**L – Radicals, Lesson 1, Operations with Radicals (r. 2018)**

RADICALS

Operations with Radicals

|  |  |  |  |
| --- | --- | --- | --- |
| **Common Core Standard**   |  | | --- | | **N-RN.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. | | **Next Generation Standard**   |  | | --- | | **AI-N.RN.3** Use properties and operations to understand the different forms of rational and irrational numbers.  a.) Perform all four arithmetic operations and apply properties to generate equivalent forms of rational numbers and square roots.  Note: Tasks include rationalizing numerical denominators of the form  where *a* is an integer and *b* is a natural number.  b.) Categorize the sum or product of rational or irrational numbers.  • The sum and product of two rational numbers is rational.  • The sum of a rational number and an irrational number is irrational.  • The product of a nonzero rational number and an irrational number is irrational.  • The sum and product of two irrational numbers could be either rational or irrational. | |

**LEARNING OBJECTIVES**

Students will be able to:

1. Perform addition, subtraction, multiplication and division with radical numbers (prior skill).
2. Identify if the sum or product of two numbers is rational or irrational and explain why.

**Overview of Lesson**

|  |  |
| --- | --- |
| **Teacher Centered Introduction**  **Overview of Lesson**  **- activate students’ prior knowledge**  **- vocabulary**  **- learning objective(s)**  **- big ideas: direct instruction**  **- modeling** | **Student Centered Activities**  **guided practice Teacher: anticipates, monitors, selects, sequences, and connects student work**  **- developing essential skills**  **- Regents exam questions**  **- formative assessment assignment (exit slip, explain the math, or journal entry)** |

**VOCABULARY**

Decimal form

Equivalent

Irrational

Perfect square

Prime number

Radical form

Rational

Simplest radical form

**BIG IDEAS**



**Is a Number Irrational or Rational?**

|  |  |
| --- | --- |
| **Irrational Numbers** | **Rational Numbers** |
| If a decimal does not repeat or terminate, it is an irrational number.  Numbers with names, such as  and are irrational. They are given names because it is impossible to state their infinitely long values.  The square roots of all numbers (that are not perfect squares) are irrational.  If a term reduced to simplest form contains an irrational number, the term is irrational. . | If a number is an integer, it is rational, since it can be expressed as a ratio with the integer as the numerator and 1 as the denominator.  If a decimal is a repeating decimal, it is a rational number.  If a decimal terminates, it is a rational number. |

**Operations with Irrational and Rational Numbers**

**Addition and Subtraction**:

When two rational numbers are added or subtracted, the result is rational.

When two irrational numbers are added or subtracted, the result is irrational.

When an irrational number and a rational number are added or subtracted, the sum is irrational.

**Multiplication and Division**:

When two rational numbers are multiplied or divided, the product is rational.

When an irrational number and a non-zero rational number are multiplied or divided, the product is irrational.

When two irrational numbers are multiplied or divided, the product is sometimes rational and sometimes irrational.

|  |  |
| --- | --- |
| Example of Rational Product | Example of Irrational Product    NOTE: Be careful using a calculator to decide if a number is irrational. The calculator stops when it runs out of room to display the numbers, and the whole number may continue beyond the calculator display. |
| Rational Quotient | Irrational Quotient |

**DEVELOPING ESSENTIAL SKILLS**

|  |  |  |
| --- | --- | --- |
| Question | Answer | Is Answer Rational or Irrational? |
| 1. Express the product of  in simplest radical form. |  |  |
| 2. The expression  written in simplest radical form is: |  |  |
| 3. The expression  written in simplest radical form is |  |  |
| 4. Express  in simplest radical form. |  |  |
| 5. Express  in simplest radical form. |  |  |
| 6. Express  in simplest radical form. |  |  |
| 7. Express  in simplest radical form. |  |  |
| 8. Perform the indicated operations and express the answer in simplest radical form. |  |  |
| 9. The expression  simplifies to |  |  |
| 10. The expression  is equivalent to |  |  |

**ANSWERS**

|  |  |  |
| --- | --- | --- |
| Question | Answer | Is Answer Rational or Irrational? |
| 1. Express the product of  in simplest radical form. |  | Irrational |
| 2. The expression  written in simplest radical form is: |  | Irrational |
| 3. The expression  written in simplest radical form is |  | Irrational |
| 4. Express  in simplest radical form. |  | Irrational |
| 5. Express  in simplest radical form. |  | Irrational |
| 6. Express  in simplest radical form. |  | Irrational |
| 7. Express  in simplest radical form. |  | Irrational |
| 8. Perform the indicated operations and express the answer in simplest radical form. |  | Irrational |
| 9. The expression  simplifies to | 48 | Rational |
| 10. The expression  is equivalent to | 4 | Rational |

**REGENTS EXAM QUESTIONS (through June 2018)**

N.RN.B.3: Operations with Radicals

381) Given: 

Which expression results in a rational number?

|  |  |  |  |
| --- | --- | --- | --- |
| 1) |  | 3) |  |
| 2) |  | 4) |  |

382) Which statement is *not* always true?

|  |  |  |  |
| --- | --- | --- | --- |
| 1) | The product of two irrational numbers is irrational. | 3) | The sum of two rational numbers is rational. |
| 2) | The product of two rational numbers is rational. | 4) | The sum of a rational number and an irrational number is irrational. |

383) Ms. Fox asked her class "Is the sum of 4.2 and  rational or irrational?" Patrick answered that the sum would be irrational. State whether Patrick is correct or incorrect. Justify your reasoning.

384) Which statement is *not* always true?

|  |  |  |  |
| --- | --- | --- | --- |
| 1) | The sum of two rational numbers is rational. | 3) | The sum of a rational number and an irrational number is irrational. |
| 2) | The product of two irrational numbers is rational. | 4) | The product of a nonzero rational number and an irrational number is irrational. |

385) For which value of *P* and *W* is  a rational number?

|  |  |  |  |
| --- | --- | --- | --- |
| 1) | and | 3) | and |
| 2) | and | 4) | and |

386) Given the following expressions:

I.  III. 

II.  IV. 

Which expression(s) result in an irrational number?

|  |  |  |  |
| --- | --- | --- | --- |
| 1) | II, only | 3) | I, III, IV |
| 2) | III, only | 4) | II, III, IV |

387) Determine if the product of  and  is rational or irrational. Explain your answer.

388) Is the sum of  and  rational or irrational? Explain your answer.

389) Jakob is working on his math homework. He decides that the sum of the expression  must be rational because it is a fraction. Is Jakob correct? Explain your reasoning.

390) State whether  is rational or irrational. Explain your answer.

391) A teacher wrote the following set of numbers on the board:

Explain why  is irrational, but  is rational.

392) The product of  and  is

|  |  |  |  |
| --- | --- | --- | --- |
| 1) | irrational because both factors are irrational | 3) | irrational because one factor is irrational |
| 2) | rational because both factors are rational | 4) | rational because one factor is rational |

393) Is the product of  and  rational or irrational? Explain your reasoning.

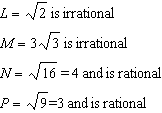
**SOLUTIONS**

381) ANS: 3

 may be expressed as the ratio of two integers.

Strategy: Recall that under the operation of addition, the addition of two irrational numbers and the addition of an irrational number and a rational number will always result in a sum that is irrational. To get a rational number as a sum, you must add two rational numbers.

STEP 1 Determine whether numbers L, M, N, and P are ratiional, then reject any answer choice that does not contain two rational numbers.



STEP 2 Reject any answer choice that does not include . Choose answer choice c.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

382) ANS: 1

Strategy: Find a counterexample to prove one of the answer choices is *not* always true.

Answer choice a is not always true because:  and  are both irrational numbers, but , and 6 is a rational number, so the product of two irrational numbers is not *always* irrational.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

383) ANS:

Patrick is correct. The sum of a rational and irrational is irrational.

Strategy: Determine whether 4.2 and  are rational or irrational numbers, then apply the rules of operations on rational and irrational numbers.

4.2 is rational because it can be expressed as , which is the ratio of two integers.

 is irrational because it cannot be expressed as the ratio of two integers.

The rules of addition and subtraction of rational and irrational numbers are:

When two rational numbers are added or subtracted, the result is rational.

When two irrational numbers are added or subtracted, the result is irrational.

When an irrational number and a rational number are added or subtracted, the sum is irrational.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

384) ANS: 2

Strategy: Find a counterexample to prove one of the answer choices is *not* always true. This will usually involve the *product* or *quotient* of two irrational numbers since the outcomes of addition and subtraction of irrational numbers are more predictable.

Answer choice b is not always true because:  and  are both irrational numbers, but , and  is an rational number, so the product of two irrational numbers is not *always* rational.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

385) ANS: 2



Strategy: Recall that under the operation of addition, the addition of two irrational numbers and the addition of an irrational number and a rational number will always result in a sum that is irrational. To get a rational number as a sum, you must add two rational numbers. Reject any answer choice that does not contain two rational numbers.

Reject answer choice a because  is irrational.

Choose answer choice b because both  and  can be expressed as rational numbers, as shown above.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

386) ANS: 1

Strategy: Eliminate wrong answers.

Expression I results in a rational number because the set of rational numbers is closed under addition. 

Expression II is is correct because the additon of a rational number and an irrational number always results in an irrational number.



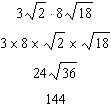
Expression III results in a rational number because , which is the ratio of two integers.

Expression IV results in a rational number because , which the ratio of two integers.

Expression II is the only expression that results in an irrational number, so Choice (a) is the correct answer.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

387) ANS:



The product is 144, which is rational, because it can be written as , a ratio of two integers.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

388) ANS:

Irrational



 is irrational because it is the product of a rational number and an irrational number.

7 is rational because it can be expressed as the ratio of two integers ()

 is irrational because the square roots of all prime numbers are irrational.

PTS: 2 NAT: N.RN.B

389) ANS:

Jakob is incorrect. The sum of a rational number and an irrational number is irrational.



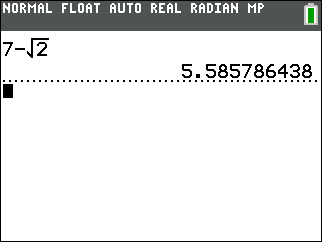
Note the square root of 5 in the sum. The square root of any prime is irrational.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

390) ANS:

Irrational

A rational number and an irrational number under addition or subtraction will always be irrational.



Note that the answer does not appear to repeat or end.

is irrational because it can not be written as the ratio of two integers.

PTS: 2 NAT: N.RN.B.3 TOP: Operations with Radicals

KEY: classify

391) ANS:

The sum of *a* and *b* is irrational because the sum of an irrational number and a rational number is always irrational.

The sum of *b* and *c* is rational because the sum of a rational number and another rational number is always rational.

 is an irrational number that can be simplified to , but cannot be expressed as the ratio of two integers or as a never-ending, never-repeating decimal.

2.5 is a rational number because it can be expressed as the ratio of two integers, such as .

 is a rational number that can be simplified to 15 and expressed as the ratio of two integers, such as .

PTS: 2 NAT: N.RN.B.3 TOP: Operations with Radicals

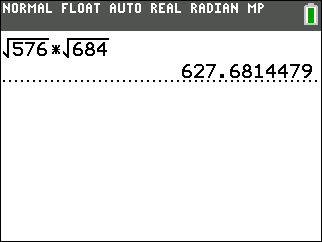
KEY: classify

392) ANS: 3

, which can be expressed as the ratio , which means that  is a rational number.

 cannot be expressed as a rational number. It can be simplified to , but it cannot be expressed as the ratio of two integers. Therefore,  is an irrational number.

The product of a rational number and an irrational number is always irrational.



Note that the product of  and  appears to be a never ending, non-repeating decimal, which indicates that the product is an irrational number.

PTS: 2 NAT: N.RN.B.3 TOP: Operations with Radicals

KEY: classify

393) ANS:

Answer: The product of  and  is rational.

Explanation: A rational number is a number that can be expressed as the ratio of two intergers, in the form of , where both *a* and *b* are integers. An irrational number is a number that cannot be expressed as the ratio of two integers.

 is a rational number because  can be expressed as , which is a ratio of two integers.

 is a rational number because it is already expressed as a ratio of two integers.

, and  is a ratio of two integers.

The product of any two rational numbers will always be a rational number.

PTS: 2 NAT: N.RN.B.3 TOP: Operations with Radicals

KEY: classify