely held contend from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude.*

The Complex Number System (N-CN)

A. Perform arithmetic operations with complex numbers.

N-CN.A.1 Know there is a complex number i such that $i^2 = –1$, and every complex number has the form *a* + *b*i with *a* and *b* real.

N-CN.A.2 Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

C. Use complex numbers in polynomial identities and equations.

N-CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.

For more information about the concepts and terms introduced in the **Number and Quantity** Domain, consult the Progression The Number System; High School, Number.

Mathematics – High School Algebra

Seeing Structure in Expressions (A-SSE)

A. Interpret the structure of expressions.

A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. For example, see x^4-y^4 as $(x^2)^2$-$(y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2$-$y^2)(x^2+y^2)$. **(Shared with AI)**

NYSED: *Includes factoring by grouping.*

PARCC: *i.) Tasks are limited to polynomial, rational, or exponential expressions. ii.) Examples: see* x^4-y^4 *as* $(x^2)^2$-$(y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2$-$y^2)(x^2+y^2)$*. In recognizing the equation* $x^2+2x+1+y^2=9$*, see and opportunity to rewrite the first three terms as* $(x+1)^2$*, thus recognizing the equation of a circle with radius 3 and center (-1,0). See* $(x^2+4)/(x^2+3)$ *as* $((x^2+3)+1)/(x2+3)$*, thus recognizing an opportunity to write it as* $1+1/(x^2+3)$*.*

B. Write expressions in equivalent forms to solve problems. ★

A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **(Shared with A1)**

c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t}$= 1.012^{12t} to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

PARCC: *i) Tasks have a real-world context. As described in the standard, there is interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation. ii) Tasks are limited to exponential expressions with rational or real exponents.*

A-SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

NYSED: *Includes using summation notation.*

Arithmetic with Polynomials and Rational Expressions (A-APR)

B. Understand the relationship between zeros and factors of polynomials.

A-APR.B.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

A-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **(Shared with A1)**

PARCC: *i) Tasks include quadratic, cubic, and quartic polynomials and polynomials for which factors are not provided. For example, find the zeros of* $(x^2-1)(x^2+1)$.

C. Use polynomial identities to solve problems.

A-APR.C.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

D. Rewrite rational expressions

A-APR.D.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

Creating Equations (A-CED) ★

A. Create equations that describe numbers or relationships.★

A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple radical and exponential functions. *(Tasks are limited to linear, quadratic, or exponential equations with integer exponents.)* **(Shared with A1)**

PARCC: *i) Tasks are limited to exponential equations with rational or real exponents and rational functions. ii) Tasks have a real-world context.*

Reasoning with Equations and Inequalities (A-REI)

A. Understand solving equations as a process of reasoning and explain the reasoning.

A-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **(Shared with A1)**

PARCC: *i) Tasks are limited to simple rational or radical equations.*

A-REI. A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

B. Solve equations and inequalities in one variable.

A-REI.B.4 Solve quadratic equations in one variable. **(Shared with A1)**

> **b.** Solve quadratic equations by inspection (e.g., for $x^2=49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$, for real numbers a and b.

PARCC: *i) In the case of equations that have roots with nonzero imaginary parts, students write the solutions as $a \pm bi$ for real numbers a and b.*

C. Solve systems of equations.

A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **(Shared with A1)**

PARCC/NYSED: *i) Tasks are limited to 3x3 systems only. Systems of 3 linear equations with 3 variables only.*

A-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

D. Represent and solve equations and inequalities graphically. ★

A-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. **(Shared with A1)**

PARCC: *i) Tasks may involve any of the function types mentioned in the standard.*

For more information about the concepts and terms introduced in the **Algebra** domain, please consult the High School Progression on Algebra.

Mathematics - High School Functions

Interpreting Functions (F-IF)

A. Understand the concept of a function and use function notation.

F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$. **(Shared with A1)**

PARCC: *i) This standard is supporting work in Algebra II. This standard should support the major work in F-BF.2 for coherence.*

B. Interpret functions that arise in applications in terms of the context. ★

F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* **(Shared with A1)**

PARCC: *i) Tasks have a real-world context. ii) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions.*

F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **(Shared with A1)**

PARCC: *i) Tasks have a real-world context. ii)Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions.*

C. Analyze functions using different representations.

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★

> **c.** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
>
> e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

> b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

NYSED: *Includes $A=Pe^{rt}$ and $A=P(1+r/n))^{nt}$*

F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum**. (Shared with A1)**

PARCC*:* *Tasks may involve polynomial, exponential, logarithmic and trigonometric functions.*

Building Functions (F-BF)

■ **A. Build a function that models a relationship between two quantities.** ★

F-BF.A.1 Write a function that describes a relationship between two quantities.

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context. **(Shared with AI)**

  PARCC*: i) Tasks have a real-world context ii) Tasks may involve linear functions, quadratic functions, and exponential functions.*

- **b**. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

○ **B. Build new functions from existing functions.**

F-BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. **(Shared with AI)**

PARCC: *i) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions ii) Tasks may involve recognizing even and odd functions.*

F-BF.B.4 Find inverse functions.

- a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x–1)$ for $x \neq 1$.

Linear, Quadratic, and Exponential Models (F-LE) ★

A. Construct and compare linear, quadratic, and exponential models and solve problems.

F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **(Shared with A1)**

PARCC: *Tasks will include solving multi-step problems by constructing linear and exponential functions.*

F-LE.A.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

B. Interpret expressions for functions in terms of the situation they model.

F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.
(Shared with A1)

PARCC: *i) Tasks have a real world context. ii) Tasks are limited to exponential functions with domains not in the integers.*

Trigonometric Functions (F-TF)

A. Extend the domain of trigonometric functions using the unit circle.

F-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

NYSED*: Includes the reciprocal trigonometric functions.*

B. Model periodic phenomena with trigonometric functions. ★

F-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

C. Prove and apply trigonometric identities.

F-TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

For more information about the concepts and terms introduced in the **Functions** domain, please consult the High School Progression on Functions.

High School – Geometry

Expressing Geometric Properties with Equations G-GPE

A. Translate between the geometric description and the equation for a conic section.

G-GPE.A.2 Derive the equation of a parabola given a focus and directrix.

High School - Statistics & Probability ★

Interpreting categorical and quantitative data (S-ID) ★

A. Summarize, represent, and interpret data on a single count or measurement variable.

S-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

B. Summarize, represent, and interpret data on two categorical and quantitative variables.

S-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

- **a.** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. **(Shared with A1)**

  PARCC: *i) Tasks have a real-world context. ii) Tasks are limited to exponential functions with domains not in the integers and trigonometric functions.*

Making Inferences and Justifying Conclusions (S-IC) ★

A. Understand and evaluate random processes underlying statistical experiments.

S-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S-IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

S-IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S-IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S-IC.B.6 Evaluate reports based on data.

Conditional Probability and the Rules of Probability (S-CP) ★

A. Understand independence and conditional probability and use them to interpret data

S-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

S-CP.A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S-CP.A.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S-CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

S-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Conditional Probability and the Rules of Probability (S-CP) ★

B. Use the rules of probability to computer probabilities of compound events in a uniform probability model.

S-CP.B.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

S-CP.B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

For more information about the concepts and terms introduced in the **Statistics & Probability** domain, please consult the High School Progression on Statistics and Probability.

Fluency Recommendations

The PARCC Model Content Frameworks recommend the following fluencies for Algebra II students:

A-APR.6 This standard sets an expectation that students will divide polynomials with remainder by inspection in simple cases. For example, one can view the rational expression(x+4)/(x+3) as = ((x+3) +1)/(x+3) = 1 + 1/(x+3).

A-SSE.2 The ability to see structure in expressions and to use this structure to rewrite expressions is a key skill in everything from advanced factoring (e.g., grouping) to summing series to the rewriting of rational expressions to examine the end behavior of the corresponding rational function.

F-IF.3 Fluency in translating between recursive definitions and closed forms is helpful when dealing with many problems involving sequences and series, with applications ranging from fitting functions to tables to problems in finance.