JMAP

Big Book of Lesson Plans

(This pdf can be navigated using bookmarks in Adobe Reader)

About this Book of Lesson Plans:

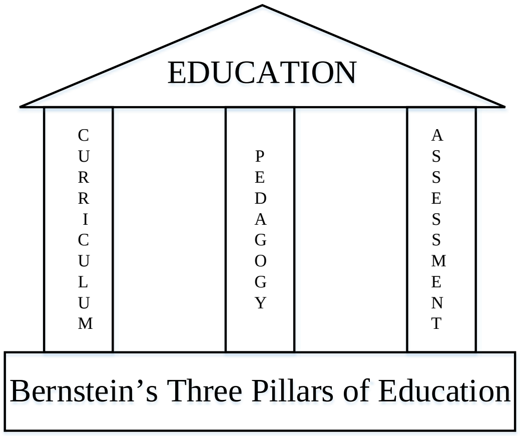
These lesson plans are suitable for use in any state with mathematics curricula aligned to the Common Core. Each lesson plan includes representative questions used by the New York State Education Department (NYSED) to assess high school students during Algebra I (Common Core) Regents mathematics examinations.

Teachers are welcome to copy, modify, and use these lesson plans and other JMAP resources for individual and classroom use, but not for profit or republication on the internet. Each lesson plan in this book is available at no cost in manipulable docx format on the JMAP website. Simply Google the name of the lesson plan and the word JMAP.

If you find errors in these lesson plans, and there undoubtedly are some errors, or if you have a recommendation for improving these resources, please let us know.

Steve and Steve
www.jmap.org
August, 2018

JMAP is a non-profit initiative working for the benefit of teachers and their students. JMAP provides free resources to teachers and receives no state or local government support. If you wish to support JMAP's efforts, please consider making a charitable donation through JMAP's website. While JMAP is not associated with NYSED or the New York City Department of Education (NYCDOE), Steve Sibol (Editor and Publisher) and Steve Watson (Principal and Cofounder) are Brooklyn public high school math teachers. Special appreciation goes to the many math teachers who have shared their ideas about how to improve JMAP.



Basil Bernstein (1924 – 2000)[1] was a British sociologist of education. He posited that there are three pillars of education: 1) curriculum; 2) pedagogy; and 3) assessment; and that all communication involves the transmission of sociolinguistic codes, which are associated with the languages that people use. This book of lesson plans is heavily influenced by Basil Bernstein's sociolinguistic theory of language codes and seeks to illuminate the hidden codes of the Common Core mathematics curriculum for teachers and students. The methodology focuses on the three pillars of education.

CURRICULUM

Curriculum in public education in New York is defined by the state, so each lesson plan begins a crosswalk between New York's current Common Core State Standards and the revised Next Generation State Standards that will be effective in 2020. These standards constitute New York State's definition of what is to be taught in Algebra I courses in public schools.

PEDAGOGY

Pedagogy in public education is defined by how teachers frame and deliver curriculum to their students. The lesson plans that follow include vocabulary and big ideas associated with the curriculum standards. This information is provided to assist the classroom teacher with ideas and tools that may be useful in the classroom. However, in the final analysis, teachers must adapt any lesson plan to their own teaching styles and to the needs of their students. To facilitate customization, every lesson plan in this book is available in Microsoft Word document format at www.jmap.org for free. Just Google JMAP and the name of the lesson, then customize at will.

ASSESSMENT

Assessment of the Algebra I curriculum in New York occurs through the Regents Examination System. Every lesson plan in this book contains every associated Algebra I Regents examination problem administered through June 2018, a total of 504 problems. By studying the entire lesson plan, teachers will gain insights into how the standards are assessed. By aligning curriculum, pedagogy, and assessment practices, teachers will assist their students in their quests to acquire the knowledge required to sustain the examinations. By working the Regents examination problems at the end of each lesson, students will become familiar with Regents assessment practices and become prepared for their own examination.

TABLE OF CONTENTS

(This pdf can be navigated using bookmarks in Adobe Reader)

Every lesson plan in this book is available in Microsoft Word document format at www.jmap.org for free. Just Google JMAP and the name of the lesson, then customize at will.

TABLE OF CONTENTS (cont.)

(This pdf can be navigated using bookmarks in Adobe Reader)

Every lesson plan in this book is available in Microsoft Word document format at www.jmap.org for free. Just Google JMAP and the name of the lesson, then customize at will.

NUMBERS, OPERATIONS AND PROPERTIES
Identifying Properties

CC Standard	**NG Standard**
A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**AI-A.REI.1a** Explain each step when solving a **linear or quadratic** equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

LEARNING OBJECTIVES

Students will be able to:

1) Use academic language to describe each step in solving an equation.
2) Use a four column strategy to show and explain each step in solving an equation.

VOCABULARY

Commutative Properties of Addition and Multiplication
Associative Properties of Addition and Multiplication
Distributive Properties of Addition and Multiplication
Addition Property of Equality
Multiplication Property of Equality
Identity Elements of Addition and Multiplication
Inverse Properties of Addition and Multiplication

BIG IDEAS

PROPERTIES

Commutative Properties of Addition and Multiplication

For all real numbers a and b:

$a+b=b+a \qquad a\cdot b=b\cdot a$

Associative Properties of Addition and Multiplication

For all real numbers a, b, and c:

$$(a+b)+c = a+(b+c) \qquad (a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Distributive Properties of Addition and Multiplication

$$a(b+c) = ab+ac \qquad a(b-c) = ab-ac$$

$$(b+c)a = ba+ca \qquad (b-c)a = ba-ca$$

Addition Property of Equality

The addition of the same number or expression to both sides of an equation is permitted.

Multiplication Property of Equality

The multiplication of both sides of an equation by the same number or expression is permitted.

IDENTITY ELEMENTS

Identity Element: The **identity element** is always associated with an *operation*. The **identity element** for a given *operation* is the element that preserves the identity of other elements under the given operation.

Addition

The **identity element** for addition is the number 0

$$a+0 = a \text{ and } 0+a = a$$

The number 0 does not change the value of other numbers under addition.

Multiplication

The **identity element** for multiplication is the number 1

$$a \cdot 1 = a \text{ and } 1 \cdot a = a$$

The number 1 does not change the value of other numbers under multiplication.

Inverse Properties of Addition and Multiplication

Inverse: The **inverse** of a number or expression under a given *operation* will result in the **identity element** for that operation. Therefore, it is necessary to know what the **identity element** of an operation is before finding the **inverse** of a given number or expression.

Addition

The additive inverse of a number or expression results in 0 under addition.

$$a+(-a=0) \text{ and } (-a)+a=0$$

$$(x+y)+(-x-y)=0 \text{ and } (-x-y)+(x+y)=0$$

Multiplication

The multiplicative inverse of a number or expression results in 1 under multiplication.

$$a \times \frac{1}{a} = 1 \text{ and } \frac{1}{a} \times a = 1 \qquad \frac{1}{a} \times a = 1$$

$$(x+y)\left(\frac{1}{(x+y)}\right) = 1 \text{ and } \left(\frac{1}{(x+y)}\right)(x+y) = 1$$

Four Column Strategy

The four column strategy focuses on organizing and documenting each step in solving an equation or inequality. Emphasis is given to explaining each step and keeping the equal signs (or inequality signs) aligned in a vertical column. The vertical and horizontal lines are simply scaffolds that can be removed as students acquire understanding and skills in solving equations.

Notes	Left Hand Expression	Sign	Right Hand Expression
Given Add (6) (Addition Property of Equality)	$2x-6$ $+6$	$=$	2 + 6
	$2x+0$	$=$	8
Divide (2) (Multiplication Property of Equality)	$\frac{2x}{2}$	$=$	$\frac{8}{2}$
Answer	x	$=$	4
Check	2(4) – 6 8-6 2	= = =	2 2 2

DEVELOPING ESSENTIAL SKILLS

Use the four column method with academic language to solve the following equations.

A	$2x+8=18$
B	$\frac{3}{4}x-7=2$
C	$3x+5=2x+10$
D	$4(x+5)-12=2x+4$

REGENTS EXAM QUESTIONS (through June 2018)

A.REI.A.1: Identifying Properties

1) When solving the equation $4(3x^2+2)-9=8x^2+7$, Emily wrote $4(3x^2+2)=8x^2+16$ as her first step. Which property justifies Emily's first step?
   1) addition property of equality
   2) commutative property of addition
   3) multiplication property of equality
   4) distributive property of multiplication over addition

2) When solving the equation $12x^2-7x=6-2(x^2-1)$, Evan wrote $12x^2-7x=6-2x^2+2$ as his first step. Which property justifies this step?
   1) subtraction property of equality
   3) associative property of multiplication

2) multiplication property of equality

4) distributive property of multiplication over subtraction

3) A part of Jennifer's work to solve the equation $2(6x^2 - 3) = 11x^2 - x$ is shown below.

Given: $2(6x^2 - 3) = 11x^2 - x$

Step 1: $12x^2 - 6 = 11x^2 - x$

Which property justifies her first step?

1) identity property of multiplication

3) commutative property of multiplication

2) multiplication property of equality

4) distributive property of multiplication over subtraction

SOLUTIONS

1) ANS: 1

Strategy: Identify what changed during Emily’s first step, then identify the property associated with what changed..

$$4(3x^2 + 2) - 9 = 8x^2 + 7$$

$$4(3x^2 + 2) = 8x^2 + 16$$

Emily moved the -9 term from the left expression of the equation to the right expression of the equation by adding +9 to both the left and right expressions.

Adding an equal amount to both sides of an equation is associated with the addition property of equality.

PTS: 2 NAT: A.REI.A.1 TOP: Identifying Properties

2) ANS: 4

Evan’s first step was to remove the parentheses from the right expression.

$$12x^2 - 7x = \quad 6 - 2(x^2 - 1)$$

$$12x^2 - 7x = \quad 6 - 2x^2 + 2$$

He removed the parentheses by using the distributive property.

PTS: 2 NAT: A.REI.A.1 TOP: Identifying Properties

3) ANS: 4

$$2(6x^2 - 3)$$

$$= 2(6x^2) + 2(-3)$$

$$= 12x^2 - 6$$

This is the distributive property of multiplication over subtraction.

PTS: 2 NAT: A.REI.A.1 TOP: Identifying Properties

B – Graphs and Statistics, Lesson 2, Central Tendency and Dispersion (r. 2018)

GRAPHS AND STATISTICS

Central Tendency and Dispersion

Common Core Standards	Next Generation Standards
S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (inter-quartile range, standard deviation) of two or more different data sets.	**AI-S.ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (inter-quartile range, sample standard deviation) of two or more different data sets. Note: Values in the given data sets will represent samples of larger populations. The calculation of standard deviation will be based on the sample standard deviation formula $s=\sqrt{\frac{\left(x-\overline{x}\right)^2}{n-1}}$. The sample standard deviation calculation will be used to make a statement about the population standard deviation from which the sample was drawn.
S-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	**AI-S.ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

LEARNING OBJECTIVES

Students will be able to:

1) Calculate measures of central tendency and dispersion for one variable data sets from a graphic representation of the data set, a table, or a context.
2) Compare measures of central tendency and dispersion for two or more one variable data sets.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

Center (measures of central tendency)
Mean
Median
Mode
Spread (measures of dispersion)
Interquartile Range

Standard Deviation Outliers (extreme data points)
Normal Curve

BIG IDEAS

Measures of Central Tendency

A **measure of central tendency** is a *summary statistic* that indicates the typical value or center of an organized data set. The three most common measures of central tendency are the *mean, median*, and *mode*.

Mean A measure of central tendency denoted by $\overline{x}$, read "x bar", that is calculated by adding the data values and then dividing the sum by the number of values. Also known as the arithmetic mean or arithmetic average. The algebraic formula for the mean is:

$$\text{Mean} = \frac{Sum\ of\ \ items}{Count} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}$$

Median A measure of central tendency that is, or indicates, the middle of a data set when the data values are arranged in ascending or descending order. ***If there is no middle number, the median is the average of the two middle numbers.***

Examples:
The **median** of the set of numbers: {2, 4, 5, 6, 7, 10, 13} is 6
The **median** of the set of numbers: {6, 7, 9, 10, 11, 17} is 9.5

Quartiles:
Q1, the **first quartile**, is the middle of the lower half of the data set.
Q2, the **second quartile**, is also known as the **median**.
Q3, the **third quartile**, is the middle of the upper half of the data set.

NOTE: To computer Q1 and Q2, find the middle numbers in the lower and upper halves of the data set. The median itself is not included in either the upper or the lower halves of the data set. When the data set contains an even number of elements, the median is the average of the two middle numbers and is excluded from the lower and upper halves of the data set.

Mode A measure of central tendency that is given by the data value(s) that occur(s) most frequently in the data set.

Examples:
The **mode** of the set of numbers {5, 6, 8, 6, 5, 3, 5, 4} is 5.
The **modes** of the set of numbers {4, 6, 7, 4, 3, 7, 9, 1,10} are 4 and 7.
The **mode** of the set of numbers {0, 5, 7, 12, 15, 3} is none or there is no mode.

Measures of Spread

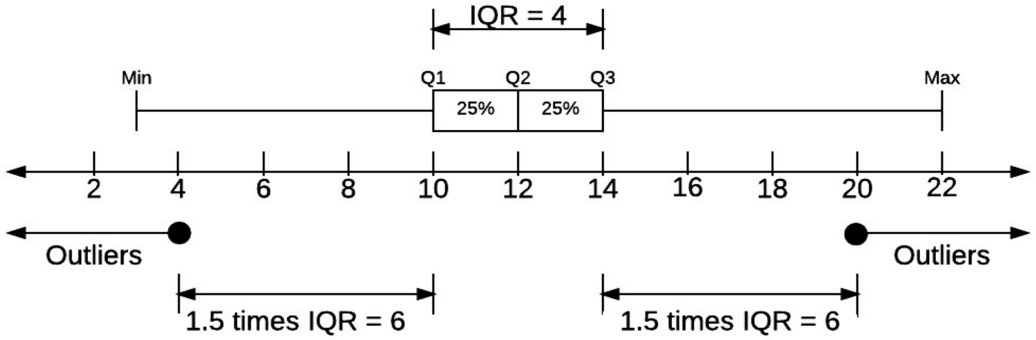
Measures of Spread indicate how the data is spread around the center of the data set. The two most common measures of spread are interquartile range and standard deviation.

Interquartile Range: The difference between the first and third quartiles; a measure of variability resistant to outliers.

$$IQR = Q3 - Q1$$

Outlier An observed value that is distant from other observations. Outliers in a distribution are 1.5 interquartile ranges (IQRs) or more below the first quartile or above the third quartile.

An **outlier** can significantly influence the measures of central tendency and/or spread in a data set.



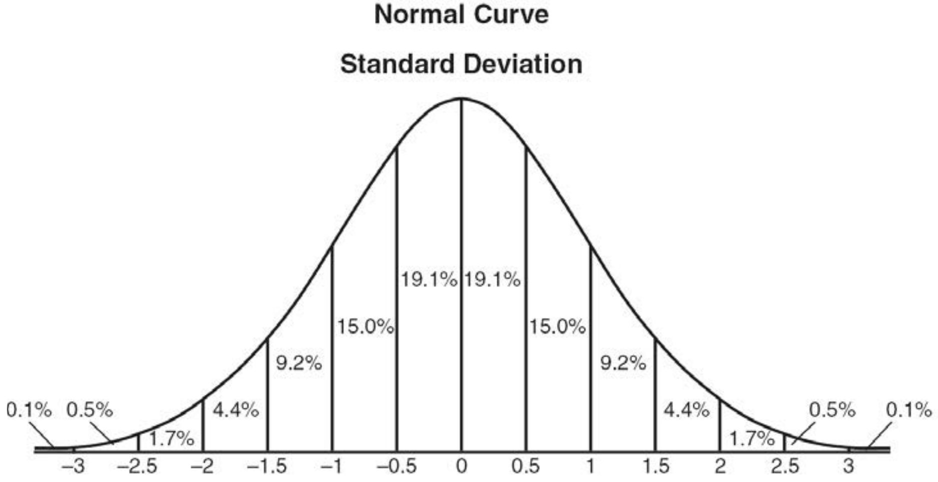
Example

In the above example, **outliers** would be any observed values less than or equal to 4 and/or any observed values greater than or equal to 20.

NOTE: Box plots, like the one above, are useful graphical representations of dispersion.

Standard Deviation: A measure of variability. **Standard deviation** measures the average distance of a data element from the mean. Typically, 98.8% of any set of univariate data can be divided into a total of six standard deviation units: three standard deviation units above the mean and three standard deviation units below the mean.

Standard Deviations and the Normal Curve



- **When a data set is normally distributed, there are more elements closer to the mean and fewer elements further away from the mean.**
- **The normal curve shows the distribution of elements based on their distance from the mean.**
- **Three standard deviation units above the mean and three standard deviation units below the mean will include approximately 98.8% of all elements in a normally distributed data set.**
- **Each standard deviation above or below the mean corresponds to a specific value in the data set.**
  - **In the above example, the distance associated with each standard deviation unit corresponds to a distance of approximately $2\frac{2}{3}$ units on the scale below the curve.**
- **Many things in nature, such as height, weight, and intelligence, are normally distributed.**

There are two types of **standard deviations**: population and sample.

Population Standard Deviation: If data is taken from the *entire population*, divide by n when averaging the squared deviations. The following is the formula for **population standard deviation**:

$$\sigma = \sqrt{\frac{\sum(x_i - \overline{x})^2}{n}}$$

NOTE: Population standard deviation not included in Next Generation Standards.

Sample Standard Deviation: If data is taken from a *sample* instead of the *entire population*, divide by $n-1$ when averaging the squared deviations. This results in a *larger* standard deviation. The following is the formula for **sample standard deviation**:

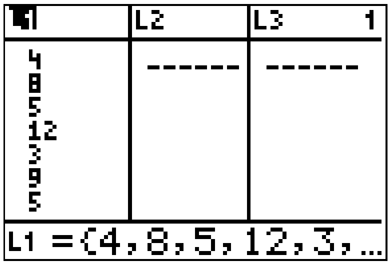
$$s = \sqrt{\frac{\sum(x_i - \overline{x})^2}{n-1}}$$

Tips for Computing Measures of Central Tendency and Dispersion:

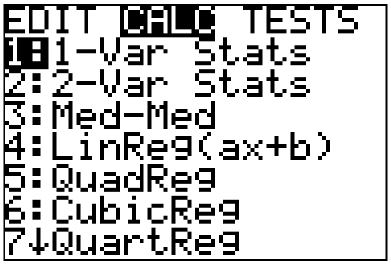
Use the STATS function of a graphing calculator to calculate measures of central tendency and dispersion.

INPUT VALUES: {4, 8, 5, 12, 3, 9, 5, 2}

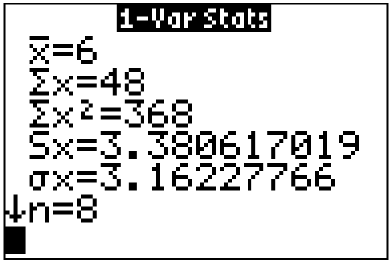
1. Use STATS EDIT to input the data set.
2. Use STATS CALC 1-Var Stats to calculate standard deviations.



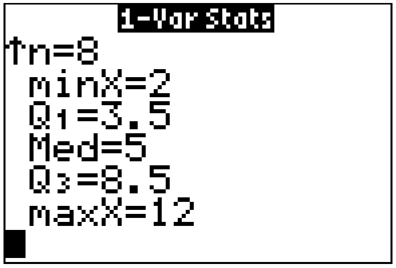
```
L1     L2      L3     1
4      ------  ------
8
5
12
3
9
5
L1 ={4,8,5,12,3,...
```



```
EDIT CALC TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7↓QuartReg
```



```
1-Var Stats
 x̄=6
 Σx=48
 Σx²=368
 Sx=3.380617019
 σx=3.16227766
↓n=8
```



```
1-Var Stats
↑n=8
 minX=2
 Q1=3.5
 Med=5
 Q3=8.5
 maxX=12
```

The outputs include:

$\overline{X}$, which is the mean (average), $\sum x$, which is the sum of the data set.

$\sum x^2$, which is the sum of the squares of the data set.
Sx, which is the **sample** standard deviation.
σx, which is the **population** standard deviation.
n, which is the number of elements in the data set

minX, which is the minimum value
Q1, which is the first quartile
Med, which is the median (second quartile)
Q3, which is the third quartile
maxX, which is the maximum value

DEVELOPING ESSENTIAL SKILLS

Use a graphing calculator to calculate one variable statistics for the following data sets:

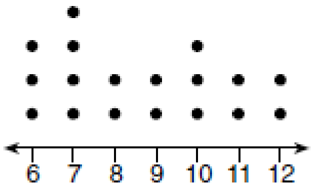
Set A

0.5	0.5	0.6	0.7	0.75	0.8
1.0	1.0	1.1	1.25	1.3	1.4
1.4	1.8	2.5	3.7	3.8	4
4.2	4.6	5.1	6	6.3	7.2

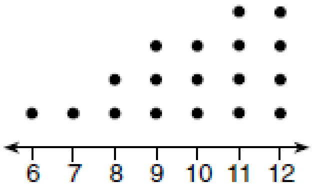
Set B

Number of Candy Bars Sold				
0	35	38	41	43
45	50	53	53	55
68	68	68	72	120

Sets C and D



Soccer Players' Ages



Basketball Players' Ages

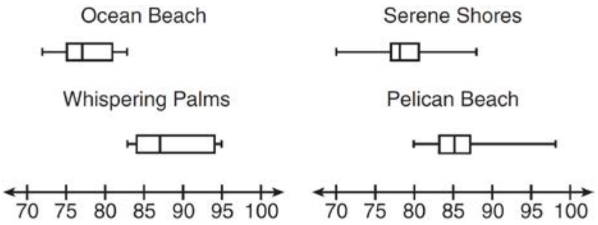
REGENTS EXAM QUESTIONS

S.ID.A.2-3: Central Tendency and Dispersion

4) Christopher looked at his quiz scores shown below for the first and second semester of his Algebra class.
Semester 1: 78, 91, 88, 83, 94
Semester 2: 91, 96, 80, 77, 88, 85, 92
Which statement about Christopher's performance is correct?

1) The interquartile range for semester 1 is greater than the interquartile range for semester 2.
2) The median score for semester 1 is greater than the median score for semester 2.
3) The mean score for semester 2 is greater than the mean score for semester 1.
4) The third quartile for semester 2 is greater than the third quartile for semester 1.

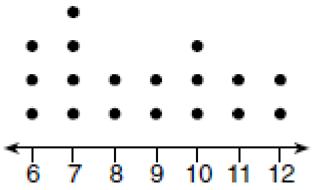
5) Corinne is planning a beach vacation in July and is analyzing the daily high temperatures for her potential destination. She would like to choose a destination with a high median temperature and a small interquartile range. She constructed box plots shown in the diagram below.



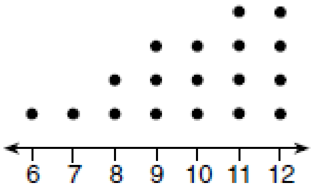
Which destination has a median temperature above 80 degrees and the smallest interquartile range?

1) Ocean Beach
2) Whispering Palms
3) Serene Shores
4) Pelican Beach

6) Noah conducted a survey on sports participation. He created the following two dot plots to represent the number of students participating, by age, in soccer and basketball.



Soccer Players' Ages



Basketball Players' Ages

Which statement about the given data sets is correct?

1) The data for soccer players are skewed right.
3) The data for basketball players have the same median as the data for soccer players.

2) The data for soccer players have less spread than the data for basketball players.

4) The data for basketball players have a greater mean than the data for soccer players.

7) The two sets of data below represent the number of runs scored by two different youth baseball teams over the course of a season.

Team *A*: 4, 8, 5, 12, 3, 9, 5, 2
Team *B*: 5, 9, 11, 4, 6, 11, 2, 7

Which set of statements about the mean and standard deviation is true?

1) mean A < mean B
standard deviation A > standard deviation B

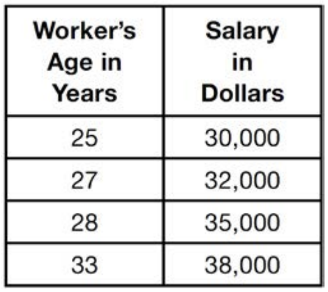
3) mean A < mean B
standard deviation A < standard deviation B

2) mean A > mean B
standard deviation A < standard deviation B

4) mean A > mean B
standard deviation A > standard deviation B

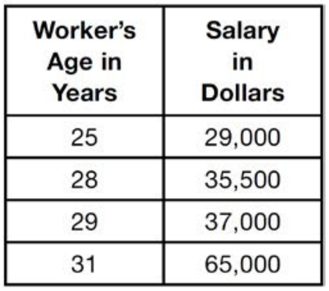
8) Isaiah collects data from two different companies, each with four employees. The results of the study, based on each worker's age and salary, are listed in the tables below.

Company 1



Worker's Age in Years	Salary in Dollars
25	30,000
27	32,000
28	35,000
33	38,000

Company 2



Worker's Age in Years	Salary in Dollars
25	29,000
28	35,500
29	37,000
31	65,000

Which statement is true about these data?

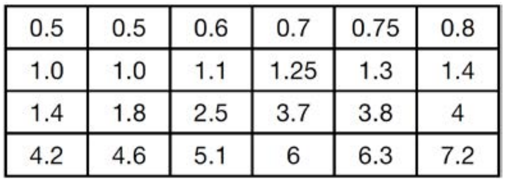
1) The median salaries in both companies are greater than $37,000.

3) The salary range in company 2 is greater than the salary range in company 1.

2) The mean salary in company 1 is greater than the mean salary in company 2.

4) The mean age of workers at company 1 is greater than the mean age of workers at company 2.

9) The table below shows the annual salaries for the 24 members of a professional sports team in terms of millions of dollars.



0.5	0.5	0.6	0.7	0.75	0.8
1.0	1.0	1.1	1.25	1.3	1.4
1.4	1.8	2.5	3.7	3.8	4
4.2	4.6	5.1	6	6.3	7.2

The team signs an additional player to a contract worth 10 million dollars per year. Which statement about the median and mean is true?

1) Both will increase.
2) Only the median will increase.
3) Only the mean will increase.
4) Neither will change.

10) The heights, in inches, of 12 students are listed below.

61,67,72,62,65,59,60,79,60,61,64,63

Which statement best describes the spread of these data?

1) The set of data is evenly spread.
2) The median of the data is 59.5.
3) The set of data is skewed because 59 is the only value below 60.
4) 79 is an outlier, which would affect the standard deviation of these data.

11) The 15 members of the French Club sold candy bars to help fund their trip to Quebec. The table below shows the number of candy bars each member sold.

Number of Candy Bars Sold				
0	35	38	41	43
45	50	53	53	55
68	68	68	72	120

When referring to the data, which statement is *false*?

1) The mode is the best measure of central tendency for the data.
2) The data have two outliers.
3) The median is 53.
4) The range is 120.

SOLUTIONS

4)ANS: 3

Strategy: Compute the mean, Q1, Q2, Q3, and interquartile range for each semester, then choose the correct answer based on the data.

	Mean	Q1	Median (Q2)	Q3	IQR
Semester 1	86.8	80.5	88	92.5	12
Semester 2	87	80	88	92	12

PTS: 2 NAT: S.ID.A.2 TOP: Central Tendency and Dispersion

5) ANS: 4

Strategy: Eliminate wrong answers based on daily high temperatures, then eliminate wrong answers based on size of interquartile ranges.

Ocean Breeze and Serene Shores can be eliminated because they *do not* have median high temperatures above 80 degrees. Whispering Palms and Pelican Beach *do* have median high temperatures above 80 degrees, so the correct answer must be either Whispering Palms or Pelican Beach.

The interquartile range is defined as the difference between the first and third quartiles. Pelican Beach has a much smaller interquartile range than Whispering Palms, so Pelican Beach is the correct choice.

PTS: 2 NAT: S.ID.A.2 TOP: Central Tendency and Dispersion

6) ANS: 4

Strategy: Determine the skew, spread, median, and mean for both data sets, then eliminate wrong answers.

	Soccer Players	Basketball Players
Skew	???	Left Skewed
Spread	$12-6=6$	$12-6=6$
Median	8.5	10
Mean	$\frac{156}{18}$	$\frac{178}{18}$

~~a)~~ The data for soccer players are skewed right. Uncertain

~~b)~~ The data for soccer players have ~~less spread~~ than the data for basketball players. Not True. Both data sets have the same spread.

~~c)~~ The data for basketball players have the ~~same median~~ as the data for soccer players. Not True $8.5 \neq 10$

d) The data for basketball players have a greater mean than the data for soccer players. Definitely True

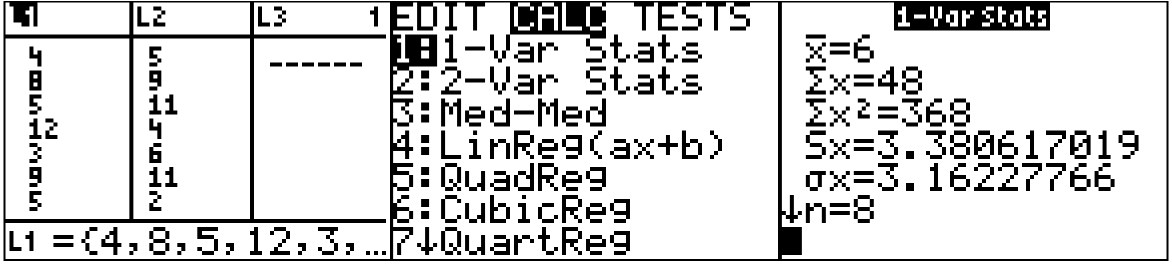
$\frac{178}{18} > \frac{156}{18}$

PTS: 2 NAT: S.ID.A.2 TOP: Central Tendency and Dispersion

7) ANS: 1

Strategy: Compute the mean and standard deviations for both teams, then select the correct answer.

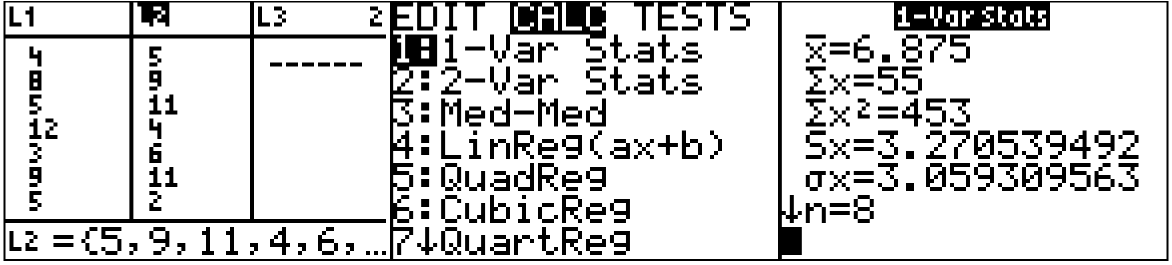
STEP 1. Enter the two sets of data into the STAT function of a graphing calculator, then select the first list (Team A) and run 1-Variable statistics, as shown below:



```
L1     L2     L3     1
4      5      ------
8      9
5      11
12     4
3      6
9      11
5      2
L1 ={4,8,5,12,3,...
```

```
x̄=6
Σx=48
Σx²=368
Sx=3.380617019
σx=3.16227766
↓n=8
```

STEP 2. Repeat STEP 1 for the second list (Team B).



```
L1     L2     L3     2
4      5      ------
8      9
5      11
12     4
3      6
9      11
5      2
L2 ={5,9,11,4,6,...
```

```
x̄=6.875
Σx=55
Σx²=453
Sx=3.270539492
σx=3.059309563
↓n=8
```

STEP 3. Use the data from the graphing calculator to choose the correct answer.

Choice a: mean A < mean B

$6 < 6.875$

standard deviation A > standard deviation B

$3.16227766 > 3.059309563$

Both statements in choice A are true.

$A: \bar{x} = 6;\ \sigma_x = 3.16 \quad B: \bar{x} = 6.875;\ \sigma_x = 3.06$

PTS: 2 NAT: S.ID.A.2 TOP: Central Tendency and Dispersion

8) ANS: 3

Strategy: Compute the median salary, mean salary, salary range, and mean age of employees for both companies, then select the correct answer.

		Company 1	Company 2
1	median salary	33,500	36,250
2	mean salary	33,750	44,125
3	salary range	8,000	36,000
4	mean age	28.25	28.25

PTS: 2 NAT: S.ID.A.2 TOP: Central Tendency and Dispersion

9) ANS: 3

Median remains at 1.4.

Strategy:
Compare the current median and mean to the new median and mean:

STEP 1. Compare the medians:
The data are already in ascending order, so the median is the middle number. In this case, the data set contains 24 elements - an even number of elements. This means there are two middle numbers, both of which are 1.4. When the data set contains an even number of elements, the median is the average of the two middle numbers, which in this case is $\frac{1.4 + 1.4}{2} = 1.4$

The new data set will contain 10 as an additional element, which brings the total number of elements to 25. The new median will be the 13th element, which is 1.4.

The current median and the new median are the same, so we can eliminate answer choices a and b.

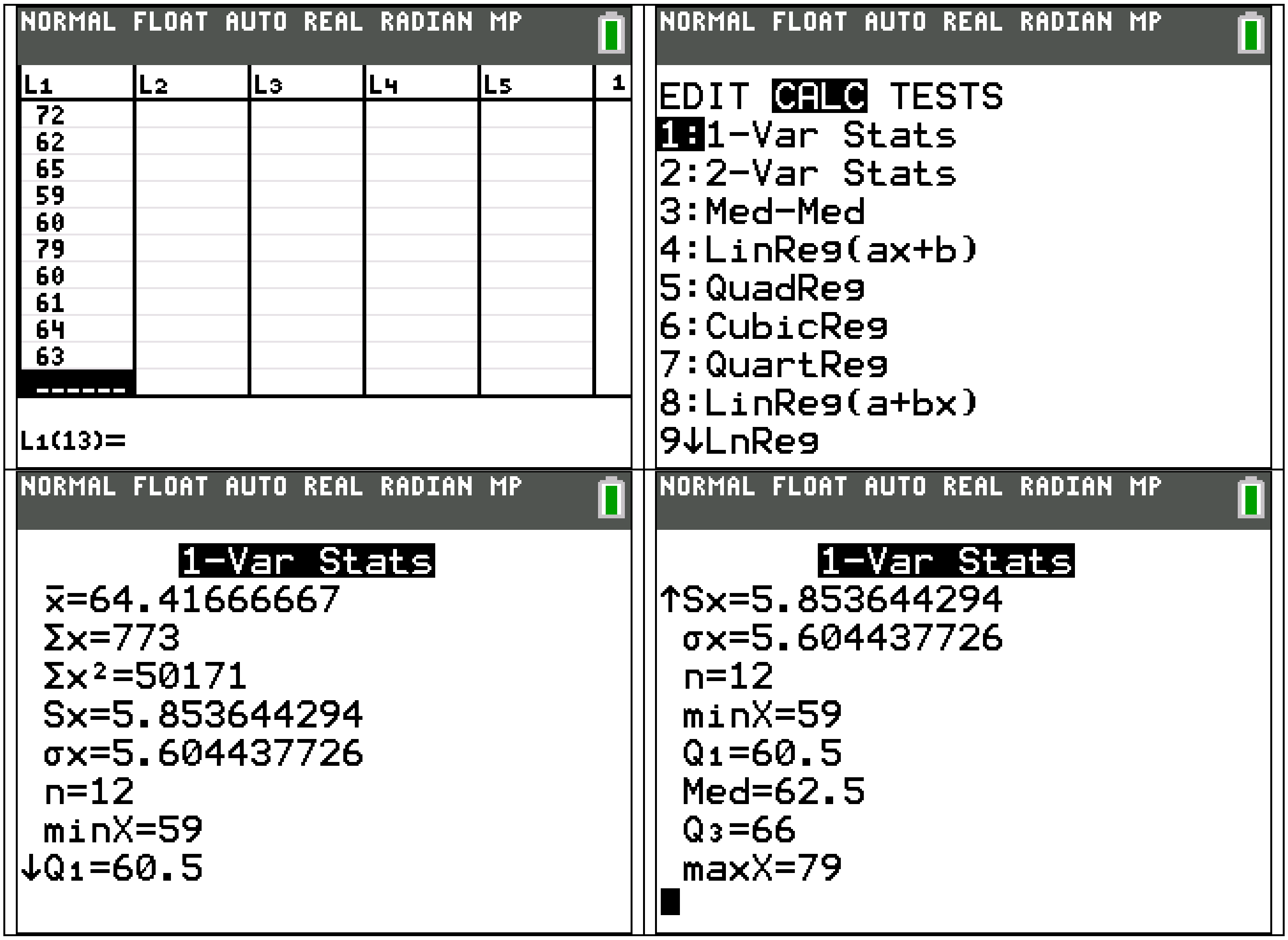
STEP 2. Compare the means:
The mean will increase because the additional element (10) is bigger than any current element. It is not necessary to do the calculations. We can eliminate answer choice d.

DIMS? Does it make sense that the answer is choice c?
Yes. The median will stay and 1.4 and only the mean will increase.

PTS: 2 NAT: S.ID.A.3 TOP: Central Tendency and Dispersion

10) ANS: 4

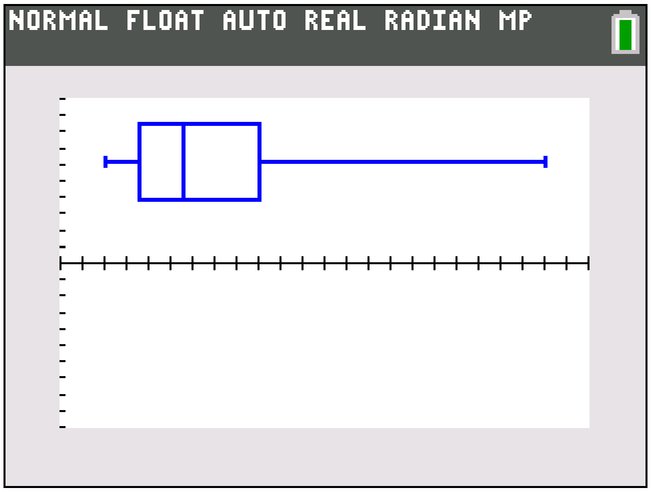
Input the data in a graphing calculator and obtain single variable statistics, then create a boxplot.



```
NORMAL FLOAT AUTO REAL RADIAN MP
L1    L2    L3    L4    L5    1
72
62
65
59
60
79
60
61
64
63
------
L1(13)=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
       1-Var Stats
 x̄=64.41666667
 Σx=773
 Σx²=50171
 Sx=5.853644294
 σx=5.604437726
 n=12
 minX=59
↓Q1=60.5
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
       1-Var Stats
↑Sx=5.853644294
 σx=5.604437726
 n=12
 minX=59
 Q1=60.5
 Med=62.5
 Q3=66
 maxX=79
```



(1) The set of data is evenly spread. Wrong. The data is not evenly spread.
(2) The median of the data is 59.5. Wrong. The median of the data is 62.5.
(3) The set of data is skewed because 59 is the only value below 60. Wrong. The data is skewed, but the reason for skewdness is that the mean does not equal the median.
(4) 79 is an outlier, which would affect the standard deviation of these. True. Any value greater than Q3 plus 1.5 times the interquartile range is an outlier.

$$Q3 + 1.5\left(IQR\right) = Upper\ Outlier\ Fence$$

$$66 + 1.5(66 - 60.5) = Upper\ Outlier\ Fence$$

$$66 + 1.5(5.5) = Upper\ Outlier\ Fence$$

$$66 + 8.25 = Upper\ Outlier\ Fence$$

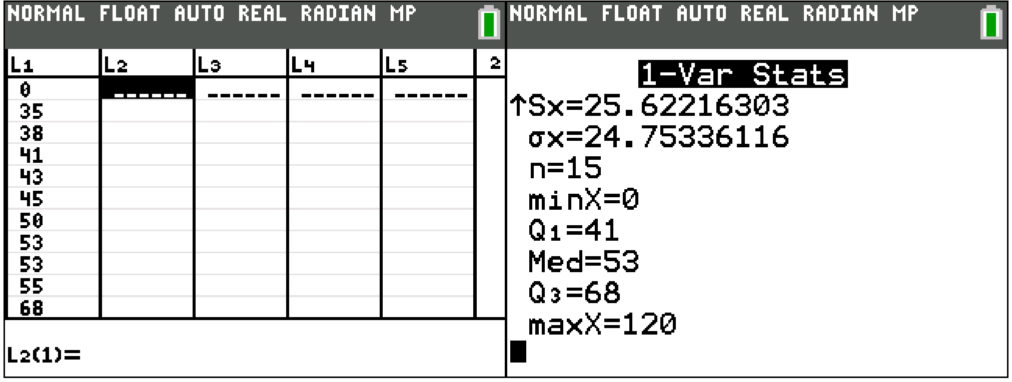
$$74.25 = Upper\ Outlier\ Fence$$

79 is beyond the upper outlier fence.

PTS: 2 NAT: S.ID.A.3 TOP: Central Tendency and Dispersion

11) ANS: 1

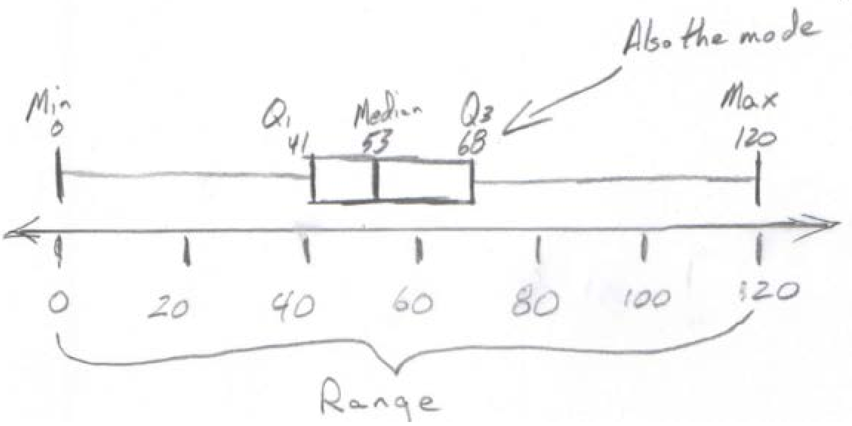
STEP 1. Insert the data into the stats editor of a graphing calculator and calculate 1 variable statistics.



```
NORMAL FLOAT AUTO REAL RADIAN MP

L1    L2     L3     L4     L5     2
0     ------ ------ ------ ------
35
38
41
43
45
50
53
53
55
68

L2(1)=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP

       1-Var Stats
↑Sx=25.62216303
 σx=24.75336116
 n=15
 minX=0
 Q1=41
 Med=53
 Q3=68
 maxX=120
```

Step 2. Construct a box plot.



STEP 3: Eliminate *true* answer choices.
It is trued that the data have two outliers. These are 0 and 120.
It is true that the median is 53.
It is true that the range is 120

PTS: 2 NAT: S.ID.A.3 TOP: Central Tendency and Dispersion

GRAPHS AND STATISTICS

Frequency Tables

Common Core Standard	Next Generation Standard
S-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	**AI-S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

LEARNING OBJECTIVES

Students will be able to:

1) Complete a two-way frequency table.
2) Calculate the percentage of data elements in a cell, row, or column of a two-way frequency table.

VOCABULARY

univariate
bivariate
frequency table

two-way frequency table
percentage
percent

BIG IDEAS

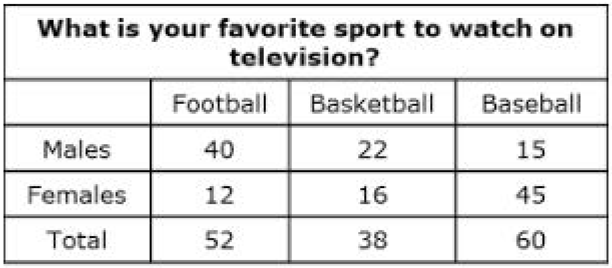
frequency table: A table that shows the observed number or frequency for a single number or range of numbers in a set of univariate data.

Example:

Interval	Tally	Frequency
1-5	~~IIII~~ I	6
6-10	~~IIII~~ II	7
11-15	II	2

two-way frequency table: A table that shows the observed number or frequency for two variables in a set of bivariate data, the rows indicating one category and the columns indicating the other category.

Example:



What is your favorite sport to watch on television?

	Football	Basketball	Baseball
Males	40	22	15
Females	12	16	45
Total	52	38	60

Calculating Percents

To calculate what **percent** A is of B, you simply divide A by B, then take that **number** and move the decimal place two spaces to the right.

Example: To find what percent 3 is of 4, simply divided 3 by 4, then take .75 and move the decimal two spaces to the right. The answer is 75%.

You can also use proportions. To find what percent 3 is of 4, set up the proportion

$$\frac{3}{4} = \frac{x\%}{100\%}$$

$$300 = 4x\%$$

$$75\% = x$$

DEVELOPING ESSENTIAL SKILLS

1) Organize Data

The senior spirit committee sold hot dogs, pizza, water, and soda at soccer games to raise money for the prom. 400 sales were made. They sold 200 sodas, 150 bottles of water, 158 hot dogs, and 182 pizzas. 50 students who bought hot dogs also bought sodas, and 58 students who bought pizzas also bought bottles of water. 30 students bought soda, but no food; and 46 students bought hot dogs, but no drink. Organize this data in a two-way frequency table.

Concession Stand Sales				
	Soda	Water	No Drink	Total
Hot Dog	50	62	46	158
Pizza	120	58	4	182
No Food	30	30	0	60
Total	200	150	50	400

2) Calculate Percentages

Calculate the percent of sales in each cell of your two-way frequency table *to the nearest tenth of a percent.*

Concession Stand Sales				
	Soda	Water	No Drink	Total
Hot Dog	12.5%	15.5%	11.5%	39.5%
Pizza	30%	14.5%	1%	45.5%
No Food	7.5%	7.5%	0%	15%
Total	50%	37.5%	12.5%	100%

REGENTS EXAM QUESTIONS

S.ID.B.5: Frequency Tables

12) The school newspaper surveyed the student body for an article about club membership. The table below shows the number of students in each grade level who belong to one or more clubs.

	1 Club	**2 Clubs**	**3 or More Clubs**
9th	90	33	12
10th	125	12	15
11th	87	22	18
12th	75	27	23

If there are 180 students in ninth grade, what percentage of the ninth grade students belong to more than one club?

13) A survey of 100 students was taken. It was found that 60 students watched sports, and 34 of these students did not like pop music. Of the students who did *not* watch sports, 70% liked pop music. Complete the two-way frequency table.

	Watch Sports	Don’t Watch Sports	Total
Like Pop			
Don’t Like Pop			
Total			

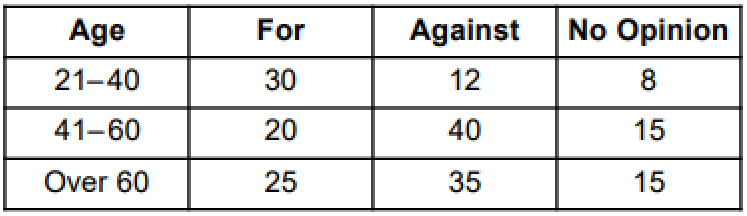
14) A statistics class surveyed some students during one lunch period to obtain opinions about television programming preferences. The results of the survey are summarized in the table below.

Programming Preferences

	Comedy	**Drama**
Male	70	35
Female	48	42

Based on the sample, predict how many of the school's 351 males would prefer comedy. Justify your answer.

15) A public opinion poll was taken to explore the relationship between age and support for a candidate in an election. The results of the poll are summarized in the table below.

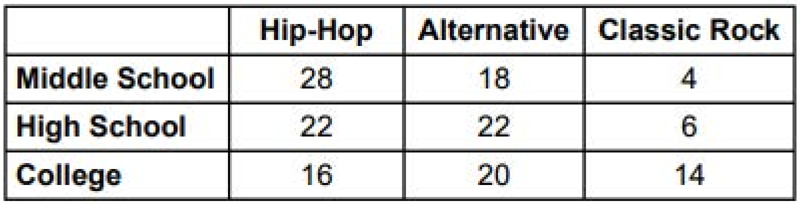


Age	For	Against	No Opinion
21–40	30	12	8
41–60	20	40	15
Over 60	25	35	15

What percent of the 21-40 age group was for the candidate?

1) 15	3) 40
2) 25	4) 60

16) A radio station did a survey to determine what kind of music to play by taking a sample of middle school, high school, and college students. They were asked which of three different types of music they prefer on the radio: hip-hop, alternative, or classic rock. The results are summarized in the table below.



	Hip-Hop	Alternative	Classic Rock
Middle School	28	18	4
High School	22	22	6
College	16	20	14

What percentage of college students prefer classic rock?

1) 14%	3) 33%
2) 28%	4) 58%

17) Students were asked to name their favorite sport from a list of basketball, soccer, or tennis. The results are shown in the table below.

	Basketball	Soccer	Tennis
Girls	42	58	20
Boys	84	41	5

What percentage of the students chose soccer as their favorite sport?

1) 39.6%	3) 50.4%
2) 41.4%	4) 58.6%

SOLUTIONS

12) ANS:
25%
Strategy: Use data from the table and information from the problem to calculate a percentage.
STEP 1. Determine the total number of students in the ninth grade who are in 2 or more clubs (33+12).
STEP 2. Divide by the total number of students in the ninth grade (180).
STEP 3. Convert the decimal to a percentage

$$\frac{33+12}{180}=\frac{45}{180}=.25$$

$$.25=25\%$$

PTS: 2 NAT: S.ID.B.5 TOP: Frequency Histograms, Bar Graphs and Tables

13) ANS:

Step 1. Fill in the known information from the problem.

	Watch Sports	Dont Watch Sports	Total
Like Pop			
Don't Like Pop	34		
Total	60		100

Step 2. Complete additional cells using given information.

	Watch Sports	Dont Watch Sports	Total
Like Pop	26		
Don't Like Pop	34		
Total	60	40	100

Step 3. Complete the "Don't Watch Sports - Like Pop" cell using information from the problem that states "Of the students who did *not* watch sports, 70% liked pop music." Compute $40 \times 70\% = 28$.

	Watch Sports	Dont Watch Sports	Total
Like Pop	26	28	
Don't Like Pop	34		
Total	60	40	100

Step 4. Complete the reamining cells.

	Watch Sports	Dont Watch Sports	Total
Like Pop	26	28	54
Don't Like Pop	34	12	46
Total	60	40	100

PTS: 2 NAT: S.ID.B.5 TOP: Frequency Tables

14) ANS:

234 of the school's 351 males prefer comedy based on the sample.

Step 1. Understand that the table is only a sample of the population, and the population of males is 351. Assume that the sample was not biased.

Step 2. Strategy. Determine the percent (or fraction) of the males in the sample that prefer comedy, then apply that percent to the total population.

Step 3. Execution of strategy.

$70+35=105$ males were surveyed.

Based on the sample, $\frac{70}{105}=\frac{2}{3}=66.67\%$ of the males preferred comedy.

$\frac{2}{3}(351)=\frac{2\times 351}{3\times 1}=\frac{702}{3}=234.$

Step 4. Does it make sense. Yes, if $\frac{2}{3}$ of the males in the sample prefer comedy, we can predict that $\frac{2}{3}$ of the males in the population will prefer comedy.

PTS: 2 NAT: S.ID.B.5 TOP: Frequency Tables

15) ANS: 4

Step 1. Understand that the problem is only interested in the percent for the candidate in the 21-40 age group. The bottom two rows of the table are not relevant to the problem.

Step 2. Strategy. Determine the total number of poll responses in the 21-40 age group and what percentage of these responses were for the candidate.
Step 3. Execute the strategy.

$$\left.\frac{\text{for}}{\text{total}}\right| \quad \frac{30}{30+12+8}=\frac{30}{50}=\frac{60}{100}=60\%$$

Step 4. Does it make sense? Yes. We know that 30 responses were for the candidate. Choices a), b), and c) are wrong because: a) 15% of 50 is $.15\times 50=7.5$; b) 25% of 50 is $.25\times 50=12.5$; and c) 40% of 50 is $.40\times 50=20$. Choice d) is the only correct answer because 60% of 50 is $.50\times 60=30$.

PTS: 2 NAT: S.ID.B.5 TOP: Frequency Tables

16) ANS: 2
Understand the Problem:
The questions asks what percentage of college students prefer classic rock. The information in the table about middle school and high school students is not important.

The total number of college students is 16 + 20 + 14 = 50.
14 out of 50 college students prefer classic rock.

Strategy: Write and solve a proportion to convert 14 out of 50 to a percentage.

$$\frac{14}{50}=\frac{x}{100}$$

$$1400=50x$$

$$28=x$$

PTS: 2 NAT: S.ID.B.5 TOP: Frequency Tables

17) ANS: 1
Strategy:
STEP 1. Find the total numbers of students who like each sport.
Basketball: A total of 126 boys and girls chose basketball.
Soccer: A total of 99 boys and girls chose soccer.
Tennis: A total of 25 boys and girls chose tennis.
STEP 2. Find the total number of students in the entire table.
Total basketball plus total soccer plus total tennis = 250
STEP 3. Write a proportion to find the percentage of students who chose soccer.

$$\frac{\text{chose soccer}}{\text{total students}}=\frac{\text{\% of students who chose socccer}}{100}\Leftrightarrow\frac{99}{250}=\frac{x}{100}$$

STEP 4. Solve the proportion for x

$$\frac{99}{250}=\frac{x}{100}$$

$$99\times 100=250x$$

$$9900=250x$$

$$\frac{9900}{250}=x$$

$$39.6=x$$

PTS: 2 NAT: S.ID.B.5 TOP: Frequency Tables

B – Graphs and Statistics, Lesson 3, Frequency Histograms, Box Plots and Dot Plots (r. 2018)

GRAPHS AND STATISTICS

Frequency Histograms, Box Plots and Dot Plots

Common Core Standard	**Next Generation Standard**
S-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).	**AI-S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

LEARNING OBJECTIVES

Students will be able to:

1) Construct and label dot plots, histograms, and box plots above a number line to represent *univariate* data sets.

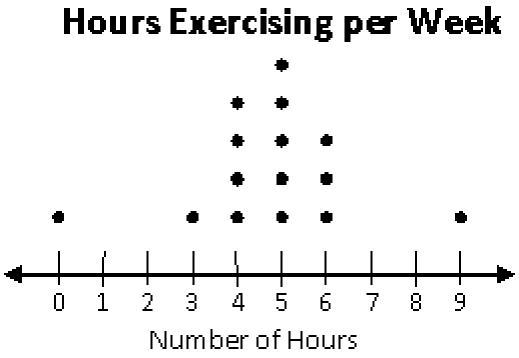
VOCABULARY

univariate
bivariate
dot plot
histogram
box plot
quartile

BIG IDEAS

Dot Plots

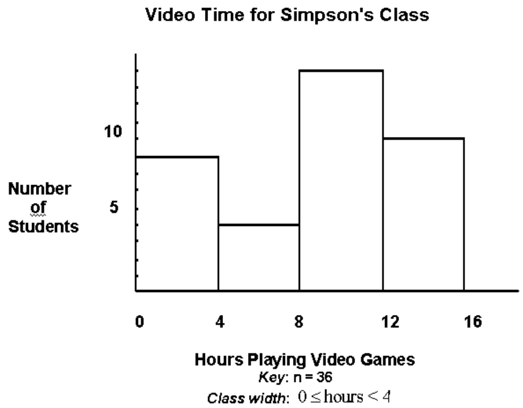
A dot plot consists of data points plotted on a simple scale. Dot plots are used for continuous, quantitative, *univariate* data. Data points may be labelled if there are few of them. The horizontal axis is a number line that displays the data in *equal intervals*. The frequency of each bar is shown by the number of dots on the vertical axis. Example: This dot plot shows how many hours students exercise each week. Fifteen students were asked how many hours they exercise in one week.



To create a dot plot, draw and number line, then draw one dot above the number line to represent each value in the data set.

Histograms

A **histogram** is a frequency distribution for continuous, quantitative, univariate data. The horizontal axis is a number line that displays the data in equal intervals. The frequency of each bar is shown on the vertical axis.
Example: This histogram shows the number of students in Simpson's class that are in each interval. The students were asked how many hours they spent playing video games in one week.

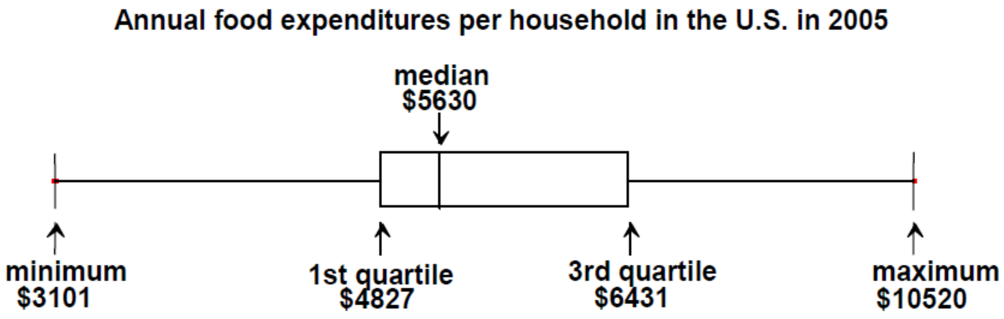


To create a histogram, first complete a frequency table to show the number of values in intervals of *equal* size. Then draw a number line with equal intervals. Then, plot the frequency for each interval on the vertical axis.

Interval	Tally	Frequency
0-4	~~IIII~~ III	8
4.01-8	IIII	4
8.01-12	~~IIII~~ ~~IIII~~ III	13
12.01-16	~~IIII~~ ~~IIII~~ I	11

Box Plots

A **box plot**, also known as a **box and whiskers chart**, is a visual display of a set of data showing the five number summary: minimum, first quartile, median, third quartile, and maximum. A **box plot** shows the range of scores within *each quarter* of the data. It is useful for examining the variation in a set of data and comparing the variation of more than one set of data.
Example:



To create a box plot, use one-variable stats in a graphing calculator and plot the minimum, Q1, Q2, Q3, and maximum values on *a number line*. Draw boxes around the middle two quartiles. Connect the boxes to the minimum and maximum using lines.

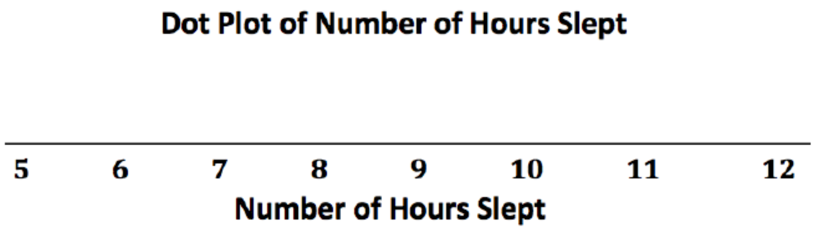
DEVELOPING ESSENTIAL SKILLS

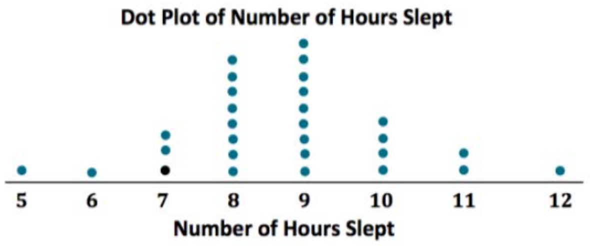
1. Create a dot plot to represent the following information.

   Robert, a sixth grader at Roosevelt Middle School, usually goes to bed around 10:00 p.m. and gets up around 6:00 a.m. to get ready for school. That means he gets about **8** hours of sleep on a school night. He decided to investigate the statistical question: How many hours per night do sixth graders usually sleep when they have school the next day?
   Robert took a survey of **29** sixth graders and collected the following data to answer the question.

   7 8 5 9 9 9 7 7 10 10 11 9 8 8 8 12 6 11 10 8 8 9 9 9 8 10 9 9 8

   Robert decided to make a dot plot of the data to help him answer his statistical question. Robert first drew a number line and labeled it from **5** to **12** to match the lowest and highest number of hours slept. Robert's datum is not included.





SOURCE: Engage New York

2. Create a histogram to represent the following data table.

   The Fahrenheit temperature readings on 30 April mornings in Stormville, New York, are shown below.

41°, 58°, 61°, 54°, 49°, 46°, 52°, 58°, 67°, 43°,
47°, 60°, 52°, 58°, 48°, 44°, 59°, 66°, 62°, 55°,
44°, 49°, 62°, 61°, 59°, 54°, 57°, 58°, 63°, 60°

Using the data, complete the frequency table below.

Interval	Tally	Frequency
40–44		
45–49		
50–54		
55–59		
60–64		
65–69		

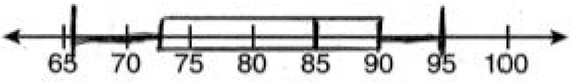
On the grid below, construct and label a frequency histogram based on the table.

3. Create a box plot to represent the following data.

The test scores from Mrs. Gray's math class are shown below.

72, 73, 66, 71, 82, 85, 95, 85, 86, 89, 91, 92

Construct a box-and-whisker plot to display these data.



REGENTS EXAM QUESTIONS

S.ID.A.l: Frequency Histograms, Box Plots and Dot Plots

18) The heights, in feet, of former New York Knicks basketball players are listed below.

6.4 6.9 6.3 6.2 6.3 6.0 6.1 6.3 6.8 6.2
6.5 7.1 6.4 6.3 6.5 6.5 6.4 7.0 6.4 6.3
6.2 6.3 7.0 6.4 6.5 6.5 6.5 6.0 6.2

Using the heights given, complete the frequency table below.

Interval	**Frequency**
6.0-6.1	
6.2-6.3	
6.4-6.5	
6.6-6.7	
6.8-6.9	
7.0-7.1	

Based on the frequency table created, draw and label a frequency histogram on the grid below.

Determine and state which interval contains the upper quartile. Justify your response.

19) Robin collected data on the number of hours she watched television on Sunday through Thursday nights for a period of 3 weeks. The data are shown in the table below.

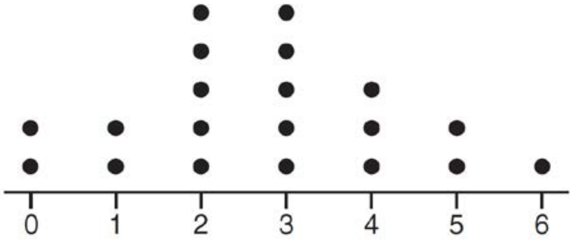
	Sun	Mon	Tues	Wed	Thurs
Week 1	4	3	3.5	2	2
Week 2	4.5	5	2.5	3	1.5
Week 3	4	3	1	1.5	2.5

Using an appropriate scale on the number line below, construct a box plot for the 15 values.

20) Which statistic can not be determined from a box plot representing the scores on a math test in Mrs. DeRidder's algebra class?

1) the lowest score	3) the highest score
2) the median score	4) the score that occurs most frequently

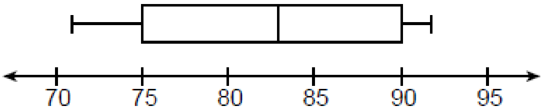
21) The dot plot shown below represents the number of pets owned by students in a class.



Which statement about the data is *not* true?

1) The median is 3.	3) The mean is 3.
2) The interquartile range is 2.	4) The data contain no outliers.

22) The box plot below summarizes the data for the average monthly high temperatures in degrees Fahrenheit for Orlando, Florida.

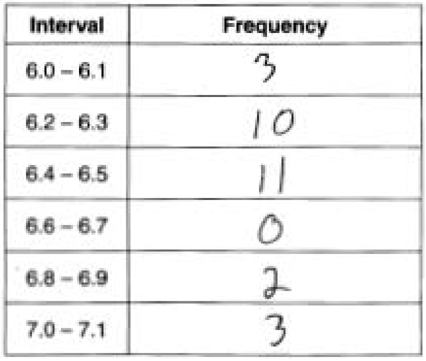


The third quartile is

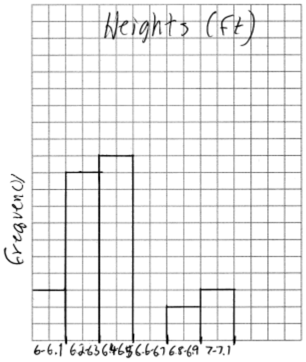
1) 92	3) 83
2) 90	4) 71

<u>SOLUTIONS</u>

18) ANS:



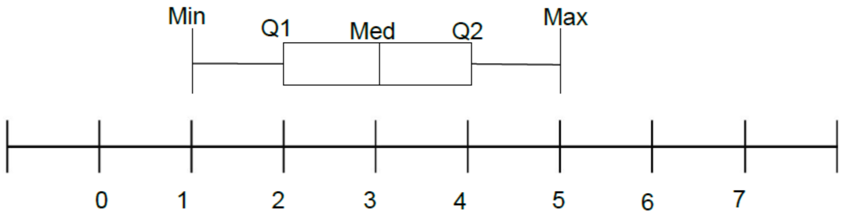
Interval	Frequency
6.0 – 6.1	3
6.2 – 6.3	10
6.4 – 6.5	11
6.6 – 6.7	0
6.8 – 6.9	2
7.0 – 7.1	3



Each quartile contains $\frac{1}{4}$ or 25% of the data values. There are a total of 29 data values in the data set, so each quartile will contain $\frac{29}{4} = 7.25$ values. The upper quartile will begin 7.25 values from the maximum, which places the upper quartile in the 6.4-6.5 interval.

PTS: 4 NAT: S.ID.A.1 TOP: Frequency Histograms
KEY: frequency histograms

19) ANS:



Strategy #1: Input all the numbers from the table in a TI 83+ graphing calculator, then calculate 1 variable stats, then use the calculator output to construct the box and whiskers plot.

Strategy #2 Follow these step-by-step procedures for creating a box and whiskers plot.

STEP 1. Organize the data set in ascending order, as follows. Be sure to include all the data.:
1, 1.5, 1.5, 2, 2, 2.5, 2.5, 3, 3, 3, 3.5, 4, 4, 4.5, 5

STEP 2. Plot a scale on the number line. In this case, the scale is 0 to five in equal intervals of .5 units.

STEP 3. Plot the minimum and maximum values: minimum = 1 and maximum = 2.

STEP 4. Identify the median. In this problem, there are fifteen numbers and the median is the middle number, which is 3. There are seven numbers to the left of 3 and seven numbers to the right of 3.

STEP 5. Plot and label the median = 3 (also known as Q2 or the second quartile).

STEP 6. Identify Q1, which is the *median of the bottom half* of the organized data set. The bottom half of the data includes all numbers below the median, which in this problem, includes the following numbers
1, 1.5, 1.5, 2, 2, 2.5, 2.5
The middle number in an organized list of seven numbers is the fourth number, which in this case is a 2.

STEP 7. Plot and label Q1 = 2.

STEP 8. Identify Q3, which is the *median of the top half* of the organized data set. The top half of the data includes all numbers above the median, which in this problem, includes the following numbers

3, 3, 3.5, 4, 4, 4.5, 5

Again, the middle number in an organized list of seven numbers is the fourth number, which in this case is a 4.

STEP 9. Plot and label Q3 = 4.

STEP 10. Finish the box plot by drawing boxes between the plotted points for Q1, Q2, and Q3.

PTS: 2 NAT: S.ID.A.1 TOP: Box Plots

20) ANS: 4

A box plot is also known as a box and whiskers chart and shows the following five statistics:

1. The minimum score.
2. Q1, which is the top of the first quartile.
3. Q2, which is also the median score and the top of the second quartile.
4. Q3, which is the top of the third quartile.
5. The maximum score.

The interquartile range can be determined by subtracting Q1 from Q2.

PTS: 2 NAT: S.ID.A.1

21) ANS: 3

Step 1. Understand that the problem is asking you to apply different statistical measures to the data in the dot plot and find the one answer choice that is not true.

Step 2. Strategy: Evaluate each answer choice and eliminate wrong answers.

Step 3. Execution of Strategy

a) To evaluate this answer choice, the median (middle) of the ordered data elements must be identified. There are 20 dots, so the middle is somewhere between the 10th and 11th dots. Counting 10 dots from either end, the median will occur in the 3 column. The median is 3, so answer a) must be eliminated.

b) To evaluate this answer, the interquartile range must be calculated. The interquartile range is defined as the distance between the first and third quartiles in an ordered distribution. The dot plot has 20 dots. Since each quartile contains 25% of the dots, each quartile will contain 25% of 20 dots, which equals 5 dots.

Q1 ends after five dots, so Q1=2.
Q2 ends after 10 dots, so Q2=10.
Q3 ends after 15 dots, so Q3=4.

The interquartile range is computed as Q3-Q2. In this dot plot, the interquartile range is 2, so answer b) is true and must be eliminated.

c) The mean for this data plot can be calculated as follows:

$$\overline{X} = \frac{0+0+1+1+2+2+2+2+2+3+3+3+3+3+4+4+4+5+5+6}{20}$$

$$\overline{X} = \frac{55}{20}$$

$$\overline{X} = 2.75$$

Answer c) is not true, because the mean of this data set is 2.75. Therefore, answer choice c) is the correct answer.

d) The data has no outliers. This is true by inspection. All the data is close together and there are no large gaps between the data. Hence, there are no outliers and choice d) must be eliminated.

Step 4. Does it make sense? Yes. Three answer choices have been shown to be true and one answer choice has been shown to be false. The statement that is not true is choice c).

$\text{median} = 3,\ \text{IQR} = 4 - 2 = 2,\ \bar{x} = 2.75.$ An outlier is outside the interval $[Q_1 - 1.5(\text{IQR}), Q_3 + 1.5(\text{IQR})]$.

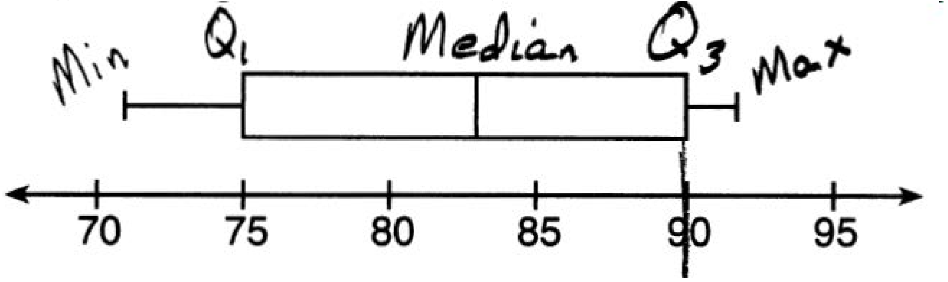
$[2 - 1.5(2), 4 + 1.5(2)]$

$[-1,7]$

PTS: 2 NAT: S.ID.A.1 TOP: Dot Plots

22) ANS: 2

Strategy: Label the five statisitcs shown by a box plot.



PTS: 2 NAT: S.ID.A.1 TOP: Box Plots KEY: interpret

23) ANS: 2

The number of pages a paper will have does not depend on how fast the student types.

PTS: 2 NAT: S.ID.C.9 TOP: Analysis of Data

24) ANS: 2

Strategy: Eliminate wrong answers.

Observe: Both variables (numer of pages and amount of ink) increase together, so the correlation is positve. Eliminate answer choices with negative correlation.

Reason: Printing causes ink to be used, so the relationship is causal. Eliminate answer choices with non-causal.

~~a)~~ positive correlation, but ~~not~~ causal
b) positive correlation, and causal
~~c)~~ ~~negative~~ correlation, but not causal
~~d)~~ ~~negative~~ correlation, and causal

PTS: 2 NAT: S.ID.C.9 TOP: Analysis of Data

GRAPHS AND STATISTICS
Analysis of Data

Common Core Standard	Next Generation Standard
S-ID.C.9 Distinguish between correlation and causation.	**AI-S.ID.9** Distinguish between correlation and causation.

LEARNING OBJECTIVES

Students will be able to:

1) Distinguish between correlation and causation in context.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

correlikely

Example of a Fallacy of Composition: Deep in the rain forest, a tribe of indigenous people live. Every year, when the days start getting longer, the shaman of the tribe does a rain dance. Soon, the spring rains come. The people of the village believe the shaman's dance caused the rain to come. Modern scientists would argue that the rains come every year because of the changing of the seasons, and the village peoples' belief is a **fallacy of composition** - the rains were not caused by the shaman's dance - they were only correlated with the timing of the dance. Such fallacies of composition can be difficult to identify, and it might be even more difficult to convince the village people that the rains are only correlated with, not caused by, the shaman's rain dance.

DEVELOPING ESSENTIAL SKILLS

Decide whether the relationships between events A and B are correlation or causation.

Event A	Causes Event B	Which is it?
I get in the bathtub.	The phone rings.	Correlation
Attendance at the baseball game goes up.	Ice cream sales increase.	Correlation
I wear these socks.	We win the soccer game.	Correlation
I stream more videos on my cell phone.	My cell phone bill goes up.	Causation
I eat more food.	My weight increases.	Uncertain
Mankind's influence on the environment.	Global warming.	Causation
I wash my car.	It rains.	Correlation
Smoking cigarettes.	Increased chances of getting lung cancer.	Causation
Junk food is sold in school to raise money.	Student obesity increases.	Uncertain
I get higher scores on exams.	My course grade increases.	Causation.
I do more homework.	My exam scores increase.	Correlation

REGENTS EXAM QUESTIONS

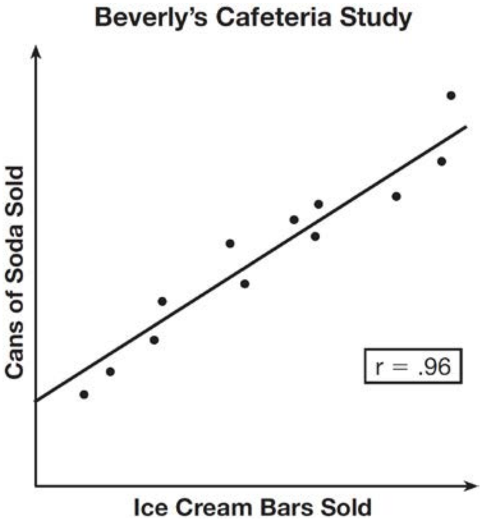
S.ID.C.9: Analysis of Data

23) Which situation does *not* describe a causal relationship?
   1) The higher the volume on a radio, the louder the sound will be.
   2) The faster a student types a research paper, the more pages the paper will have.
   3) The shorter the distance driven, the less gasoline that will be used.
   4) The slower the pace of a runner, the longer it will take the runner to finish the race.

24) What type of relationship exists between the number of pages printed on a printer and the amount of ink used by that printer?
   1) positive correlation, but not causal
   2) positive correlation, and causal
   3) negative correlation, but not causal

4) negative correlation, and causal

25) Beverly did a study this past spring using data she collected from a cafeteria. She recorded data weekly for ice cream sales and soda sales. Beverly found the line of best fit and the correlation coefficient, as shown in the diagram below.



Given this information, which statement(s) can correctly be concluded?
I. Eating more ice cream causes a person to become thirsty.
II. Drinking more soda causes a person to become hungry.
III. There is a strong correlation between ice cream sales and soda sales.
1) I, only
2) III, only
3) I and III
4) II and III

SOLUTIONS

23) ANS: 2
The number of pages a paper will have does not depend on how fast the student types.

PTS: 2 NAT: S.ID.C.9 TOP: Analysis of Data

24) ANS: 2
Strategy: Eliminate wrong answers.
Observe: Both variables (numer of pages and amount of ink) increase together, so the correlation is positve. Eliminate answer choices with negative correlation.
Reason: Printing causes ink to be used, so the relationship is causal. Eliminate answer choices with non-causal.
~~a)~~ positive correlation, but~~ not ~~causal
b) positive correlation, and causal
~~c)~~ ~~negative~~ correlation, but not causal
~~d)~~ ~~negative~~ correlation, and causal

PTS: 2 NAT: S.ID.C.9 TOP: Analysis of Data

25) ANS: 2
Strategy: Determine the truth value of each statement, then determine which of the four answer choices best matches the truth values of the three statements.

STEP 1. Determine the truth values of each statement:

Statement I is **false**. Eating more ice cream **does not necessarily cause** a person to become thirsty.

Statement II is **false**. Drinking more soda **does not necessarily cause** a person to become hungry.

Statement III is **true**. **There is a strong correlation** between ice cream sales and soda sales.

STEP 2. Use knowledge of correlation and causation to select the correct answer.

Statement III is the only statement than can be correctly concluded. The answer is choice b.

PTS: 2 NAT: S.ID.C.9 TOP: Analysis of Data

GRAPHS AND STATISTICS

Regression

Common Core Standard	**Next Generation Standard**
S-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	**AI-S.ID.6** Represent bivariate data on a scatter plot, and describe how the variables' values are related. **Note: It's important to keep in mind that the data must be linked to the same "subjects," not just two unrelated quantitative variables; being careful not to assume a relationship between the actual variables (correlation/causation issue).**
S-ID.B.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. PARCC: Tasks have real world context. Exponential functions are limited to those with domains in the integers. NYSED: Includes the regression capabilities of the calculator.	**AI-S.ID.6a** Fit a function to real-world data; use functions fitted to data to solve problems in the context of the data. (Shared standard with Algebra II) Note: **Algebra I emphasis is on linear models** and includes the regression capabilities of the calculator.

LEARNING OBJECTIVES

Students will be able to:

1) Draw an approximate line of best fit through a scatterplot.
2) Use a graphing calculator to find the equation of the line of best fit for a given set of data.
3) Make a prediction using a linear regression equation.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

Regression
Line of Best Fit
Scatterplot
Data Cloud

BIG IDEAS

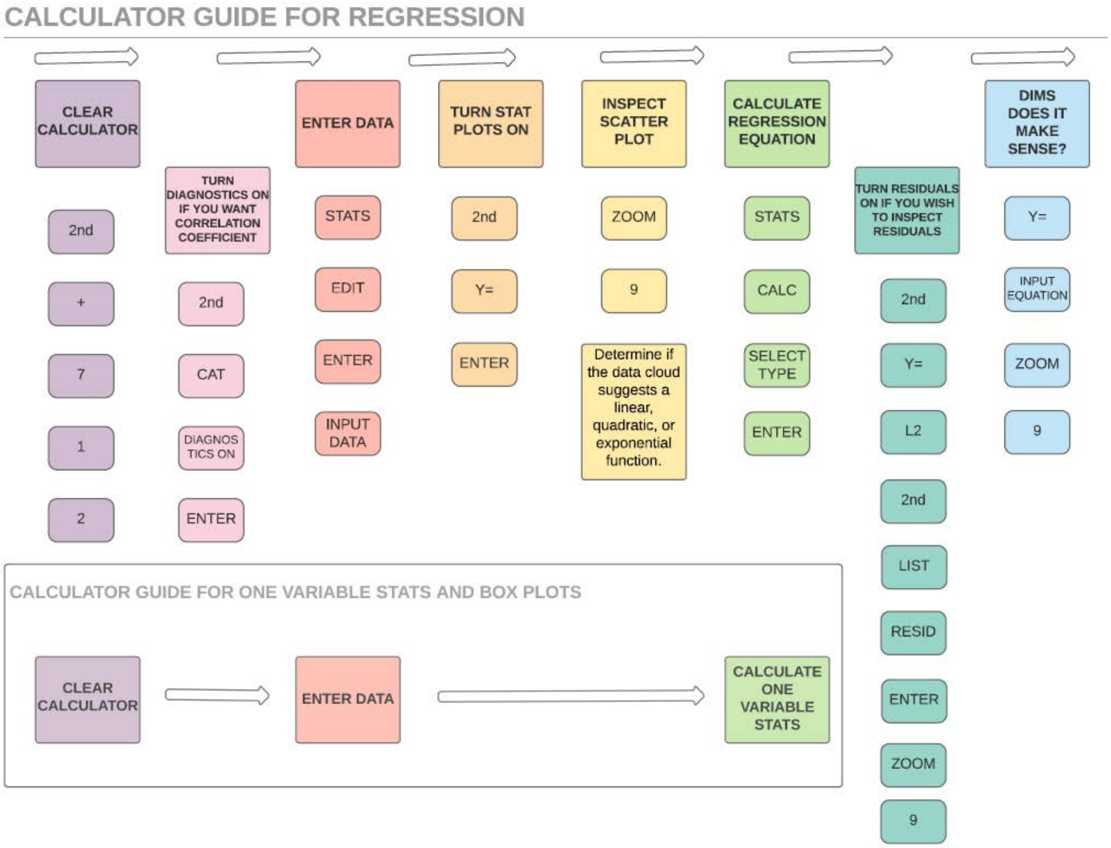
Regression Model: A function (e.g., linear, exponential, power, logarithmic) that fits a set of paired data. T*he model may enable other values of the dependent variable to be predicted.*

Big Ideas

The **individual data points** in a **scatterplot** form **data clouds** with shapes that suggest relationships between dependent and independent variables.

A **line of best fit** divides the data cloud into two equal parts with about the same number of data points on each side of the line. A line of best fit can be a straight line or a curved line, depending on the shape of the data cloud.

Overview of Regression Using TI 83/83 Family of Graphing Calculators

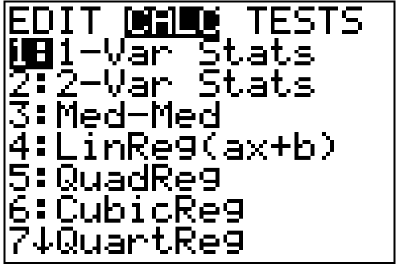


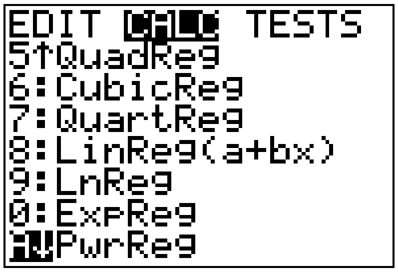
Calculating Regression Equations. Technology is almost always used to calculate regression equations. .

- **STEP 1**. Use STATS EDIT to Input the data into a graphing calculator.
- **STEP 2**. Use 2nd STAT PLOT to turn on a data set, then ZOOM 9 to inspect the graph of the data and determine which regression strategy will best fit the data.
- **STEP 3**. Use STAT CALC and the appropriate regression type to obtain the regression equation.
- **STEP 4**. Ask the question, "Does it Make Sense (DIMS)?"

DIFFERENT TYPES OF REGRESSION

The graphing calculator can calculate numerous types of regression equations, but it must be told which type to calculate. All of the calculator procedures described above can be used with various types of regression. The following screenshots show some of the many regressions that can be calculated on the TI-83/84 family of graphing calculators.





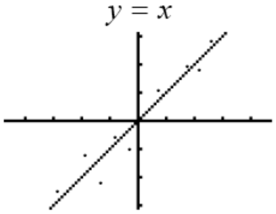
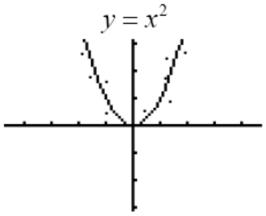
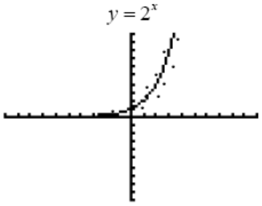
The general purpose of linear regression is to make predictions based on a line of best fit.

Choosing the Correct Type of Regression to Calculate

There are two general approaches to determining the type of regression to calculate:

- The decision of which type of regression to calculate can be made based on visual examination of the data cloud, or.
- On Regents examinations, the wording of the problem often specifies a particular type of regression to be used.

Using the Data Cloud to Select the Correct Regression Calculation Program

 y = x If the data cloud takes the general form of a straight line, use **linear regression**.	 $y = x^2$ If the data cloud takes the general form of a parabola, use **quadratic regression**.	 $y = 2^x$ If the data cloud takes the general form of an exponential curve, use **exponential regression**.
Note: The general forms of some data clouds are difficult to interpret. In difficult to interpret cases, the strength of the correlation coefficient can be used to determine which type of regression best fits the data. See lesson for standard S.ID.C.8,		

Drawing a Line of Best Fit on a Scatterplot

A line of best fit may be drawn on a scatterplot of data by using values from the regression equation.
STEP 1. Input the regression equation in the y-editor of a graphing calculator
STEP2. Use ordered pairs of coordinates from the table of values to plot the line of best fit.

- In linear regression, the line of best fit will always go through the point $\left(\bar{x},\bar{y}\right)$, where $\bar{x}$ is the mean of all values of *x*, and $\bar{y}$ is the mean of all values of *y*. For example, the line of best fit for a scatterplot with points (2,5), (4,7) and (8,11) must include the point $\left(x=\frac{14}{3},y=\frac{23}{3}\right)$, because these *x* and *y* values are the averages of all the x-values and all the y-values.

- If the regression equation is linear and in $y = mx + b$ form, the y-intercept and slope can be used to plot the line of best fit.

Making Predictions Based on a Line of Best Fit

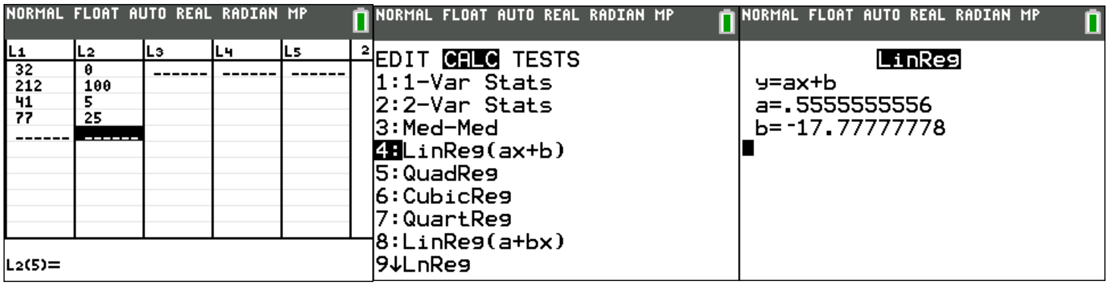
Predictions may be made based on a line of best fit.
STEP 1. Input the regression equation in the y-editor of a graphing calculator
STEP2. Use ordered pairs of coordinates from the table of values to identify expected values of the dependent (y) variable for any desired value of the independent (x) variable.

DEVELOPING ESSENTIAL SKILLS – Class Assignment

Nazmun and Daniel came to America from two different parts of the world. Nazmun measures temperature in degrees Celsius, while Daniel measures temperature in degrees Fahrenheit. They want to understand the relationship between these two different ways of measuring temperature. They each know the temperature when water freezes, when water boils, the temperature outside today, and the temperature inside their very warm classroom. They record these temperatures in the following table.

Comparison Table	Fahrenheit Degrees	Celsius Degrees
Water Freezes	32	0
Water Boils	212	100
Today's Outdoor Temperature	41	5
Temperature in Classroom	77	25

Use linear regression to find the mathematical relationship between degrees Fahrenheit and degrees Celsius. Then, use your regression equation to add three new rows to the comparison table.



```
L1     L2     L3     L4     L5     2
32     0      ------ ------ ------
212    100
41     5
77     25
------ ------

L2(5)=
```

```
LinReg
y=ax+b
a=.5555555556
b=⁻17.77777778
```

The linear regression equation is

$$y = .\overline{55}x - 17.\overline{77}$$
$$C = .\overline{55}F - 17.\overline{77}$$

.

This regression equation can be transformed to a more familiar formula as follows:

$$C = .\overline{55}F - 17.\overline{77}$$

$$C = \frac{5}{9}F - 17\frac{7}{9}$$

$$C = \frac{5}{9}F - \frac{160}{9}$$

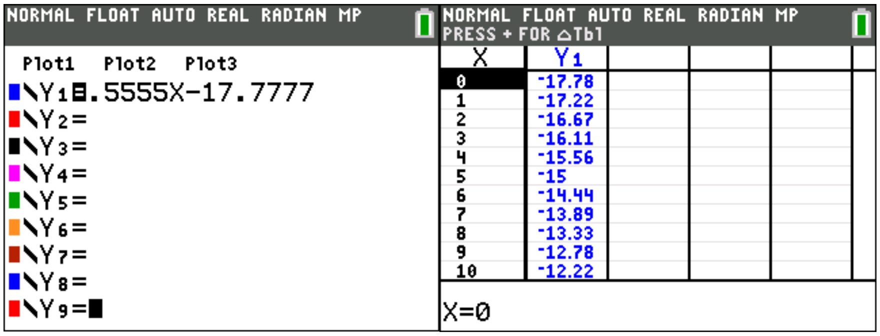
$$C = \frac{5}{9}F - \left(\frac{5}{9}\right)\left(\frac{\cancel{9}}{5}\right)\frac{160}{\cancel{9}}$$

$$C = \frac{5}{9}F - \left(\frac{5}{9}\right)\frac{160}{5}$$

$$C = \frac{5}{9}F - \left(\frac{5}{9}\right)32$$

$$C = \frac{5}{9}(F - 32)$$

To add three new rows to the comparison table, input the regression formula into the y-editor of a graphing calculator and use the table of values.



```
NORMAL FLOAT AUTO REAL RADIAN MP

 Plot1  Plot2  Plot3
▪\Y1=.5555X−17.7777
▪\Y2=
▪\Y3=
▪\Y4=
▪\Y5=
▪\Y6=
▪\Y7=
▪\Y8=
▪\Y9=█
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl
```

X	Y1
0	-17.78
1	-17.22
2	-16.67
3	-16.11
4	-15.56
5	-15
6	-14.44
7	-13.89
8	-13.33
9	-12.78
10	-12.22

X=0

REGENTS EXAM QUESTIONS (through June 2018)

S.ID.B.6: Regression

26) Emma recently purchased a new car. She decided to keep track of how many gallons of gas she used on five of her business trips. The results are shown in the table below.

Miles Driven	Number of Gallons Used
150	7
200	10
400	19
600	29
1000	51

Write the linear regression equation for these data where miles driven is the independent variable. (Round all values to the *nearest hundredth*.)

27) About a year ago, Joey watched an online video of a band and noticed that it had been viewed only 843 times. One month later, Joey noticed that the band’s video had 1708 views. Joey made the table below to keep track of the cumulative number of views the video was getting online.

Months Since First Viewing	Total Views
0	843
1	1708
2	forgot to record
3	7124
4	14,684
5	29,787
6	62,381

a) Write a regression equation that best models these data. Round all values to the *nearest hundredth*. Justify your choice of regression equation.

b) As shown in the table, Joey forgot to record the number of views after the second month. Use the equation from part *a* to estimate the number of full views of the online video that Joey forgot to record.

28) The table below shows the number of grams of carbohydrates, x, and the number of Calories, y, of six different foods.

Carbohydrates (x)	**Calories** (y)
8	120
9.5	138
10	147
6	88
7	108
4	62

Which equation best represents the line of best fit for this set of data?

1) $y = 15x$
2) $y = 0.07x$
3) $y = 0.1x - 0.4$
4) $y = 14.1x + 5.8$

29) An application developer released a new app to be downloaded. The table below gives the number of downloads for the first four weeks after the launch of the app.

Number of Weeks	1	2	3	4
Number of Downloads	120	180	270	405

Write an exponential equation that models these data. Use this model to predict how many downloads the developer would expect in the 26th week if this trend continues. Round your answer to the nearest download. Would it be reasonable to use this model to predict the number of downloads past one year? Explain your reasoning.

30) The data table below shows the median diameter of grains of sand and the slope of the beach for 9 naturally occuring ocean beaches.

Median Diameter of Grains of Sand in Millimeters (x)	0.17	0.19	0.22	0.235	0.235	0.3	0.35	0.42	0.85
Slope of Beach in Degrees (y)	0.63	0.7	0.82	0.88	1.15	1.5	4.4	7.3	11.3

Write the linear regression equation for this set of data, rounding all values to the *nearest thousandth*. Using this equation, predict the slope of a beach, to the *nearest tenth of a dregree*, on a beach with grains of sand having a median diameter of 0.65 mm.

31) Omar has a piece of rope. He ties a knot in the rope and measures the new length of the rope. He then repeats this process several times. Some of the data collected are listed in the table below.

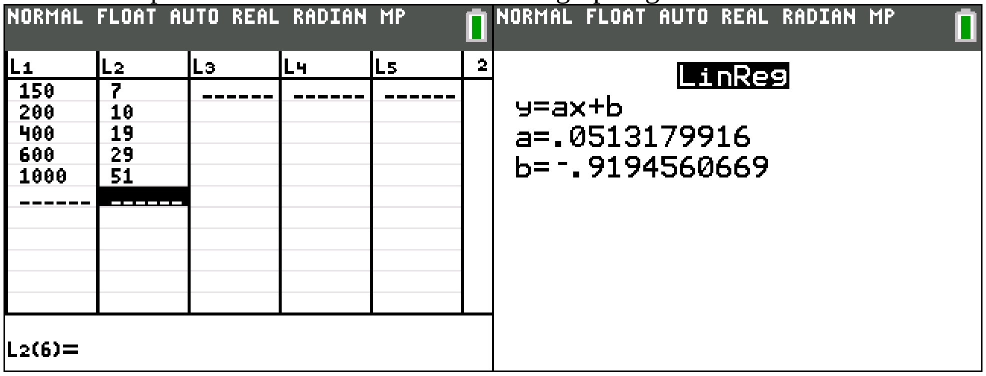
Number of Knots	4	5	6	7	8
Length of Rope (cm)	64	58	49	39	31

State, to the *nearest tenth*, the linear regression equation that approximates the length, y, of the rope after tying x knots. Explain what the y-intercept means in the context of the problem. Explain what the slope means in the context of the problem.

SOLUTIONS

26) ANS:

STEP 1: Input the data in the stats editor of a graphing calculator and calculate linear regression.



```
NORMAL FLOAT AUTO REAL RADIAN MP
L1      L2      L3      L4      L5      2
 150     7      ------  ------  ------
 200     10
 400     19
 600     29
 1000    51
 ------  ------

L2(6)=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
        LinReg
y=ax+b
a=.0513179916
b=⁻.9194560669
```

$$\boxed{y = 0.05x - 0.92}$$

PTS: 2 NAT: S.ID.B.6a TOP: Regression

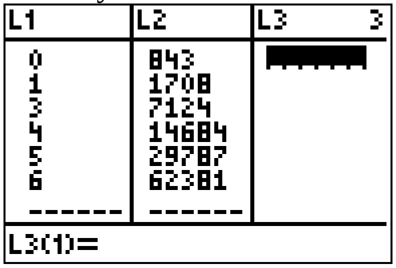
27) ANS:

Part a: $f(x) = 836.47(2.05)^x$ The data appear to grow at an exponential rate.

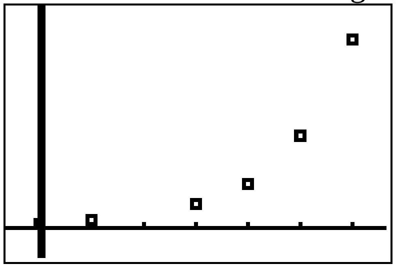
Part b: $f(2) = 836.47(2.05)^2 = 3515$

Strategy: Input the data into a graphing calculator, inspect the data cloud, and find a regression equation to model the data table, input the regression equation into the y-editor, predict the missing value.

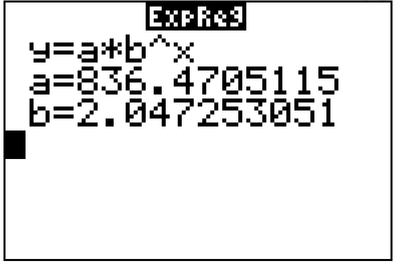
- **STEP 1**. Input the data into a graphing calculator or plot the data cloud on a graph, if necessary, so that you can look at the data cloud to see if it has a recognizable shape.



```
L1      L2      L3      3
 0       843
 1       1708
 3       7124
 4       14684
 5       29787
 6       62381
 ------  ------
L3(1)=
```



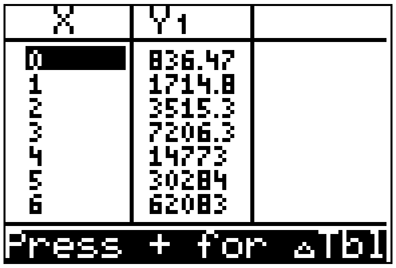
- **STEP 2**. Determine which regression strategy will best fit the data. The graph looks like the graph of an exponential function, so choose exponential regression.
- **STEP 3**. Execute the appropriate regression strategy in the graphing calculator.



```
ExpReg
y=a*b^x
a=836.4705115
b=2.047253051
```

Round all values to the nearest hundredth: $y = 836.47(2.05)^x$

- **STEP 4**. Input the regression equation into the y-editor feature of the graphing calculator and view the associated table of values to find the value of y when x equals 2.



```
X    Y1
0    836.47
1    1714.8
2    3515.3
3    7206.3
4    14773
5    30284
6    62083
Press + for ΔTbl
```

Round 3515.3 to 3515.

- **STEP 4**. In

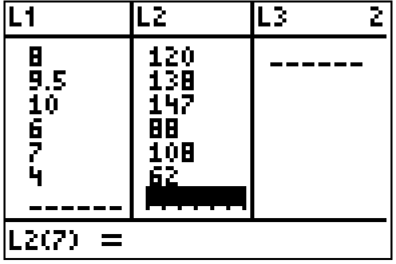
Ask the question, “Does it Make Sense (DIMS)?” that the missing total number of views in month 2 would be around 3515 views?

PTS: 4 NAT: S.ID.B.6a TOP: Regression

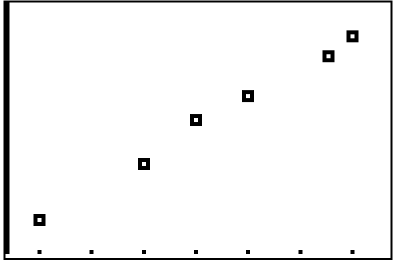
28) ANS: 4

Strategy: Input the data into a graphing calculator, inspect the data cloud, and find a regression equation to model the data table, input the regression equation into the y-editor, predict the missing value.

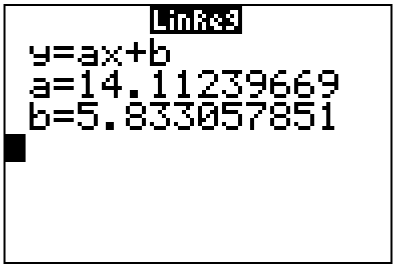
- **STEP 1**. Input the data into a graphing calculator or plot the data cloud on a graph, if necessary, so that you can look at the data cloud to see if it has a recognizable shape.



```
L1      L2      L3     2
8       120     ------
9.5     138
10      147
6       88
7       108
4       62
------
L2(7) =
```



- **STEP 2**. Determine which regression strategy will best fit the data. The graph looks like the graph of an linear function, so choose linear regression.
- **STEP 3**. Execute the appropriate regression strategy in the graphing calculator.



```
LinReg
y=ax+b
a=14.11239669
b=5.833057851
```

Write the regression equation in a format that can be compared to the answer choices: $y = 14.11x + 5.83$

- **STEP 4**. Compare the answer choices to the regression equation and select choice d.

PTS: 2 NAT: S.ID.B.6a TOP: Regression

29) ANS:

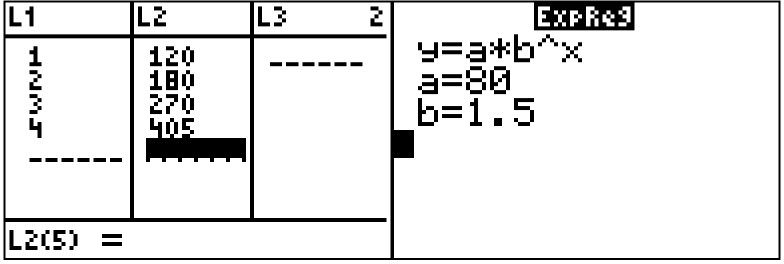
a) $y = 80(1.5)^x$

b) $80(1.5)^{26} \approx 3,030,140.$

c) No, because the prediction at $x = 52$ is already too large.

Strategy: Use data from the table and exponential regression in a graphing calculator.

STEP 1: Model the function in a graphing calculator using exponential regression.

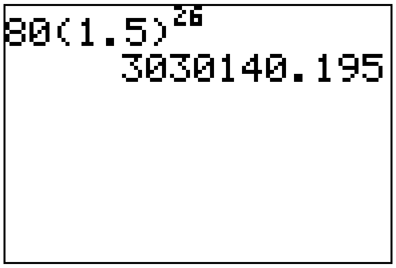


```
L1      L2      L3     2
1       120     ------
2       180
3       270
4       405
------  ------
L2(5) =
```

```
ExpReg
y=a*b^x
a=80
b=1.5
```

The exponential regression equation is $y = 80(1.5)^x$

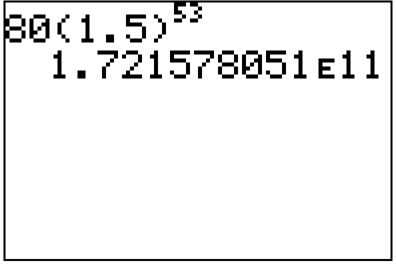
STEP 2. Use the equation to predict the number of downloads when $x = 26$.



```
80(1.5)^26
         3030140.195
```

Rounded to the nearest download, the answer is 3,030,140.

STEP 3. Determine if it would be reasonable to use the model to predict downloads past one year.



```
80(1.5)^53
   1.721578051E11
```

It would not be reasonable to use this model to make predictions past one year. The number of predicted downloads is more 170 billion downloads, which is more than 20 downloads in one week for every person in the world.

DIMS? Does It Make Sense? For near term predicitons, yes. For long term predictions, no.

PTS: 4 NAT: S.ID.B.6a TOP: Regression NOT: NYSED classifies this as A.CED.A.2

30) ANS:

$y = 17.159x - 2.476$

$y = 17.159(0.65)x - 2.476$

$y \approx 8.7$

y = 17.159x ? . 2.476. y = 17.159(.65) ? . 2.476 ? . 8.7

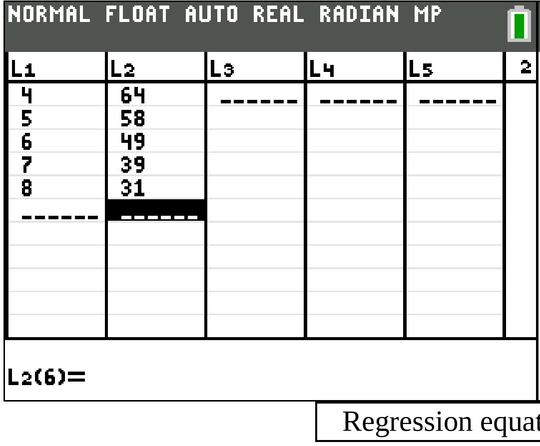
Strategy: Input the table of values into the stats-editor of a graphing calculator, then use the stats-calc-linear regression with “diagnostics on” to obtain the linear regression equation, then use the linear regression equation to calculate the value of y when $x = 0.65$.

PTS: 4 NAT: S.ID.B.6

31) ANS:

STEP 1.

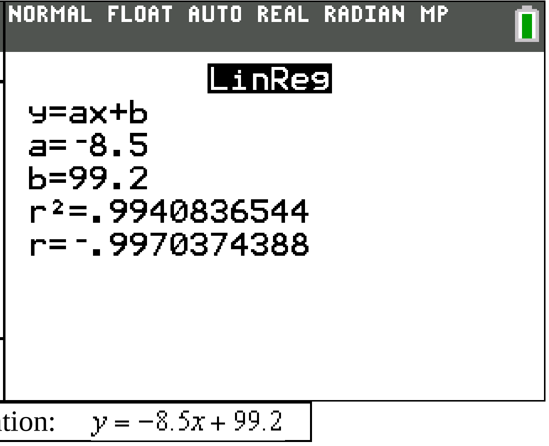
Input values from the table in the stats editor of a graphing calculator, then calculate linear regression.



```
NORMAL FLOAT AUTO REAL RADIAN MP
```

L1	L2	L3	L4	L5	2
4	64	------	------	------	
5	58				
6	49				
7	39				
8	31				
------	------				

```
L2(6)=
```



```
NORMAL FLOAT AUTO REAL RADIAN MP
LinReg
y=ax+b
a=⁻8.5
b=99.2
r²=.9940836544
r=⁻.9970374388
```

Regression equation: $y = -8.5x + 99.2$

STEP 2. Explain what the y-intercept means in this equation.

When there are no knots, the rope is 99.2 cm. long.

STEP 3. Explain what the slope means in the context of this problem.

The slope is -8.5 and means there is a negative relationship between the number of knots and the length of the rope. Each knot makes the rope 8.5 cm. shorter.

$y = -8.5x + 99.2$ The y-intercept represents the length of the rope without knots. The slope represents the decrease in the length of the rope for each knot.

PTS: 4 NAT: S.ID.B.6 TOP: Regression KEY: linear

B – Graphs and Statistics, Lesson 6, Correlation Coefficient (r. 2018)

GRAPHS AND STATISTICS

Correlation Coefficient

Common Core Standard	Next Generation Standard
S-ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.	**AI-S.ID.8** Calculate (using technology) and interpret the correlation coefficient of a linear fit.

LEARNING OBJECTIVES

Students will be able to:

1) Calculate the correlation coefficient of a linear fit.
2) Interpret the meaning of a correlation coefficient.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

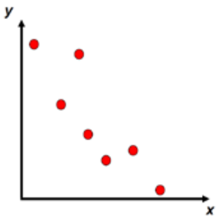
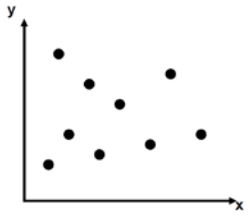
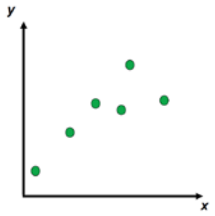
VOCABULARY

correlation coefficient A number between -1 and 1 that indicates the strength and direction of the linear relationship between two sets of numbers. The letter "r" is used to represent correlation coefficients. In all cases, $-1 \le r \le 1$.

BIG IDEAS

SIGNS OF CORRELATIONS

The **sign of the correlation** tells you if two variables increase or decrease together (positive); or if one variable increase when the other variable decreases (negative). The **sign of the correlation** also provides a general idea of what the graph will look like.

 **Negative Correlation** In general, one set of data decreases as the other set increases. An example of a negative correlation between two variables would be the relationship between absentiism from school and class grades. As one variable increases, the other would be expected to decrease.	 **No Correlation** Sometimes data sets are not related and there is no general trend. A correlation of zero does not always mean that there is no relationship between the variables. It could mean that the relationship is not **linear**. For example, the correlation between points on a circle or a regular polygon would be zero or very close to zero, but the points are very predictably related.	 **Positive Correlation** In general, both sets of data increase together. An example of a positive correlation between two variables would be the relationship between studying for an examination and class grades. As one variable increases, the other would also be expected to increase.

STRENGTH OF CORRELATIONS

	-1	Perfect	+1	
N	-.9		.9	P
E	-.8	Strong	.8	O
G	-.7		.7	S
A	-.6		.6	I
T	-.5	Moderate	.5	T
I	-.4		.4	I
V	-.3		.3	V
E	-.2	Weak	.2	E
	-.1		.1	
	0	None		

- The closer the absolute value of the correlation is to 1, the stronger the correlation between the variables.
- The closer the absolute value of the correlation is to zero, the weaker the correlation between the variables.
- In a perfect correlation, when $r = \pm 1$, all data points balance the equations and also lie on the graph of the equation.

How to Calculate a Correlation Coefficient Using a Graphing Calculator:

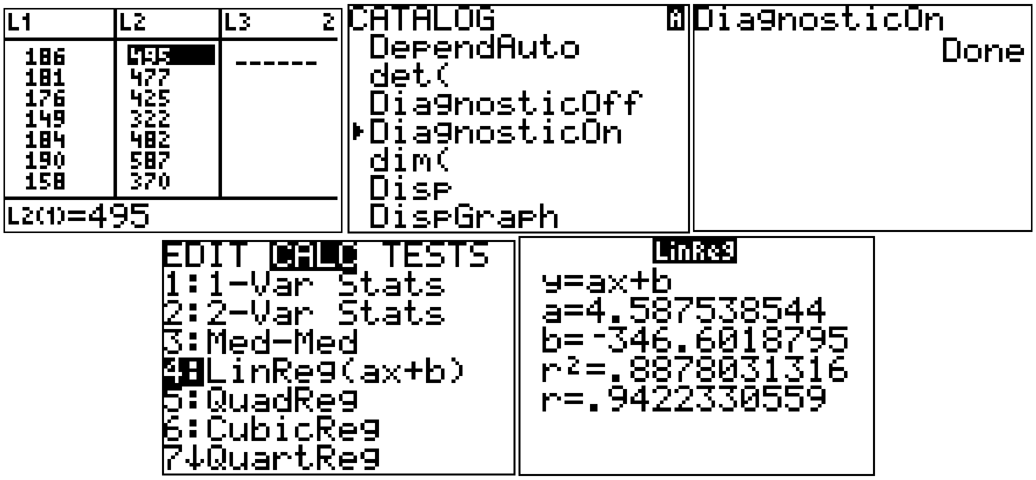
STEP 1. Press STAT EDIT 1:Edit .

STEP 2. Enter bivariate data in the L1 and L2 columns. All the x-values go into L1 column and all the Y values go into L2 column. Press ENTER after every data entry.

STEP 3. Turn the diagnostics on by pressing 2ND CATALOG and scrolling down to DiagnosticsOn . Then, press ENTER ENTER . The screen should respond with the message Done . NOTE: If Diagnostics are turned *off,* the correlation coefficient will not appear beneath the regression equation.

Step 4. Press STAT CALC 4:4-LinReg (ax+b) ENTER ENTER

Step 5. The *r* value that appears at the bottom of the screen is the correlation coefficient.



```
L1     L2     L3     2
186    495    -------
181    477
176    425
149    322
184    482
190    587
158    370
L2(1)=495
```

```
CATALOG
 DependAuto
 det(
 DiagnosticOff
▸DiagnosticOn
 dim(
 Disp
 DispGraph
```

```
DiagnosticOn
            Done
```

```
EDIT CALC TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7↓QuartReg
```

```
LinReg
y=ax+b
a=4.587538544
b=⁻346.6018795
r²=.8878031316
r=.9422330559
```

DEVELOPING ESSENTIAL SKILLS

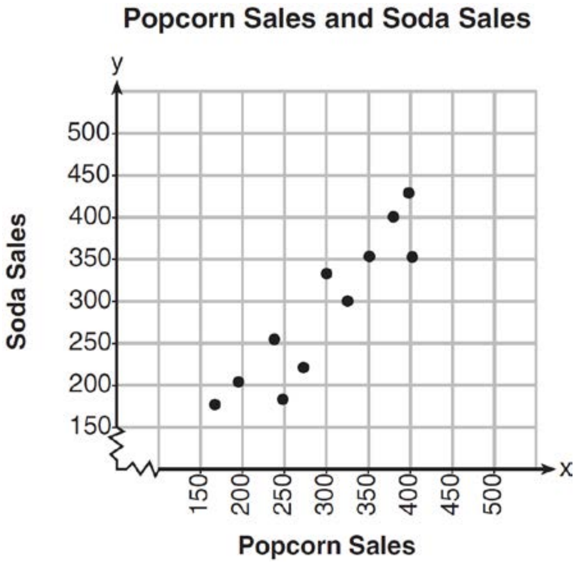
Interpret the following correlation coefficients:

Correlation Coefficient	Interpretation (must include strength and direction)
r = .5	Moderate Positive
r = -.6	Moderate Negative
r = -1	Strong Negative (Perfect)
r = .7	Strong Positive
r = -.9	Strong Negative
r = .0	No Correlation
r = .2	Weak Positive

REGENTS EXAM QUESTIONS (through June 2018)

S.ID.C.8: Correlation Coefficients

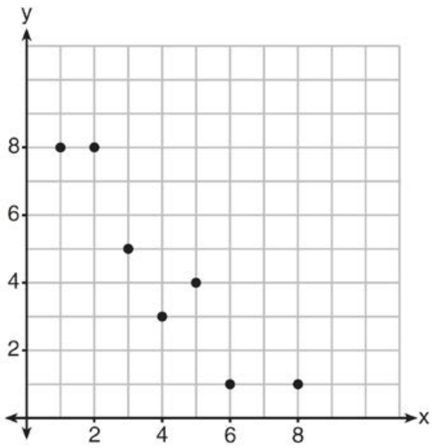
32) The scatterplot below compares the number of bags of popcorn and the number of sodas sold at each performance of the circus over one week.



Which conclusion can be drawn from the scatterplot?

1) There is a negative correlation between popcorn sales and soda sales.
2) There is a positive correlation between popcorn sales and soda sales.
3) There is no correlation between popcorn sales and soda sales.
4) Buying popcorn causes people to buy soda.

33) What is the correlation coefficient of the linear fit of the data shown below, to the *nearest hundredth*?



1) 1.00
2) 0.93
3) −0.93
4) −1.00

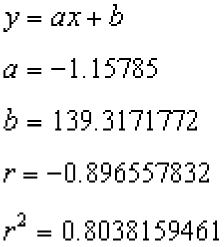
34) Analysis of data from a statistical study shows a linear relationship in the data with a correlation coefficient of -0.524. Which statement best summarizes this result?

1) There is a strong positive correlation between the variables.
2) There is a strong negative correlation between the variables.
3) There is a moderate positive correlation between the variables.
4) There is a moderate negative correlation between the variables.

35) Bella recorded data and used her graphing calculator to find the equation for the line of best fit. She then used the correlation coefficient to determine the strength of the linear fit. Which correlation coefficient represents the strongest linear relationship?

1) 0.9
2) 0.5
3) -0.3
4) -0.8

36) The results of a linear regression are shown below.



$y = ax + b$

$a = -1.15785$

$b = 139.3171772$

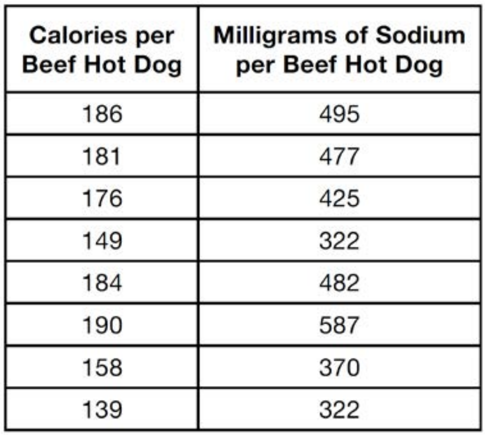
$r = -0.896557832$

$r^2 = 0.8038159461$

Which phrase best describes the relationship between x and y?

1) strong negative correlation
2) strong positive correlation
3) weak negative correlation
4) weak positive correlation

37) A nutritionist collected information about different brands of beef hot dogs. She made a table showing the number of Calories and the amount of sodium in each hot dog.



Calories per Beef Hot Dog	Milligrams of Sodium per Beef Hot Dog
186	495
181	477
176	425
149	322
184	482
190	587
158	370
139	322

a) Write the correlation coefficient for the line of best fit. Round your answer to the *nearest hundredth.*
b) Explain what the correlation coefficient suggests in the context of this problem.

38) The table below shows 6 students' overall averages and their averages in their math class.

Overall Student Average	92	98	84	80	75	82
Math Class Average	91	95	85	85	75	78

If a linear model is applied to these data, which statement best describes the correlation coefficient?

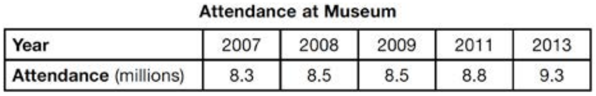
1) It is close to -1.
2) It is close to 1
3) It is close to 0.
4) It is close to 0.5.

39) At Mountain Lakes High School, the mathematics and physics scores of nine students were compared as shown in the table below.

Mathematics	55	93	89	60	90	45	64	76	89
Physics	66	89	94	52	84	56	66	73	92

State the correlation coefficient, to the *nearest hundredth*, for the line of best fit for these data. Explain what the correlation coefficient means with regard to the context of this situation.

40) The table below shows the attendance at a museum in select years from 2007 to 2013.



Attendance at Museum

Year	2007	2008	2009	2011	2013
Attendance (millions)	8.3	8.5	8.5	8.8	9.3

State the linear regression equation represented by the data table when $x = 0$ is used to represent the year 2007 and y is used to represent the attendance. Round all values to the *nearest hundredth*. State the correlation coefficient to the *nearest hundredth* and determine whether the data suggest a strong or weak association.

41) Erica, the manager at Stellarbeans, collected data on the daily high temperature and revenue from coffee sales. Data from nine days this past fall are shown in the table below.

	Day 1	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 6**	**Day 7**	**Day 8**	**Day 9**
High Temperature, t	54	50	62	67	70	58	52	46	48
Coffee Sales, f(t)	\$2900	\$3080	\$2500	\$2380	\$2200	\$2700	\$3000	\$3620	\$3720

State the linear regression function, $f(t)$, that estimates the day's coffee sales with a high temperature of t. Round all values to the *nearest integer*.

State the correlation coefficient, r, of the data to the *nearest hundredth*. Does r indicate a strong linear relationship between the variables? Explain your reasoning.

42) The percentage of students scoring 85 or better on a mathematics final exam and an English final exam during a recent school year for seven schools is shown in the table below.

Percentage of Students Scoring 85 or Better	
Mathematics, x	**English, y**
27	46
12	28
13	45
10	34
30	56
45	67
20	42

Write the linear regression equation for these data, rounding all values to the *nearest hundredth*. State the correlation coefficient of the linear regression equation, to the *nearest hundredth*. Explain the meaning of this value in the context of these data.

SOLUTIONS

32) ANS: 2
Strategy: Eliminate wrong answers.

a) Eliminate choice (1) because a negative correlation is a relationship where the dependent (*y*) values decrease as independent (*x*) values increase. A graph showing negative correlation would go down from left to right. The graph in this problem does not go down from left to right.
b) Select choice (2) because a positive correlation is a relationship where the dependent (*y*) values increase as independent values (*x*) increase. A graph showing positive correlation would go up from left to right, like the graph in this problem.
c) Eliminate choice (3) because no correlation occurs when there is no relationship between the dependent (*y*) values and independent (*x*) values. A graph showing no correlation would not appear to go up or down or have any pattern.
d) Eliminate choice (4) because there is no evidence that buying a bag of popcorn causes someone to buy a soda. Causation only occurs when a change in one quantity causes a change in another quantity. For example, doubling the number of cookies baked causes more cookie dough to be used.

PTS: 2 NAT: S.ID.C.8 TOP: Correlation Coefficient

33) ANS: 3
Strategy #1: This problem can be answered by looking at the scatterplot.

The slope of the data cloud is negative, so answer choices *a* and *b* can be eliminated because both are positive.

The data cloud suggests a linear relationship, put the dots are not in a perfect line. A perfect correlation of ± 1 would show all the dots in a perfect line. Therefore, we can eliminate asnswer choice d.

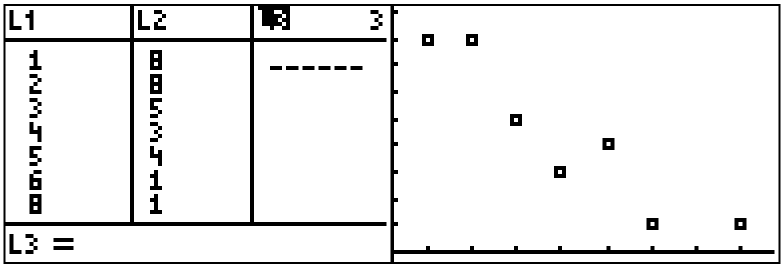
The correct answer is choice c.

DIMS: Does it make sense? Yes. The data cloud shows a negative correlation that is strong, but not perfect. Choice c is the best answer.

Strategy #2: Input the data from the chart in a graphing calculator and calculate the correlation coeffient using linear regression and the diagnostics on feature.

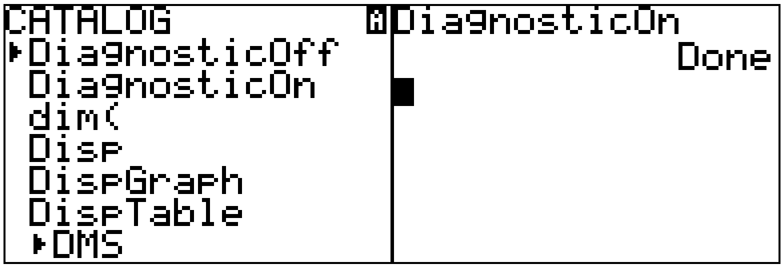
STEP 1. Creat a table of values from the graphing view of the function and input it into the graphing calculator.

x	y
1	8
2	8
3	5
4	3
5	4
6	1
8	1



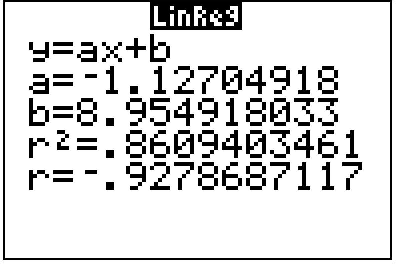
```
L1      L2      L3     3
1       8       ------
2       8
3       5
4       3
5       4
6       1
8       1
L3 =
```

- **STEP 2**. Turn diagnostics on using the catalog.



```
DiagnosticOn
                Done
```

- **STEP 3**. Determine which regression strategy will best fit the data. The graph looks like the graph of an linear function, so choose linear regression.
- **STEP 4**. Execute the appropriate regression strategy with diagnositcs on in the graphing calculator.



```
LinReg
y=ax+b
a=-1.12704918
b=8.954918033
r²=.8609403461
r=-.9278687117
```

Round the correlation coefficient to the nearest hundredth: $r = -.93$
- **STEP 4**. Select answer choice c.

DIMS: Does it make sense? Yes. The data cloud shows a negative correlation that is strong, but not perfect. Choice c is the best answer.

PTS: 2 NAT: S.ID.C.8 TOP: Correlation Coefficient and Residuals

34) ANS: 4
A correlation coefficient of -0.524 is both <u>negative</u> and <u>moderate</u>.
A perfect correlation is ± 1 and no correlation is 0.

Strategy: Eliminate wrong answers
~~a)~~ There is a strong ~~positive~~ correlation between the variables.
~~b)~~ There is a ~~strong~~ negative correlation between the variables.
~~c)~~ There is a moderate ~~positive~~ correlation between the variables.
d) There is a moderate negative correlation between the variables.

PTS: 2 NAT: S.ID.C.8 TOP: Correlation Coefficient

35) ANS: 1
The correlation coefficient with the absolute value closest to 1 indicates the strongest relationship.

$$|.9| > |-0.8| > |0.5| > |-0.3|$$

PTS: 2 NAT: S.ID.C.8 TOP: Correlation Coefficient

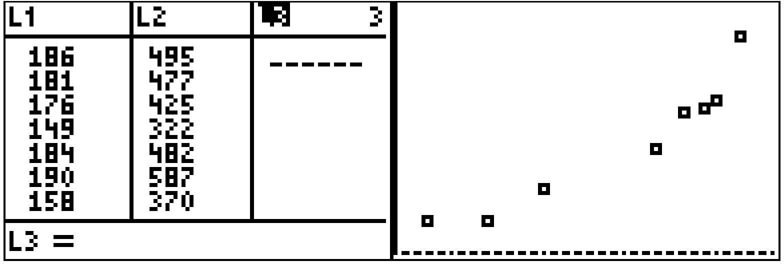
36) ANS: 1
The correlation coefficient $r = -0.896557832$ indicates a strong negative correlation.

PTS: 2 NAT: S.ID.C.8 TOP: Correlation Coefficient

37) ANS:
$r \approx 0.94$. The correlation coefficient suggests that as calories increase, so does sodium.

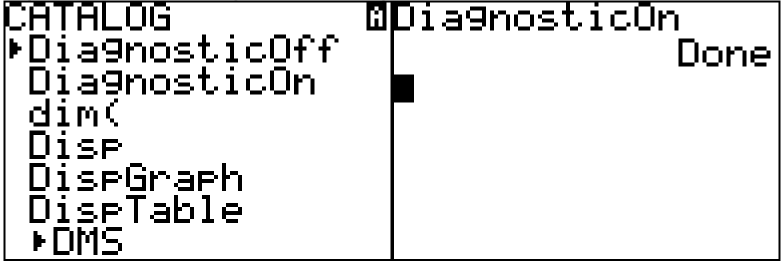
Strategy: Use data from the table and a graphing calculator to find both the regression equation and its correlation coefficient.

STEP 1. Input the data from the table in a graphing calculator and look at the data cloud.



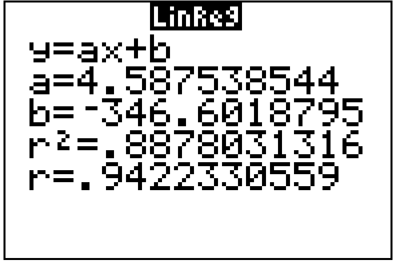
```
L1      L2      L3    3
186     495     ------
181     477
176     425
149     322
184     482
190     587
158     370
L3 =
```

- **STEP 2**. Turn diagnostics on using the catalog.



```
CATALOG              DiagnosticOn
▸DiagnosticOff                   Done
 DiagnosticOn
 dim(
 Disp
 DispGraph
 DispTable
 ▸DMS
```

- **STEP 3**. Determine which regression strategy will best fit the data. The graph looks like the graph of an linear function, so choose linear regression.
- **STEP 4**. Execute the appropriate regression strategy with diagnositcs on in the graphing calculator.



```
LinReg
y=ax+b
a=4.587538544
b=-346.6018795
r²=.8878031316
r=.9422330559
```

Round the correlation coefficient to the nearest hundredth: $r = .94$

DIMS: Does it make sense? Yes. The data cloud and the table show a positive correlation that is strong, but not perfect. A correlation coefficient of .94 is positive, but not a perfectly straight line.

PTS: 4 NAT: S.ID.C.8 TOP: Correlation Coefficient and Residuals

38) ANS: 2

A simple inspection of the table shows that high overall student averages are highly correlated in a positive way with math class averages. The actual correlation coefficient for this table is

$r = .924771\ldots$

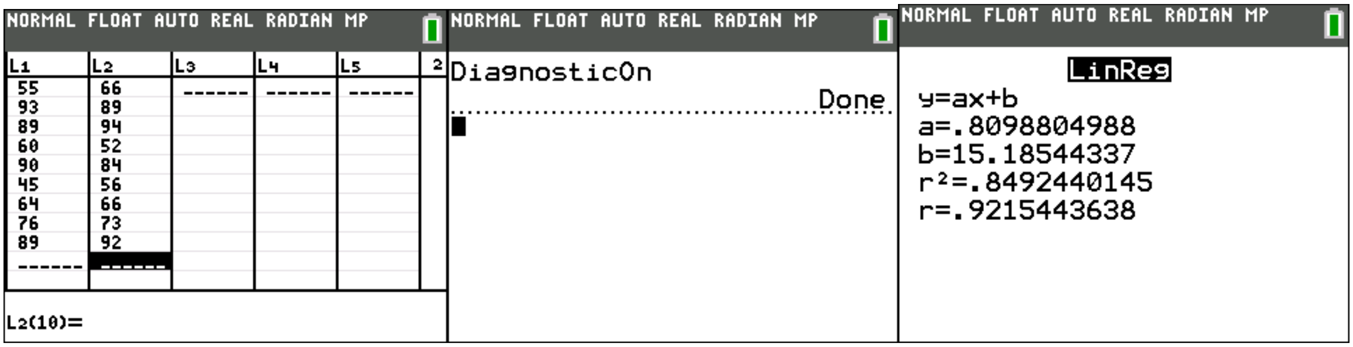
PTS: 2 NAT: S.ID.C

39) ANS:

$r \approx 0.92$.

The correlation coefficient suggests a strong positive correlation between a student's mathematics and physics scores.

Solution Strategy: Input the data into the stats editor of a graphing calculator and calculate linear regression with diagnostics on.



```
NORMAL FLOAT AUTO REAL RADIAN MP
L1     L2     L3     L4     L5     2
55     66     ------ ------ ------
93     89
89     94
60     52
90     84
45     56
64     66
76     73
89     92
------ ------
L2(10)=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
DiagnosticOn
                                Done
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
            LinReg
 y=ax+b
 a=.8098804988
 b=15.18544337
 r²=.8492440145
 r=.9215443638
```

PTS: 2 NAT: S.ID.C.8 TOP: Correlation Coefficient

40) ANS:
$y = 0.16x + 8.27 \quad r = 0.97$, which suggests a strong association.

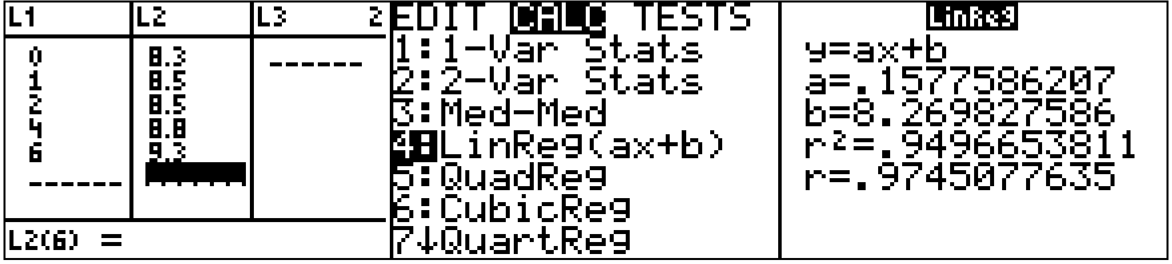
Strategy: Convert the table to data that can be input into a graphing calculator, then use the linear regression feature of the graphing calculator to respond to the question.

STEP 1. Convert the table for input into the calculator.

Attendance at Museum					
Year (L1)	0	1	2	4	6
Attendance (L2)	8.3	8.5	8.5	8.8	9.3

STEP 2. Make sure that STAT DIAGNOSTICS is set to “On” in the mode feature of the graphing calculator. Setting STAT DIAGNOSTICS to on causes the correlation coefficient (*r*) to appear with the linear regression output.

STEP 3. Use the linear regression feature of the graphing calculator.



```
L1     L2     L3     2
0      8.3    ------
1      8.5
2      8.5
4      8.8
6      9.3
------
L2(6) =
```

```
EDIT CALC TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7↓QuartReg
```

```
   LinReg
 y=ax+b
 a=.1577586207
 b=8.269827586
 r²=.9496653811
 r=.9745077635
```

NOTE: Round the graphing calculator output to the *nearest hundredth* as required in the problem.

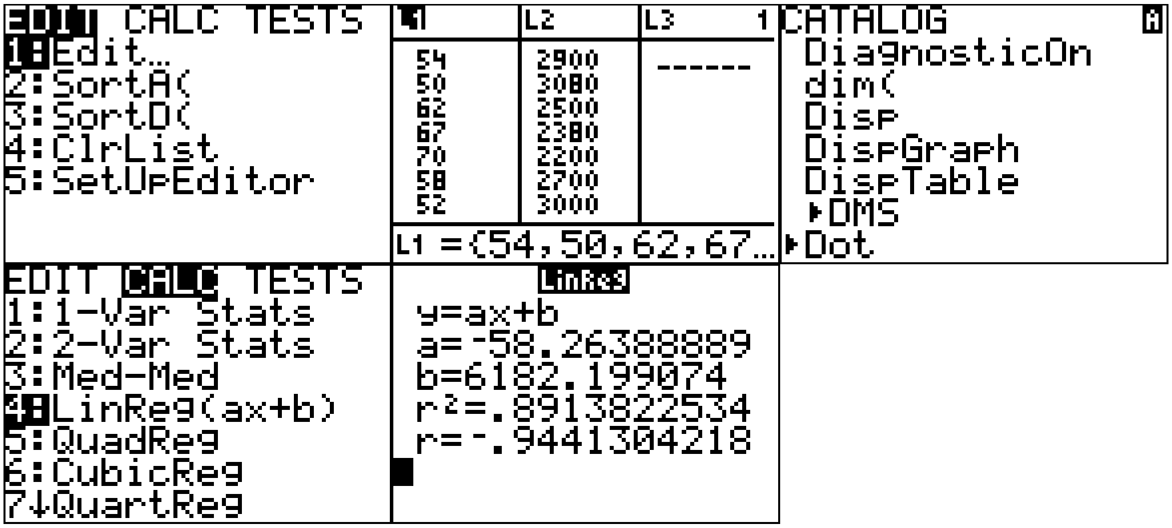
STEP 4. Record your solution.

PTS: 4 NAT: S.ID.C.8 TOP: Regression KEY: linear
NOT: NYSED classifies as S.ID.B.6a

41) ANS:
$f(t) = -58t + 6182$
$r = -.94$

The correlation coefficient indicates a strong linear relationship because the absolute value of *r* is close to 1.

Strategy: Input the table of values into the stats-editor of a graphing calculator, then use the stats-calc-linear regression with “diagnostics on” to obtain both the linear regression equation and the correlation coeffient (*r*). The following screenshots illustrate the solution using a TI-84 family graphing calculator.



L1	L2	L3 1
54	2900	------
50	3080	
62	2500	
67	2380	
70	2200	
58	2700	
52	3000	

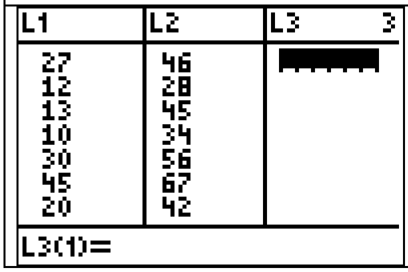
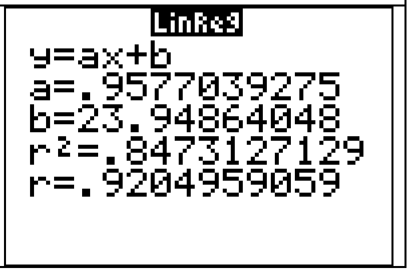
L1 ={54,50,62,67…

```
LinReg
y=ax+b
a=⁻58.26388889
b=6182.199074
r²=.8913822534
r=⁻.9441304218
```

PTS: 4 NAT: S.ID.C.8 TOP: Regression KEY: linear
NOT: NYSED classifies as S.ID.B.6

42) ANS:
$y = 0.96x + 23.95$, 0
A correlation coefficient value of .92 indicates a strong positive correlation between scores 85 or better on the math and English exams for the seven schools.

Strategy: Use the linear regression function of a graphing calculator to determine the equation and the correlation coefficient for the data in the table.

STEP 1. Input the values from the table into the stats editor of a graphing calculator.	STEP 2. Turn diagnostics on to calculate the correlation coefficient with the regression equation.	STEP 3. Calculate the regression equation and correlation coefficient.
 L1 L2 L3 3 27 46 12 28 13 45 10 34 30 56 45 67 20 42 L3(1)=	 DiagnosticOn Done	 LinReg y=ax+b a=.9577039275 b=23.94864048 r²=.8473127129 r=.9204959059

PTS: 4 NAT: S.ID.B.6 TOP: Regression KEY: linear with correlation coefficient

B – Graphs and Statistics, Lesson 7, Residuals (r. 2018)

GRAPHS AND STATISTICS

Residuals

Common Core Standard	Next Generation Standard
S-ID.B.6b Informally assess the fit of a function by plotting and analyzing residuals. NYSED: Includes creating residual plots using the capabilities of the calculator (not manually).	**STANDARD REMOVED**
S-ID.B.6c Fit a linear function for a scatter plot that suggests a linear association. NYSED: Both correlation coefficient and residuals will be addressed in this standard.	**STANDARD REMOVED**

LEARNING OBJECTIVES

Students will be able to:

1) Understand residuals as the difference between actual and predicted y-values based on a line of best fit.
2) Create residual plots given a table of residual values.
3) Interpret patterns in residual plots as an indication that the regression equation does not fit the data.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

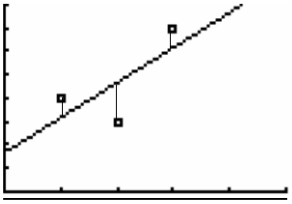
VOCABULARY

actual y-value
predicted y-value
residual
residual plot

line of best fit
pattern
fit the data

BIG IDEAS

A **residual** is the vertical distance between where a regression equation predicts a point will appear on a graph and the actual location of the point on the graph (scatterplot). If there is no difference between where a regression equation places a point and the actual position of the point, the **residual** is zero.



A **residual** can also be understood as the difference in predicted and actual y-values (dependent variable values) for a given value of x (the independent variable).

Residual = (actual y-value)-(predicted y-value)

A **residual plot** is a scatter plot that shows the residuals as points on a vertical axis (y-axis) above corresponding (paired) values of the independent variable on the horizontal axis (x-axis).

Any *pattern* in a residual plot suggests that the regression equation is *not appropriate* for the data.

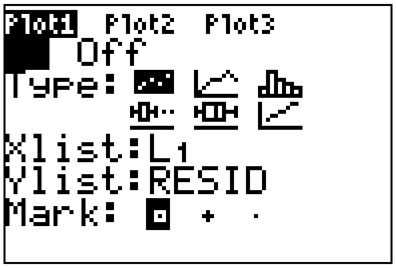
Patterns in residual plots are bad.

Residual plots with patterns indicate the regression equation is not a good fit.

Residual plots without patterns indicate the regression equation is a good fit.

A **residual plot** without a *pattern* and with a near equal distribution of points above and below the x-axis suggests that the regression equation is a *good fit* for the data.

Residuals are automatically stored in graphing calculators when regression equations are calculated. To view a residuals scatterplot in the graphing calculator, you must use 2nd LIST to set the Y list variable to RESID, then use Zoom 9 to plot the residuals.



Xlist:L_1
Ylist:RESID
Mark: ▣ + ·

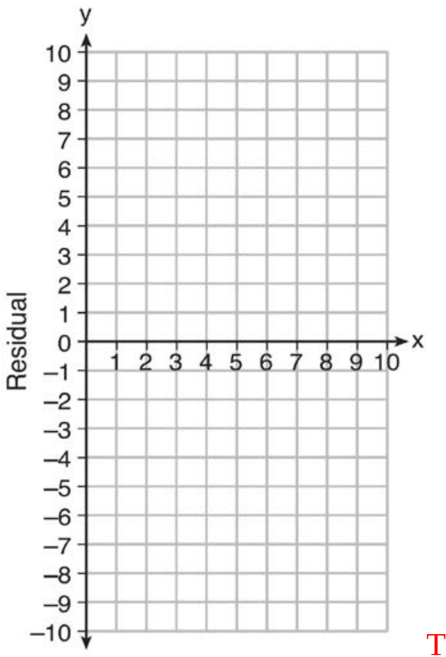
DEVELOPING ESSENTIAL SKILLS

Calculate the residual values:

x	Actual y-value	Predicted y-value	Residual
0	4	-14	10
1	6	-2	8
2	8	2	6
3	10	4	4
4	12	10	2
5	14	14	0

6	16	15	0
7	18	16	2
8	20	15	5
9	22	13	9
10	24	14	10

Plot the residuals and determine if they indicate a good fit or a bad fit.

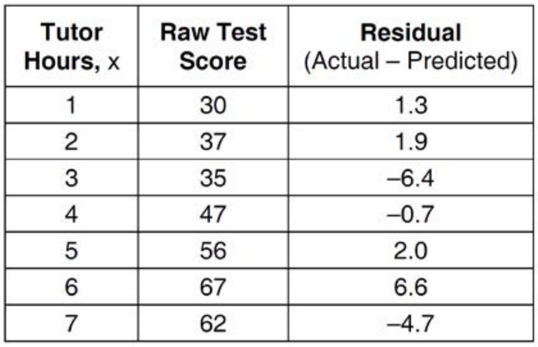


The residuals form a pattern, so the fit is bad.

REGENTS EXAM QUESTIONS (through June 2018)

S.ID.B.6b: Residuals

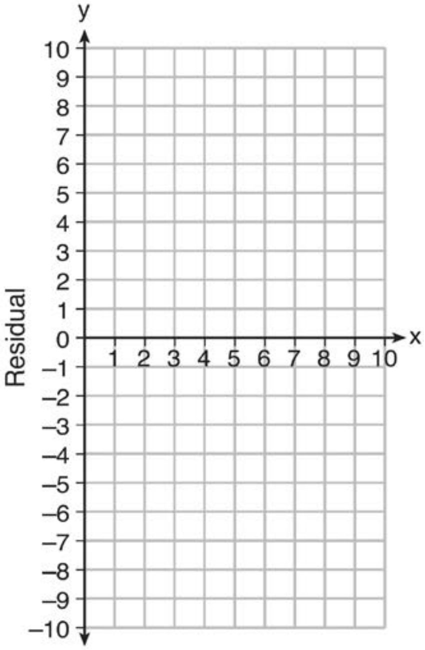
43) Use the data below to write the regression equation ($y = ax + b$) for the raw test score based on the hours tutored. Round all values to the *nearest hundredth*.



Tutor Hours, x	Raw Test Score	Residual (Actual – Predicted)
1	30	1.3
2	37	1.9
3	35	–6.4
4	47	–0.7
5	56	2.0
6	67	6.6
7	62	–4.7

Equation: ______________________________

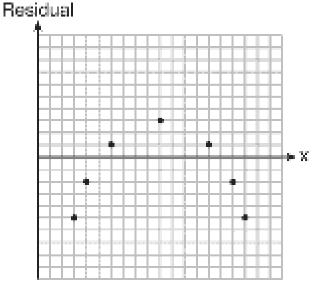
Create a residual plot on the axes below, using the residual scores in the table above.



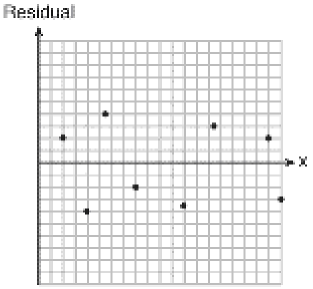
Based on the residual plot, state whether the equation is a good fit for the data. Justify your answer.

44) Which statistic would indicate that a linear function would *not* be a good fit to model a data set?

1) $r = -0.93$

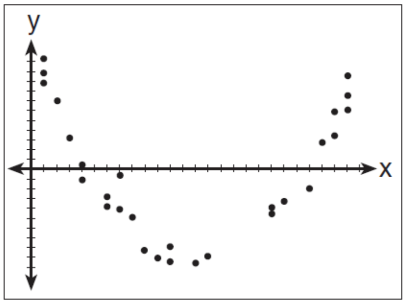
3)


2) $r = 1$

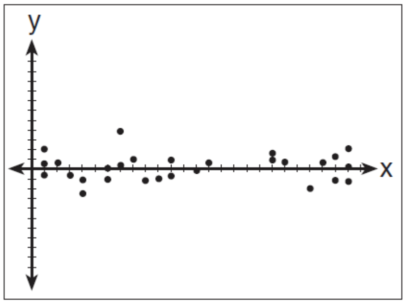
4)


45) After performing analyses on a set of data, Jackie examined the scatter plot of the residual values for each analysis. Which scatter plot indicates the best linear fit for the data?

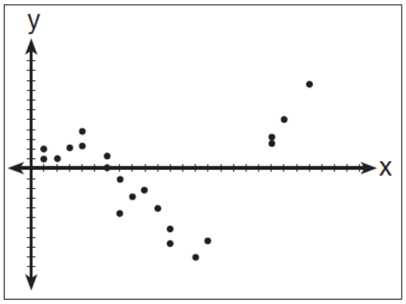
1)



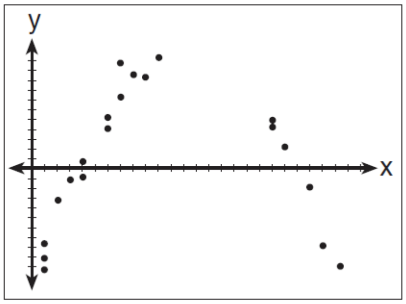
3)



2)



4)



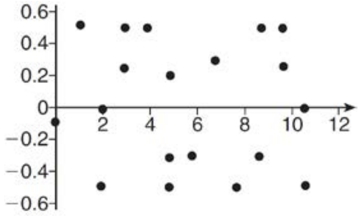
46) The table below represents the residuals for a line of best fit.

x	2	3	3	4	6	7	8	9	9	10
Residual	2	1	−1	−2	−3	−2	−1	2	0	3

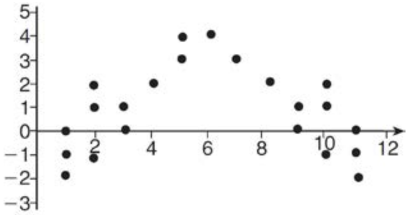
Plot these residuals on the set of axes below.

Using the plot, assess the fit of the line for these residuals and justify your answer.

47) The residual plots from two different sets of bivariate data are graphed below.



Graph A

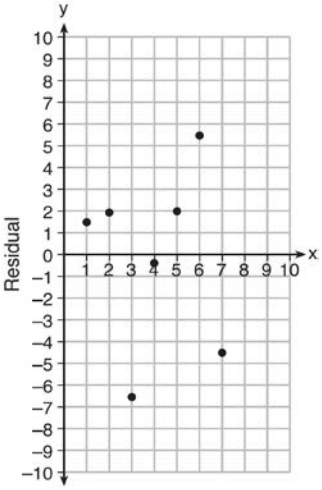


Graph B

Explain, using evidence from graph *A* and graph *B*, which graph indicates that the model for the data is a good fit.

SOLUTIONS

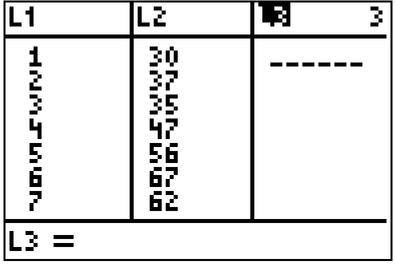
43) ANS:
$y = 6.32x + 22.43$



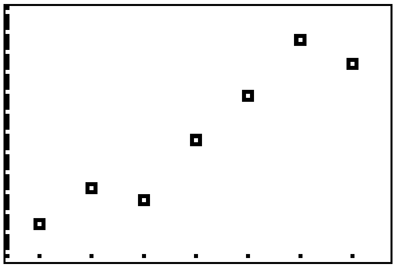
Based on the residual plot, the equation is a good fit for the data because the residual values are scattered without a pattern and are fairly evenly distributed above and below the *x*-axis.

Strategies:
Use linear regression to find a regression equation that fits the first two columns of the table, then create a residuals plot using the first and third columns of the table to see if there is a pattern in the residuals.

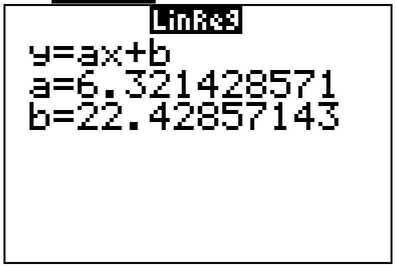
- **STEP 1**. Input the data from the first two columns of the table into a graphing calculator.



```
L1      L2      L3      3
 1      30      ------
 2      37
 3      35
 4      47
 5      56
 6      67
 7      62
L3 =
```



- **STEP 2**. Determine which regression strategy will best fit the data. The problem states that the regression equation should be in the form ($y = ax + b$), which means linear reression. The scatterplot produced by the graphing calculator also suggests linear regression.
- **STEP 3**. Execute the lionear regression strategy in the graphing calculator.

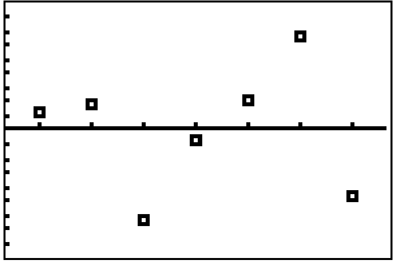


```
LinReg
y=ax+b
a=6.321428571
b=22.42857143
```

Round all values to the nearest hundredth: $y = 6.32x + 22.43$

- **STEP 4**. Plot the residual values on the graph provided using data from the first and third columns of the table. The graph shows a near equal number of points above the line and below the line, and the graph shows no pattern. The regression equation appears to be a good fit.

NOTE: The graphing calculator will also produce a residuals plot.



DIMS: Ask the question, “Does It Make Sense (DIMS)?” Yes. The regression equation produces the same residuals as shown in the table.

PTS: 4 NAT: S.ID.B.6b TOP: Correlation Coefficient and Residuals

44) ANS: 3

Strategy: Use knowledge of correlation coefficients and residual plots to determine which answer choice is **not** a good fit to model a data set.

STEP 1. A correlation coefficient close to –1 or 1 indicates a good fit, so answer choices a and b can be eliminated. Both suggest a good fit.

STEP 2. For a residual plot, there should be no observable pattern and a similar distribution of residuals above and below the *x*-axis. The residual plot in answer choice d shows a good fit, so answer choice d can be eliminated, leaving answer choice c as the correct answer.

DIMS? Does it make sense? Yes. The clear pattern in answer choice c tells us that the linear function is **not** a good fit to model the data set.

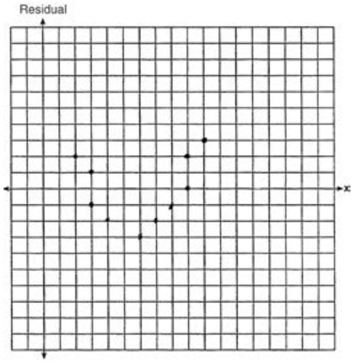
PTS: 2 NAT: S.ID.B.6b TOP: Correlation Coefficient and Residuals

45) ANS: 3

For a residual plot, there should be no observable pattern and about the same number of dots above and below the x axis. Any pattern in a residual plot means that line is **not** a good fit for the data.

PTS: 2 NAT: S.ID.B.6b TOP: Correlation Coefficient and Residuals

46) ANS:



The line is a poor fit because the residuals form a pattern.

PTS: 2 NAT: S.ID.B.6b TOP: Correlation Coefficient and Residuals

47) ANS:
Graph A is a good fit because it does not have a clear pattern, whereas Graph B does have a clear pattern..

PTS: 2 NAT: S.ID.6b TOP: Correlation Coefficient and Residuals

EXPRESSIONS AND EQUATIONS

Dependent and Independent Variables

Common Core Standards	**Next Generation Standards**
A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.	**AI-A.SSE.1** Interpret expressions that represent a quantity in terms of its context.
A-SSE.A.1a ~~Interpret parts of an expression, such as terms, factors, and coefficients.~~ ~~NYSED: The "such as" listed are not the only parts of an expression students are expected to know; others include, but are not limited to, degree of a polynomial, leading coefficient, constant term, and the standard form of a polynomial (descending exponents).~~	**AI-A.SSE.1a** Write the standard form of a given polynomial and identify the terms, coefficients, degree, leading coefficient, and constant term.
A-SSE.A.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*	**AI-A.SSE.1b** Interpret expressions by viewing one or more of their parts as a single entity. e.g., Interpret $P(1+r)^n$ as the product of P and a factor not depending on P. Note: This standard is a fluency expectation for Algebra I. Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square, and other mindful algebraic calculations.

LEARNING OBJECTIVES

Students will be able to:

1) Identify which terms in a mathematical relationship involving two variables are associated with independent and dependent variables.

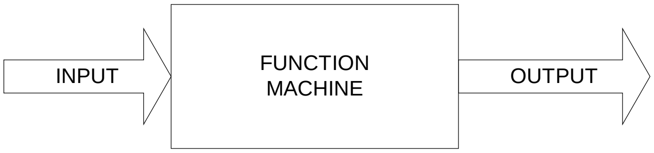
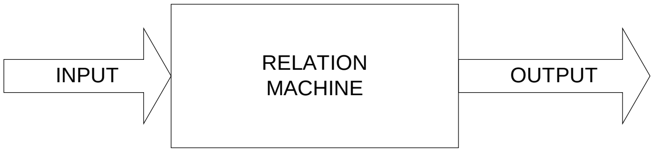
Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

dependent variable
independent variable
term
variable
variable expression

BIG IDEAS

	**Function**: A **function** is a relation that assigns exactly one value of the dependent variable to each value of the independent variable. A **function** is always a relation. Example: y=2x
	**Relation**: A relation may produce more than one output for a given input. A relation may or may not be a function. Example: $y^2 = x$ $y = \sqrt{x}$ This is not a function, because when x=16, there is more than one y-value. $\sqrt{16} = \pm 4$.

The **input variable** is the independent variable.
- It can be any value in the domain of the mathematical relation.
- It is plotted on the x-axis in graphs.

The **output variable** is the dependent variable.
- Its value depends upon what is input.
- It is plotted on the y-axis.

A **term** is a *number*, a *variable*, or the *product* of numbers and variables.
- **Terms** in an expression are always separated by a plus sign or minus sign.
- **Terms** in an expression are always either positive or negative.
- Numbers and variables connected by the operations of division and multiplication are parts of the same **term**.
- **Terms**, together with their signs, can be moved around within the same expression without changing the value of the expression. If you move a **term** from the left expression to the right expression, or from the right expression to the left expression (across the equal sign), the plus or minus sign associated with the term must be changed.

DEVELOPING ESSENTIAL SKILLS

Identify the dependent and independent variables in the following mathematical relationships.

Mathematical Relationship	Independent Variable	Dependent Variable
$y = 2x + 5$	x	y
$C = \frac{5}{9}(F - 32)$	F	C
$\frac{9}{5}C + 32 = F$	C	F
$A = \pi r^2$	r	A
$f(t) = t^2 + 4t + 57$	t	$f(t)$

REGENTS EXAM QUESTION (through June 2018)

A.SSE.A.1: Dependent and Independent Variables

48) The formula for the surface area of a right rectangular prism is $A = 2lw + 2hw + 2lh$, where l, w, and h represent the length, width, and height, respectively. Which term of this formula is *not* dependent on the height?

1) A
2) $2lw$
3) $2hw$
4) $2lh$

SOLUTION

48) ANS: 2

The problem asks "Which term of this formula is *not* dependent on the height."

Term #1	Sign	Term #2	Term #3	Term #4
A	=	+2lw	+2hw	+2lh
This term is the dependent variable in the equation, which is influenced by the height of the rectangular prism.		This is the only term that is *not* dependent on height.	Height is a variable in this term.	Height is a variable in this term.

PTS: 2 NAT: A.SSE.A.1 TOP: Dependent and Independent Variables

C – Expressions and Equations, Lesson 2, Modeling Expressions (r. 2018)

EXPRESSIONS AND EQUATIONS

Modeling Expressions

Common Core Standards	Next Generation Standards
A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.	**AI-A.SSE.1** Interpret expressions that represent a quantity in terms of its context.
A-SSE.A.1a ~~Interpret parts of an expression, such as terms, factors, and coefficients.~~ ~~NYSED: The "such as" listed are not the only parts of an expression students are expected to know; others include, but are not limited to, degree of a polynomial, leading coefficient, constant term, and the standard form of a polynomial (descending exponents).~~	**AI-A.SSE.1a** Write the standard form of a given polynomial and identify the terms, coefficients, degree, leading coefficient, and constant term.
A-SSE.A.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)_n$ *as the product of P and a factor not depending on P.*	**AI-A.SSE.1b** Interpret expressions by viewing one or more of their parts as a single entity. e.g., Interpret $P(1 + r)_n$ as the product of P and a factor not depending on P. **Note: This standard is a fluency expectation for Algebra I. Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square, and other mindful algebraic calculations.**

LEARNING OBJECTIVES

Students will be able to:

1) Use academic language to identify the terms, coefficients, degree, leading coefficient, and constant term of a mathematical statement.
2) Relate parts of equations and expressions to real world contexts.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

coefficient
constant
degree
degree of an equation
equation

expression
leading coefficient
leading term
monomial
polynomial

standard form
term
variable
variable expression

BIG IDEAS

Important skills in mathematics involve recognizing and using academic vocabulary to:

1) communicate the structure of mathematics and;
2) relate parts of mathematical equations and expressions to real world contexts.

Equation An equation consists of two *expressions* connected by an equal sign. The equal sign indicates that both *expressions* have the same (equal) value. The two expressions in an equation are typically called the *left expression* and the *right expression*.

Expression An expression is a mathematical statement or phrase consisting of one or more *terms*. *Terms* are the building blocks of expressions, similar to the way that letters are the building blocks of words. An expression will always be either a monomial or a polynomial.

- Monomial expressions have only one term.
- Polynomial expressions have two or more terms.

Term A term is a *number*, a *variable*, or the *product* of numbers and variables.

- Terms in an expression are always separated by a plus sign or minus sign.
- Terms in an expression are always either positive or negative.
- Numbers and variables connected by the operations of division and multiplication are parts of the same term.
- Terms, together with their signs, can be moved around within the same expression without changing the value of the expression. If you move a term from the left expression to the right expression, or from the right expression to the left expression (across the equal sign), the plus or minus sign associated with the term must be changed.

Leading Term: The leading term in a polynomial expression is the highest degree term.

Variable A variable is a quantity whose value can change or vary. In algebra, a letter is typically used to represent a variable. The value of the letter can change. The letter x is commonly used to represent a variable, but other letters can also be used. The letters s, o, and sometimes l are avoided by some students because they are easily confused in equations with numbers.

- **Independent Variable**: Always shown on the x-axis, the independent variable is the input for an equation.
- **Dependent Variable**: Always shown on the y-axis, the dependent variable is the output of the equation.
- **Variable Term**: A term than contains at least one variable.
- **Variable Expression**: A mathematical phrase that contains at least one variable.
  Example: The equation 2x+3 = 5 contains a left expression and a right expression. The two expressions are connected by an equal sign. The expression on the left is a polynomial variable expression containing two terms, which are +2x and +3. The expression on the right is monomial that contains only one term, which is the constant +5.

Coefficient: A coefficient is the numerical factor of a term in a polynomial. It is typically thought of as the number in front of a variable.

Example: 14 is the coefficient in the term $14x^3y$.

- **Leading Coefficient:** The leading coefficient of a polynomial is the coefficient of the leading term.

Constant: A constant is a number with a constant value (ie. not a variable).

Standard Form of a Polynomial: A polynomial is in standard from when the degrees of its terms are in descending order.

Examples: $3x^3 + 5x^2 + 7x$ is in standard form.
$5x^2 + 3x^3 + 7x$ is *not* in standard..

DEVELOPING ESSENTIAL SKILLS

Answer each question about the following mathematical statements:

Mathematical Statements	$y = x + 3 - 2x^2$	$4x^4 - 6x^2 + 3x^3 + 2x - 2$	$0 = x + 4$
Is this mathematical statement an expression or an equation?	Equation	Expression	Equation
How many terms are in this mathematical statement?	4	5	3
What is the leading term?	$-2x^2$	$4x^4$	$-2x^2$
What is the degree of this mathematical statement?	Second	Fourth	First
What is the coefficient of the lowest variable term?	1	2	1
What is the constant?	3	-2	0 and 4
Write this mathematical statement in standard form.	$y = -2x^2 + x + 3$	$4x^4 + 3x^3 - 6x^2 + 2x - 2$	$x = -4$

REGENTS EXAM QUESTIONS (through June 2018)

A.SSE.A.1: Modeling Expressions

49) To watch a varsity basketball game, spectators must buy a ticket at the door. The cost of an adult ticket is \$3.00 and the cost of a student ticket is \$1.50. If the number of adult tickets sold is represented by *a* and student tickets sold by *s*, which expression represents the amount of money collected at the door from the ticket sales?

1) $4.50as$
2) $4.50(a + s)$
3) $(3.00a)(1.50s)$
4) $3.00a + 1.50s$

50) An expression of the fifth degree is written with a leading coefficient of seven and a constant of six. Which expression is correctly written for these conditions?

1) $6x^5 + x^4 + 7$
2) $7x^6 - 6x^4 + 5$
3) $6x^7 - x^5 + 5$
4) $7x^5 + 2x^2 + 6$

51) When multiplying polynomials for a math assignment, Pat found the product to be $-4x + 8x^2 - 2x^3 + 5$. He then had to state the leading coefficient of this polynomial. Pat wrote down -4. Do you agree with Pat's answer? Explain your reasoning.

52) Andy has \$310 in his account. Each week, *w*, he withdraws \$30 for his expenses. Which expression could be used if he wanted to find out how much money he had left after 8 weeks?

1) $310 - 8w$
2) $280 + 30(w - 1)$
3) $310w - 30$
4) $280 - 30(w - 1)$

53) Konnor wants to burn 250 Calories while exercising for 45 minutes at the gym. On the treadmill, he can burn 6 Cal/min. On the stationary bike, he can burn 5 Cal/min. If *t* represents the number of minutes on the treadmill and *b* represents the number of minutes on the stationary bike, which expression represents the number of Calories that Konnor can burn on the stationary bike?

1) b
2) $5b$
3) $45 - b$
4) $250 - 5b$

54) Mrs. Allard asked her students to identify which of the polynomials below are in standard form and explain why.

I. $15x^4 - 6x + 3x^2 - 1$
II. $12x^3 + 8x + 4$
III. $2x^5 + 8x^2 + 10x$

Which student's response is correct?

1) Tyler said I and II because the coefficients are decreasing.
2) Susan said only II because all the numbers are decreasing.
3) Fred said II and III because the exponents are decreasing.
4) Alyssa said II and III because they each have three terms.

SOLUTIONS

49) ANS: 4

Strategy: Translate the words into mathematical expressions.

$$\text{The cost of an } \overset{a}{\underline{\text{adult ticket}}} \overset{\times}{\text{ is }} \overset{3.00}{\underline{\$3.00}} \overset{+}{\underline{\text{ and }}} \text{ the cost of a } \overset{s}{\underline{\text{student ticket}}} \overset{\times}{\text{ is }} \overset{1.50}{\underline{\$1.50}}.$$

$$a(3.00) + s(1.50)$$

$$3.00a + 1.50s$$

PTS: 2 NAT: A.SSE.A.1 TOP: Modeling Linear Equations

50) ANS: 4

The degree of a polynomial is determined by the largest exponent of a term within a polynomial. A polynomial expression of the fifth degree can have not exponent larger than 5, so choices b and c can be eliminated.

A leading coefficient is the coefficient of the first term of a polynomial written in descending order of exponents. Since the leading coefficient is seven, choices a and c can be eliminated, leaving choice d as the only possible answer.

Does it make sense? Yes. $7x^5 + 2x^2 + 6$ has a leading coefficient of seven, is a fifth degree polynomial because 5 is the highest exponent, and a constant term of six.

PTS: 2 NAT: A.SSE.A.1 TOP: Modeling Expressions

51) ANS:

I disagree. The leading coefficient of a polynomial is the coefficient of the term with the highest exponent when all of the terms are arranged in descending order by exponents.

$$-2x^3 + 8x^2 - 4x + 5$$

Pat should have written that the leading coefficient is -2.

PTS: 2 NAT: A.SSE.A.1

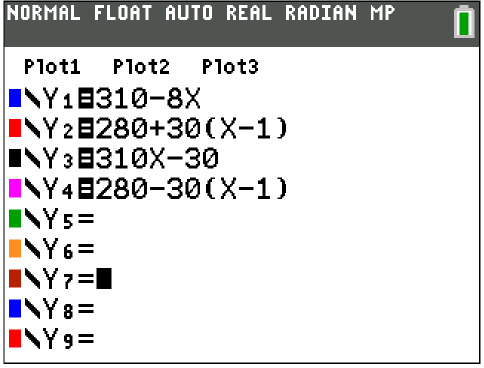
52) ANS: 4

Strategy:

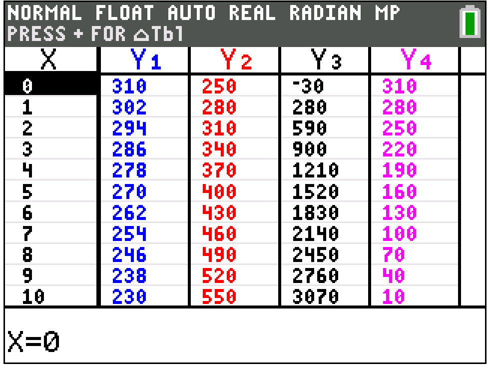
Step 1. Create a table of values by starting at week 0 with $310 dollars.

Week	$$$
x	y
0	310
1	280
2	250
3	220
4	190
5	160
6	130
7	100
8	70

Step 2. Use a graphing calcualtor to determine which expression reproduces the table of values.



```
NORMAL FLOAT AUTO REAL RADIAN MP

 Plot1  Plot2  Plot3
Y1=310-8X
Y2=280+30(X-1)
Y3=310X-30
Y4=280-30(X-1)
Y5=
Y6=
Y7=
Y8=
Y9=
```



```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl
  X     Y1    Y2    Y3     Y4
  0     310   250   -30    310
  1     302   280   280    280
  2     294   310   590    250
  3     286   340   900    220
  4     278   370   1210   190
  5     270   400   1520   160
  6     262   430   1830   130
  7     254   460   2140   100
  8     246   490   2450   70
  9     238   520   2760   40
  10    230   550   3070   10
X=0
```

PTS: 2 NAT: A.SSE.A.1 TOP: Modeling Expressions

53) ANS: 2

On the stationary bike, Konnor can burn 5 Cal/min.

b represents the number of minutes Konnor spends on the stationary bike.

5 times *b* represents the number of Calories that Konnor can burn on the stationary bike.

PTS: 2 NAT: A.SSE.A.1 TOP: Modeling Expressions

54) ANS: 3

Strategy: Find the polynomials that have the exponents decreasing from left to right. This is the definition of standard form.

I. $15x^4 - 6x + 3x^2 - 1$ is not in standard form because the exponent of the middle term is less than the exponent of the third term.

II. $12x^3 + 8x + 4$ is in standard form because the exponents decrease from left to right.

III. $2x^5 + 8x^2 + 10x$ is in standard form because the exponents decrease from left to right.

Fred is correct. II and III are in standard form.

PTS: 2 NAT: A.SSE.A.1 TOP: Modeling Expressions

C – Expressions and Equations, Lesson 3, Solving Linear Equations (r. 2018)

EXPRESSIONS AND EQUATIONS

Solving Linear Equations

Common Core Standard	**Next Generation Standard**
A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**AI-A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **Note: Algebra I tasks do not involve solving compound inequalities**.

LEARNING OBJECTIVES

Students will be able to:

1) Solve one step and multiple step equations.
2) Explain each step involved in solving one step and multiple step equations.
3) Do a check to see if the solution is correct.

Overview of Lesson

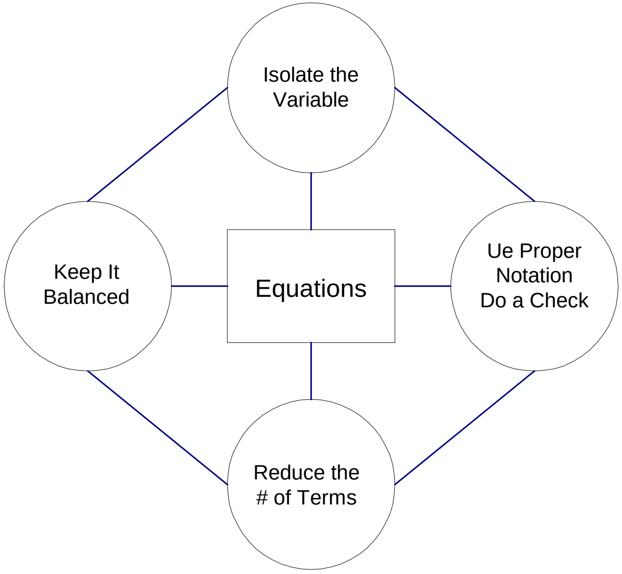
Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

balance
check
common sense
DIMS
four column strategy
four general rules
isolate
plug
proper notation
substitute

BIG IDEAS

The Four General Rules
(for solving equations and inequalities)



Isolate the Variable: The goal of solving any equation is to isolate the desired variable in either the left or right expression.
Keep It Balanced: During each step of the equation solving process, the left and right expressions must equal one another.
Reduce the Number of Terms: Any step that reduces the number of terms in an equation is usually a good step.
Use Proper Notation and Do a Check: You check your answers in algebra on two levels: first, you see if the answer actually makes sense, and then you plug your answer back into the problem to see if it works.

- **Proper Notation** involves making short notes that describe the action taken during each step of solving an equation. Academic language is sometimes required.
- **Does It Make Sense (DIMS)**
  The first step in checking a solution is to use "common sense." For example, if your solution is $x = 5$, and you are solving for a football player's weight in pounds, you have probably made a mistake because it does not make sense that a football player weighs only five pounds. On the other hand, if you are solving for the number of pennies in a nickel, it makes perfect sense.
- **Plug (substitute) the answer back into the problem to see if it works.**
  The second step in checking a solution is to substitute your solution into the original equation and solve the equation once again with your solution in it. If the left expression is equal to the right expression, the equation balances and your solution is correct.

The Four Column Strategy

The four column strategy focuses on organizing and documenting each step in solving an equation or inequality. Emphasis is given to explaining each step and keeping the equal signs (or inequality signs) aligned in a vertical column. The vertical and horizontal lines are simply scaffolds that can be removed as students acquire understanding and skills in solving equations.

((keep the equation/inequality signs aligned vertically)

Notes	Left Hand Expression	Sign	Right Hand Expression
Given Add (6)	$2x-6$ $+6$	=	2 $+\ 6$
	$2x+0$	=	8
Divide (2)	$\frac{2x}{2}$	=	$\frac{8}{2}$
Answer	x	=	4
Check	2(4) – 6 8-6 2	= = =	2 2 2

DEVELOPING ESSENTIAL SKILLS

Use the four general rules and the four column strategy to solve the following problems:

If $3(x-2)=2x+6$, the value of x is 12

Notes	Left Hand Expression	Sign	Right Hand Expression
Given	$3(x-2)$	=	$2x+6$
Distributive Property	$3x-6$	=	$2x+6$
Subtract 2x	$-2x$		$-2x$
Simplify	$x-6$	=	6
Add 6	$+6$		$+6$
Solution	x	=	12
Check	$3(x-2)=2x+6$ $3(12-2)=2(12)+6$ $3(10)=24+6$ $30=30$		

What is the value of x in the equation $\frac{3}{4}x+2=\frac{5}{4}x-6$? 16

Notes	Left Hand Expression	Sign	Right Hand Expression
Given	$\frac{3}{4}x+2$	=	$\frac{5}{4}x-6$
Multiply by 4	$3x+8$	=	$5x-24$
Subtract 3x	8	=	$2x-24$
Add 24	32	=	2x

Divide by 2	16	$=$	X
Check	$\frac{3}{4}x+2=\frac{54}{}x-6$ $\frac{3}{4}(16)+2=\frac{5}{4}(16)-6$ $\frac{48}{4}+2=\frac{80}{4}-6$ $12+2=20-6$ $14=14$		

REGENTS EXAM QUESTIONS (through June 2018)

A.REI.B.3: Solving Linear Equations

55) Which value of x satisfies the equation $\frac{7}{3}\left(x+\frac{9}{28}\right)=20$?

1) 8.25
2) 8.89
3) 19.25
4) 44.92

56) What is the value of x in the equation $\frac{x-2}{3}+\frac{1}{6}=\frac{5}{6}$?

1) 4
2) 6
3) 8
4) 11

57) An equation is given below.

$$4(x-7)=0.3(x+2)+2.11$$

The solution to the equation is

1) 8.3
2) 8.7
3) 3
4) -3

58) Which value of x satisfies the equation $\frac{5}{6}\left(\frac{3}{8}-x\right)=16$?

1) -19.575
2) -18.825
3) -16.3125
4) -15.6875

59) The value of x which makes $\frac{2}{3}\left(\frac{1}{4}x-2\right)=\frac{1}{5}\left(\frac{4}{3}x-1\right)$ true is

1) -10
2) -2
3) $-9.\overline{09}$
4) $-11.\overline{3}$

60) Solve the equation below algebraically for the exact value of x.

$$6-\frac{2}{3}(x+5)=4x$$

SOLUTIONS

55) ANS: 1

Strategy: Use the four column method.

Notes	Left Expression	Sign	Right Expression
Given	$\frac{7}{3}\left(x+\frac{9}{28}\right)$	=	20
Divide both expressions by $\frac{7}{3}$ (Division property of equality)	$\dfrac{\frac{7}{3}\left(x+\frac{9}{28}\right)}{\frac{7}{3}}$	=	$\dfrac{\frac{20}{1}}{\frac{7}{3}}$
Cancel and Simplify	$x+\frac{9}{28}$	=	$\frac{60}{7}$
Subtract $\frac{9}{28}$ from both expressions (Subtraction property of equality)	x	=	$\frac{60}{7}-\frac{9}{28}$
Simplify	x	=	$\frac{231}{28}$
Simplify	x	=	8.25

or

Notes	Left Expression	Sign	Right Expression
Given	$\frac{7}{3}\left(x+\frac{9}{28}\right)$	=	20
Distributive Property	$\frac{7}{3}x+\frac{7}{3}\left(\frac{9}{28}\right)$	=	20
Cancellation	$\frac{7}{3}x+\frac{1}{3}\left(\frac{9}{4}\right)$	=	20
Simplification	$\frac{7}{3}x+\frac{3}{4}$	=	20
Subtract $\frac{3}{4}$ from both expressions (Subtraction Property of Equality)	$\frac{7}{3}x$	=	$20-\frac{3}{4}$
Simplification	$\frac{7}{3}x$	=	$\frac{77}{4}$
Multiply both expressions by 12 (Multiplication property of equality)	$\frac{12}{1}\left(\frac{7x}{3}\right)$	=	$\frac{12}{1}\left(\frac{77}{4}\right)$
Cancel	$\frac{4}{1}\left(\frac{7x}{1}\right)$	=	$\frac{3}{1}\left(\frac{77}{1}\right)$
Siomplify	$28x$	=	231
Divide both expressions by 28 (Division property of equality)	$\frac{28x}{28}$	=	$\frac{231}{28}$

Simplify	x	=	8.25

PTS: 2 NAT: A.REI.B.3 TOP: Solving Linear Equations
KEY: fractional expressions

56) ANS: 1
Strategy: Use the four column method.

Notes	Left Expression	Sign	Right Expression
Given:	$\frac{x-2}{3}$	=	$\frac{4}{6}$
Multiply both expressions by 6 (Multiplication property of equality)	$\frac{6}{1}\left(\frac{x-2}{3}\right)$	=	$\frac{6}{1}\left(\frac{4}{6}\right)$
Cancel and Simplify	$\frac{2}{1}\left(\frac{x-2}{1}\right)$	=	$\frac{1}{1}\left(\frac{4}{1}\right)$
Simplify	$2x-4$	=	4
Add +4 to both expressions (Addition property of equality)	$2x$	=	8
Divide both expressions by 2 (Division property of equality)	x	=	4

PTS: 2 NAT: A.REI.B.3 TOP: Solving Linear Equations
KEY: fractional expressions

57) ANS: 1

$$4(x-7)=0.3(x+2)+2.11$$
$$4x-28=.3x+2.71$$
$$4x-.3x=2.71+28$$
$$3.7x=30.71$$
$$x=8.3$$

PTS: 2 NAT: A.REI.B.3 TOP: Solving Linear Equations
KEY: decimals

58) ANS: 2

$$\frac{5}{6}\left(\frac{3}{8}-x\right)=16$$

$$5\left(\frac{3}{8}-x\right)=96$$

$$\frac{3}{8}-x=\frac{96}{5}$$

$$-x=\frac{96}{5}-\frac{3}{8}$$

$$-x=18.825$$

$$x=-18.825$$

PTS: 2 NAT: A.REI.B.3 TOP: Solving Linear Equations
KEY: fractional expressions

59) ANS: 4

Solve for x:

$$\frac{2}{3}\left(\frac{1}{4}x-2\right)=\frac{1}{5}\left(\frac{4}{3}x-1\right)$$

Multiply by 3 to clear the first fraction.

$$\left(\frac{3}{1}\right)\frac{2}{3}\left(\frac{1}{4}x-2\right)=\left(\frac{3}{1}\right)\frac{1}{5}\left(\frac{4}{3}x-1\right)$$

$$2\left(\frac{1}{4}x-2\right)=\frac{3}{5}\left(\frac{4}{3}x-1\right)$$

Multiply by 5 to clear the remaining fraction.

$$(5)2\left(\frac{1}{4}x-2\right)=\left(\frac{5}{1}\right)\frac{3}{5}\left(\frac{4}{3}x-1\right)$$

$$10\left(\frac{1}{4}x-2\right)=3\left(\frac{4}{3}x-1\right)$$

Use distributive property to clear parentheses.

$$\frac{10}{4}x-20=4x-3$$

Multiply by 4 to clear fraction.

$$(4)\,\frac{10}{4}x-(4)20=(4)4x-(4)3$$

$$10x-80=16x-12$$

Transpose and solve for x.

$$-6x=68$$

$$\frac{-6x}{-6}=\frac{68}{-6}$$

$$x=-11.\overline{33}$$

PTS: 2 NAT: A.REI.B.3 TOP: Solving Linear Equations
KEY: fractional expressions

60) ANS:

Answer: $\frac{4}{7}$

Strategy: Solve algebraically (without a calculator).

Notes	Left Expression	Sign	Right Expression
Given	$6-\frac{2}{3}(x+5)$	=	$4x$
Multiply by 3	$18-2(x+5)$	=	$12x$
Distributive Property	$18-2x-10$	=	$12x$
Add 2x	$18-10$	=	$14x$
Simplify	8	=	$14x$
Divide by 14	$\frac{8}{14}$	=	x
Simplify	$\frac{4}{7}$	=	x

Check.is Optional

Notes	Left Expression	Sign	Right Expression
Given	$6-\frac{2}{3}(x+5)$	=	4x
Evaluate for $x=\frac{4}{7}$	$6-\frac{2}{3}\left(\frac{4}{7}+5\right)$	=	$4\left(\frac{4}{7}\right)$
Get a Common Denominator Inside Parentheses	$6-\frac{2}{3}\left(\frac{4}{7}+\frac{35}{7}\right)$	=	$4\left(\frac{4}{7}\right)$
Do Addition Inside Parentheses	$6-\frac{2}{3}\left(\frac{39}{7}\right)$	=	$4\left(\frac{4}{7}\right)$
Remove Parentheses Using Multiplication of Fractions	$6-\frac{78}{21}$	=	$\frac{16}{7}$
Get a Common Denominator	$\frac{126}{21}-\frac{78}{21}$	=	$\frac{48}{21}$
Simplify	$\frac{48}{21}$	=	$\frac{48}{21}$

PTS: 3 NAT: A.REI.B.3 TOP: Solving Linear Equations
KEY: fractional expressions

C – Expressions and Equations, Lesson 4, Modeling Linear Equations (r. 2018)

EXPRESSIONS AND EQUATIONS

Modeling Linear Equations

Common Core Standards	**Next Generation Standards**
A-CED.A.1 Create equations and inequalities in one variable ~~and use them to solve problems~~. *Include equations arising from linear and quadratic functions, and ~~simple rational and~~ exponential functions.* PARCC: Tasks are limited to linear, quadratic, or exponential equations with integer exponents.	**AI-A.CED.1** Create equations and inequalities in one variable to **represent a real-world context**. (Shared standard with Algebra II) Notes: • **This is strictly the development of the model (equation/inequality).** • **Limit equations to linear, quadratic, and exponentials of the form $f(x) = a(b)_x$ where $a > 0$ and $b > 0$ ($b \neq 1$).** • **Work with geometric sequences may involve an exponential equation/formula of the form $a_n = ar_{n-1}$, where a is the first term and r is the common ratio.** • **Inequalities are limited to linear inequalities.** • Algebra I tasks do not involve compound inequalities.
A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; ~~graph equations on coordinate axes with labels and scales~~.	**AI-A.CED.2** Create equations and linear inequalities in two variables to represent a **real-world context**. Notes: • **This is strictly the development of the model (equation/inequality).** • **Limit equations to linear, quadratic, and exponentials of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).**
A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	**AI-A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.

LEARNING OBJECTIVES

Students will be able to:

1) Model real-world word problems as mathematical expressions and equations.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

See key words and their mathematical translations under big ideas.

BIG IDEAS

Translating words into mathematical expressions and equations is an important skill.

General Approach

The general approach is as follows:

1. Read and understand the entire problem.
2. Underline key words, focusing on variables, operations, and equalities or inequalities.
3. Convert the key words to mathematical notation (consider meaningful variable names other than x and y).
4. Write the final expression or equation.
5. Check the final expression or equation for reasonableness.

Key English Words and Their Mathematical Translations

These English Words	Usually Mean	*Examples: English becomes math*
sum, plus, and	addition	*the sum of 5 and x becomes* $5 + x$
minus, less, take away, difference of	subtraction	*5 minus x becomes* $5 - x$ *the difference of x and 5 becomes* $x - 5$
less than	subtraction	*3 less than x becomes* $x - 3$
product, times, multiplied by	multiplication	*the product of five times two becomes* 5 x 2 *x multiplied by 4 becomes* $4x$
fraction of, percent of	multiplication	*one half of x becomes* $\frac{1}{2}x$ *33 percent of y becomes* $.33y$
quotient, divided by, ratio of	Division	*the quotient of x and y becomes* $\frac{x}{y}$ *the ratio of two times y and 4 becomes* $\frac{2y}{4}$
is, are	equals	*the sum of 5 and x is 20 becomes* $5 + x = 20$

Examples of Modeling Specific Types of Equations

Age Problems

Typical Problem in English	Mathematical Translation	Hints and Strategies
Tamara has two sisters. One of the sisters is 7 years older than Tamara. The other sister is 3 years younger than Tamara. The *product of Tamara's sisters' ages* is 24. How old is Tamara?	Let x represent Tamara's age. Let x+7 represent the older sister's age. Let x-3 represent the younger sister's age. Write: $\left(x+7\right)\left(x-7\right)=24$ Solve for x. x = 5	Define your variables. Check your answers. Remember than "is" means =.

Area, Volume and Perimeter Problems

Typical Problem in English	Mathematical Translation	Hints and Strategies
If the length of a rectangular prism is doubled, its width is tripled, and its height remains the same, what is the volume of the new rectangular prism in relation to the volume of the original rectangular prism?	Use the formula $V = lwh$. Let the volume of the original rectangular prism be represented by *lwh*. Let the volume of the new rectangular prism be represented by $2l \times 3w \times h$, which simplifies to 6 times *lwh*. The new rectangular prism has six times the volume of the original rectangular prism.	Use a geometric formula as a guide.

Coin Problems

Typical Problem in English	Mathematical Translation	Hints and Strategies
Byron has 72 coins in his piggy bank. The piggy bank contains only dimes and quarters. If he has \$14.70 in his piggy bank, write an equation that can be used to determine *q*, the number of quarters he has?	The total value of all coins is 1470 cents. Let the number of quarters be represented by *q* and the value of quarters be represented by 25*q*. Let the number of dimes be represented by 72 – *q* and the value of dimes be represented by 10(75 – *q*) Write: $25q+10(72-q)=1470$ Solve for *q*. $q=30$	Work with cents as units. Remember that each coin has a specific value in cents

Consecutive Integer Problems

Typical Problem in English	Mathematical Translation	Hints and Strategies

The sum of three consecutive odd integers is 18 less than five times the middle number. Find the three integers. [Only an algebraic solution can receive full credit.]	Let x represent the first integer. Let x + 2 represent the middle integer. Let x + 4 represent the 3rd integer. Write: $\left(x+x+2+x+4\right)=5\left(x+2\right)-18$ Solve for x, x +2, and x + 4. 7, 9, 11	For consecutive integer problems, define your variables as x, x + 1, and x + 2 For consecutive *even or odd* integer problems, define your variables as x, x +2, and x + 4.

Missing Number in the Average Problems

Typical Problem in English	Mathematical Translation	Hints and Strategies
TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is \$360. If the weekly salaries of four of the employees are \$340, \$340, \$345, and \$425, what is the salary of the fifth employee?	Let x_5 represent the missing salary Write: $360=\dfrac{340+340+345+425+x_5}{5}$ Solve for x_5. $x_5=\$350$	Substitute given values into the following formula for finding the average. $\overline{x}=\dfrac{x_1+x_2+...+x_n}{n}$, then solve for the missing value.

Number Problems

Typical Problem in English	Mathematical Translation	Hints and Strategies
Twice the larger of two numbers is ten more than five times the smaller, and the sum of four times the larger and three times the smaller is 39. What are the numbers?	Let x represent the larger #. Let y represent the smaller #. Write two equations: $2x=10+5y$ And $4x+3y=46$ Solve as a system of equations. x = 10 and y = 2	Define your variables. Check your answers. Remember that "is" means =.

DEVELOPING ESSENTIAL SKILLS

Write equations or expressions that model each of the following word problems.

1. The length of a rectangular window is 5 feet more than its width, *w*. The area of the window is 36 square feet. Write an equation that could be used to find the dimensions of the window?	$w\left(w+5\right)=36$ or $w^2+5w-36=0$
2. Rhonda has \$1.35 in nickels and dimes in her pocket. If she has six more dimes than nickels, write an equation that can be used to determine *x*, the number of nickels she has?	$0.05x+0.10(x+6)=1.35$ or $5x+10(x+6)=135$
3. If *h* represents a number, write an equation that is a correct translation of "Sixty more than 9 times a number is 375"?	$9h+60=375$

4. The ages of three brothers are consecutive even integers. Three times the age of the youngest brother exceeds the oldest brother's age by 48 years. Write an equation that could be used to find the age of the youngest brother?	$3x=48+(x+4)$ or $3x-(x+4)=48$
5. The width of a rectangle is 3 less than twice the length, x. If the area of the rectangle is 43 square feet, write an equation that can be used to find the length, in feet?	$x(2x-3)=43$
6. If n is an odd integer, write an equation that can be used to find three consecutive odd integers whose sum is -3?	$n+(n+2)+(n+4)=-3$
7. The width of a rectangle is 4 less than half the length. If ℓ represents the length, write an equation that could be used to find the width, w?	$w=\frac{1}{2}l-4$
8. Three times the sum of a number and four is equal to five times the number, decreased by two. If x represents the number, write an equation that is a correct translation of the statement?	$3(x-4)=5x-2$
9. The product of a number and 3, increased by 5, is 7 less than twice the number. Write an equation that can be used to find this number, n?	$3n+5=2n-7$

REGENTS EXAM QUESTIONS

A.CED.A.1: Modeling Linear Equations

61) Donna wants to make trail mix made up of almonds, walnuts and raisins. She wants to mix one part almonds, two parts walnuts, and three parts raisins. Almonds cost \$12 per pound, walnuts cost \$9 per pound, and raisins cost \$5 per pound. Donna has \$15 to spend on the trail mix. Determine how many pounds of trail mix she can make. [Only an algebraic solution can receive full credit.]

62) Kendal bought x boxes of cookies to bring to a party. Each box contains 12 cookies. She decides to keep two boxes for herself. She brings 60 cookies to the party. Which equation can be used to find the number of boxes, x, Kendal bought?

1) $2x-12=60$
2) $12x-2=60$
3) $12x-24=60$
4) $24-12x=60$

63) John has four more nickels than dimes in his pocket, for a total of \$1.25. Which equation could be used to determine the number of dimes, x, in his pocket?

1) $0.10(x+4)+0.05(x)=\$1.25$
2) $0.05(x+4)+0.10(x)=\$1.25$
3) $0.10(4x)+0.05(x)=\$1.25$
4) $0.05(4x)+0.10(x)=\$1.25$

64) A gardener is planting two types of trees:

Type A is three feet tall and grows at a rate of 15 inches per year.
Type B is four feet tall and grows at a rate of 10 inches per year.

Algebraically determine exactly how many years it will take for these trees to be the same height.

65) A parking garage charges a base rate of \$3.50 for up to two hours, and an hourly rate for each additional hour. The sign below gives the prices for up to 5 hours of parking.

Parking Rates	
2 hours	\$3.50
3 hours	\$9.00
4 hours	\$14.50
5 hours	\$20.00

Which linear equation can be used to find x, the additional hourly parking rate?

1) $9.00 + 3x = 20.00$
2) $9.00 + 3.50x = 20.00$
3) $2x + 3.50 = 14.50$
4) $2x + 9.00 = 14.50$

66) Sandy programmed a website's checkout process with an equation to calculate the amount customers will be charged when they download songs. The website offers a discount. If one song is bought at the full price of \$1.29, then each additional song is \$.99. State an equation that represents the cost, *C*, when *s* songs are downloaded. Sandy figured she would be charged \$52.77 for 52 songs. Is this the correct amount? Justify your answer.

67) A cell phone company charges \$60.00 a month for up to 1 gigabyte of data. The cost of additional data is \$0.05 per megabyte. If *d* represents the number of additional megabytes used and *c* represents the total charges at the end of the month, which linear equation can be used to determine a user's monthly bill?

1) $c = 60 - 0.05d$
2) $c = 60.05d$
3) $c = 60d - 0.05$
4) $c = 60 + 0.05d$

68) A typical cell phone plan has a fixed base fee that includes a certain amount of data and an overage charge for data use beyond the plan. A cell phone plan charges a base fee of \$62 and an overage charge of \$30 per gigabyte of data that exceed 2 gigabytes. If *C* represents the cost and *g* represents the total number of gigabytes of data, which equation could represent this plan when more than 2 gigabytes are used?

1) $C = 30 + 62(2 - g)$
2) $C = 30 + 62(g - 2)$
3) $C = 62 + 30(2 - g)$
4) $C = 62 + 30(g - 2)$

SOLUTIONS

61) ANS:
Donna can make 2 pounds of trail mix.

Strategy 1: Determine the costs of six pounds of mix, then scale the amount down to \$15 of mix.

STEP 1. The mix will have six parts. If each part is 1 pound, the costs of the mix can be determined as follows:
\$12 for one part almonds @ \$12 per pound,
\$18 for two parts walnuts @ \$9 per pound, and
\$15 for three parts raisins @ \$5 per pound.
\$45 for six pounds of mix.

STEP 2: Scale the amount down to \$15 of mix

$$\frac{Cost}{Pounds}\Bigg|\ \frac{\$45}{6}=\frac{\$15}{x}$$

$$45x = 6(15)$$

$$45x = 90$$

$$x = 2$$

Donna can make 2 pounds of trail mix.

DIMS? Does It Make Sense? Yes. If 2 pounds of the mix cost \$15, 3 times as much should cost \$45.

Strategy 2. Write an expression that scales the costs of the mix to \$15.

Let x represent the scale factor.

Write $\begin{bmatrix}\text{(1lb. almonds @ \$12 per lb.)} \times \text{scale factor} + \\ \text{(2lbs. walnuts @ \$9 per lb.)} \times \text{scale factor} + \\ \text{(3lbs. raisins @ \$5 per lb.)} \times \text{scale factor}\end{bmatrix} = \$15$

$$12x + (2 \times 9)x + (3 \times 5)x = 15$$

$$12x + 18x + 15x = 15$$

$$45x = 15$$

$$x = \frac{15}{45}$$

$$x = \frac{1}{3}$$

The scale factor is $\frac{1}{3}$. If an entire batch of trail mix contains 6 pounds of ingredients, Donna needs to scale the recipe down and make only $\frac{1}{3}$ of that amount. In other words, Donna needs to make $\frac{1}{3} \times 6 = 2$ pounds of trail mix if she only has \$15 to spend.

PTS: 2 NAT: A.CED.A.1 TOP: Modeling Linear Equations

62) ANS: 3

STEP 1. Underline key words.

Kendal bought <u>*x* boxes</u> of cookies to bring to a party. <u>Each box contains 12 cookies</u>. She decides to keep two boxes for herself. She <u>brings 60 cookies to the party</u>. Which equation can be used to find the <u>number of boxes, *x*</u>, Kendal bought?

STEP 2. Define key terms.

Let 12 x represent the total number of cookies Kendal Bought.

Let 24 represent the total number of cookies Kendal kept for herself.

Let 60 represent the total number of cookies Kendal took to school.

STEP 3. Write

$$12x - 24 = 60$$

PTS: 2 NAT: A.CED.A.1

63) ANS: 2

Strategy: This is a coin problem, and the value of each coin is important.

Let x represent the number of dimes, as required by the problem.
Let .10x represent the value of the dimes. (A dime is worth $0.10)

The problem says that John has 4 more nickels than dimes.
Let $(x+4)$ represent the number of nickels that John has.
Let .05$(x+4)$ represent the value of the nickles. (A nickel is worth $0.05)

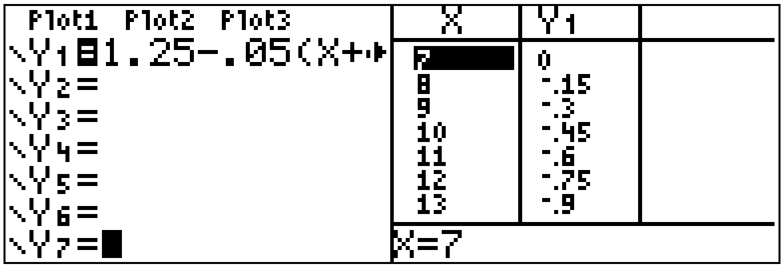
The total amount of money that John has is $1.25.
The total amount of money that John has can also be represented by $.10x+.05(x+4)$
These two expressions are both equal, so write:

$$.10x+.05(x+4)=\$1.25$$

This is not an answer choice, but using the commutative property, we can rearrange the order of the terms in the left expression $.05(x+4)+.10x=\$1.25$, which is the same as answer choice b.

DIMS? Does It Make Sense? Yes. Transform the equation for input into a graphing calculator as follows:
$.05(x+4)+.10x=\$1.25$

$0=\$1.25-.05(x+4)-.10x$



```
 Plot1  Plot2  Plot3
\Y1■1.25-.05(X+▸
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=■
```

```
   X      Y1
   7      0
   8      -.15
   9      -.3
   10     -.45
   11     -.6
   12     -.75
   13     -.9
X=7
```

John has 7 dimes and 11 nickles. The dimes are worth 70 cents and the nickels are word 55 cents. In total, John has $1.25.

PTS: 2 NAT: A.CED.A.1 TOP: Modeling Linear Equations

64) ANS:
2.4 years

Strategy: Convert all measurements to inches per year, then write two equations, then write and solve a new equation by equating the right expressions of the two equations.

STEP 1: Convert all measurements to inches per year.
Type *A* is 36 inches tall and grows at a rate of 15 inches per year.
Type *B* is 48 inches tall and grows at a rate of 10 inches per year.

STEP 2: Write 2 equations

$$G(A)=36+15t$$

$$G(B)=48+10t$$

STEP 3: Write and solve a break-even equation from the right expressions.

$$36 + 15t = 48 + 10t$$

$$15t - 10t = 48 - 36$$

$$5t = 12$$

$$t = \frac{12}{5}$$

$$t = 2.4 \text{ years}$$

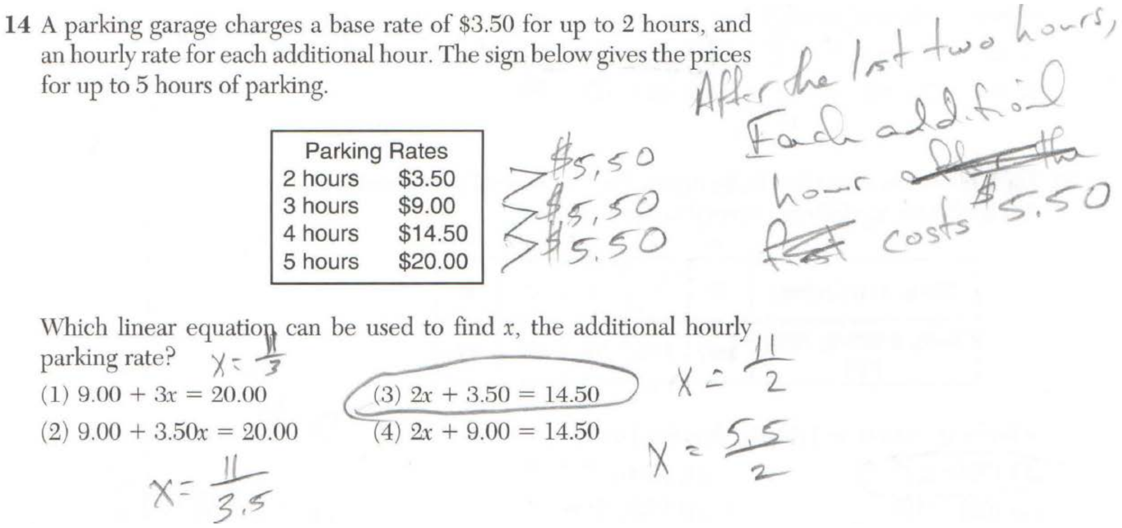
DIMS? Does It Make Sense? Yes. After 2.4 years, the type A trees and the type B trees will both be 72 inches tall.

$$G(A) = 36 + 15(2.4) = 36 + 36 = 72$$

$$G(B) = 48 + 10(2.4) = 48 + 24 = 72$$

PTS: 2 NAT: A.REI.C.6 TOP: Modeling Linear Equations
NOT: NYSED classifies this problem as A.CED.1: Create Inequations and Inequalities

65) ANS: 3



14 A parking garage charges a base rate of \$3.50 for up to 2 hours, and an hourly rate for each additional hour. The sign below gives the prices for up to 5 hours of parking.

Parking Rates	
2 hours	\$3.50
3 hours	\$9.00
4 hours	\$14.50
5 hours	\$20.00

\$5.50
\$5.50
\$5.50

After the 1st two hours, Each additional hour ~~after the~~ ~~first~~ costs \$5.50

Which linear equation can be used to find x, the additional hourly parking rate?

(1) $9.00 + 3x = 20.00$ $x = \frac{11}{3}$
(2) $9.00 + 3.50x = 20.00$ $x = \frac{11}{3.5}$
[✓ (3) $2x + 3.50 = 14.50$] $x = \frac{11}{2}$
(4) $2x + 9.00 = 14.50$ $x = \frac{5.5}{2}$

PTS: 2 NAT: A.CED.A.1

66) ANS:
$C(s) = 1.29 + .99(s - 1)$
Sandy is not correct. She used the wrong equation.

# Songs (s)	Correct Costs $C(s) = 1.29 + .99(s - 1)$	Sandy's Costs $C(s) = 1.29 + .99s$
1	1.29	2.28
2	2.28	3.27
3	3.27	4.26
----	----	----
52	51.76	52.77

PTS: 2 NAT: A.CED.A.2 TOP: Modeling Linear Equations

67) ANS: 4
Strategy: Translate the words into algebraic terms and expressions. Then eliminate wrong answers.

The problem tells us to:
Let *c* represent the total charges at the end of the month.
Let 60 represent the cost of 1 gigabyte of data.
Let d represent the cost of each megabyte of data after the first gigabyte.

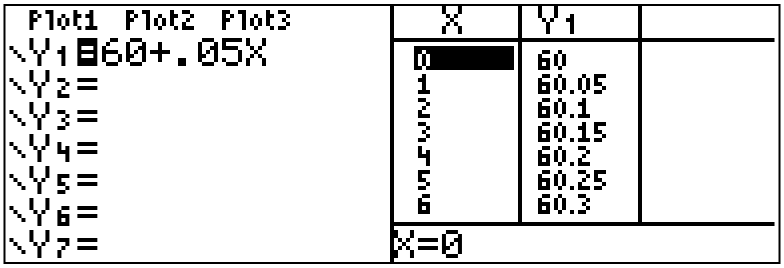
The total charges equal 60 plus .05d.
Write $c = 60 + .05d$. This is answer choice d.

DIMS? Does It Make Sense? Yes. $c = 60 + .05d$ could be used to represent the user's monthly bill. First, transpose the formula for input into the graphing calculator:

$$c = 60 + .05d$$

$$0 = 60 + .05x$$

$$Y_1 = 60 + .05x$$



```
 Plot1  Plot2  Plot3
\Y1=60+.05X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
   X   |  Y1   |
-------+-------+------
   0   |  60   |
   1   |  60.05|
   2   |  60.1 |
   3   |  60.15|
   4   |  60.2 |
   5   |  60.25|
   6   |  60.3 |
X=0
```

The table of values shows that the monthly charges increase 5 cents for every additional megabyte of data.

PTS: 2 NAT: A.CED.A.1 TOP: Modeling Linear Equations

68) ANS: 4
Strategy: Translate the words into algebraic terms and expressions. Then eliminate wrong answers.

The problem tells us to:
Let *C* represent the total cost.
Let *g* represent the number of gigabytes used.

The first sentence, "*A typical cell phone plan has a* ***fixed base fee*** *that includes a certain amount of data and an* ***overage charge*** *for data use beyond the plan*." tells us that total cost equals a base fee plus an overage charge. From this, we know that the basic equation will look something like

$$C = fixed\ base\ fee + overage\ ch\arg e$$

The second sentence tells us that "A cell phone plan charges a base fee of \$62 ...." so we can substitute this specific information into our general equation and we have

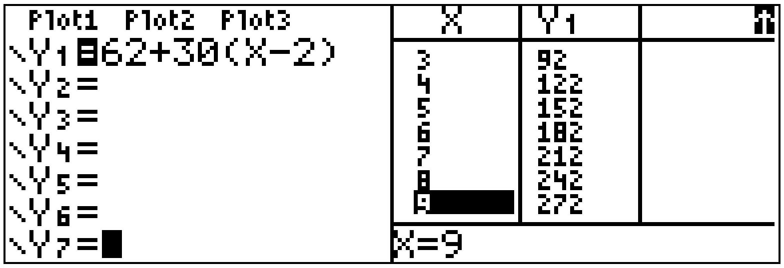
$$C = \$62 + overage\ ch\arg e$$

We can eliminate answer choices *a* and *b*. The correct answer is either *c* or d.

The second sentence also tells us that the overage charge is "...\$30 per gigabyte of data that exceed 2 gigabytes." We can use this information to choose between answer choices *c* and *d*.

Answer choice *c* is $C = 62 + 30(2 - g)$. This doesn't make sense, because the value of the term $30\left(2 - g\right)$ becomes negative if the number of gigabytes used is greater than 2, and the total cost becomes negative if the number of gigabytes used is 5 or more. Answer choice c can be eliminated. Answer choice d is the only choice left, and is the correct answer.

DIMS? Does It Make Sense? Yes. $C = 62 + 30(g - 2)$ could represent the plan *when more than 2 gigabytes are used*, as shown in the following table of values for this function..



Plot1 Plot2 Plot3
\Y1=62+30(X−2)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=

X	Y1
3	92
4	122
5	152
6	182
7	212
8	242
9	272

X=9

PTS: 2 NAT: A.CED.A.1 TOP: Modeling Linear Functions

C – Expressions and Equations, Lesson 5, Transforming Formulas (r. 2018)

EXPRESSIONS AND EQUATIONS

Transforming Formulas

Common Core Standard	Next Generation Standard
A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V=IR to highlight resistance R.*	**AI-A.CED.4** Rewrite formulas to highlight a quantity of interest, using the same reasoning as in solving equations. e.g., Rearrange Ohm's law $V = IR$ to highlight resistance R.

LEARNING OBJECTIVES

Students will be able to:

1) rewrite (transform) formulas to isolate specific variables.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

formula transform transformation isolate

BIG IDEAS

Properties and operations can be used to transform **formulas** to isolate different variables in the same ways that equations are manipulated to isolate a variable.

Example: The **formula** $P = 2l + 2w$ can be used to find the perimeter of a rectangle. In English, $P = 2l + 2w$ translates as "The ***p****erimeter equals two times the length plus two times the width.*" In the **formula** $P = 2l + 2w$, the P variable is already isolated. You can isolate the l variable or the w variables, as follows. (*Note that the steps and operations are the same as with regular equations.*)

To isolate the l variable:	To isolate the w variable:
Start with the formula:	Start with the formula:
$P = 2l + 2w$	$P = 2l + 2w$
Move the term 2w to the left expression.	Move the term $2l$ to the left expression.
$P - 2w = 2l$	$P - 2l = 2w$
Divide both sides of the equation by 2.	Divide both sides of the equation by 2.

$\frac{P-2w}{2}=l$ You now have a formula for *l* in terms of *P* and *w*.	$\frac{P-2l}{2}=w$ You now have a formula for *l* in terms of *P* and *w*.

DEVELOPING ESSENTIAL SKILLS

Isolate each variable in the Volume formula for a rectangular prism $V = lwh$.

$$V = lwh$$
$$\frac{V}{wh} = l$$
$$\frac{V}{lh} = w$$
$$\frac{V}{lw} = h$$

Isolate each variable in the slope intercept formula of a line $y = mx + b$.

$$y = mx + b$$
$$\frac{y-b}{x} = m$$
$$\frac{y-b}{m} = x$$
$$y - mx = b$$

REGENTS EXAM QUESTIONS

A.CED.A.4: Transforming Formulas

69) The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. The radius, *r*, of the cone may be expressed as

1) $\sqrt{\frac{3V}{\pi h}}$

2) $\sqrt{\frac{V}{3\pi h}}$

3) $3\sqrt{\frac{V}{\pi h}}$

4) $\frac{1}{3}\sqrt{\frac{V}{\pi h}}$

70) The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$. Express b_1 in terms of *A*, *h*, and b_2. The area of a trapezoid is 60 square feet, its height is 6 ft, and one base is 12 ft. Find the number of feet in the other base.

71) The equation for the volume of a cylinder is $V = \pi r^2 h$. The positive value of *r*, in terms of *h* and *V*, is

1) $r = \sqrt{\frac{V}{\pi h}}$

3) $r = 2V\pi h$

2) $r = \sqrt{V\pi h}$ 4) $r = \dfrac{V}{2\pi}$

72) The distance a free falling object has traveled can be modeled by the equation $d = \frac{1}{2}at^2$, where a is acceleration due to gravity and t is the amount of time the object has fallen. What is t in terms of a and d?

1) $t = \sqrt{\dfrac{da}{2}}$ 3) $t = \left(\dfrac{da}{d}\right)^2$

2) $t = \sqrt{\dfrac{2d}{a}}$ 4) $t = \left(\dfrac{2d}{a}\right)^2$

73) The volume of a large can of tuna fish can be calculated using the formula $V = \pi r^2 h$. Write an equation to find the radius, r, in terms of V and h. Determine the diameter, to the nearest inch, of a large can of tuna fish that has a volume of 66 cubic inches and a height of 3.3 inches.

74) Michael borrows money from his uncle, who is charging him simple interest using the formula $I = \mathrm{Pr}t$. To figure out what the interest rate, r, is, Michael rearranges the formula to find r. His new formula is r equals

1) $\dfrac{I-P}{t}$ 3) $\dfrac{I}{Pt}$

2) $\dfrac{P-I}{t}$ 4) $\dfrac{Pt}{I}$

75) The formula for the sum of the degree measures of the interior angles of a polygon is $S = 180(n-2)$. Solve for n, the number of sides of the polygon, in terms of S.

76) Solve the equation below for x in terms of a.

$$4(ax+3) - 3ax = 25 + 3a$$

77) Boyle's Law involves the pressure and volume of gas in a container. It can be represented by the formula $P_1V_1 = P_2V_2$. When the formula is solved for P_2, the result is

1) $P_1V_1V_2$ 3) $\dfrac{P_1V_1}{V_2}$

2) $\dfrac{V_2}{P_1V_1}$ 4) $\dfrac{P_1V_2}{V_1}$

78) The formula for blood flow rate is given by $F = \dfrac{p_1 - p_2}{r}$, where F is the flow rate, p_1 the initial pressure, p_2 the final pressure, and r the resistance created by blood vessel size. Which formula can *not* be derived from the given formula?

1) $p_1 = Fr + p_2$ 3) $r = F(p_2 - p_1)$

2) $p_2 = p_1 - Fr$ 4) $r = \dfrac{p_1 - p_2}{F}$

79) Using the formula for the volume of a cone, express r in terms of V, h, and π.

80) The formula $F_g = \frac{GM_1M_2}{r^2}$ calculates the gravitational force between two objects where *G* is the gravitational constant, M_1 is the mass of one object, M_2 is the mass of the other object, and *r* is the distance between them. Solve for the positive value of *r* in terms of F_g, *G*, M_1, and M_2.

81) Students were asked to write a formula for the length of a rectangle by using the formula for its perimeter, $p = 2\ell + 2w$. Three of their responses are shown below.

I. $\ell = \frac{1}{2}p - w$

II. $\ell = \frac{1}{2}(p - 2w)$

III. $\ell = \frac{p - 2w}{2}$

Which responses are correct?

1) I and II, only
2) II and III, only
3) I and III, only
4) I, II, and III

SOLUTIONS

69) ANS: 1

Strategy: Use the four column method.

Notes	Left Expression	Sign	Right Expression
Given	V	=	$\frac{1}{3}\pi r^2 h$
Multiply both expressions by 3	$3V$	=	$\pi r^2 h$
Divide both expressions by πh	$\frac{3V}{\pi h}$	=	$\frac{\pi r^2 h}{\pi h}$
Simplify	$\frac{3V}{\pi h}$	=	r^2
Take square root of both sides.	$\sqrt{\frac{3V}{\pi h}}$	=	r

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

70) ANS:

a) $b_1 = \frac{2A}{h} - b_2$

b) The other base is 8 feet.

Strategy: Use the four column method to isolate b_1 and create a new formula, then use it to find the length of the other base.

Notes	Left Expression	Sign	Right Expression
Given	A	=	$\frac{1}{2}h(b_1 + b_2)$
Multiply both expressions by 2	$2A$	=	$h(b_1 + b_2)$

Divide both expressions by *h*	$\frac{2A}{h}$	=	$\frac{h(b_1+b_2)}{h}$
Simplify	$\frac{2A}{h}$	=	b_1+b_2
Subtract b_2 from both expressions	$\frac{2A}{h}-b_2$	=	b_1

Substitute the values stated in the problem in the formula.

$$A = 60,\ h = 6,\ b_2 = 12$$

$$b_1 = \frac{2A}{h} - b_2$$

$$b_1 = \frac{2(60)}{6} - 12$$

$$b_1 = \frac{120}{6} - 12$$

$$b_1 = 20 - 12$$

$$b_1 = 8 \text{ feet}$$

PTS: 4 NAT: A.CED.A.4 TOP: Transforming Formulas

71) ANS: 1

Strategy: Use the four column method to isolate *r*.

Notes	Left Expression	Sign	Right Expression
Given	V	=	$\pi r^2 h$
Divide both expressions by πh	$\frac{V}{\pi h}$	=	$\frac{\pi r^2 h}{\pi h}$
Simplify	$\frac{V}{\pi h}$	=	r^2
Take square root of both expressions.	$\sqrt{\frac{V}{\pi h}}$	=	r

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

72) ANS: 2

Strategy: Use the four column method. Isolate *t*.

Notes	Left Expression	Sign	Right Expression
Given	d	=	$\frac{1}{2}at^2$
Multiply both expressions by 2	$2d$	=	at^2
Divide both expressions by *a*	$\frac{2d}{a}$	=	$\frac{at^2}{a}$
Simplify	$\frac{2d}{a}$	=	t^2

Take square root of both expressions	$\sqrt{\frac{2d}{a}}$	=	t

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

73) ANS:

a) $r = \sqrt{\frac{V}{\pi h}}$

b) 5 inches

Strategy: Use the four column method to isolate *r* and create a new formula, then use the new formula to answer the problem.

Notes	Left Expression	Sign	Right Expression
Given	V	=	$\pi r^2 h$
Divide both expressions by πh	$\frac{V}{\pi h}$	=	$\frac{\pi r^2 h}{\pi h}$
Simplify	$\frac{V}{\pi h}$	=	r^2
Take square root of both expressions.	$\sqrt{\frac{V}{\pi h}}$	=	r

Substitute the values from the problem into the new equation.

$$V = 66,\ \ h = 3.3$$

$$r = \sqrt{\frac{V}{\pi h}}$$

$$r = \sqrt{\frac{66}{\pi(3.3)}}$$

$$r = \sqrt{\frac{20}{\pi}}$$

$$r \approx \sqrt{6.4}$$

$$r \approx 2.52$$

If the radius is approximately 2.5 inches, the diameter is approximately 5 inches.

PTS: 4 NAT: A.CED.A.4 TOP: Transforming Formulas

74) ANS: 3

Strategy: Isolate r, as follows:

$$I = \mathrm{Pr}t$$

$$I = Pt(r)$$

$$\frac{I}{Pt} = r$$

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

75) ANS:

$$S = 180(n-2)$$

$$S = 180n - 360$$

$$S + 360 = 180n$$

$$\frac{S+360}{180} = n$$

or

$$\frac{S}{180} + 2 = n$$

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

76) ANS:

$x = \frac{13}{a} + 3$

$$4(ax+3) - 3ax = 25 + 3a$$

$$4ax + 12 - 3ax = 25 + 3a$$

$$ax + 12 = 25 + 3a$$

$$ax = 13 + 3a$$

$$ax - 3a = 13$$

$$a(x-3) = 13$$

$$x - 3 = \frac{13}{a}$$

$$x = \frac{13}{a} + 3$$

PTS: 2 NAT: A.CED.A.4

77) ANS: 3

Given	P_1V_1	=	P_2V_2
Divide by V_2	$\frac{P_1V_1}{V_2}$	=	$\frac{P_1\cancel{V_2}}{\cancel{V_2}}$
Simplify	$\frac{P_1V_1}{V_2}$	=	P_1

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

78) ANS: 3

$$F = \frac{p_1 - p_2}{r}$$

$$rF = p_1 - p_2$$

$$r = \frac{p_1 - p_2}{F}$$

If $r = \dfrac{p_1 - p_2}{F}$, then $r = F(p_2 - p_1)$ cannot be true.

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

79) ANS:

$$V = \frac{1}{3}\pi r^2 h.$$

$$3V = \pi r^2 h$$

$$\frac{3V}{\pi h} = \frac{\pi r^2 h}{\pi h}$$

$$\frac{3V}{\pi h} = r^2$$

$$\sqrt{\frac{3V}{\pi h}} = r$$

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

80) ANS:

$$F_g = \frac{GM_1M_2}{r^2}$$

$$r^2F_g = GM_1M_2$$

$$r^2 = \frac{GM_1M_2}{F_g}$$

$$r = \sqrt{\frac{GM_1M_2}{F_g}}$$

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

81) ANS: 4

Strategy: Transform the formula to isolate the l variable.

$$p = 2\ell + 2w$$

$$p - 2w = 2l$$

$$\frac{p - 2w}{2} = l$$

This is solution III.

NOTE that solution III can also be expressed as:

$$\frac{1}{2}(p - 2w) = l$$

This is solution II.

NOTE also that the distributive property of multiplication can transform solution II into:

$$\frac{1}{2}p - w = l$$

This is solution I.

The correct answer choice is I, II, and III.

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

D – Rate, Lesson 1, Conversions (r. 2018)

RATE
Conversions

Common Core Standard	Next Generation Standard
N.Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	**AI-N.Q.1** Select quantities and use units as a way to: i) interpret and guide the solution of multi-step problems; ii) choose and interpret units consistently in formulas; and iii) choose and interpret the scale and the origin in graphs and data displays.

LEARNING OBJECTIVES

Students will be able to:

1) Understand units as essential to understanding and interpreting graphs.
2) Understand units as essential to problem solving.
3) Use and convert units when problem solving.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

cancellation
convert / conversion
cross-multiplication
denominator
numerator
per
proportion
rate
ratio
scale

BIG IDEAS

Big Units - Small Units

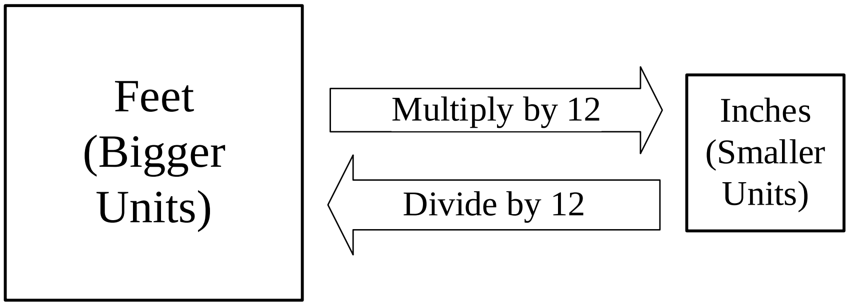
As a general rule, big units are used to measure big things and small units are used to measure small things.
For example,

- the distance from New York City to San Francisco is a big distance, so it would be measured in big units, like miles or kilometers.
- the distance from a student's elbow to the tip of his or her finger is a small distance, so it would be measured in small units, like inches or centimeters.

Since both big and small units can be used to measure the same thing, it is sometimes desirable to change from one unit of measurement to another. When different size units are used to measure the same thing:

- Changing from a big unit to a small unit involves multiplication.
- Changing from a small unit to a big unit involves division.

For example, 1 foot = 12 inches. The following diagram shows the mathematical operations involved in converting feet to inches and inches to feet.



Ratios, Rates, and Proportions

A **ratio** is a simple comparison of two numbers, such as 12:1 or 1:12.

A **rate** is a ratio that includes units, such as $\frac{12 \text{ inches}}{1 \text{ foot}}$, which is read as “Twelve inches *per* foot,” or $\frac{1 \text{ foot}}{12 \text{ inches}}$, which is read as “One foot *per* twelve inches.” The vinculum (fraction bar) is read as the word “*per*.”

A **unit rate** is a rate with a denominator of 1 unit. $\frac{12 \text{ inches}}{1 \text{ foot}}$ is a unit rate. $\frac{1 \text{ foot}}{12 \text{ inches}}$ is *not* a unit rate.

NOTE: When working with problems that involve rates, it is common practice to omit the unit labels and manipulate ratios instead of rates. While this increases computational efficiency, it can lead to conceptual errors. While $\frac{12 \text{ inches}}{1 \text{ foot}}$ and $\frac{1 \text{ foot}}{12 \text{ inches}}$ express the same mathematical relationship between inches and feet, this is because they are rates – not ratios. When the units are omitted, these rates become the ratios, $\frac{12}{1}$ and $\frac{1}{12}$, which are *not* the same mathematical relationship. After a long series of computations with ratios, it is easy to mislabel the units. Therefore, a good practice is to always make notes about the units.

A **proportion** is an equation with two ratios and an equal sign between them. For example, $\frac{1}{4} = \frac{4}{16}$ is a proportion.

- Every proportion has four parts: two numerators and two denominators.

- When three of the four parts in a proportion are given, it is possible to solve for the fourth part using **cross-multiplication**. An example of using cross-multiplication to solve a proportion in which one part is unknown follows:

Notes	Left Expression	Sign	Right Expression
Given	$\frac{1}{4}$	=	$\frac{4}{x}$
Cross Multiply	$1(x)$	=	$4(4)$
Solution	x	=	16

- When using proportions to solve unit conversion problems, it is important to label the units to avoid conceptual errors. This is done by simply adding units notation when setting up the proportion. In the following example, the rate of 4 quarts per 1 gallon is used to find the number of quarts in 4 gallons. All the numerators are gallon units and all the denominators are quarts units. Since the x is in a denominator, the answer will be in quarts units.

Notes	Left Expression	Sign	Right Expression
Given	$\left.\frac{gallons}{quarts}\right\vert\frac{1}{4}$	=	$\frac{4}{x}$
Cross Multiply	$1(x)$	=	$4(4)$
Solution	x	=	16 *quarts*

Cancellation of Units (Factor-Unit Conversion)

Cancellation can be used to simplify units within a single expression. The general approach is to consider the units as factors, which can be cancelled using the same rules that are used for cancellation of fractions.

In the following example, cancellation is to find the number of seconds in a year.

$$\frac{60\text{ seconds}}{1\ \cancel{\text{minute}}}\times\frac{60\ \cancel{\text{minutes}}}{1\ \cancel{\text{hour}}}\times\frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{day}}}\times\frac{365\ \cancel{\text{days}}}{1\text{ year}}=\frac{60\times60\times24\times365\text{ seconds}}{1\text{ year}}=\frac{30{,}536{,}000\text{ seconds}}{1\text{year}}$$

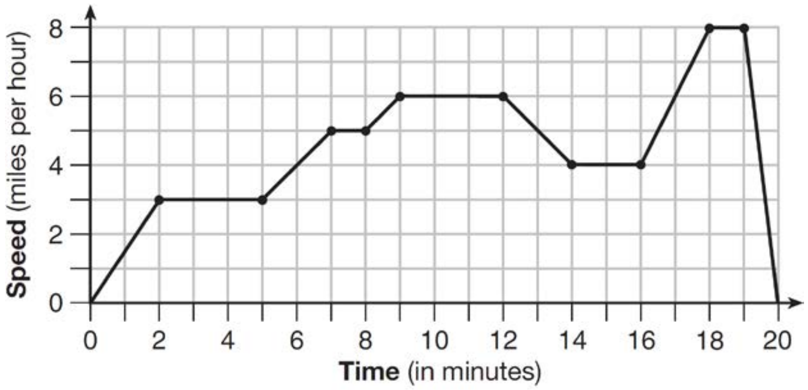
Another example of using cancellation is converting 10 miles per hour to meters per second.

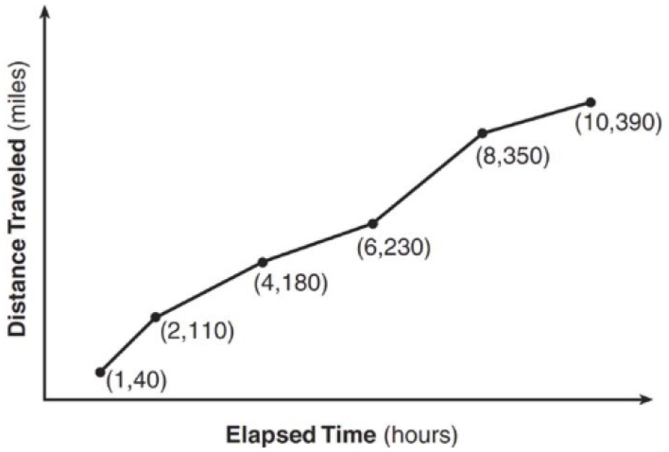
$$\frac{10\ \cancel{\text{miles}}}{1\ \cancel{\text{hours}}}\times\frac{1609.344\text{ meters}}{1\ \cancel{\text{miles}}}\times\frac{1\ \cancel{\text{hours}}}{3600\text{ seconds}}=\frac{16093.44\text{ miles}}{3600\text{ seconds}}=\frac{4.4704\text{ meters}}{1\text{ second}}$$

Units and Graphs

A graph is one view of the relationship between two variables. The variables are measured in specific units, which are very important to understanding the meaning of the graph.

Example: The two graphs below are from different Regents problems. The units for the *x*-axis both measure time, but the units are different. The units for the y-axis are totally different kinds of measurements. The different units used require different interpretations of the two graphs.





Conversions Chart Used in Regents Algebra 1 (Common Core) Exams

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilogram
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

DEVELOPING ESSENTIAL SKILLS

Convert the following:	Solutions
20 kilometers to feet	$\frac{1\ \cancel{\text{mile}}}{1.609\ \cancel{\text{kilometers}}}\times\frac{5280\text{ feet}}{1\ \cancel{\text{mile}}}\times\frac{20\ \cancel{\text{kilometers}}}{1}=\frac{1\times5280\times20\text{feet}}{1.609\times1\times1}$ $=\frac{105{,}600\text{feet}}{1.609}$ $\approx 65{,}630$ feet

30 kilometers per hour to miles per hour	$\frac{30\ \cancel{\text{kilometers}}}{1\ \text{hour}} \times \frac{1\ \text{mile}}{1.609\ \cancel{\text{kilometers}}} = \frac{30\times1\ \text{miles}}{1\times1.609\ \text{hours}} = \frac{30\text{miles}}{1.609\text{hours}} \approx \frac{18.65\ \text{miles}}{1\ \text{hour}}$
1 cubic foot to cubic centimeters	$\frac{1\ \text{cubic foot}}{12\times12\times12\ \cancel{\text{cubic inches}}} \times \frac{1\ \cancel{\text{cubic inch}}}{2.54\times2.54\times2.54\ \text{cubic centimeters}} = \frac{1\times1\ \text{cubic foot}}{12\times12\times12\times2.54\times2.54\times2.54\ \text{cubic centimeters}} \approx \frac{1\ \text{cubic foot}}{28{,}316.8\ \text{cubic centimeters}}$

REGENTS EXAM QUESTIONS (through June 2018)

N.Q.A.l: Conversions

82) Peyton is a sprinter who can run the 40-yard dash in 4.5 seconds. He converts his speed into miles per hour, as shown below.

$$\frac{40\text{ yd}}{4.5\text{ sec}} \cdot \frac{3\text{ ft}}{1\text{ yd}} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{60\text{ sec}}{1\text{ min}} \cdot \frac{60\text{ min}}{1\text{ hr}}$$

Which ratio is *incorrectly* written to convert his speed?

1) $\frac{3\text{ ft}}{1\text{ yd}}$
2) $\frac{5280\text{ ft}}{1\text{ mi}}$
3) $\frac{60\text{ sec}}{1\text{ min}}$
4) $\frac{60\text{ min}}{1\text{ hr}}$

83) Dan took 12.5 seconds to run the 100-meter dash. He calculated the time to be approximately

1) 0.2083 minute
2) 750 minutes
3) 0.2083 hour
4) 0.52083 hour

84) Faith wants to use the formula $C(f) = \frac{5}{9}(f - 32)$ to convert degrees Fahrenheit, f, to degrees Celsius, $C(f)$.

If Faith calculated $C(68)$, what would her result be?

1) 20° Celsius
2) 20° Fahrenheit
3) 154° Celsius
4) 154° Fahrenheit

85) A typical marathon is 26.2 miles. Allan averages 12 kilometers per hour when running in marathons. Determine how long it would take Allan to complete a marathon, to the *nearest tenth of an hour*. Justify your answer.

86) A construction worker needs to move 120 ft^3 of dirt by using a wheelbarrow. One wheelbarrow load holds 8 ft^3 of dirt and each load takes him 10 minutes to complete. One correct way to figure out the number of hours he would need to complete this job is

1) $\frac{120\text{ ft}^3}{1} \bullet \frac{10\text{ min}}{1\text{ load}} \bullet \frac{60\text{ min}}{1\text{ hr}} \bullet \frac{1\text{ load}}{8\text{ ft}^3}$
3) $\frac{120\text{ ft}^3}{1} \bullet \frac{1\text{ load}}{10\text{ min}} \bullet \frac{8\text{ ft}^3}{1\text{ load}} \bullet \frac{1\text{ hr}}{60\text{ min}}$

2) $\frac{120\text{ ft}^3}{1} \bullet \frac{60\text{ min}}{1\text{ hr}} \bullet \frac{8\text{ ft}^3}{10\text{ min}} \bullet \frac{1}{1\text{ load}}$

4) $\frac{120\text{ ft}^3}{1} \bullet \frac{1\text{ load}}{8\text{ ft}^3} \bullet \frac{10\text{ min}}{1\text{ load}} \bullet \frac{1\text{ hr}}{60\text{ min}}$

87) The Utica Boilermaker is a 15-kilometer road race. Sara is signed up to run this race and has done the following training runs:

I. 10 miles
II. 44,880 feet
III. 15,560 yards

Which run(s) are at least 15 kilometers?

1) I, only
2) II, only
3) I and III
4) II and III

SOLUTIONS

82) ANS: 2

Strategy: Work through each step of the problem and ask the DIMS question. Does It Make Sense.

STEP 1. $\frac{40\text{ yards}}{4.5\text{ seconds}} \times \frac{3\text{ feet}}{1\text{ yard}} = \frac{120\text{ feet}}{4.5\text{ seconds}}$ This makes sense. The yard units cancel and Peyton's speed becomes measured in feet per second instead of yards per second. We take the ratio of $\frac{120\text{ feet}}{4.5\text{ seconds}}$ to the next step in our analysis.

STEP 2. $\frac{120\text{ feet}}{4.5\text{ seconds}} \times \frac{5280\text{ feet}}{1\text{ mile}} = \frac{120 \times 5280\text{ feet}^2}{4.5\text{ second miles}}$. This does not make sense. The speed of a runner would not be measured in feet^2 per second miles. The problem is that the numerator and denominator are switched. It should be $\frac{1\text{ mile}}{5280\text{ feet}}$. When the numerator and denominator are changed, the problem becomes $\frac{120\text{ feet}}{4.5\text{ seconds}} \times \frac{1\text{ mile}}{5280\text{ feet}} = \frac{120\text{ miles}}{23{,}760\text{ seconds}}$. The feet units cancel and our measurement of Peyton's speed has distance over time, which makes sense. Answer choice b is selected to show that this ratio is *incorrectly* written.

STEP 3. Though we have solved the problem, we can continue our step by step analysis by taking the ratio of $\frac{120\text{ miles}}{23{,}760\text{ seconds}}$ to the next step in our analysis. The problem now becomes $\frac{120\text{ miles}}{23{,}760\text{ seconds}} , \frac{60\text{ seconds}}{1\text{ minute}} = \frac{120 \times 60\text{ miles}}{23{,}760 \times 1\text{minutes}} = \frac{72{,}000\text{ miles}}{23{,}760\text{ minutes}}$. This makes sense. The seconds units cancel and we again have distance over miles. We take the ratio $\frac{72{,}000\text{ miles}}{23{,}760\text{ minutes}}$ to the next step.

STEP 4. $\frac{72{,}000\text{ miles}}{23{,}760\text{ minutes}} \times \frac{60\text{ minutes}}{1\text{ hour}} = \frac{72{,}000 \times 60\text{ miles}}{23{,}760 \times 1\text{ hours}} = \frac{432{,}000\text{ miles}}{23{,}760\text{ hours}} = 18\frac{2}{11}$ miles per hour. This makes sense. Peyton is a fast sprinter.

PTS: 2 NAT: N.Q.A.1 TOP: Conversions

83) ANS: 1

Step 1. Read both the question and the answers. Understand that the problem is asking you to convert seconds into either minutes or hours. The 100 meters is constant, so it is not important to the problem of converting time into minutes or hours.
Step 2. Create two proportions using the conversion rates of 1) 60 seconds per minute; and 2) 3600 seconds per hour, to express 12.5 seconds in minutes and hours.
Step 3. Execute the strategy.

12.5 second equals how many minutes?	12.5 second equals how many hours?
$\left.\frac{\text{seconds}}{\text{minutes}}\right\vert \quad \frac{12.5}{x} = \frac{60}{1}$ $12.5 = 60x$ $\frac{12.5}{60} = x$ $.208\overline{3}\text{ minutes} = x$	$\left.\frac{\text{seconds}}{\text{hours}}\right\vert \quad \frac{12.5}{x} = \frac{3600}{1}$ $12.5 = 3600x$ $\frac{12.5}{3600} = x$ $.00347\overline{2}\text{ hours} = x$

The correct choice is a), 12.5 seconds equals 0.2083 minutes.
4. Does it make sense? Yes. It is obvious that 12.5 seconds does not equal 750 minutes (choice b) and it is also obvious that 12.5 seconds is not more than half an hour (choice d). The only choice that is less than a minute is choice a), and 12.5 seconds is definitely less than a minute.

PTS: 2 NAT: N.Q.A.1 TOP: Conversions KEY: dimensional analysis

84) ANS: 1

Given Find $C(68)$	$C(f)$	=	$\frac{5}{9}(f-32)$
Substitute 68 for *f*	$C(68)$	=	$\frac{5}{9}(68-32)$
Solve inside parentheses	$C(68)$	=	$\frac{5}{9}(36)$
Simplify Fraction Using Cancellation	$C(68)$	=	$\frac{5}{1}(4)$
Simplify Right Expression	$C(68)$	=	20

PTS: 2 NAT: N.Q.A.1 TOP: Conversions KEY: formula

85) ANS:
3.5 hours

Note: 1 kilometer = 0.62 miles
Step 1. Convert 12 kilometers per hour to miles per hour.

$$\left.\frac{\text{Miles}}{\text{Kilometers}}\right\vert \quad \frac{.62}{1} = \frac{x}{12}$$

Allan averages 7.44 miles per hour.

$$12(.62) = 7.44$$

Step 2. Use the speed formula to find time.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$7.44 = \frac{26.2}{\text{time}}$$

$$\text{time} = \frac{26.2}{7.44}$$

$$\text{time} = 3.52 \text{ hours}$$

Step 3. Round to the nearest tenth of an hour.

$$3.52 \approx 3.5$$

PTS: 2 NAT: N.Q.A.1 TOP: Conversions KEY: dimensional analysis

86) ANS: 4

The units for the correct solution must be in hours.

$\frac{120 \text{ ft}^3}{1} \bullet \frac{10 \text{ min}}{1 \text{ load}} \bullet \frac{60 \text{ min}}{1 \text{ hr}} \bullet \frac{1 \text{ load}}{8 \text{ ft}^3}$: Wrong. After cancellations, the remaining units are $\frac{\text{min}^2}{hr}$

$\frac{120 \text{ ft}^3}{1} \bullet \frac{60 \text{ min}}{1 \text{ hr}} \bullet \frac{8 \text{ ft}^3}{10 \text{ min}} \bullet \frac{1}{1 \text{ load}}$: Wrong. After cancellations, the remaining units are $\frac{ft^3}{load}$

$\frac{120 \text{ ft}^3}{1} \bullet \frac{1 \text{ load}}{10 \text{ min}} \bullet \frac{8 \text{ ft}^3}{1 \text{ load}} \bullet \frac{1 \text{ hr}}{60 \text{ min}}$: Wrong. After cancellations, the remaining units are $\frac{ft^3 hr}{\text{min}^2}$

$\frac{120 \text{ ft}^3}{1} \bullet \frac{1 \text{ load}}{8 \text{ ft}^3} \bullet \frac{10 \text{ min}}{1 \text{ load}} \bullet \frac{1 \text{ hr}}{60 \text{ min}}$: Correct. After cancellations, the remaining units are $\frac{hr}{1}$.

PTS: 2 NAT: N.Q.A.1 TOP: Conversions KEY: dimensional analysis

87) ANS: 1

Strategy: Convert the distance of each training run to kilometers.

I. 1 kilometer equals approximately 0.62 miles, so 1 mile equals approximately $\frac{1}{0.62} = 1.61$ kilometers.

Ten miles equals approximately $10 \times 1.61 = 16.1$ kilometers. Therefore, ten miles is greater than 15 kilometers.

II. One mile contains 5,280 feet, so 44,880 feet equals $\frac{44,880}{5280} = 8.5$ miles. 8.5 miles times 1.61 kilometers per mile equals approximately $8.5 \times 1.61 = 13.69$ kilometers. Therefore, 44,880 feet is less than 15 kilometers.

III. One yard contains three feet, so 15,560 yards equals $15,560 \times 3 = 46,680$ feet. 46,680 feet equals approximately $\frac{44,680}{5,280} = 8.84$ miles. 8.84 miles equals $8.84 \times 1.61 = 14.23$ kilometers. Therefore, 15,560 yards is less than 15 kilometers.

The only training run that is longer than 15 kilometers is the ten mile training run.

PTS: 2 NAT: N.Q.A.1 TOP: Conversions KEY: dimensional analysis

D – Rate, Lesson 2, Using Rate (r. 2018)

RATE

Using Rate

Common Core Standard	**Next Generation Standard**
N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling. PARCC: In Algebra I, this standard will be assessed by ensuring that some modeling tasks (involving Algebra I content or securely held content from grades 6-8) require the student to create a quantity of interest in the situation being described. For example, a quantity of interest is not selected for the student by the task. For example, In a situation involving data, the student might autonomously decide that a measure of center is a key variable in a situation, and then choose to work with the mean.	**STANDARD REMOVED**

LEARNING OBJECTIVES

Students will be able to:

1) Use conversion rates to solve problems involving scale.
2) Use unit conversion rates and the operations of multiplication and division to convert units.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

conversion rate	proportion	scale	unit

BIG IDEAS

It is important to understand the units and scales used in mathematical representations. As a general rule, big units should be used to measure big things and small units are used to measure small things. Real world events are often modeled using scaled representations.

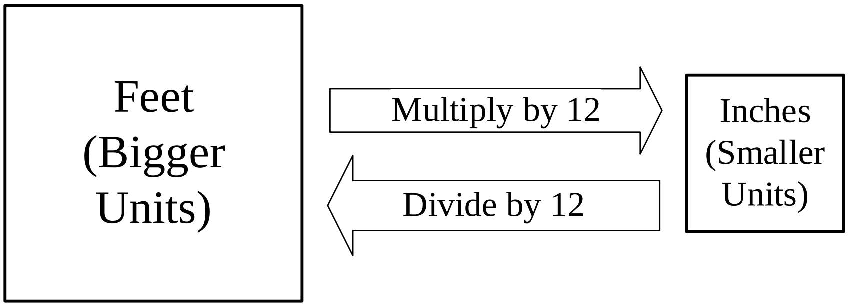
A **scale** is a ratio of the $\frac{\text{measurement of a model}}{\text{measurement of the real thing}}$.

Example. A toy car is 1 foot long. The real car it represents is 20 feet long. The scale of the model is:

$$\frac{\text{measurement of toy car}}{\text{measurement of real car}} = \frac{1 \text{ feet}}{20 \text{ feet}} = \frac{1}{20} \text{ or } 1{:}20$$

Scales may also be expressed in rates. For example, a map might have a scale expressed as $\frac{\text{1 inch}}{\text{5 miles}}$, or a graph might use scaled intervals of various units on the x-axis and y-axis.

When using scales for representation, it is important to know whether you are going from smaller units to larger units, or from larger units to smaller units, as shown in the following graphic.



A unit conversion rate because it states the value of 1 unit in terms of another unit. Unit conversion rates are typically used in conversion tables. For example, 1 inch = 2.54 centimeters. Proportions and cross multiplication can be used to convert a unit conversion rate for one unit into to a unit conversion rate for the other unit. For example:

$$\left.\frac{inches}{centimeters}\right|\frac{1}{2.54}=\frac{\mathrm{x}}{1}$$

$$1 = 2.54x$$

$$\frac{1}{2.54} = x$$

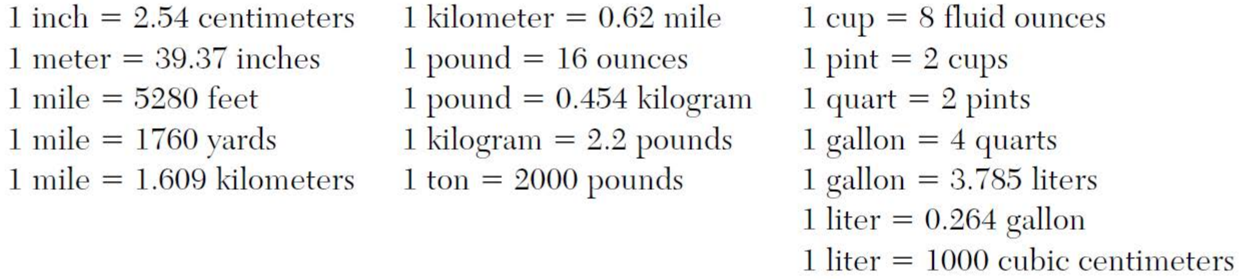
$$0.39 = x$$

This tells us that 1 centimeter = 0.39 inches.

DEVELOPING ESSENTIAL SKILLS

Use the conversion chart to state whether multiplication or division should be used when converting from one unit to the other unit. Specify the multiplicand or divisor for each operation.

Conversions Chart Used in Regents Algebra 1 (Common Core) Exams



1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilogram
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

From	To	Operation Used
inches	centimeters	multiply by 2.54
centimeters	inches	divide by 2.54
meters	inches	multiply by 39.37
inches	meters	divide by 39.37
miles	feet	multiply by 5280
Feet	miles	divide by 5280
miles	kilometers	multiply by 0.62
kilometers	miles	divide by 0.62
pounds	ounces	multiply by 16
ounces	pounds	divide by 16
pounds	kilograms	divide by 2.2 or multiply by 0.454
kilograms	pounds	multiply by 2.2
ton	pound	multiply by 2.2
pound	ton	divide by 2000
cup	fluid ounces	multiply by 8
fluid ounces	cups	divide by 8
pint	cups	multiply by 2
cups	pints	divide by 2
quart	pints	multiply by 2
pints	quarts	divide by 2
gallons	quarts	multiply by 4
quarts	gallons	divide by 4
gallons	liters	multiply by 3.785
liters	gallons	divide by 3.785
liters	cubic centimeters	multiply by 1000
centimeters	liters	divide by 1000

REGENTS EXAM QUESTIONS (through June 2018)

N.Q.A.2: Using Rate

88) Patricia is trying to compare the average rainfall of New York to that of Arizona. A comparison between these two states for the months of July through September would be best measured in

1) feet per hour	3) inches per month
2) inches per hour	4) feet per month

89) A two-inch-long grasshopper can jump a horizontal distance of 40 inches. An athlete, who is five feet nine, wants to cover a distance of one mile by jumping. If this person could jump at the same ratio of body-length to jump-length as the grasshopper, determine, to the *nearest jump*, how many jumps it would take this athlete to jump one mile.

90) The distance traveled is equal to the rate of speed multiplied by the time traveled. If the distance is measured in feet and the time is measured in minutes, then the rate of speed is expressed in which units? Explain how you arrived at your answer.

SOLUTIONS

88) ANS: 3

Rainfall is not typically measured in feet, so eliminate choices *a* and *b*.
An hourly rate would not be meaningful.

PTS: 2 NAT: N.Q.A.1

89) ANS:

Strategy 1: Use proportional reasoning and work with inch units.
If a 2 inch long grasshopper can jump 40 inches, the grasshopper can jump 20 times its body length.
If a 5-feet nine-inch person could jump 20 times his body length, he could jump $69 \times 20 = 1380$ inches.
A mile is 5,280 feet long, or $5280 \times 12 = 63,360$ inches.

$$\frac{63,360 \text{ inches}}{1380 \text{ inches per jump}} = 45.913\ldots \approx 46 \text{ jumps}$$

Strategy 2: Use proportional reasoning and work with feet units

$$\left.\frac{\text{Body Length}}{\text{Horizontal Jump}}\right|\ \frac{2 \text{ inches}}{40 \text{ inches}} = \frac{5.75 \text{ feet}}{x \text{ feet}}$$

$$2x = 40 \times 5.75$$

$$2x = 230$$

$$x = 115 \text{ feet}$$

$$\left.\frac{\text{One Mile}}{\text{One Jump}}\right|\ \frac{5,280 \text{ feet}}{115 \text{ feet}} \approx 46 \text{ jumps}$$

PTS: 2 NAT: N.Q.A.2 TOP: Using Rate

90) ANS:

Speed would be measured in feet per minute.
Explanation:
The problem tells us that distance (d) equals speed (s) multiplied by time (t).

Therefore:

$$d = st$$

and

$$s = \frac{d}{t}$$

If distance units are measured in feet and time units are measured in minutes, then:

$$s = \frac{\text{d feet}}{\text{t minutes}}$$

PTS: 2 NAT: N.Q.A.2 TOP: Using Rate

D – Rate, Lesson 3, Speed (r. 2018)

RATE
Speed

Common Core Standard	**Next Generation Standard**
A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; ~~graph equations on coordinate axes with labels and scales~~.	**AI-A.CED.2** Create equations and linear inequalities in two variables to represent a **real-world context**. Notes: • This is strictly the development of the model (equation/inequality). • Limit equations to linear, quadratic, and exponentials of the form $f(x) = a(b)_x$ where $a > 0$ and $b > 0$ ($b \neq 1$).

LEARNING OBJECTIVES

Students will be able to:

1) Solve problems involving the speed formula: $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

distance	speed	time

BIG IDEAS

$$\text{speed} = \frac{\text{distance}}{\text{time}} \qquad s = \frac{d}{t}$$

$$\text{distance} = \text{speed} \times \text{time} \qquad d = st$$

$$\text{time} = \frac{\text{distance}}{\text{speed}} \qquad t = \frac{d}{s}$$

DEVELOPING ESSENTIAL SKILLS

Questions	**Answers**
An airplane travels 700 miles in two hours. What is its average speed?	$s=\frac{d}{t}=\frac{700\text{ miles}}{2\text{ hours}}=350\text{ miles per hour}$
A train travel 400 miles in 8 hours. What is its average speed?	$s=\frac{d}{t}=\frac{400\text{ miles}}{8\text{ hours}}=50\text{ miles per hour}$
A car's average speed is 60 miles per hour. How far has it travelled after 6 hours?	$s=\frac{d}{t}$ $st=d$ $\frac{60\text{ miles}}{1\ \cancel{\text{hour}}}\times\frac{6\ \cancel{\text{hours}}}{1}=\text{distance}$ $\frac{60\times 6\text{ miles}}{1\times 1}=\text{distance}$ $360\text{ miles}=\text{distance}$
A car averages 55 miles per hour. How long will it take to travel 300 miles, to the hour and *nearest minute*?	$s=\frac{d}{t}$ $t=\frac{d}{s}$ $t=\frac{300\ \cancel{\text{miles}}}{55\ \cancel{\text{miles}}\text{ per hour}}$ $t=5.\overline{45}\text{ hours}$ $\frac{\text{hours}}{\text{minutes}}\Big\vert\frac{1}{60}=\frac{0.\overline{45}}{x}$ $x=60\times 0.\overline{45}$ $x\approx 27\text{ minutes}$ total time: 5 hours and 27 minutes

REGENTS EXAM QUESTIONS

A.CED.A.2: Speed

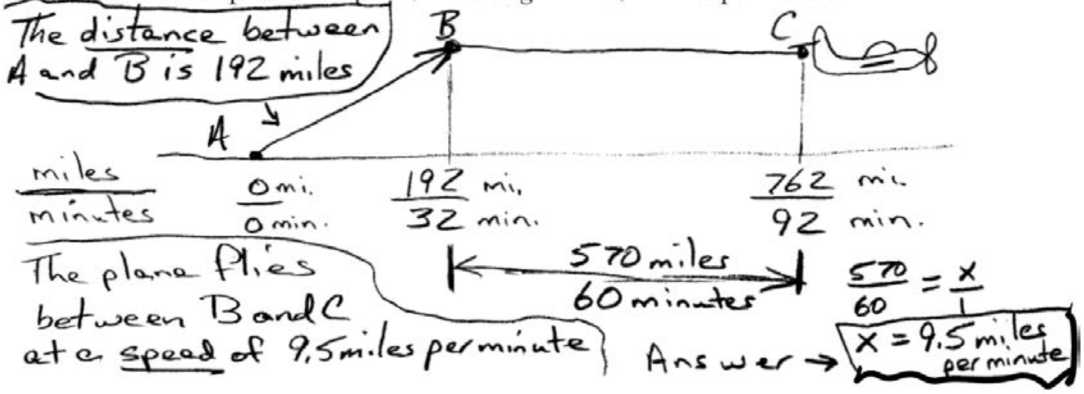
91) An airplane leaves New York City and heads toward Los Angeles. As it climbs, the plane gradually increases its speed until it reaches cruising altitude, at which time it maintains a constant speed for several hours as long as it stays at cruising altitude. After flying for 32 minutes, the plane reaches cruising altitude and has flown 192 miles. After flying for a total of 92 minutes, the plane has flown a total of 762 miles. Determine the speed of the plane, at cruising altitude, in miles per minute. Write an equation to represent the number of miles the plane has flown, y, during x minutes at cruising altitude, only. Assuming that the plane maintains its speed at cruising altitude, determine the total number of miles the plane has flown 2 hours into the flight.

92) Loretta and her family are going on vacation. Their destination is 610 miles from their home. Loretta is going to share some of the driving with her dad. Her average speed while driving is 55 mph and her dad's average speed while driving is 65 mph. The plan is for Loretta to drive for the first 4 hours of the trip and her dad to drive for the remainder of the trip. Determine the number of hours it will take her family to reach their destination. After Loretta has been driving for 2 hours, she gets tired and asks her dad to take over. Determine, to the *nearest tenth of an hour*, how much time the family will save by having Loretta's dad drive for the remainder of the trip.

SOLUTIONS

91) ANS:
Strategy: Draw a picture to model the problem.

Determine the speed of the plane, at cruising altitude, in miles per minute.



At cruising altitude, the plane is flying at the speed of 9.5 miles per minute.

Write an equation to represent the number of miles the plane has flown, *y*, during *x* minutes at cruising altitude, only. (NOTE: This is line segment $\overline{BC}$ in the above picture.

$$y = 9.5x$$

Assuming that the plane maintains its speed at cruising altitude, determine the total number of miles the plane has flown 2 hours into the flight.

Let *M* represent the total miles flown. Let *t* represent the number of minutes flown.

$$M(t) = 9.5(t-32) + 192$$

$$M(120) = 9.5(120-32) + 192$$

$$M(120) = 9.5(88) + 192$$

$$M(120) = 836 + 192$$

$$M(120) = 1028$$

2 hours into the flight, the plane has flown 1,028 miles.

PTS: 4 NAT: A.CED.A.2 TOP: Speed

92) ANS:
10 hours and .3 hours.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{distance} = \text{speed}'\text{time}$$

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

If Loretta drives at an average speed of 55 miles per hour for the first 4 hours of the trip, she will drive $55 \times 4 = 220$ miles. Since the total distance is 610 miles, this leaves $610 - 220 = 390$ miles for her dad to drive. If her dad drives at an average speed of 65 miles per hour, it will take him $\frac{390}{65} = 6$ hours to drive 390 miles. If Loretta drives 4 hours and her dad drives 6 hours, the total trip will take 10 hours.

If Loretta gets tired after two hours of driving at an average speed of 55 miles per hour, she will have driven $55 \times 2 = 110$ miles, leaving $610 - 110 = 500$ miles for her dad to drive. At an average speed of 65 miles per hour, it will take her dad $\frac{500}{65} \approx 7.7$ hours to drive 500 miles. The family will save approximately $10 - 7.7 = .3$ hours by having Loretta's dad drive for the remainder of the trip.

PTS: 4 NAT: A.CED.A.2 TOP: Speed

D – Rate, Lesson 4, Rate of Change (r. 2018)

RATE

Rate of Change

Common Core Standard	Next Generation Standard
F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. PARCC: Tasks have a real-world context. Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piece-wise defined functions (including step functions and absolute value functions) and exponential functions with domains in the integers.	**AI-F.IF.6** Calculate and interpret the average rate of change of a function over a specified interval. (Shared standard with Algebra II) Notes: • Functions may be presented by function notation, a table of values, or graphically. • Algebra I tasks have a real-world context and are limited to the following functions: linear, quadratic, square root, piece-wise defined (including step and absolute value), and exponential functions of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$, $(b \neq 1)$.

LEARNING OBJECTIVES

Students will be able to:

1) Calculate and interpret the average rate of change of a function over a specified interval.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

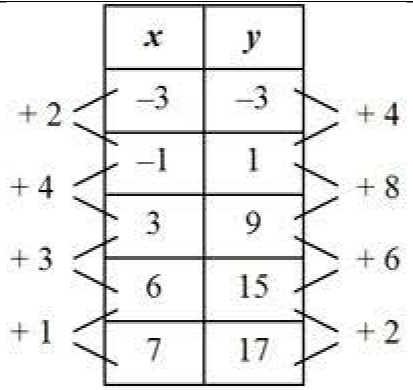
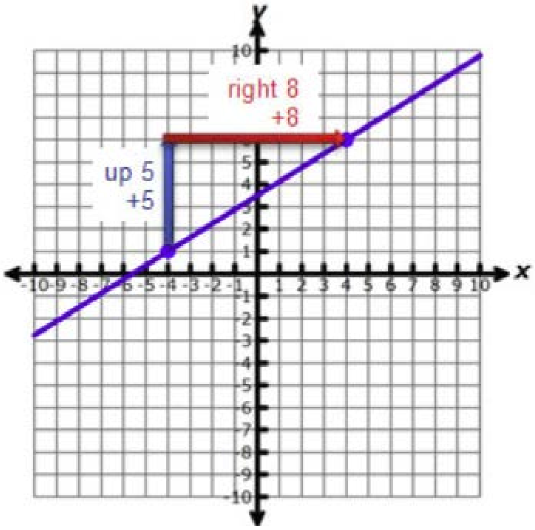
average rate of change
coordinate pair
interval
rate of change
slope
slope formula
steepness

BIG IDEAS

Rate of Change is a measure of how two variables are related to one another. Rate of change can be expressed in many ways:

$$\text{rate of change} = \text{slope} = \text{m} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in y}}{\text{change in x}}$$

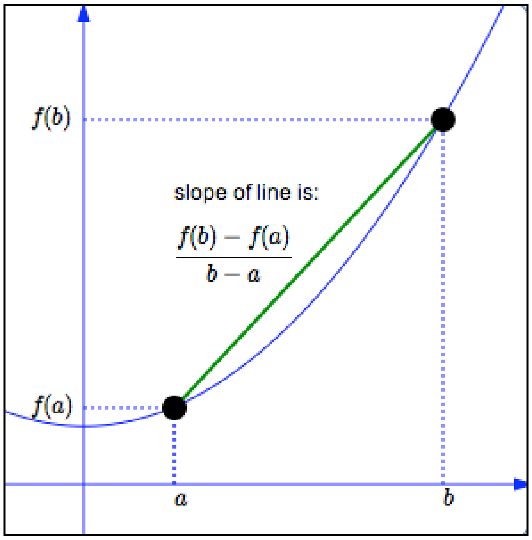
Rate of change can also be found by inspecting a table or graph of a function.

<table>
<tr><th>Table</th><th>Graph</th></tr>
<tr>
<td>

<table>
<tr><th></th><th>x</th><th>y</th><th></th></tr>
<tr><td></td><td>–3</td><td>–3</td><td></td></tr>
<tr><td>+ 2</td><td></td><td></td><td>+ 4</td></tr>
<tr><td></td><td>–1</td><td>1</td><td></td></tr>
<tr><td>+ 4</td><td></td><td></td><td>+ 8</td></tr>
<tr><td></td><td>3</td><td>9</td><td></td></tr>
<tr><td>+ 3</td><td></td><td></td><td>+ 6</td></tr>
<tr><td></td><td>6</td><td>15</td><td></td></tr>
<tr><td>+ 1</td><td></td><td></td><td>+ 2</td></tr>
<tr><td></td><td>7</td><td>17</td><td></td></tr>
</table>
By finding the *change in y* and the *change in x* between each coordinate pair in a table, you can see if the rate of change is constant or variable. This table shows a *linear function* because the rate of change is constant.
$$\frac{2}{4}=\frac{4}{8}=\frac{3}{6}=\frac{1}{2}$$
</td>
<td>


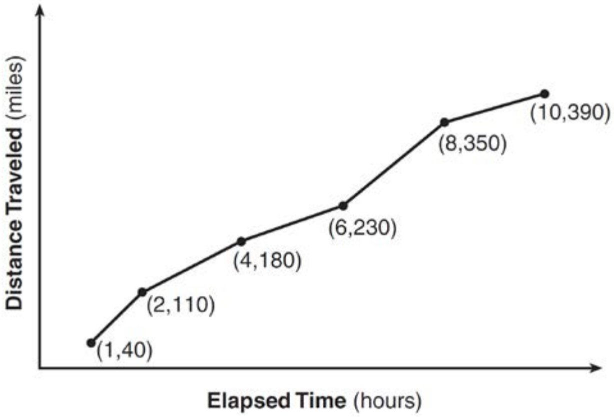
By picking two known points on the graph of a line, you can build a right triangle to find the *change in y* and the *change in x*. The rate of change for this linear function is $\frac{5}{8}$. Always read from left to right when using this method.
</td>
</tr>
</table>

- The rate of change in a *linear function* is constant between any two points anywhere on the straight line.
- The rate of change in a *non-linear* function varies.
- The rate of change can be either positive or negative.
  - A positive rate of change indicates that both variables are either increasing together or decreasing together, though not necessarily at the same rate.
  - A negative rate of change indicates that one variable increases while the other variable decreases.
- The rate of change in a function can be estimated by the steepness of a graph.

Average Rate of Change is used primarily with non-linear functions, since all linear functions have a constant rate of change. Average rate of change can be defined as the *slope* of the straight line that connects the end points of the interval over which the average rate of change is measured.



Example: The graph below records the number of hours and the distance travelled by a family on a trip. The graph is non-linear, so the rate of change is not constant.



To compute the average rate of change for the entire trip, use the end points of the intervals of the graph. Include the origin (0,0) as their starting point. Ten hours later, they had travelled 390 miles (10,390). Their average rate of change for the entire trip is the slope of the line connecting points (0, 0) and (10, 390), which can be calculated as follows:

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

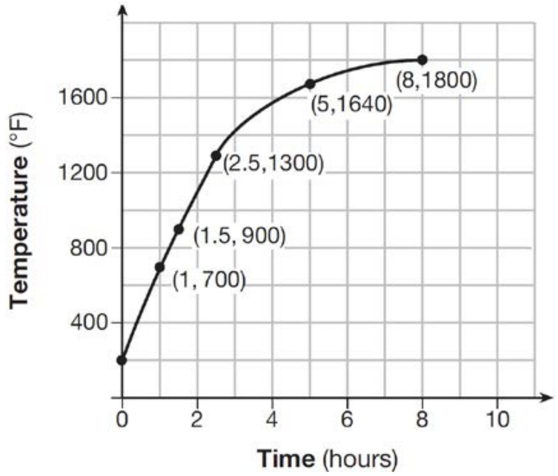
$$slope = \frac{\text{390-0 miles}}{\text{10-0 hours}}$$

$$slope = 39 \text{ miles per hour}$$

In this example, the average rate of change measures speed, the relationship between distance and time. Notice from the graph that their speed was not constant. Sometimes they went faster than 39 mph and sometimes slower than 39 mph, but their average speed over the entire trip was 39 mph.

DEVELOPING ESSENTIAL SKILLS

1. Find the average rate of change between the first and fifth 5 hours.



$$\text{rate of change} = \frac{y_2 - y_1}{x_2 - x_1} == \frac{1640\text{-}700}{5\text{-}1} = \frac{850}{4} = \frac{212.5\ °\text{F}}{1\ \text{hour}}$$

2. Use rate of change to determine if the table below represents a linear or a non-linear function. Explain your answer.

Table of Values

Year	1898	1971	1985	2006	2012
Cost (¢)	1	6	14	24	35

Interval	Δx	Δy	$\frac{\Delta y}{\Delta x}$
Rate of change between 1898 and 1971	73	5	$\frac{73}{5}$
Rate of change between 1971 and 1985	14	8	$\frac{14}{8}$

The table is non-linear because the rate of change is not constant.

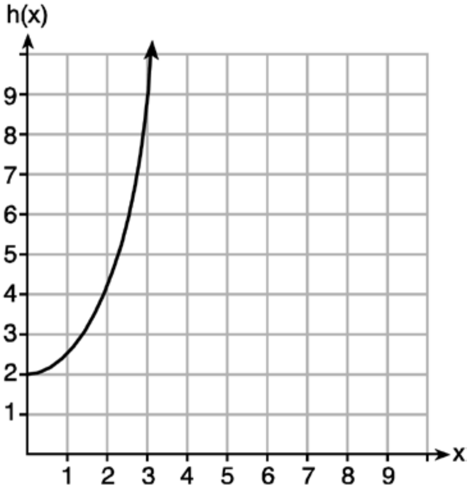
REGENTS EXAM QUESTIONS (through June 2018)

F.IF.B.6: Rate of Change

93) Given the functions $g(x)$, $f(x)$, and $h(x)$ shown below:

$$g(x) = x^2 - 2x$$

x	f(x)
0	1
1	2
2	5
3	7



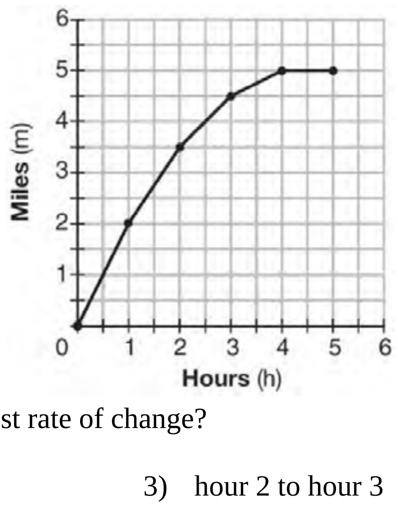
The correct list of functions ordered from greatest to least by average rate of change over the interval $0 \le x \le 3$ is

1) f(x), g(x), h(x)
2) h(x), g(x), f(x)
3) g(x), f(x), h(x)
4) h(x), f(x), g(x)

94) An astronaut drops a rock off the edge of a cliff on the Moon. The distance, $d(t)$, in meters, the rock travels after t seconds can be modeled by the function $d(t) = 0.8t^2$. What is the average speed, in meters per second, of the rock between 5 and 10 seconds after it was dropped?

1) 12
2) 20
3) 60
4) 80

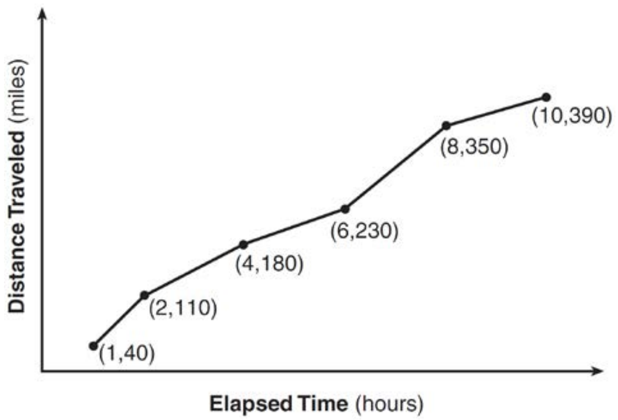
95) The graph below shows the distance in miles, *m*, hiked from a camp in *h* hours.



Which hourly interval had the greatest rate of change?

1) hour 0 to hour 1
2) hour 1 to hour 2
3) hour 2 to hour 3
4) hour 3 to hour 4

96) The Jamison family kept a log of the distance they traveled during a trip, as represented by the graph below.



During which interval was their average speed the greatest?

1) the first hour to the second hour
2) the second hour to the fourth hour
3) the sixth hour to the eighth hour
4) the eighth hour to the tenth hour

97) The table below shows the average diameter of a pupil in a person’s eye as he or she grows older.

Age (years)	**Average Pupil Diameter** (mm)
20	4.7
30	4.3
40	3.9
50	3.5
60	3.1
70	2.7
80	2.3

What is the average rate of change, in millimeters per year, of a person’s pupil diameter from age 20 to age 80?

1) 2.4
2) 0.04
3) -2.4
4) -0.04

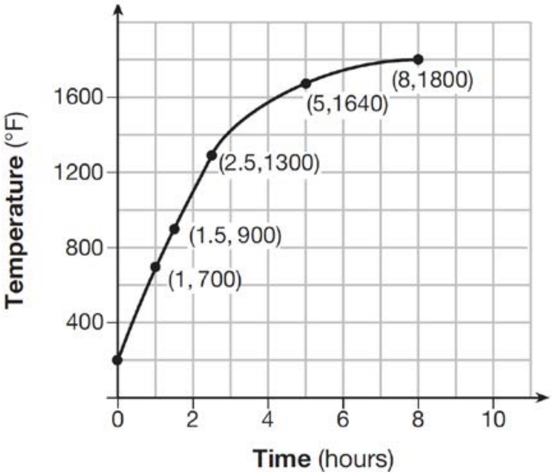
98) Joey enlarged a 3-inch by 5-inch photograph on a copy machine. He enlarged it four times. The table below shows the area of the photograph after each enlargement.

Enlargement	0	1	2	3	4
Area (square inches)	15	18.8	23.4	29.3	36.6

What is the average rate of change of the area from the original photograph to the fourth enlargement, to the *nearest tenth*?

1) 4.3
2) 4.5
3) 5.4
4) 6.0

99) Firing a piece of pottery in a kiln takes place at different temperatures for different amounts of time. The graph below shows the temperatures in a kiln while firing a piece of pottery after the kiln is preheated to 200ºF.



During which time interval did the temperature in the kiln show the greatest average rate of change?

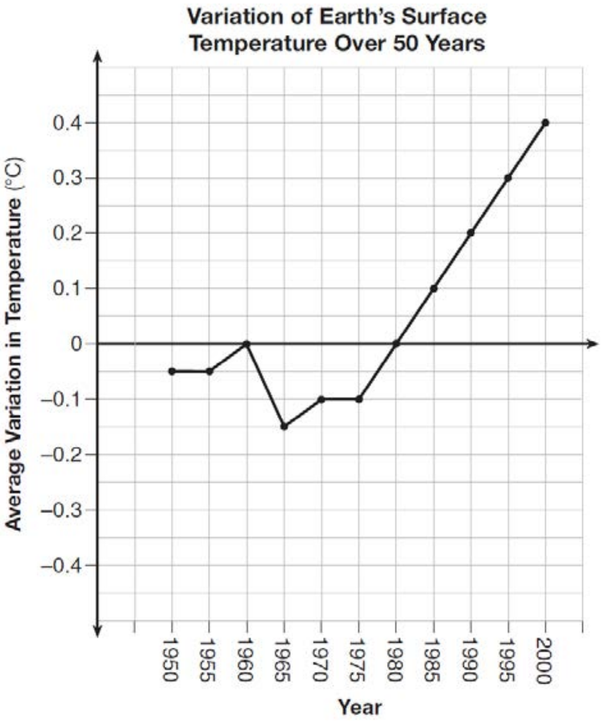
1) 0 to 1 hour
2) 1 hour to 1.5 hours
3) 2.5 hours to 5 hours
4) 5 hours to 8 hours

100) The table below shows the cost of mailing a postcard in different years. During which time interval did the cost increase at the greatest average rate?

Year	1898	1971	1985	2006	2012
Cost (¢)	1	6	14	24	35

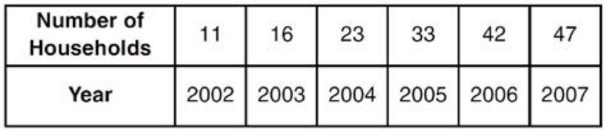
1) 1898-1971
2) 1971-1985
3) 1985-2006
4) 2006-2012

101) The graph below shows the variation in the average temperature of Earth's surface from 1950-2000, according to one source.



During which years did the temperature variation change the most per unit time? Explain how you determined your answer.

102) The table below shows the year and the number of households in a building that had high-speed broadband internet access.



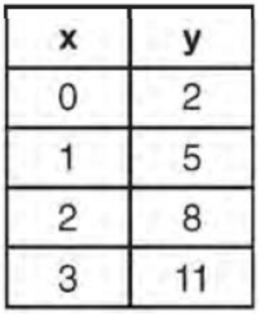
Number of Households	11	16	23	33	42	47
Year	2002	2003	2004	2005	2006	2007

For which interval of time was the average rate of change the *smallest*?

1) 2002 - 2004
2) 2003 - 2005
3) 2004 - 2006
4) 2005 - 2007

103) Which function has a constant rate of change equal to -3?

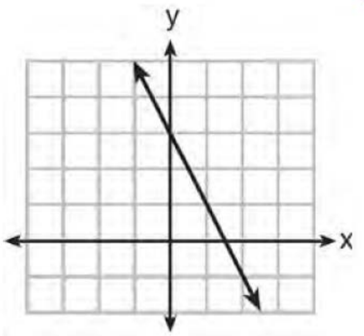
1)



x	y
0	2
1	5
2	8
3	11

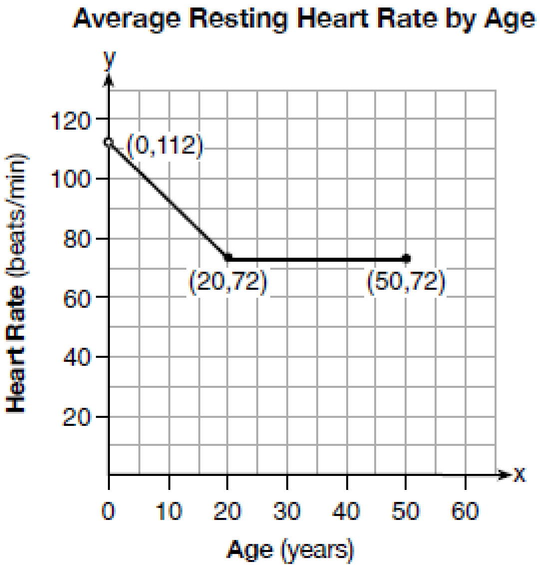
2) {(1,5), (2,2), (3,-5), (4,4)}

3)



4) $2y = -6x + 10$

104) A graph of average resting heart rates is shown below. The average resting heart rate for adults is 72 beats per minute, but doctors consider resting rates from 60-100 beats per minute within normal range.



Which statement about average resting heart rates is *not* supported by the graph?

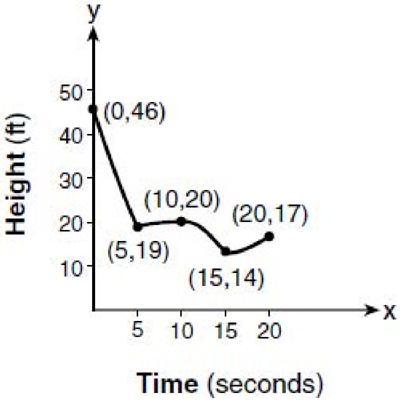
1) A 10-year-old has the same average resting heart rate as a 20-year-old.
2) A 20-year-old has the same average resting heart rate as a 30-year-old.
3) A 40-year-old may have the same average resting heart rate for ten years.
4) The average resting heart rate for teenagers steadily decreases.

105) A family is traveling from their home to a vacation resort hotel. The table below shows their distance from home as a function of time.

Time (hrs)	0	2	5	7
Distance (mi)	0	140	375	480

Determine the average rate of change between hour 2 and hour 7, including units.

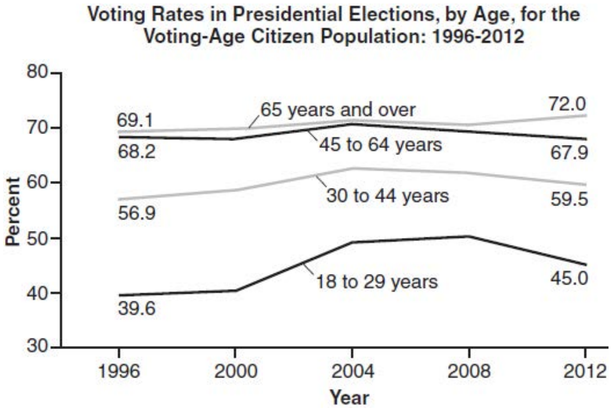
106) The graph below models the height of a remote-control helicopter over 20 seconds during flight.



Over which interval does the helicopter have the *slowest* average rate of change?

1) 0 to 5 seconds
2) 5 to 10 seconds
3) 10 to 15 seconds
4) 15 to 20 seconds

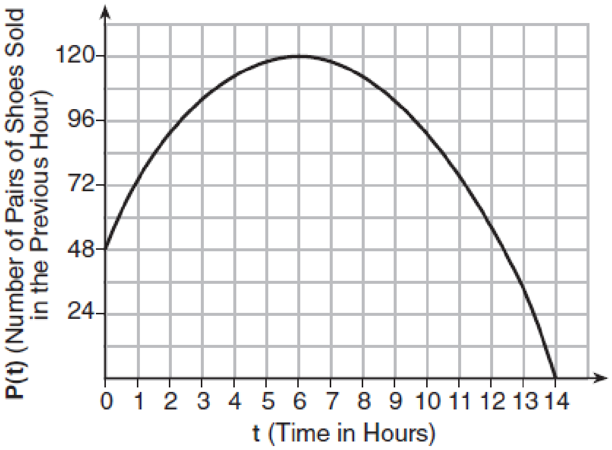
107) Voting rates in presidential elections from 1996-2012 are modeled below.



Which statement does *not* correctly interpret voting rates by age based on the given graph?

1) For citizens 18-29 years of age, the rate of change in voting rate was greatest between years 2000-2004.
2) From 1996-2012, the average rate of change was positive for only two age groups.
3) About 70% of people 45 and older voted in the 2004 election.
4) The voting rates of eligible age groups lies between 35 and 75 percent during presidential elections every 4 years from 1996-2012.

108) A manager wanted to analyze the online shoe sales for his business. He collected data for the number of pairs of shoes sold each hour over a 14-hour time period. He created a graph to model the data, as shown below.



The manager believes the set of integers would be the most appropriate domain for this model. Explain why he is *incorrect*. State the entire interval for which the number of pairs of shoes sold is increasing. Determine the average rate of change between the sixth and fourteenth hours, and explain what it means in the context of the problem.

109) A population of rabbits in a lab, $p(x)$, can be modeled by the function $p(x) = 20(1.014)^x$, where x represents the number of days since the population was first counted. Explain what 20 and 1.014 represent in the context of the problem. Determine, to the *nearest tenth*, the average rate of change from day 50 to day 100.

SOLUTIONS

93) ANS: 4

Over the interval $0 \le x \le 3$, the average rate of change for $h(x) = \frac{9-2}{3-0} = \frac{7}{3}$, $f(x) = \frac{7-1}{3-0} = \frac{6}{3} = 2$, and $g(x) = \frac{3-0}{3-0} = \frac{3}{3} = 1$.

PTS: 2 NAT: F.IF.B.6 TOP: Rate of Change

94) ANS: 1

Strategy: Use the formula for speed: $\text{speed} = \frac{\text{distance}}{\text{time}}$ and information from the problem to calculate average speed.

STEP 1. Calculate $d(t)$ for $t = 5$ and $t = 10$.

$d(t) = 0.8t^2$ and $d(t) = 0.8t^2$

$d(5) = 0.8(5)^2$ $d(10) = 0.8(10)^2$

$d(5) = 0.8(25)$ $d(10) = 0.8(100)$

$d(5) = 20$ $d(5) = 80$

The rock had fallen 20 meters after 5 seconds and 80 meters after 10 seconds.
The total distance traveled was 60 meters in 5 seconds.

STEP 2: Use the speed formula to find average speed.

Substituting distance and time in the speed formula, $\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{60 \text{ meters}}{5 \text{ seconds}} = \frac{12 \text{ meters}}{1 \text{ second}}$.

The rock's average speed between 5 and 10 seconds after being dropped was 12 meters per second.

DIMS? Does it make sense? Yes. The speed formula makes sense and the answer is expressed in meters per second as required by the problem.

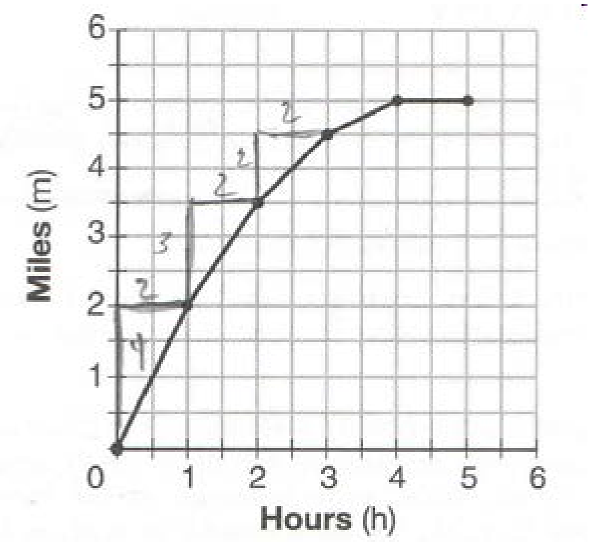
PTS: 2 NAT: F.IF.B.6 TOP: Rate of Change

95) ANS: 1

Step 1. Understand that the problem is asking which hourly interval has the greatest rate of change (slope).

Step 2. Strategy: Calculate $\frac{rise}{run}$ for each hourly interval and select the largest ratio.

Step3. Execution of Strategy



a) between hour 0 and hour 1, $\frac{rise}{run} = \frac{2 \text{ miles}}{1 \text{ hour}}$.

b) between hour 1 and hour 2, $\frac{rise}{run} = \frac{1.5 \text{ miles}}{1 \text{ hour}}$.

c) between hour 2 and hour 3, $\frac{rise}{run} = \frac{1 \text{ mile}}{1 \text{ hour}}$.

d) between hour 3 and hour 4, $\frac{rise}{run} = \frac{.5 \text{ mile}}{1 \text{ hour}}$.

Answer choice a) has the greatest rate of change.

Step 4. Does it make sense? Yes. The graph starts out steep and gradually gets less steep. The steepest part is between hour 0 and hour 1, which corresponds to the calculated rates of change (slopes).

PTS: 2 NAT: F.IF.B.6

96) ANS: 1

Strategy: Equate speed with rate of change. $speed = \frac{\Delta y}{\Delta x} = \frac{rise}{run} = slope = rate\ of\ change$

Make a visual estimate of the steepest line segment on the graph, then use the slope formula to calculate the exact rates of change.

STEP 1. The line segment from (1, 40) to (2, 110) appears to be the steepest line segment in the graph. The line segment from (6, 230) to (8, 350) also seems very steep.

STEP 2. Use $slope = \frac{y_2 - y_1}{x_2 - x_1}$

The line segment from (1, 40) to (2, 110) has $slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{110 - 40}{2 - 1} = \frac{70\ miles}{1\ hour}$.

The line segment from (6, 230) to (8, 350) has $slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{350 - 230}{8 - 6} = \frac{120}{2} = \frac{60\ miles}{1\ hour}$.

PTS: 2 NAT: F.IF.B.6 TOP: Rate of Change

97) ANS: 4

Strategy: Rate of change is the same as slope. Use the slope formula to find the rate of change between (20, 4.7) and (80, 2.3).

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.3 - 4.7}{80 - 20} = \frac{-2.4}{60} = -0.04.$$

DIMS? Does it make sense? Yes. The average pupil diameter gets smaller very very slowly. Choices a and c are way too big and choices a and b indicate that the average pupil size is getting bigger rather than smaller.

PTS: 2 NAT: F.IF.B.6 TOP: Rate of Change

98) ANS: 3

Strategy: Use the slope formula and data from the table to calculate the exact rate of change over four enlargements.

STEP 1. Use $slope = \frac{y_2 - y_1}{x_2 - x_1}$ to compute the rate of change between (0, 15) and (4, 36.6).

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{36.6 - 15}{4 - 0} = \frac{21.6}{4} = 5.4.$$

DIMS? Does it make sense? Yes. If you start with 15 and add 5.4 + 5.4 +5.4 + 5.4, you end up with 36.6. There were four enlargements and the average increase of each enlargement was 5.4 square inches.

PTS: 2 NAT: F.IF.B.6 TOP: Rate of Change

99) ANS: 1

Strategy: Equate rate of change with slope. Make a visual estimate of the steepest line segment on the graph, then use the slope formula to calculate the exact rates of change over given intervals.

STEP 1. The line segment from (0, 200) to (1, 700) appears to be the steepest line segment in the graph. The line segment from (1, 700) to (1.5, 900) also seems very steep. The rate of change gets slower as the temperature of the kiln gets hotter.

STEP 2. Use $slope = \frac{y_2 - y_1}{x_2 - x_1}$

The line segment from (0, 200) to (1, 700) has

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{700 - 200}{1 - 0} = \frac{500 \text{ degrees}}{1 \text{ hour}}.$$

The line segment from (1, 700) to (1.5, 900) has $slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{900 - 700}{1.5 - 1} = \frac{200}{0.5} = \frac{400 \text{ degrees}}{1 \text{ hour}}$

The rate of change was greatest in the first hour.

DIMS? Does it make sense? Yes. The graph shows that rate of change slows down as time increases, so the first hour would have the greatest rate of change.

PTS: 2 NAT: F.IF.B.6 TOP: Rate of Change

100) ANS: 4

Strategy: Find the average rate of change using the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

(a) $\frac{6 - 1}{1971 - 1898} = \frac{5}{73} \approx .07$

(b) $\frac{14-6}{1985-1971}=\frac{8}{14}\approx .57$

(c) $\frac{24-14}{2006-1985}=\frac{10}{21}\approx .48$

(d) $\frac{35-24}{2012-2006}=\frac{11}{6}\approx 1.83$

PTS: 2 NAT: F.IF.B.6 TOP: Rate of Change

101) ANS:

During 1960-1965, because the graph has the steepest slope during these years.

PTS: 2 NAT: F.IF.B.6 TOP: Rate of Change

102) ANS: 1

Step 1. Understand the problem as asking for the *smallest* average rate of change over an interval. The number of households depends on the year, so number of households is the y (dependent) variable and year is the x (independent) variable.
Step 2. Strategy. Average rate of change over an interval is found using the end points of the interval and the slope formula.
Step 3. Execution.

Average rate of change for choice 1: $m=\frac{y_2-y_1}{x_2-x_1}=\frac{23-11}{2004-2002}=\frac{12}{2}=6$

Average rate of change for choice 2: $m=\frac{y_2-y_1}{x_2-x_1}=\frac{33-16}{2005-2003}=\frac{17}{2}=8\frac{1}{2}$

Average rate of change for choice 3: $m=\frac{y_2-y_1}{x_2-x_1}=\frac{42-23}{2006-2004}=\frac{19}{2}=9\frac{1}{2}$

Average rate of change for choice 4: $m=\frac{y_2-y_1}{x_2-x_1}=\frac{47-33}{2007-2005}=\frac{14}{2}=7$

Step 4. Does it make sense? Yes. Choice (1) has the *smallest* average rate of change.

PTS: 2 NAT: F.IF.B.6 TOP: Rate of Change

103) ANS: 4

$y=mx+b$, where m equals slope and b equals y-intercept

$2y=-6x+10$

$y=-3x+10$

$m=-3$

PTS: 2 NAT: F.LE.A.1

104) ANS: 1

The graph shows that a newborn child has age zero and an average resting heartbeat of 112 (0, 112). A 20 year old has an average resting heartbeat of 72 (20, 72). From birth to age 20, the average resting heartbeat decreases at a constant rate. After age 20, the average resting heartbeat stays the same until age 50 (50, 72). The problem wants to know which answer choice is *not* supported by the graph.
Strategy: Eliminate wrong answers.
a) A 10-year-old has the same average resting heart rate as a 20-year-old. This is not supported by the graph. The graph indicates that the average resting heartbeat of a 10-year-old is 92. $72\neq 92$. This is the correct answer.

~~b)~~ A 20-year-old has the same average resting heart rate as a 30-year-old. This is supported by the graph. $72 = 72$, so it is a wrong answer.
~~c)~~ A 40-year-old may have the same average resting heart rate for ten years. This is supported by the graph. $72 = 72$, so it is a wrong answer.
~~d)~~ The average resting heart rate for teenagers steadily decreases. This is supported by the graph, so it is a wrong answer. From birth to age 20, the average resting heartbeat decreases at a constant rate.

PTS: 2 NAT: F.IF.B.6 TOP: Rate of Change

105) ANS:
68 miles per hour

Strategy: The average rate of change is the slope of the straight line between the two points at the ends of the interval. Find the slope of the straight line between (2, 140) and (7, 480) using the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{480 - 140 \text{ miles}}{7 - 2 \text{ hours}}$$

$$m = \frac{340 \text{ miles}}{5 \text{ hours}}$$

$$m = 68 \text{ miles per hour}$$

PTS: 2 NAT: F.IF.B.6 TOP: Rate of Change

106) ANS: 2
The average rate of change is slowest between the coordinates (5, 10) and (10, 20), which corresponds to the interval 5 to 10 seconds.

PTS: 2 NAT: F.IF.B.6 TOP: Rate of Change
KEY: AI

107) ANS: 2
From 1996-2012, the average rate of change was positive for three age groups.

PTS: 2 NAT: F.IF.B.6 TOP: Rate of Change

108) ANS:
PART 1

The set of integers includes negative numbers, so it is not an appropriate domain for time.

PART 2

$$\boxed{0 < t < 14}$$

The total number of shoes sold increases every hour that shoes are sold.

or

$$\boxed{0 < t < 6}$$

The hourly rate of shoe sales increases from 0 hours to 6 hours.

PART 3: Calculate the average rate of change and explain what it means.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 120}{14 - 6} = \frac{-120}{8} = -15$$

The average rate of change is -15, which means that, on average, 15 fewer shoes were sold each hour between the sixth and fourteenth hours.

PTS: 4 NAT: F.IF.B.6 TOP: Rate of Change

109) ANS:

20 represents the initial number of rabbits.
1.014 represents the rate of population growth.
The average rate of change from day 50 to day 100 is 0.8 rabbits per day.

Strategy: Use the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to compute the average rate of change between day 50 and day 100.

STEP 1. Input $p(x) = 20(1.014)^x$ in a graphing calculator.

STEP 2. Use the table of values view to afind the number of rabbits on day 50 and dat 100.

Day (x)	Number of Rabbits $p(x)$
50	40.08
100	80.32

STEP 3. Input (50, 40.08) and (100, 80.32) into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{80.32 - 40.08}{100 - 50}$$

$$m = \frac{40.24}{50}$$

$$m \approx 0.8$$

PTS: 2 NAT: F.IF.B.6 TOP: Rate of Change

LINEAR EQUATIONS

Modeling Linear Equations

Common Core Standards	Next Generation Standards
F-BF.A.1 Write a function that describes a relationship between two quantities.	**F-BF.1** Write a function that describes a relationship between two quantities.
F-LE.A.2 Construct linear and exponential functions, ~~including arithmetic and geometric sequences~~, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). PARCC: Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).	**AI-F.LE.2** Construct a linear or exponential function **symbolically** given: i) a graph; ii) a description of the relationship; iii) two input-output pairs (include reading these from a table). (Shared standard with Algebra II) Note: Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).
F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context. PARCC: Tasks have a real-world context. Exponential functions are limited to those with domains in the integers.	**AI-F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context. (Shared standard with Algebra II) Note: Tasks have a real-world context. **Exponential functions are limited to those with domains in the integers and are of the form** $f(x) = a(b)^x$ **where** $a > 0$ **and** $b > 0$ $(b \neq 1)$**.**
S-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	**AI-S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

LEARNING OBJECTIVES

Students will be able to:

1)

Overview of Lesson

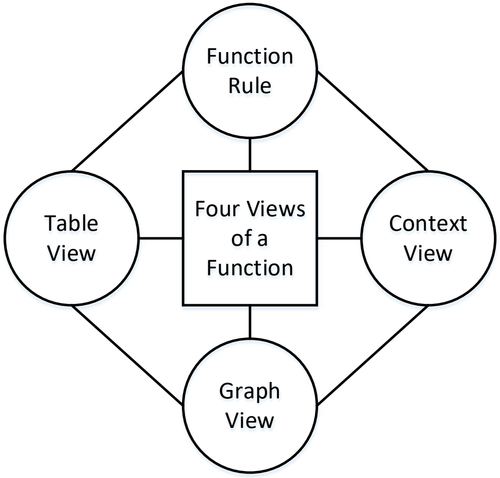
Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

BIG IDEAS

Linear functions are modelled using the same general approaches used in modelling linear equations (see Expressions and Equations, Lesson 4, Modelling Linear Equations). When modelling linear functions, however, any one of the four views of a function may describe the mathematical relationship between the variables and function notation may be required.

There are **four views of a function**: 1) the function rule; 2) the table of values; 3) the graph; and 4) the narrative or "context" view. All four views can be used to help understand a function, and the ability to move from one view to another is important.



NOTE: Graphing calculators will produce table and graph views of a function after inputting the function rule. The context view cannot be modelled with a graphing calculator. Regression can be used to find a function rule from a table or graph (see Graphs and Statistics, Lesson 5, Regression).

Function Notation

$$f\left(\overset{\substack{\text{input value}\\ \downarrow}}{x}\right) = \underbrace{2x+3}_{\text{output value}}$$

Function notation is a language for writing functions. It provides simple, but important information about the mathematical relationship between the variables in a function.

- Function notation identifies the mathematical relationship as a function.
  - A function has one and only one output (y-value) for each input (x-value).
  - Function notation should *not* be used with mathematical relationships that are not functions.
- Function notation identifies both the output (dependent variable) and the intput (independent variables) in a mathematical relationship
- The most common function notation is $f(x)$, which is read as "*f of x*".
  - $f(x)$ is used to represent the dependent variable (y-value) of the function, and x is used to represent the independent variable (x-value) of the function.
    - In practice, any equation describing a function can be changed to function notation by substituting $f(x)$ for y.
    - The y-axis of a graph is often labeled $f(x)$ axis.
    - Ordered pairs may be written as $\left(x, f\left(x\right)\right)$
  - Other letters besides f and x may be used with function notation.
    - In practice, letters are often chosen to be descriptive of the variables involved.

    - Examples are the function rules for describing how to convert degrees Fahrenheit to degrees Celsius, and back.

Fahrenheit to Celsius	**Celsius to Fahrenheit**
$C(f)=\frac{5}{9}(f-32)$	$F(c)=\frac{9}{5}c+32$
This function rule can be interpreted as “degrees Celsius is a function of degrees Fahrenheit”.	This function rule can be interpreted as “degrees Fahrenheit is a function of degrees Celsius”.

- Function notation can be used to identify which value of the independent variable is to be used as an input.
    - For example, if $f(x)=3x+7$, then $f(5)=22$
    - $f(5)$ says that the output of the function should be evaluated when the input is $x = 5$.

$$f(5)=3x+7$$
$$f(5)=3\times 5+7$$
$$f(5)=15+7$$
$$f(5)=22$$

Modeling a Sample Function

Context View: The inside of a freezer is kept at a constant temperature of 15 degrees Fahrenheit. When a quart of liquid water is placed in the freezer, its Fahrenheit temperature drops by one-half every 20 minutes until it turns into ice and reaches a constant temperature of 15 degrees.

Table View: The tables views below model what the temperatures of two different quarts of water with different initial temperatures would be after *m* minutes in the freezer.

Initial Temperature = 80 degrees

Minutes in Freezer (*m*)	0	20	40	60	80
Temperature *f(m)*	80	40	20	15	15

Initial Temperature = 120 degrees

Minutes in Freezer (*m*)	0	20	40	60	80
Temperature *f(m)*	120	60	30	15	15

Function Rule View

The narrative view and the table views suggest that the temperature drops exponentially at first, then stays at a constant temperature of 15 degrees.

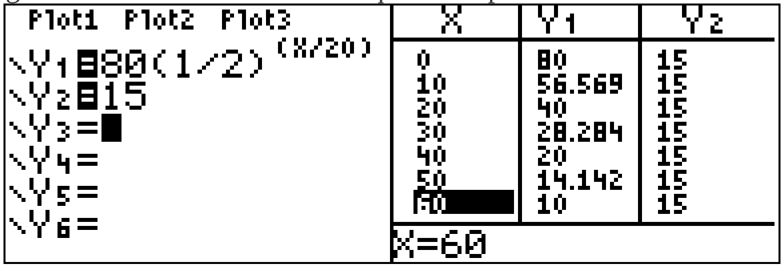
Exponential growth or decay can be modeled by the function $A-P(1\pm r)^t$, where:
A represents the current amount,
P represents the starting amount,
$(1\pm r)$ represents the rate of growth or decay per cycle, and
t represents the number of cycles (usually measured as time)

The temperature of the water can be modeled using the formula for exponential decay, as follows:

$$A - P(1 - \frac{1}{2})^{\frac{\text{time (in minutes)}}{20}}$$

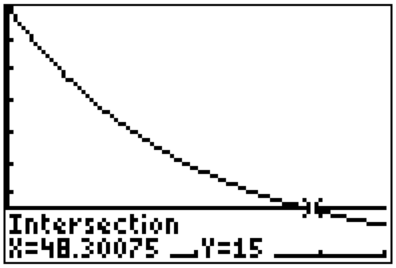
A represents the temperature of the water after m minutes in the freezer.
P represents the initial temperature of the water.
$(\frac{1}{2})$ represents the exponential rate of decay.
$\frac{\text{time (in minutes)}}{20}$ represents time.
The range of the function would be limited to $212 \geq f(m) \geq 15$

Check: Input the system of equations in a graphing calculator for a quart of water with an initial temperature of 80 degrees Fahrenheit. The second equation represents the lower limit of 15 degrees.



```
 Plot1  Plot2  Plot3          X      Y1       Y2
\Y1=80(1/2)^(X/20)            0      80       15
\Y2=15                        10     56.569   15
\Y3=                          20     40       15
\Y4=                          30     28.284   15
\Y5=                          40     20       15
\Y6=                          50     14.142   15
                              60     10       15
                             X=60
```

Graph View



DIMS - Does It Make Sense? Yes, all four views of the function show that the water cools down quickly at first, then more slowly, then reaches a final temperature of 15 degrees. The graph view shows that it would take about 48 minutes for a quart of liquid water with an initial temperature of 80 degrees to reach a frozen temperature of 15 degrees.

DEVELOPING ESSENTIAL SKILLS

The table below represents the number of hours a student worked and the amount of money the student earned.

Number of Hours (h)	Dollars Earned (d)
8	\$50.00
15	\$93.75
19	\$118.75
30	\$187.50

Write a function rule that represents the number of dollars, *d*, earned in terms of the number of hours, *h*, worked.

$$d(h) = 6.25h$$

Bob sells appliances. He gets paid a fixed salary plus a fee for every appliance he sells. His total weekly compensation in dollars is modelled by the function $c(a) = 50a + 250$. Explain what each of the three terms in this function means in the context of Bob's compensation.

1. *c(a)* Bob's compensation is a function of the number of appliances he sells.
2. *50a* Bob gets $50 for every appliance he sells.
3. *250* Bob gets $250 even if he doesn't sell any appliances.

REGENTS EXAM QUESTIONS (through June 2018)

F.BE.A.1, F.LE.A.2, F.LE.B.5, S.ID.C.7: Modeling Linear Functions

110) Caitlin has a movie rental card worth $175. After she rents the first movie, the card's value is $172.25. After she rents the second movie, its value is $169.50. After she rents the third movie, the card is worth $166.75. Assuming the pattern continues, write an equation to define $A(n)$, the amount of money on the rental card after *n* rentals. Caitlin rents a movie every Friday night. How many weeks in a row can she afford to rent a movie, using her rental card only? Explain how you arrived at your answer.

111) In 2013, the United States Postal Service charged $0.46 to mail a letter weighing up to 1 oz. and $0.20 per ounce for each additional ounce. Which function would determine the cost, in dollars, $c(z)$, of mailing a letter weighing *z* ounces where *z* is an integer greater than 1?

1) $c(z) = 0.46z + 0.20$
2) $c(z) = 0.20z + 0.46$
3) $c(z) = 0.46(z - 1) + 0.20$
4) $c(z) = 0.20(z - 1) + 0.46$

112) Alex is selling tickets to a school play. An adult ticket costs $6.50 and a student ticket costs $4.00. Alex sells *x* adult tickets and 12 student tickets. Write a function, $f(x)$, to represent how much money Alex collected from selling tickets.

113) Jackson is starting an exercise program. The first day he will spend 30 minutes on a treadmill. He will increase his time on the treadmill by 2 minutes each day. Write an equation for $T(d)$, the time, in minutes, on the treadmill on day *d*. Find $T(6)$, the minutes he will spend on the treadmill on day 6.

114) Last weekend, Emma sold lemonade at a yard sale. The function $P(c) = .50c - 9.96$ represented the profit, $P(c)$, Emma earned selling *c* cups of lemonade. Sales were strong, so she raised the price for this weekend by 25 cents per cup. Which function represents her profit for this weekend?

1) $P(c) = .25c - 9.96$
2) $P(c) = .50c - 9.71$
3) $P(c) = .50c - 10.21$
4) $P(c) = .75c - 9.96$

115) Which chart could represent the function $f(x) = -2x + 6$?

1)

x	f(x)
0	6
2	10
4	14
6	18

2)

x	f(x)
0	4
2	6
4	8
6	10

3)

x	f(x)
0	8
2	10
4	12
6	14

4)

x	f(x)
0	6
2	2
4	–2
6	–6

116) Jim is a furniture salesman. His weekly pay is \$300 plus 3.5% of his total sales for the week. Jim sells x dollars' worth of furniture during the week. Write a function, $p(x)$, which can be used to determine his pay for the week. Use this function to determine Jim's pay to the *nearest cent* for a week when his sales total is \$8250.

117) Each day Toni records the height of a plant for her science lab. Her data are shown in the table below.

Day (n)	1	2	3	4	5
Height (cm)	3.0	4.5	6.0	7.5	9.0

The plant continues to grow at a constant daily rate. Write an equation to represent $h(n)$, the height of the plant on the nth day.

118) Tanya is making homemade greeting cards. The data table below represents the amount she spends in dollars, $f(x)$, in terms of the number of cards she makes, x.

x	f(x)
4	7.50
6	9
9	11.25
10	12

Write a linear function, $f(x)$, that represents the data. Explain what the slope and y-intercept of $f(x)$ mean in the given context.

119) A company that manufactures radios first pays a start-up cost, and then spends a certain amount of money to manufacture each radio. If the cost of manufacturing r radios is given by the function $c(r) = 5.25r + 125$, then the value 5.25 best represents

1) the start-up cost
2) the profit earned from the sale of one radio
3) the amount spent to manufacture each radio
4) the average number of radios manufactured

120) A satellite television company charges a one-time installation fee and a monthly service charge. The total cost is modeled by the function $y = 40 + 90x$. Which statement represents the meaning of each part of the function?

1) y is the total cost, x is the number of months of service, \$90 is the installation fee, and \$40 is the service charge per month.
2) y is the total cost, x is the number of months of service, \$40 is the installation fee, and \$90 is the service charge per month.
3) x is the total cost, y is the number of months of service, \$40 is the installation fee, and \$90 is the service charge per month.
4) x is the total cost, y is the number of months of service, \$90 is the installation fee, and \$40 is the service charge per month.

121) The owner of a small computer repair business has one employee, who is paid an hourly rate of \$22. The owner estimates his weekly profit using the function $P(x) = 8600 - 22x$. In this function, x represents the number of

1) computers repaired per week
2) hours worked per week
3) customers served per week
4) days worked per week

122) The cost of airing a commercial on television is modeled by the function $C(n) = 110n + 900$, where n is the number of times the commercial is aired. Based on this model, which statement is true?

1) The commercial costs \$0 to produce and \$110 per airing up to \$900.
2) The commercial costs \$110 to produce and \$900 each time it is aired.
3) The commercial costs \$900 to produce and \$110 each time it is aired.
4) The commercial costs \$1010 to produce and can air an unlimited number of times.

123) The cost of belonging to a gym can be modeled by $C(m) = 50m + 79.50$, where $C(m)$ is the total cost for m months of membership. State the meaning of the slope and y-intercept of this function with respect to the costs associated with the gym membership.

124) A car leaves Albany, NY, and travels west toward Buffalo, NY. The equation $D = 280 - 59t$ can be used to represent the distance, D, from Buffalo after t hours. In this equation, the 59 represents the

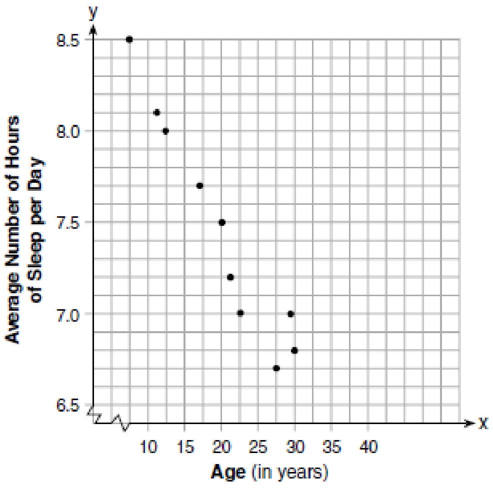
1) car's distance from Albany
2) speed of the car
3) distance between Buffalo and Albany
4) number of hours driving

125) A plumber has a set fee for a house call and charges by the hour for repairs. The total cost of her services can be modeled by $c(t) = 125t + 95$. Which statements about this function are true?

I. A house call fee costs \$95.
II. The plumber charges \$125 per hour.
III. The number of hours the job takes is represented by t.

1) I and II, only
2) I and III, only
3) II and III, only
4) I, II, and III

126) A student plotted the data from a sleep study as shown in the graph below.



The student used the equation of the line $y = -0.09x + 9.24$ to model the data. What does the rate of change represent in terms of these data?

1) The average number of hours of sleep per day increases 0.09 hour per year of age.
2) The average number of hours of sleep per day decreases 0.09 hour per year of age.
3) The average number of hours of sleep per day increases 9.24 hours per year of age.
4) The average number of hours of sleep per day decreases 9.24 hours per year of age.

127) During a recent snowstorm in Red Hook, NY, Jaime noted that there were 4 inches of snow on the ground at 3:00 p.m., and there were 6 inches of snow on the ground at 7:00 p.m. If she were to graph these data, what does the slope of the line connecting these two points represent in the context of this problem?

128) The amount Mike gets paid weekly can be represented by the expression $2.50a + 290$, where *a* is the number of cell phone accessories he sells that week. What is the constant term in this expression and what does it represent?

1) $2.50a$, the amount he is guaranteed to be paid each week
2) $2.50a$, the amount he earns when he sells *a* accessories
3) 290, the amount he is guaranteed to be paid each week
4) 290, the amount he earns when he sells *a* accessories

SOLUTIONS

110) ANS:
63 weeks

Strategy: Model the problem with a linear function.

$A(n) = \$175 - \$2.75n$
Each movie rental costs $2.75
Let *n* represent the number of rentals.
Let $A(n)$ represent the amount of money on the rental card after *n* rentals.
Caitlin can rent a movie for 63 weeks in a row.

Explanation:
Caitlin has $175.
Each movie rental costs $2.75
$175 divided by $2.75 equals 63.6, so $2.75 times 63.6 equals $175.

Caitlin has enough money to rent 63 videos. After 63 weeks, Caitlin will not have enough money to rent another movie.

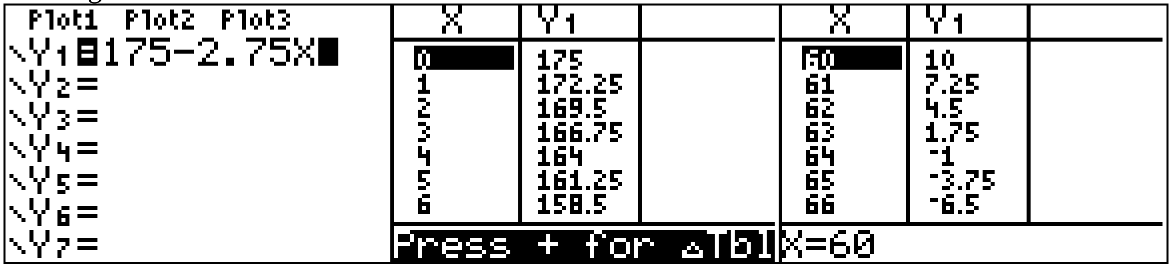
$A(63) = \$175 - \$2.75(63)$

$A(63) = \$175 - \$173.25$

$A(63) = \$1.75$

After 63 weeks, Caitlin will have \$1.75 on her rental card, which is not enough to rent another movie.

Check using a table of values:



```
Plot1 Plot2 Plot3
\Y1=175-2.75X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
X    Y1
0    175
1    172.25
2    169.5
3    166.75
4    164
5    161.25
6    158.5
Press + for ΔTbl
```

```
X    Y1
60   10
61   7.25
62   4.5
63   1.75
64   ⁻1
65   ⁻3.75
66   ⁻6.5
X=60
```

PTS: 4 NAT: F.BF.A.1 TOP: Modeling Linear Equations

111) ANS: 4

Strategy: Eliminate wrong answers.

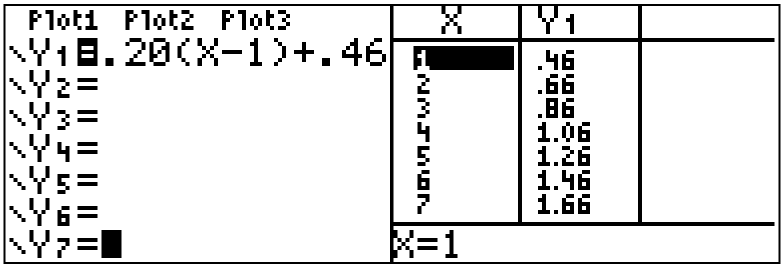
The problem states that there is a flat charge of \$0.46 to mail a letter. This flat charge applies regardless of what the letter weighs. Eliminate any answer that multiplies this flat charge by the weight of the letter. Eliminate answer choices *a* and *c*.

The difference between answer choices b and d is in the terms $0.20z$ and $0.20(z-1)$, where z represents the weight of the letter in ounces. Choice b charges 20 cents for every ounce. Choice d charges 20 cents for every ounce in excess of the first ounce. Choice d is the correct answer.

DIMS? Does It Make Sense? Yes. Transform answer choice c for input into the graphing calculator.

$$c(z) = 0.20(z-1) + 0.46$$

$$Y_1 = 0.20(x-1) + 0.46$$



```
Plot1 Plot2 Plot3
\Y1=.20(X-1)+.46
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
X    Y1
1    .46
2    .66
3    .86
4    1.06
5    1.26
6    1.46
7    1.66
X=1
```

The table shows \$0.46 to mail a letter weighing up to 1 oz. and \$0.20 per ounce for each additional ounce.

PTS: 2 NAT: A.CED.A.2 TOP: Modeling Linear Equations

112) ANS:

$f(x) = 6.50x + 4(12)$

Strategy: Translate the words into math.

\$6.50 per adult ticket plus \$4.00 per student ticket equals total money collected.
\$6.50 times x plus \$4.00 times 12 students equals total money collected

$$\$6.50x+4(12)=f(x)$$

PTS: 2 NAT: F.BF.A.1 TOP: Modeling Linear Equations

113) ANS:
$T(d)=2d+28$
Jackson will spend 40 minutes on the treadmill on day 6.

Strategy: Start with a table of values, then write an equation that models both the table view and the narrative view of the function. Then, use the equation to determine the number of minutes Jackson will spend on the treadmill on day 6.

STEP 1: Model the narrative view with a table view.

d	1	2	3	4	5	6	7	8	9
$T(d)$	30	32	34	36	38	40	42	44	46

STEP 2: Write an equation.

$$T(d)=30+2(d-1)$$
$$T(d)=30+2d-2$$
$$T(d)=28+2d$$

STEP 3: Use the equation to find the number of minutes Jackson will spend on the treadmill on day 6.

$$T(d)=28+2d$$
$$T(6)=28+2(6)$$
$$T(6)=40$$

DIMS? Does It Make Sense? Yes. Both the equation and the table of values predict that Jackson will spend 40 minutes on the treadmill on day 6.

PTS: 2 NAT: A.CED.A.1 TOP: Modeling Linear Functions

114) ANS: 4
The problem asks us to change the function $P(c)=.50c-9.96$ to reflect a 25 cents increase in the price for a cup of coffee. To do so, we must understand each part of the equation.

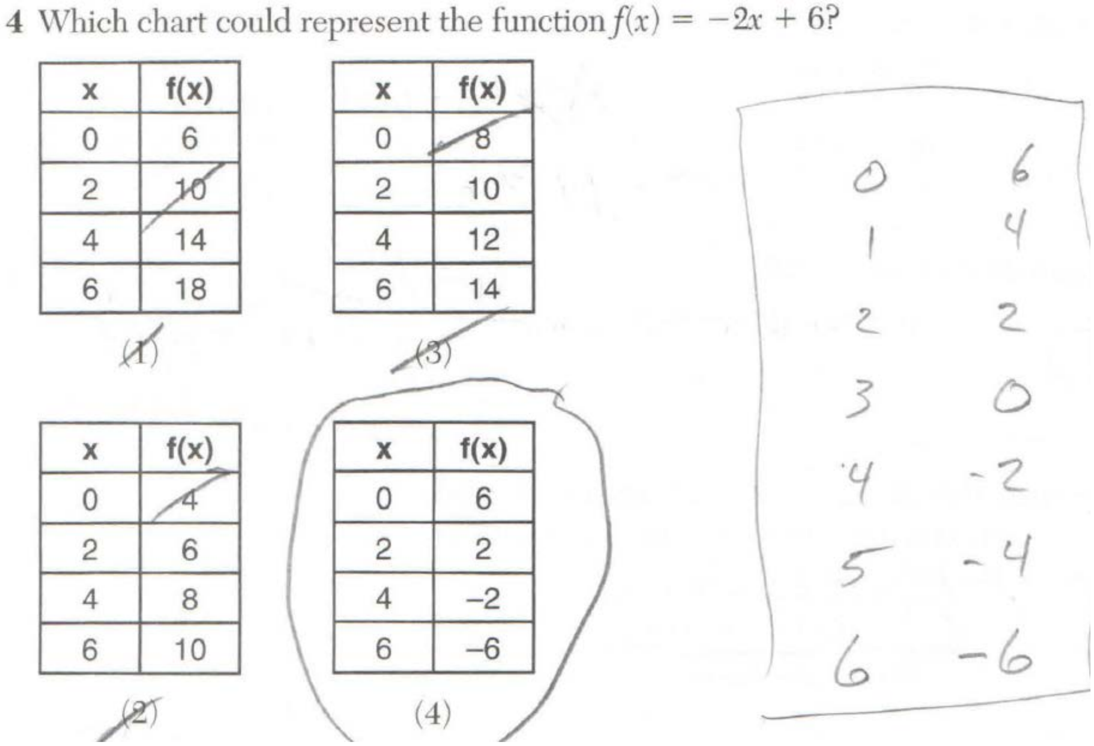
$P(c)$ represents the total profits.
$.50c$ represents the price for each cup of coffee times the numbers of cups sold.
9.96 represents fixed costs, such as the price of the coffee beans used.

If Emma increases the price of coffee by 25 cents, the term .50*c* will change to .75*c*. Everything else will stay the same.

The new function will be $P(c)=.75c-9.96$.

PTS: 2 NAT: F.BF.A.1 TOP: Modeling Linear Functions

115) ANS: 4



4 Which chart could represent the function $f(x) = -2x + 6$?

x	f(x)
0	6
2	~~10~~
4	14
6	18

~~(1)~~

x	f(x)
0	~~4~~
2	6
4	8
6	10

~~(2)~~

x	f(x)
0	~~8~~
2	10
4	12
6	14

~~(3)~~

x	f(x)
0	6
2	2
4	–2
6	–6

(4)

0	6
1	4
2	2
3	0
4	-2
5	-4
6	-6

PTS: 2 NAT: F.LE.A.2

116) ANS:

STEP 1: Define variables and write a function rule.
Let $p(x)$ represent Jim's total pay for a week.
Let 300 represent Jim's fixed pay in dollars.
Let .035 represent the additional pay that Jim receives for furniture sales.
Let x represent Jims dollars of furniture sales during the week.
Write the function rule:

$$p(x) = 300 + 0.035x$$

STEP 2: Use the function rule to determine Jim's pay if he has \$8,250 in furniture sales.

$$p(x) = 300 + 0.035x$$

$$p(8250) = 300 + 0.035(8250)$$

$$p(8250) = 300 + 288.75$$

$$p(8250) = 588.75$$

$$\boxed{\$588.75}$$

PTS: 4 NAT: F.BF.A.1 TOP: Modeling Linear Functions

117) ANS:
$y = 1.5x + 1.5$

Strategy 1: The problem states that the plant grows at a constant daily rate, so the rate of change is constant. Use the slope-intercept form of a line, $y = mx + b$, and data from the table to identify the slope and y-intercept.

STEP 1: Extend the table to show the y-intercept, as follows:

Day (n)	0	1	2	3	4	5

Height (cm)	1.5	3	4.5	6	7.5	9

The y-intercept is 1.5, so we can write $y = mx + 1.5$.

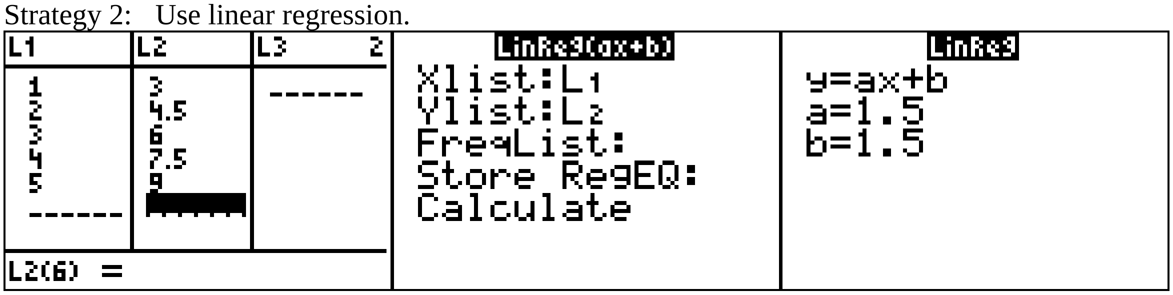
STEP 2. Use the slope formula and any two pairs of data to find the slope. In the following calculation, the points (1,3) and (5,9) are used.

$$y = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-3}{5-1} = \frac{6}{4} = \frac{3}{2} = 1.5$$

The slope is 1.5, so we can write $y = 1.5x + 1.5$

DIMS?: See below.

Strategy 2: Use linear regression.



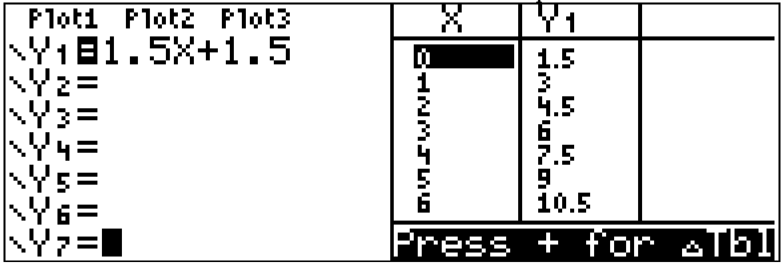
```
L1     L2     L3     2
1      3      ------
2      4.5
3      6
4      7.5
5      9
------ ------
L2(6) =
```

```
LinReg(ax+b)
Xlist:L1
Ylist:L2
FreqList:
Store RegEQ:
Calculate
```

```
LinReg
y=ax+b
a=1.5
b=1.5
```

The equation is $y = 1.5x + 1.5$

DIMS? Does It Make Sense? Yes. The equation can be used to reproduce the table view, as follows:



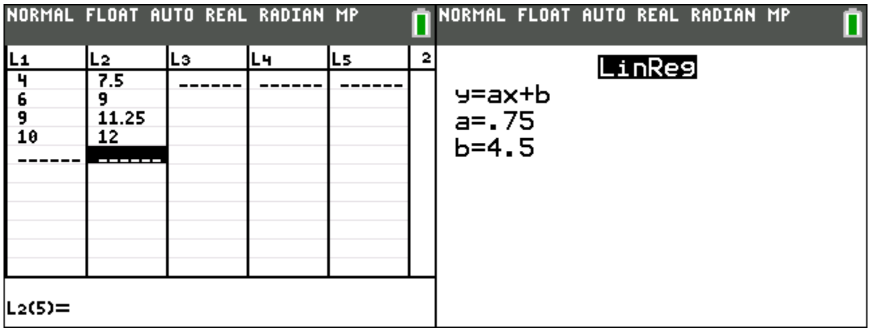
```
Plot1 Plot2 Plot3
\Y1=1.5X+1.5
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
X      Y1
0      1.5
1      3
2      4.5
3      6
4      7.5
5      9
6      10.5
Press + for ΔTbl
```

PTS: 2 NAT: F.BF.A.1 TOP: Modeling Linear Functions

118) ANS:
$f(x) = 0.75x + 4.50$. Each card costs 75¢ and start-up costs were $4.50.

Strategy: Input the table of values in a graphing calculator and use linear regression to write the function rule.



```
NORMAL FLOAT AUTO REAL RADIAN MP
L1     L2     L3     L4     L5     2
4      7.5    ------ ------ ------
6      9
9      11.25
10     12
------ ------
L2(5)=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
LinReg
y=ax+b
a=.75
b=4.5
```

PTS: 4 NAT: F.BF.A.1 TOP: Modeling Linear Functions

119) ANS: 3

Strategy: Interpret the the function $c(r) = 5.25r + 125$ in narrative (word) form.

$c(r)$	$=$	5.25	r	$+$	125
the cost of manufacturing r radios	=	$5.25	for each radio	plus	a start-up cost of $125

$5.25 for each radio represents the amount spent to manufacture each radio, which is answer choice c.

PTS: 2 NAT: F.LE.B.5 TOP: Modeling Linear Equations

120) ANS: 2

Strategy: Interpret the the function $y = 40 + 90x$ in narrative (word) form.

y	$=$	40	$+$	90	x
total cost	= a one time installation fee of	$40	plus a	$90	service charge times the number of months

PTS: 2 NAT: F.LE.B.5 TOP: Modeling Linear Equations

121) ANS: 2

The problem states that the employee is paid an hourly rate of $22.
In the equation $P(x) = 8600 - 22x$, the hourly rate of $22 appears next to the letter x, which is a *variable* representing the number of hours that the employee works.

DIMS (Does it Make Sense?)
Yes. The equation $P(x) = 8600 - 22x$ says that the owner's profit (P) is a function of how much the employee gets paid. As the value of x increases, the employee gets paid more and the owner's profits get smaller.

PTS: 2 NAT: A.SSE.A.1 TOP: Modeling Linear Equations

122) ANS: 3

Strategy: Interpret the the function $C(n) = 110n + 900$ in narrative (word) form, then eliminate wrong answers.

$C(c)$	$=$	110	n	$+$	900
The costs of a commercial	=	$110 times the	number of times the commerical airs	plus	a production cost of $900

Answer choice a is wrong because the the production costs are not $0.
Answer choice b is wrong because the production costs and costs per airing are reversed.
Answer choice c is correct.
Answer choice d in wrong because it makes no sense.

PTS: 2 NAT: F.LE.B.5 TOP: Modeling Linear Equations

123) ANS:

$$y = mx + b$$

$$y = (\text{slope})x + (y\text{-intercept})$$

$$C(x) = 50(m) + (79.50)$$

The slope is 50 and represents the amount paid each month for membewrship in the gym.
The y-intercept is 79.50 and represents the initial cost of membership.

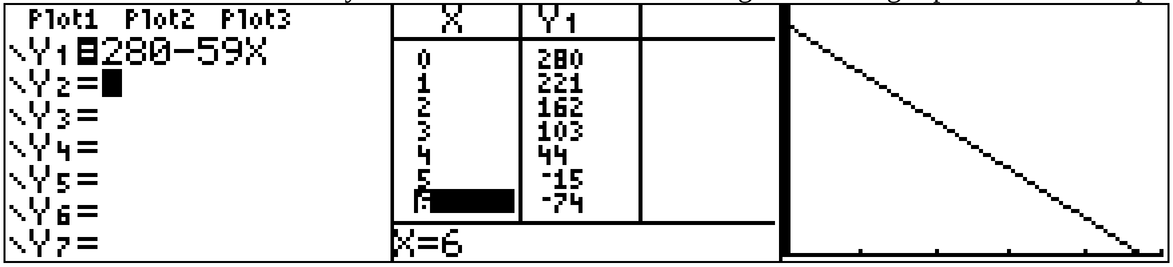
PTS: 2 NAT: F.LE.B.5 TOP: Modeling Linear Functions

124) ANS: 2

$$\text{speed}=\frac{\text{distance}}{\text{time}}$$

$$\text{speed}\cdot\text{time}=\text{distance}$$

The equation $D = 280 - 59t$ models the distance from Buffalo for a car after *t* hours. 280 represents the distance in miles from Albany to Buffalo. The car is travelling at an average speed of 59 miles per hour.



```
Plot1  Plot2  Plot3
\Y1=280-59X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

X	Y1
0	280
1	221
2	162
3	103
4	44
5	-15
6	-74

X=6

The car's distance from Albany decreases by 59 miles every hour, so 59 represents the speed of the car.

PTS: 2 NAT: F.LE.B.5 TOP: Modeling Linear Functions

125) ANS: 4

The function $c(t) = 125t + 95$ can be interpreted as follows: Cost is a function of time and is equal to $125 times the number of hours plus a set fee of $95. All three statements are true.

PTS: 2 NAT: F.LE.B.5 TOP: Modeling Linear Functions

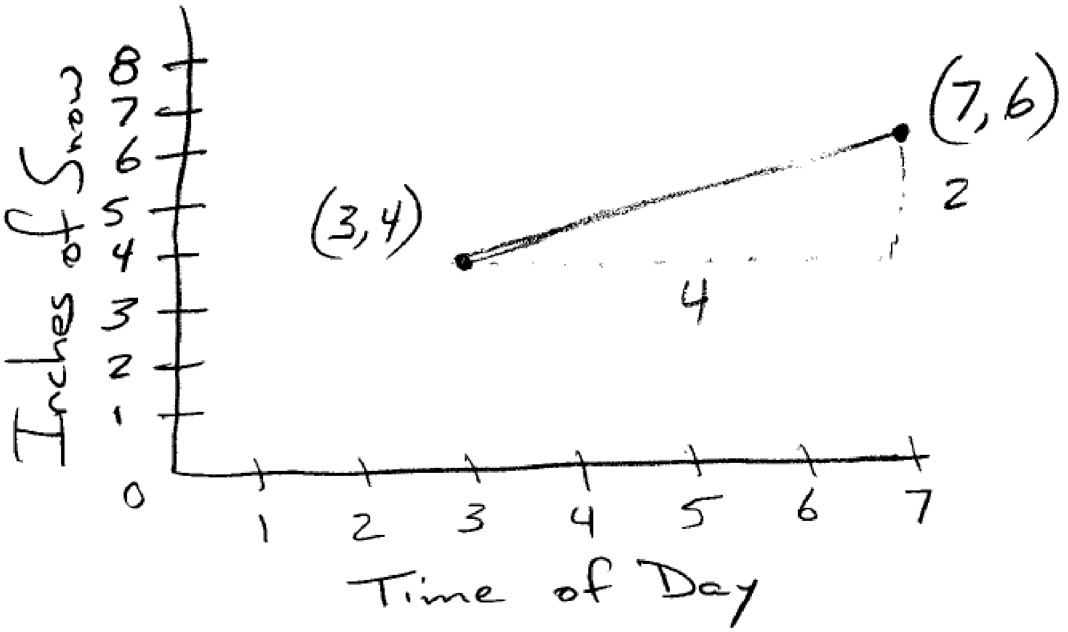
126) ANS: 2

The graph shows that the average number of hours of sleep per day *decreases* as age *increases*. The correlation is negative.

a) The average number of hours of sleep per day ~~*increases*~~ 0.09 hour per year of age.
b) The average number of hours of sleep per day *decreases* 0.09 hour per year of age.
c) The average number of hours of sleep per day ~~*increases* 9.24~~ hours per year of age.
d) The average number of hours of sleep per day decreases ~~9.24~~ hours per year of age.

PTS: 2 NAT: S.ID.C.7 TOP: Modeling Linear Functions

127) ANS:



$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-4}{7-3} = \frac{2 \text{ inches of snow}}{4 \text{ hours}}$$

The slope represents the rate of snowfall, which is 2 inches of snow every 4 hours.

PTS: 2 NAT: F.IF.B.6 TOP: Modeling Linear Functions

128) ANS: 3

Strategy: Identify the constant term in the expression $2.50a + 290$, what it means, and eliminate wrong answers..

STEP 1. *2.50a* is a variable term and 290 is a constant term. Eliminate the two answer choices that start with *2.50a*.

STEP 2. The term 290 can represent the amount Mike earns when he sells a accessories, since the term does not contain a. Eliminate this choice.

Does It Make Sense? Yes. Mike gets $2.50 for every cell phone accessory he sells plus a constant amount of $290 each week.

PTS: 2 NAT: F.LE.B.5 TOP: Modeling Linear Functions

E – Linear Equations, Lesson 2, Graphing Linear Functions (r. 2018)

LINEAR EQUATIONS

Graphing Linear Functions

Common Core Standards	**Next Generation Standards**
A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; ~~graph equations on coordinate axes with labels and scales~~.	**AI-A.CED.2** Create equations and linear inequalities in two variables to represent a **real-world context**. Notes: • **This is strictly the development of the model (equation/inequality).** • **Limit equations to linear, quadratic, and exponentials of the form** $f(x) = a(b)^x$ **where** $a > 0$ **and** $b > 0$ **(**$b \neq 1$**).**
F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* PARCC: Tasks have a real-world context. Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piece-wise defined functions (including step functions and absolute value functions) and exponential functions with domains in the integers.	**AI-F.IF.4** For a function that models a relationship between two quantities: i) interpret key features of graphs and tables in terms of the quantities; and ii) sketch graphs showing key features given a verbal description of the relationship. (Shared standard with Algebra II) Notes: • Algebra I key features include the following: intercepts, **zeros**; intervals where the function is increasing, decreasing, positive, or negative; maxima, minima; and symmetries. • Tasks have a real-world context and are limited to the following functions: linear, quadratic, square root, piecewise defined (including step and absolute value), and **exponential functions of the form** $f(x) = a(b)^x$ **where** $a > 0$ **and** $b > 0$ **(**$b \neq 1$**)**.

LEARNING OBJECTIVES

Students will be able to:

1) Write function rules that represent table and/or context views of a mathematical relationship between two variables.
2) Graph functions based on function rules, tables, or contexts.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

context
coordinate pair
function rule
graph
plot point
rate of change
table of values
x-axis intercept
y-axis intercept

BIG IDEAS

Three Facts About Graphs and Their Equations

1. The graph of an equation represents the set of all points that satisfy the equation (make the equation balance).

2. Each and every point on the graph of an equation represents a coordinate pair that can be substituted into the equation to make the equation true.

3. If a point is on the graph of the equation, the point is a solution to the equation.

How to Graph Any Equation:

Table of Values Method: Given a table of values for a function, simply plot each coordinate pair on a coordinate plane, then sketch the line that connects the plotted points. If given a function rule or context view, first create a table of values either manually or using a graphing calculator, then plot enough coordinate pairs to sketch the graph.

Minimum Number of Plot Points Required: Equations can be classified as either *linear* or *non-linear*. All linear equations, *except* vertical lines, are functions.

- **To graph a *linear* equation**, you need to plot a minimum of *two* points using either of the following methods:
  - **Two Points Method**: If you know two points on the line, simply plot both of them and draw a straight line passing through the two points.
  - **One Point and the Slope Method**: If you know one point on the line and the slope of the line, plot the point and use the slope to draw a right triangle to find a second point. Then, draw a straight line passing through the two points.
- **To graph a *non-linear* equation**, you need a minimum of *three* plot points. More plot points are better.

How to Find Intercepts of x and y-Axes

The ***x*-axis intercept** is the x-value of the point at which the graph of a relation intercepts the *x*-axis. The ordered pair for any point of the x-axis will always have a value of $y = 0$.

Example: The equation y = 2x + 8 has an *x*-intercept of -4. This can be found algebraically by substituting a value of 0 for y.

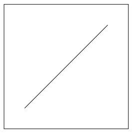
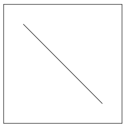
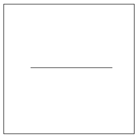
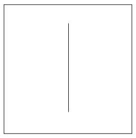
$$y = 2x + 8$$
$$0 = 2x + 8$$
$$-8 = 2x$$
$$\frac{-8}{2} = x$$
$$-4 = x$$

The ***y*-axis intercept** is the y-value of the point at which a graph of a relation intercepts the *y*-axis. The ordered pair for this point has a value of $x = 0$.

Example: The equation y = 8 + 2x has a y-intercept of 8. This can be found algebraically by substituting a value of zero for x.

$$y = 2x + 8$$
$$y = 2(0) + 8$$
$$y = 0 + 8$$
$$y = 8$$

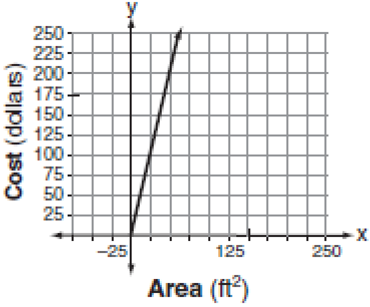
Rate of Change/Slope

 **Positive Slope** Goes up from left to right.	 **Negative Slope**. Goes down from left to right.
 **Zero Slope**. A horizontal line has a slope of zero.	 **Undefined Slope**. A vertical line has an undefined slope.

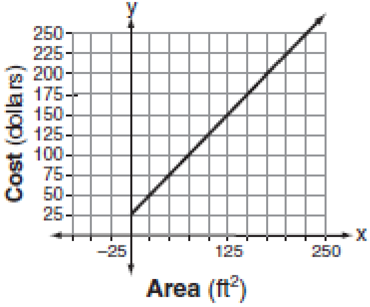
DEVELOPING ESSENTIAL SKILLS

Super Painters charges $1.00 per square foot plus an additional fee of $25.00 to paint a living room. If *x* represents the area of the walls of Francesca's living room, in square feet, and *y* represents the cost, in dollars, which graph best represents the cost of painting her living room?

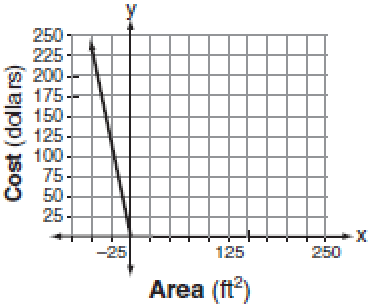
a)



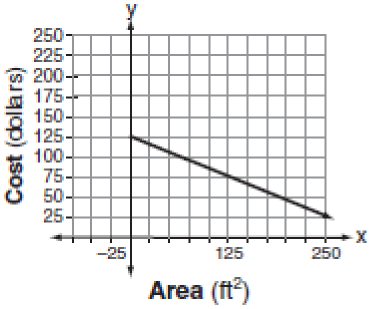
c)



b)



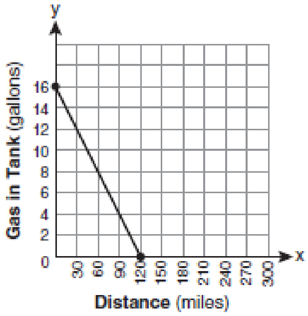
d)



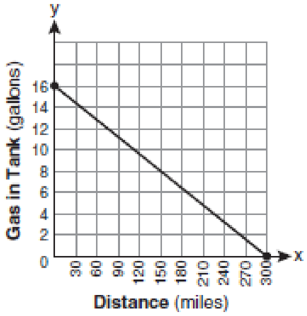
Answer: c: This graph has a *y*-intercept of 25 and a slope of 1.

The gas tank in a car holds a total of 16 gallons of gas. The car travels 75 miles on 4 gallons of gas. If the gas tank is full at the beginning of a trip, which graph represents the rate of change in the amount of gas in the tank?

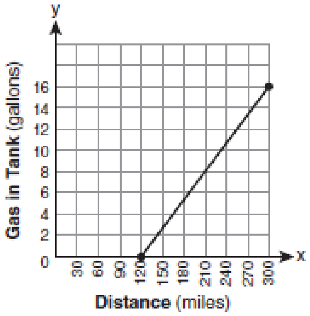
a)



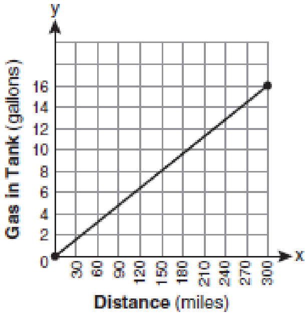
b)



c)



d)



Answer: b: If the car can travel 75 miles on 4 gallons, it can travel 300 miles on 16 gallons.

$$\frac{74}{4}=\frac{x}{16}$$

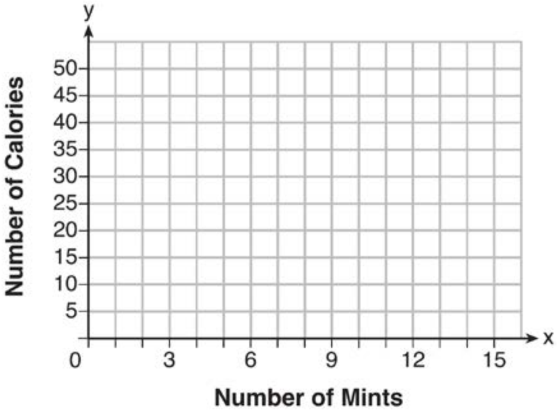
$$74\times16=4x\ .$$

$$300=x$$

REGENTS EXAM QUESTIONS (through June 2018)

A.CED.A.2, F.IF.B.4: Graphing Linear Functions

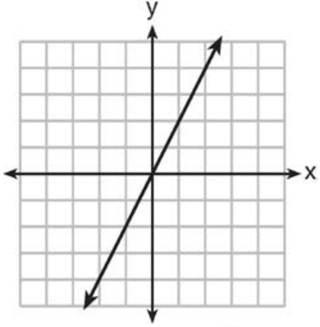
129) Max purchased a box of green tea mints. The nutrition label on the box stated that a serving of three mints contains a total of 10 Calories. On the axes below, graph the function, *C*, where $C(x)$ represents the number of Calories in *x* mints.



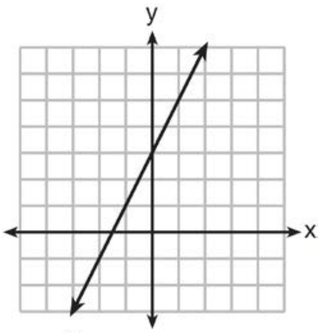
Write an equation that represents $C(x)$. A full box of mints contains 180 Calories. Use the equation to determine the total number of mints in the box.

130) Which graph shows a line where each value of y is three more than half of x?

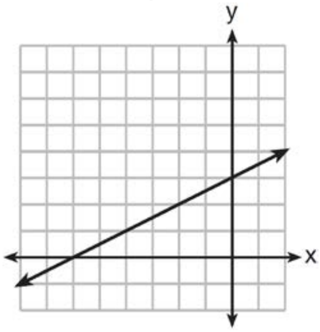
1)



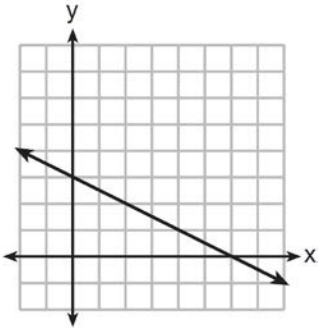
3)



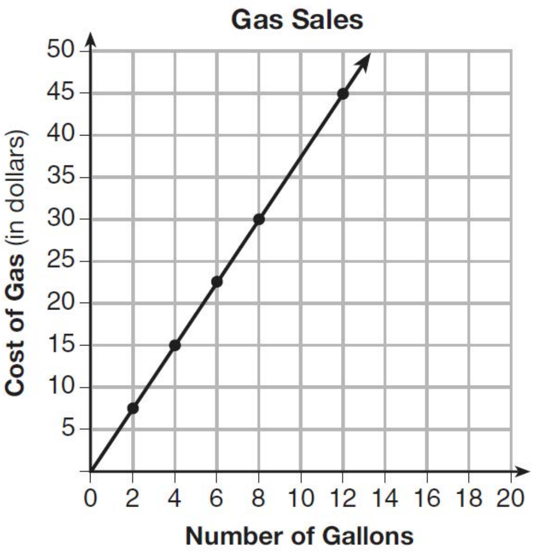
2)



4)



131) The graph below was created by an employee at a gas station.



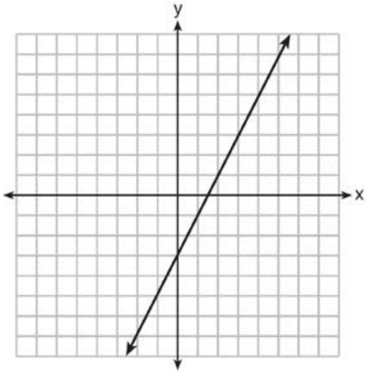
Which statement can be justified by using the graph?

1) If 10 gallons of gas was purchased, \$35 was paid.
2) For every gallon of gas purchased, \$3.75 was paid.
3) For every 2 gallons of gas purchased, \$5.00 was paid.
4) If zero gallons of gas were purchased, zero miles were driven.

132) The value of the x-intercept for the graph of $4x - 5y = 40$ is

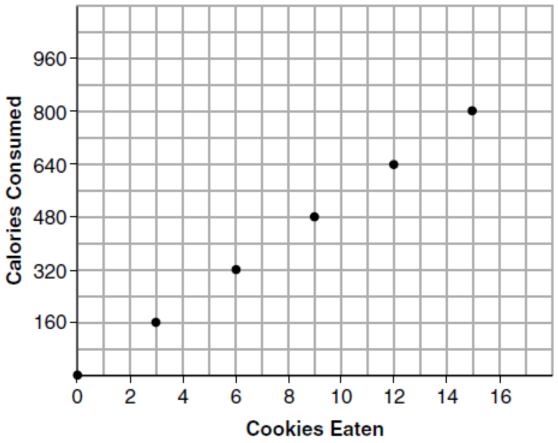
1) 10
2) $\frac{4}{5}$
3) $-\frac{4}{5}$
4) -8

133) Which function has the same y-intercept as the graph below?



1) $y = \frac{12 - 6x}{4}$
2) $27 + 3y = 6x$
3) $6y + x = 18$
4) $y + 3 = 6x$

134) Samantha purchases a package of sugar cookies. The nutrition label states that each serving size of 3 cookies contains 160 Calories. Samantha creates the graph below showing the number of cookies eaten and the number of Calories consumed.

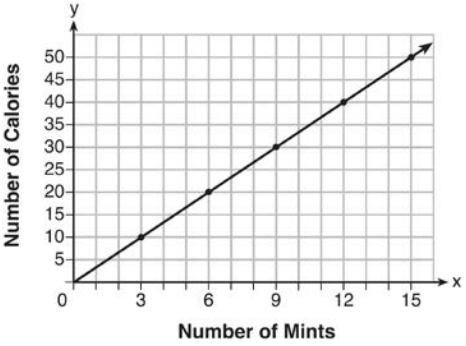


Explain why it is appropriate for Samantha to draw a line through the points on the graph.

SOLUTIONS

129) ANS:

a)



b) $C(x) = \frac{10}{3}x$

c) A full box contains 54 mints.

Strategy: Write the equation, then graph the equation, then use the equation and 180 calories to determine the number of mints in a full box.

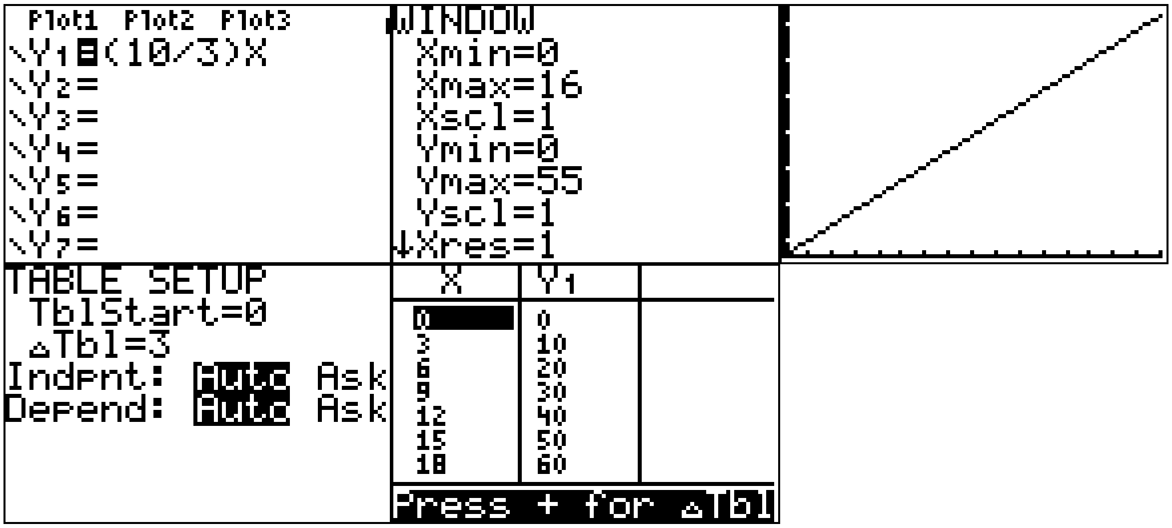
STEP 1. Write the equation.

If 3 mints contain ten calories, then one mint contains $\frac{10}{3}$ calories, and x number of mints contains $\frac{10}{3}x$ calories. Therefore: $C(x) = \frac{10}{3}x$.

STEP 2: Transform the equation and input the equation into a graphing calculator.

$$C(x) = \frac{10}{3}x$$

$$Y_1 = \frac{10}{3}x$$



```
Plot1 Plot2 Plot3
\Y1=(10/3)X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
WINDOW
 Xmin=0
 Xmax=16
 Xscl=1
 Ymin=0
 Ymax=55
 Yscl=1
↓Xres=1
```

```
TABLE SETUP
 TblStart=0
 ΔTbl=3
Indpnt: Auto Ask
Depend: Auto Ask
```

X	Y1	
0	0	
3	10	
6	20	
9	30	
12	40	
15	50	
18	60	

Press + for ΔTbl

STEP 3. Transfer the graph from the calculators table of values to the paper graph and complete the graph.

STEP 4. Substitute 180 calories for $C(x)$ in the equation and solve for *x* (the number of mints)

$$C(x) = \frac{10}{3}x$$

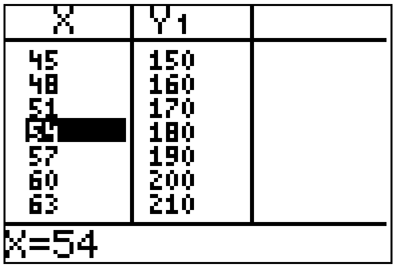
$$180 = \frac{10}{3}x$$

$$540 = 10x$$

$$54 = x$$

There are 54 mints in a full box.

DIMS: Does It Make Sense? Yes. The table view of the function also shows that 180 calories is paired with 54 mints.



X	Y1	
45	150	
48	160	
51	170	
54	180	
57	190	
60	200	
63	210	

X=54

PTS: 4 NAT: A.CED.A.2 TOP: Graphing Linear Functions

130) ANS: 2

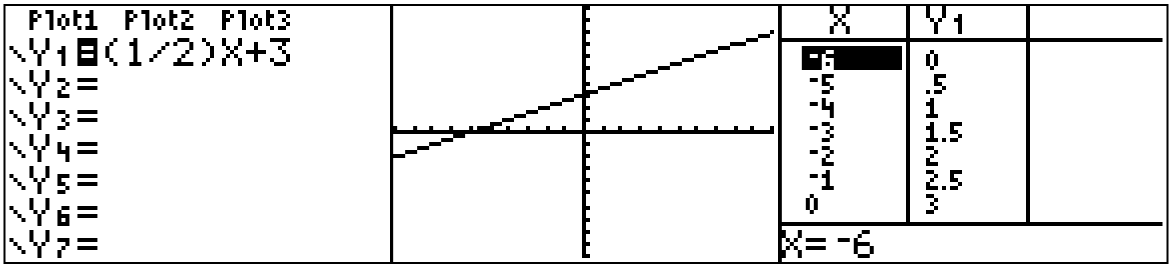
Strategy: Convert the narrative view to a function rule, then graph it.

STEP 1. Write the function rule.

$$\begin{array}{cccc} y & = & +3 & +\frac{1}{2}x \\ \hline \multicolumn{4}{c}{\text{(each value of y) is (three more) than (half of x)}} \end{array}$$

$$y = \frac{1}{2}x + 3$$

STEP 2. Input the function rule in a graphing calculator and compare the graph view of the function to the answer choices.



```
\Y1=(1/2)X+3
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

X	Y1	
-6	0	
-5	.5	
-4	1	
-3	1.5	
-2	2	
-1	2.5	
0	3	

X= -6

Answer choice *b* is correct.

DIMS? Does It Make Sense? Yes. The x and y intercepts are reflected in both the graph and the table of values.

PTS: 2 NAT: A.CED.A.2 TOP: Graphing Linear Functions
KEY: bimodalgraph

131) ANS: 2

Strategy #1: Use the slope of the line to determine the cost per gallon of gas. Select any two points that are on intersections of vertical and horizontal gridlines, then substitute them into the slope formula to determine the rate of change, which is the cost per gallon of gas.

$$\textit{Select } (8,30) \textit{ and } (4,15)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30-15}{8-4} = \frac{15}{4} = \$3.75$$

or

$$\textit{Select } (12,45) \textit{ and } (8,30)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{45-30}{12-8} = \frac{15}{4} = \$3.75$$

For every gallon of gas purchased. $3.75 was paid.

Strategy #2. Eliminate wrong answers.
Choice (a) is wrong because the chart shows that 10 gallongs of gas costs $37.50, not $35.00.
Choice (b) is correct.
Choice (c) is wrong because the chart shows that 2 gallons of gas cost $7.50, not $5.00.
Choice (d) is wrong because the chart says nothing about the number of miles driven.

PTS: 2 NAT: A.CED.A.2 TOP: Graphing Linear Functions

132) ANS: 1

Strategy: Find the value of x when y equals 0. NOTE: The x-intercept can also be defined as the x-value of the coordinate where the graph intercepts (passes through) the x-axis.

$$4x - 5y = 40$$
$$4x + 5(0) = 40$$
$$4x = 40$$
$$x = 10$$

The value of the x-intercept is 10.

PTS: 2 NAT: F.IF.C.9 TOP: Graphing Linear Functions

133) ANS: 4

Strategy: Identify the y-intercept in the graph, then test each answer choice to see if it has the same y-intercept.

STEP 1. Identify the y-intercept in the graph.

The y-intercept is can be defined as the y-value of the coordinate where the graph intercepts (passes through) the y-axis. The graph shows that the function passes through the y-axis at the point $(0,-3)$, so the value of the y-intercept is -3.

STEP 2. Test the other equations to see if the point $(0,-3)$works.

a	$y = \frac{12-6x}{4}$ Does not work $-3 = \frac{12-6(0)}{4}$ $-3 = \frac{12}{4}$ $-3 \neq 3$	c	$6y + x = 18$ Does not work $6(-3) + (0) = 18$ $-18 \neq 18$
b	$27 + 3y = 6x$ Does not work $27 + 3(-3) = 6(0)$ $27 - 9 = 0$ $18 \neq 0$	d	$y + 3 = 6x$ $(0,-3)$works! $(-3) + 3 = 6(0)$ $0 = 0$

PTS: 2 NAT: F.IF.C.9 TOP: Graphing Linear Functions

134) ANS:

A line is appropriate because the data is continuous. Samantha could eat any fraction of a cookie and fill in the line between the points for whole cookies.

PTS: 2 NAT: F.IF.B.4 TOP: Graphing Linear Functions

E – Linear Equations, Lesson 3, Writing Linear Equations (r. 2018)

LINEAR EQUATIONS

Writing Linear Equations

Common Core Standard	**Next Generation Standard**
A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, ~~often forming a curve (which could be a line)~~.	**AI-A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. **Note: Graphing linear equations is a fluency recommendation for Algebra I. Students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity; as well as modeling linear phenomena.**

LEARNING OBJECTIVES

Students will be able to:

1) Determine if different equations represent the same mathematical relationship between two variables.
2) Write the equation of a line of a line given two points on the line or one point and the slope of the line.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

transform	equivalent	relationship
isolate	$y = mx + b$ form	

BIG IDEAS

Three Facts About Graphs and Their Equations

1. The graph of an equation represents the set of all points that satisfy the equation (make the equation balance).

2. Each and every point on the graph of an equation represents a coordinate pair that can be substituted into the equation to make the equation true.

3. If a point is on the graph of the equation, the point is a solution to the equation.

Equivalent Forms of Equations

An equation represents a mathematical relationship between variables. The same relationship between the variables can be represented in many different ways. For example, y = 2x, 2y = 4x, and 3y = 6x all represent the same idea that y is 2 times x.

To determine if different equations represent the same mathematical relationship between variables, use one or more of the following strategies.

- transform the different equations into equivalent forms. If the equations can be transformed into identical forms, the equations represent the same mathematical relationship between the variables.
- isolate the same variable in all the equations and input the equations in a graphing calculator. If the tables of values and graphs are identical, the equations represent the same mathematical relationship between the variables.

Given Two Points on a Line, or One Point and the Slope of a Line, How to Write the Equation of the Line

STEP 1. First, find the slope. If not given, use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

STEP 2. Set up and label three columns, as follows:

Write what you are given in this column.	**y = mx + b**	**y = mx + b**
y = **m =** **x =** **b =**	*Substitute the values from the first column into the formula and solve for the unknown b value in this column.*	*Use this column to write the final equation by substituting m and b in the slope-intercept form.*

STEP 3. Complete each column, left to right. The last column will be the equation of the line.

Example:

Write the equation of the line that passes through (-5, 6) and (7, 2).

Step 1. Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2-6}{7-(-5)}$$

$$m = \frac{-4}{12}$$

$$m = -\frac{1}{3}$$

Write what you are given in this column.	**y = mx + b**	**y = mx + b**

y = 2 **m** = $-\frac{1}{3}$ **x** = 7 **b** = b	$2=-\frac{1}{3}(7)+b$ $2=-\frac{7}{3}+b$ $2+\frac{7}{3}=b$ $4\frac{1}{3}=b$	$y=\frac{1}{3}x+4\frac{1}{3}$

DEVELOPING ESSENTIAL SKILLS

Which of the following equations represent the same mathematical relationship between the variables? Justify your answer.

$$y=3x+6$$

$$y=3(x+2)$$

$$y-4=3x+2$$

$$\frac{1}{3}y=x+2$$

All of the equations represent the same mathematical relationship.

$$y=3x+6$$

Use distributive property $y=3(x+2)$

$$\boxed{y=3x+6}$$

Add 4 to both expressions $y-4=3x+2$

$$\boxed{y=3x+6}$$

Multiply both expressions by 3 $\frac{1}{3}y=x+2$

$$\boxed{y=3x+6}$$

Write the equation of the line that passes through the points (-2, -8) and (6, 16).

$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$m=\frac{16-(-8)}{6-(-2)}$$
$$m=\frac{24}{8}$$
$$m=3$$

Write what you are given in this column. **y** = 16 **m** = 3 **x** = 6 **b** = b	**y = mx + b** $16=3(6)+b$ $16=18+b$ $-2=b$	**y = mx + b** $y=3x-2$

REGENTS EXAM QUESTIONS (through June 2018)

A.REI.D.10: Writing Linear Equations

135) The graph of a linear equation contains the points $(3,11)$ and $(-2,1)$. Which point also lies on the graph?

1) $(2,1)$
2) $(2,4)$
3) $(2,6)$
4) $(2,9)$

136) Sue and Kathy were doing their algebra homework. They were asked to write the equation of the line that passes through the points $(-3,4)$ and $(6,1)$. Sue wrote $y-4=-\frac{1}{3}(x+3)$ and Kathy wrote $y=-\frac{1}{3}x+3$. Justify why both students are correct.

137) How many of the equations listed below represent the line passing through the points $(2,3)$ and $(4,-7)$?

$$5x+y=13$$
$$y+7=-5(x-4)$$
$$y=-5x+13$$
$$y-7=5(x-4)$$

1) 1
2) 2
3) 3
4) 4

SOLUTIONS

135) ANS: 4

Strategy: Find the slope of the line between the two points, then use $y-mx+b$ to find the y-intercept, then write the equation of the line and determine which answer choice is also on the line.

STEP 1. Find the slope of the line that passes through the points $(3,11)$ and $(-2,1)$.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{1-11}{-2-3}=\frac{-10}{-5}=2$$

Write $y = 2x + b$

STEP 2. Use either given point and the equation $y = 2x + b$ to solve for *b*, the y-intercept. The following calculation uses the point (3,11).

$$y = 2x + b$$

$$11 = 2(3) + b$$

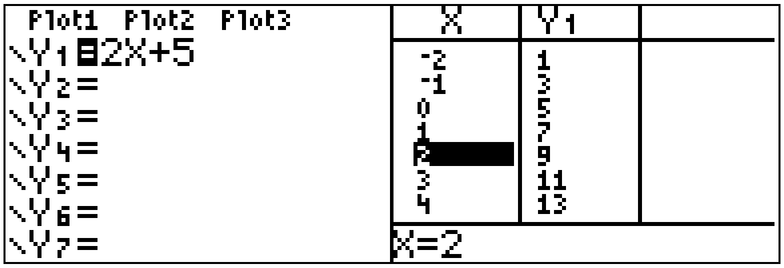
$$11 = 6 + b$$

$$5 = b$$

Write $y = 2x + 5$

STEP 3 Determine which answer choice balances the equation $y = 2x + 5$.

Use a graphing calculator



```
Plot1 Plot2 Plot3
\Y1=2X+5
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

X	Y1
-2	1
-1	3
0	5
1	7
2	9
3	11
4	13

X=2

or simply solve the equation $y = 2x + 5$ for *y* when $x = 2$.

$$y = 2x + 5$$

$$y = 2(2) + 5$$

$$y = 4 + 5$$

$$y = 9$$

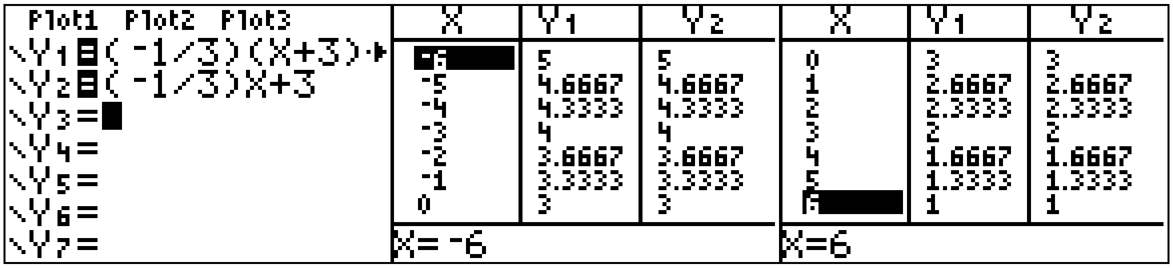
The point (2, 9) is also on the line.

PTS: 2 NAT: A.REI.D.10 TOP: Graphing Linear Functions

136) ANS:

Strategy: Input both equations in a graphing calculator and see if they produce the same outputs.

Sue's Equation y_1	Kathy's Equation y_2
$y_1 - 4 = -\frac{1}{3}(x + 3)$ $y_1 = -\frac{1}{3}(x + 3) + 4$	$y_2 = -\frac{1}{3}x + 3$



```
Plot1 Plot2 Plot3
\Y1=(-1/3)(X+3)▸
\Y2=(-1/3)X+3
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

X	Y1	Y2
-6	5	5
-5	4.6667	4.6667
-4	4.3333	4.3333
-3	4	4
-2	3.6667	3.6667
-1	3.3333	3.3333
0	3	3

X=-6

X	Y1	Y2
0	3	3
1	2.6667	2.6667
2	2.3333	2.3333
3	2	2
4	1.6667	1.6667
5	1.3333	1.3333
6	1	1

X=6

Both students are correct because both equations pass through the points $(-3, 4)$ and $(6, 1)$.

Alternate justification: Show that the points $(-3, 4)$ and $(6, 1)$ satisfy both equations.

Sue’s Equation y_1		Kathy’s Equation y_2	
$y-4=-\frac{1}{3}(x+3)$		$y=-\frac{1}{3}x+3$	
$(-3,4)$	$(6,1)$	$(-3,4)$	$(6,1)$
$4-4=-\frac{1}{3}(-3+3)$	$1-4=-\frac{1}{3}(6+3)$	$y=-\frac{1}{3}x+3$	$y=-\frac{1}{3}x+3$
$0=-\frac{1}{3}(0)$	$-3=-\frac{1}{3}(9)$	$4=-\frac{1}{3}(-3)+3$	$1=-\frac{1}{3}(6)+3$
$0=0$	$-3=-3$	$4=1+3$	$1=-2+3$
		$4=4$	$1=1$

Both students are correct because the points $(-3,4)$ and $(6,1)$ satisfy both equations.

PTS: 2 NAT: A.REI.D.10 TOP: Writing Linear Equations
KEY: other forms

137) ANS: 3

Step 1. Transform each equation for input into a graphing calculator.

Original	Input in Calculator
$5x+y=13$	$y=13-5x$
$y+7=-5(x-4)$	$y=-5(x-4)-7$
$y=-5x+13$	$y=-5x+13$
$y-7=5(x-4)$	$y=5(x-4)+7$

Step 2. Input each equation in a graphing calculator and inspect the tables of values for the points $(2,3)$ and $(4,-7)$.

INEQUALITIES
Solving Linear Inequalities

Common Core Standard	Next Generation Standard
A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**AI-A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **Note: Algebra I tasks do not involve solving compound inequalities**.

NOTE: This lesson is closely related to, and builds upon, Expressions and Equations, Lesson 3, Solving Linear Equations.

LEARNING OBJECTIVES

Students will be able to:

1) Solve one step and multiple step inequalities.
2) Explain each step involved in solving one step and multiple step inequalities.
3) Do a check to see if the solution is correct.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

big rule of inequalities
equality
four column strategy
four general rules
greater than
greater than or equal to
inequality
inequality sign
less than
less than or equal to
not equal to
solution set

BIG IDEAS

The Big Rule for Solving Inequalities:
All the rules for solving equations apply to inequalities – plus one:
When an inequality is multiplied or divided by any negative number, the direction of the inequality sign changes.

Inequality Symbols:

$<$	less than	$>$	greater than
$\leq$	less than or equal to	$\geq$	greater than or equal to
$\neq$	not equal to		

The **solution of an inequality** includes any values that make the inequality true. Solutions to inequalities can be graphed on a number line using open and closed dots.

Checking Solutions to Inequalities

To check the **solution** to an **inequality**, replace the **variable** in the inequality with a value in the solution set. If the value selected is a correct solution, the simplified inequality will produce a true statement.
NOTE: The value selected *must* be in the solution set.

DEVELOPING ESSENTIAL SKILLS

Solve for x: $4+\frac{2}{5}x>3+x$

Notes	Left Hand Expression	Sign	Right Hand Expression
Given	$4+\frac{2}{5}x$	$>$	3+x
Multiply by 5	$20+2x$	$>$	$15+5x$
Subtract 2x	20	$>$	15+3x
Subtract 15	5	$>$	3x
Divide by 3	$\frac{5}{3}$	$>$	x
Check	Select $\frac{4}{3}$, which is less than $\frac{5}{3}$, to test the solution. $4+\frac{2}{5}x>3+x$ $4+\frac{2}{5}\left(\frac{4}{3}\right)>3+\left(\frac{4}{3}\right)$ $4+\frac{8}{15}>3+\frac{20}{15}$ $\frac{60}{15}+\frac{8}{15}>\frac{45}{15}+\frac{20}{15}$ $\frac{68}{15}>\frac{65}{15}$ *true*		

REGENTS EXAM QUESTIONS (through June 2018)

A.REI.B.3: Solving Linear Inequalities

138) The inequality $7-\frac{2}{3}x<x-8$ is equivalent to

1) $x>9$	3) $x<9$
2) $x>-\frac{3}{5}$	4) $x<-\frac{3}{5}$

139) Given that $a>b$, solve for *x* in terms of *a* and *b*:

$$b(x-3)\geq ax+7b$$

140) When $3x+2\leq 5(x-4)$ is solved for *x*, the solution is

1) $x\leq 3$	3) $x\leq -11$
2) $x\geq 3$	4) $x\geq 11$

141) What is the solution to $2h+8>3h-6$?

1) $h<14$	3) $h>14$
2) $h<\frac{14}{5}$	4) $h>\frac{14}{5}$

142) Solve the inequality below:

$$1.8-0.4y\geq 2.2-2y$$

143) What is the solution to the inequality $2+\frac{4}{9}x\geq 4+x$?

1) $x\leq -\frac{18}{5}$	3) $x\leq \frac{54}{5}$
2) $x\geq -\frac{18}{5}$	4) $x\geq \frac{54}{5}$

144) The solution to $4p+2<2(p+5)$ is

1) $p>-6$	3) $p>4$
2) $p<-6$	4) $p<4$

SOLUTIONS

138) ANS: 1

Strategy: Use the four column method for solving and documenting an equation or inequality.

Notes	Left Expression	Sign	Right Expression
Given:	$7-\frac{2}{3}x$	<	$x-8$
Add +8 to both expressions (Addition property of equality)	$15-\frac{2}{3}x$	<	x
Add $+\frac{2}{3}x$ to both expressions (Addition property of equality)	15	<	$x+\frac{2}{3}x$
Simplify	15	<	$\frac{5}{3}x$

Divide both expressions by $\frac{5}{3}$ (Division property of equality)	$\frac{\frac{15}{1}}{\frac{5}{3}}$	<	$\frac{\frac{5}{3}x}{\frac{5}{3}}$
Simplify	9	<	x
Rewrite	x	>	9

PTS: 2 NAT: A.REI.B.3 TOP: Solving Linear Inequalities

139) ANS:

$x \le \frac{10b}{b-a}$

Strategy: Use the four column method. Remember that $a > b$.

Notes	Left Expression	Sign	Right Expression
Given	$b(x-3)$	$\ge$	$ax+7b$
Distributive Property	$bx-3b$	$\ge$	$ax+7b$
Transpose	$bx-ax$	$\ge$	$10b$
Factor	$x(b-a)$	$\ge$	$10b$
Divide by $(b-a)$	x	$\le$ See NOTE below	$\frac{10b}{b-a}$

NOTE: Since $a > b$, the expression $(b-a)$ must be a negative number. When dividing an inequality by a negative number, the direction of the inequality sign must be reversed.

PTS: 2 NAT: A.REI.B.3 TOP: Solving Linear Inequalities

140) ANS: 4

$$3x+2 \le 5(x-4)$$
$$3x+2 \le 5x-20$$
$$2+20 \le 5x-3x$$
$$22 \le 2x$$
$$11 \le x$$
$$x \ge 11$$

PTS: 2 NAT: A.REI.B.3 TOP: Solving Linear Inequalities

141) ANS: 1

$$2h+8 > 3h-6$$
$$2h+14 > 3h$$
$$14 > h$$

PTS: 2 NAT: A.REI.B.3

142) ANS:

$y \ge \frac{1}{4}$

Given	$1.8-0.4y$	$\ge$	$2.2-2y$

Add (2y)	$+2y$		$+2y$
Simplify	$1.8+1.6y$	$\geq$	2.2
Subtract (1.8)	-1.8		-1.8
Simplify	$1.6y$	$\geq$	0.4
Divide (1.6)	$\frac{1.6y}{1.6}$	$\geq$	$\frac{0.4}{1.6}$
Simplify	y	$\geq$	$\frac{1}{4}$

$$1.8-0.4y\geq 2.2-2y$$
$$1.6y\geq 0.4$$
$$y\geq 0.25$$

PTS: 2 NAT: A.REI.B.3 TOP: Solving Linear Inequalities

143) ANS: 1

$$2+\frac{4}{9}x\geq 4+x$$
$$18+4x\geq 36+9x$$
$$-5x\geq 18$$
$$x\leq\frac{18}{-5}$$
$$x\leq-\frac{18}{5}$$

Remember to change the direction of the inequality sign when multiplying or dividing by a negative number.

PTS: 2 NAT: A.REI.B.3 TOP: Solving Linear Inequalities

144) ANS: 4

Strategy: Use order of operations.

Notes	Left Expression	Sign	Right Expression
Given	$4p+2$	<	$2(p+5)$
Divide by 2	$2p+1$	<	$p+5$
Subtract p	$p+1$	<	5
Subtract 1	p	<	4

PTS: 2 NAT: A.REI.B.3 TOP: Solving Linear Inequalities

INEQUALITIES
Interpreting Solutions

Common Core Standard	**Next Generation Standard**
A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**AI-A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **Note: Algebra I tasks do not involve solving compound inequalities**.

LEARNING OBJECTIVES

Students will be able to:

1) Identify solutions to inequalities as sets of solutions that can be plotted on a number line.
2) Use proper notation to define solution sets.
3) Identify integer values within solution sets.
4) Determine if a specified integer value is within a solution set.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

integer
solution set
open dot
closed dot
curved parentheses
square parentheses
number line

BIG IDEAS

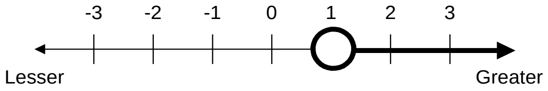
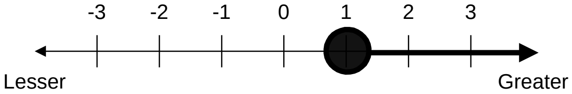
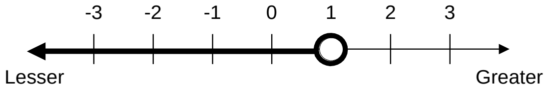
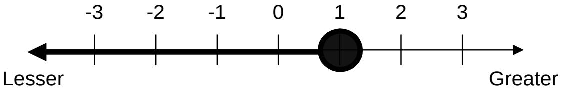
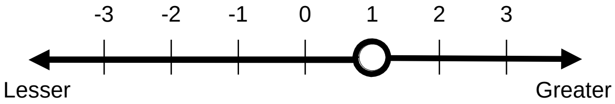
Inequality Symbols:

$<$ less than
$>$ greater than
$\leq$ less than or equal to
$\geq$ greater than or equal to
$\neq$ not equal to

The **solution of an inequality** includes any values that make the inequality true. Solutions to inequalities can be graphed on a number line using open and closed dots.

Open Dots v Closed Dots

Square vs Curved Parentheses

When the inequality sign does not contain an equality bar beneath it, the dot is open.	When the inequality sign contains includes an equality bar beneath it, the dot is closed, or shaded in.
Graph of $x > 1$ or (1... means 1 *is not* included in the solution set. 	Graph of $x \geq 1$ or [1... means 1 *is* included in the solution set 
Graph of $x < 1$ or ...1) means 1 *is not* included in the solution set 	Graph of $x \leq 1$ or ...1] means 1 *is* included in the solution set 
Graph of $x \neq 1$ 	

DEVELOPING ESSENTIAL SKILLS

Solve for the smallest integer value of x: $3+\frac{2}{5}x \geq 4-6x$

Notes	Left Hand Expression	Sign	Right Hand Expression
Given	$3+x$	$\geq$	$4-6x$
Add 6x	3+7x	$\geq$	4
Subtract 3	7x	$\geq$	1
Divide by 7	x	$\geq$	$\frac{1}{7}$
Answer	1 is the smallest integer that is in the solution set.		
Check	0 is less than $\frac{1}{7}$ and should *not* be in the solution set. $3+x \geq 4-6x$ $3+(0) \geq 4-6(0)$ $3 \geq 4$ *not* true		1 is greater than or equal to $\frac{1}{7}$ and *should* be in the solution set. $3+x \geq 4-6x$ $3+(1) \geq 4-6(1)$ $4 \geq 4-6$ $4 \geq -2$ true

REGENTS EXAM QUESTIONS (through June 2018)

A.REI.B.3: Interpreting Solutions

145) Given $2x + ax - 7 > -12$, determine the largest integer value of *a* when $x = -1$.

146) Solve the inequality below to determine and state the smallest possible value for *x* in the solution set.

$$3(x+3) \le 5x - 3$$

147) Determine the smallest integer that makes $-3x + 7 - 5x < 15$ true.

148) Solve for *x* algebraically: $7x - 3(4x - 8) \le 6x + 12 - 9x$
If *x* is a number in the interval $[4, 8]$, state all integers that satisfy the given inequality. Explain how you determined these values.

149) Which value would be a solution for *x* in the inequality $47 - 4x < 7$?

1) -13	3) 10
2) -10	4) 11

150) Given the set $\{x \mid -2 \le x \le 2, \text{where } x \text{ is an integer}\}$, what is the solution of $-2(x-5) < 10$?

1) $0, 1, 2$	3) $-2, -1, 0$
2) 1, 2	4) $-2, -1$

SOLUTIONS

145) ANS:
The largest integer value for *a* is 2.
Strategy: Use the four column method.

Notes	Left Expression	Sign	Right Expression
Given	$2x + ax - 7$	>	-12
Substitute -1 for *x*	$2(-1) + a(-1) - 7$	>	-12
Simplify	$-2 - a - 7$	>	-12
Combine like terms	$-a - 9$	>	-12
Add +9 to both expressions (Addition property of equality)	$-a$	>	-3
Divide both expressions by -1 and reverse the sign	a	<	3

The largest integer value that is less the 3 is 2.

PTS: 2 NAT: A.REI.B.3 TOP: Solving Linear Inequalities

146) ANS:
6 is the smallest possible value for *x* in the solution set.

Strategy: Use the four column method.

Notes	Left Expression	Sign	Right Expression
Given	$3x + 9$	$\le$	$5x - 3$
Subtract 3x from both expressions	9	$\le$	$2x - 3$

(Subtraction property of equality)			
Add +3 to both expressions (Addition Property of equality)	12	$\le$	$2x$
Divide both expressions by 2 (Division property of equality)	6	$\le$	x
Rewrite	x	$\ge$	6

PTS: 2 NAT: A.REI.B.3 TOP: Solving Linear Inequalities

147) ANS:
0 is the smallest integer in the solution set.
.
Strategy: Use the four column method to solve the inequality, then interpret the solution.

STEP 1: Solve the inequality.

Notes	Left Expression	Sign	Right Expression
Given	$-3x+7-5x$	<	15
Simplify (Combine like terms)	$-8x+7$	<	15
Add +8x to both expressions (Addition Property of Equality)	7	<	8x+15
Subtract 15 from both expressions (Subtraction Property of Equality)	-8	<	8x
Divide both expressions by +8 (Division property of equality)	-1	<	x
Rewrite	x	>	-1

STEP 2: Interpret the solution set for the smallest integer.
The smallest integer greater than -1 is 0.

PTS: 2 NAT: A.REI.B.3 TOP: Solving Linear Inequalities

148) ANS:
6, 7, 8 are the numbers greater than or equal to 6 in the interval.

Strategy: Use the four column method to solve the inequality, then interpret the solution.

STEP 1: Solve the inequality.

Notes	Left Expression	Sign	Right Expression
Given	$7x-3(4x-8)$	$\le$	$6x+12-9x$
Clear parentheses	$7x-12x+24$	$\le$	$6x+12-9x$

(Distributive property)			
Simplify (Combine like terms)	$-5x+24$	$\leq$	$-3x+12$
Add 5x to both expressions (Addition property of equality)	24	$\leq$	$2x+12$
Subtract 12 from both expressions (Subtraction property of equality)	12	$\leq$	2x
Divide both expressions by 2 (Division property of equality)	6	$\leq$	x
Rewrite	x	$\geq$	6

STEP 2: Interpret the solution set for the interval $[4, 8]$.
The interval $[4, 8]$ contains the integers 4, 5, 6, 7, and 8.
If $x \geq 6$, then the solution set of integers is $\{6, 7, 8\}$.

PTS: 4 NAT: A.REI.B.3 TOP: Solving Linear Inequalities

149) ANS: 4

$$47 - 4x < 7$$

$$-4x < -40$$

Remember to change the direction of the sign when multiplying or dividing an inequality by a negative number.

$$x > \frac{-40}{-4}$$

$$x > 10$$

11 is the only answer choice that is greater than 10.

PTS: 2 NAT: A.REI.B.3 TOP: Interpreting Solutions

150) ANS: 2

STEP 1: Solve the inequality $-2(x-5) < 10$

$$-2(x-5) < 10$$

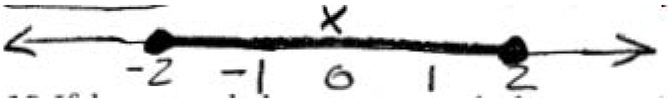
$$\frac{-2(x-5)}{-2} < \frac{10}{-2}$$

$$x - 5 > -5$$

$$x > 0$$

STEP 2: Select integers from the interval $\{x \mid -2 \leq x \leq 2, \text{where } x \text{ is an integer}\}$ that satisfy the inequality.
The integers in the interval are: {-2, -1, 0, 1, 2}.

-2 is not greater than 0
-1 is not greater than 0
0 is not greater than 0
1 is greater than 0
2 is greater than zero.



PTS: 2 NAT: A.REI.B.3 TOP: Interpreting Solutions

F – Inequalities, Lesson 3, Modeling Linear Inequalities (r. 2018)

INEQUALITIES
Modeling Linear Inequalities

Common Core Standards	**Next Generation Standards**
A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* PARCC: Tasks are limited to linear, quadratic, or exponential s with integer exponents.	**AI-A.CED.1** Create equations and inequalities in one variable to **represent a real-world context**. (Shared standard with Algebra II) Notes: • **This is strictly the development of the model (equation/inequality).** • **Limit equations to linear, quadratic, and exponentials of the form $f(x) = a(b)_x$ where $a > 0$ and $b > 0$ ($b \neq 1$).** • **Work with geometric sequences may involve an exponential equation/formula of the form $a_n = ar_{n-1}$, where a is the first term and r is the common ratio.** • **Inequalities are limited to linear inequalities.** • Algebra I tasks do not involve compound inequalities.
A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	**AI-A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.

NOTE: This lesson is related to Expressions and Equations, Lesson 4, Modeling Linear Equations

LEARNING OBJECTIVES

Students will be able to:

1) Model real-world word problems as mathematical inequalities.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

See key words and their mathematical translations under big ideas.

BIG IDEAS

Translating words into mathematical expressions and equations is an important skill.

General Approach

The general approach is as follows:

1. Read and understand the entire problem.
2. Underline key words, focusing on variables, operations, and equalities or inequalities.
3. Convert the key words to mathematical notation (consider meaningful variable names other than x and y).
4. Write the final expression or equation.
5. Check the final expression or equation for reasonableness.

The Solution to a Linear Inequality Can Represent a Part of a Number Line.

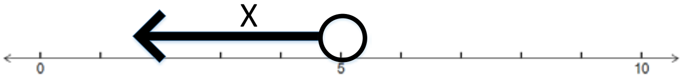
A linear inequality describes a part of a number line with either: 1) an upper limit; 2) a lower limit; or 3) both upper and lower limits.

Example - Upper Limit

Let A represent age.

A playground for little kids will not allow children older than four years. If A represents age in years, this can be represented as

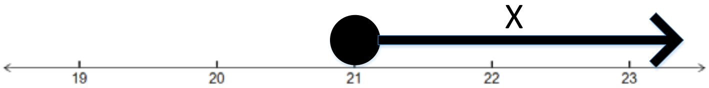
$$A < 5$$



Example - Lower Limit

A state will not allow persons below the age of 21 to drink alcohol. If A represents age in years, the legal drinking age can be represented as

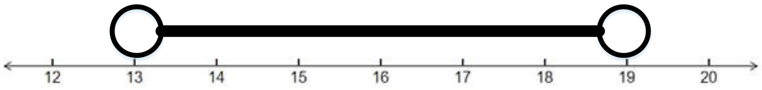
$$A \geq 21$$



Example - Both Upper and Lower Limits

A high school football team limits participation to students from 14 to 18 years old. If A represents age in years, participation on the football team can be represented as

$$13 < A < 19.$$



Key English Words and Their Mathematical Translations

These English Words	Usually Mean	*Examples: English becomes math*
is, are	equals	*the sum of 5 and x is 20 becomes 5 + x = 20*
more than, greater than	inequality >	*x is greater than y becomes x > y* *x is more than 5 becomes x > 5* *5 is more than x becomes 5 > x*
greater than or equal to, a minimum of, at least	inequality ≥	*x is greater than or equal to y becomes* *the minimum of x is 5 becomes* *x is at least 20 becomes*
less than	inequality <	*x is less than y becomes* *x is less than 5 becomes* *5 is less than x becomes*
less than or equal to, a maximum of, not more than	Inequality ≤	*X is less than or equal to y becomes* *The maximum of x is 5 becomes* *X is not more than becomes*

Examples of Modeling Specific Types of Inequality Problems

Spending Related Inequalities

Typical Problem in English	**Mathematical Translation**	**Hints and Strategies**
Mr. Braun has $75.00 to spend on pizzas and soda pop for a picnic. Pizzas cost $9.00 each and the drinks cost $0.75 each. Five times as many drinks as pizzas are needed. What is the maximum number of pizzas that Mr. Braun can buy?	$75 is the most that can be spent, so start with the idea that $75 \ge \text{something}$ • Let P represent the # of Pizzas and 9P represent the cost of pizzas. • Let 5P represent the number of drinks and .75(5P) represent the cost of drinks. Write the expression for total costs: $9P + .75(5P)$ Combine the left expression, inequality sign, and right expression into a single inequality. $75 \ge 9P + .75(5P)$ Solve the inequality for P. $75 \ge 9P + .75(5P)$ $75 \ge 9P + 3.75P$ $75 \ge 12.75P$ $\frac{75}{12.75} \ge P$ $5.9 \ge P$	1. Identify the minimum or maximum amount on one side of the inequality. 2. Pay attention to the direction of the inequality and whether the boundary is included or not included in the solution set. 3. Develop the other side of the inequality as an expression.

	It does not make sense to order 5.9 pizzas, and there is not enough money to buy six pizzas, so round down. Mr. Braun has enough money to buy 5 pizzas.	

How Many? Type of Inequalities

Typical Problem in English	Mathematical Translation	Hints and Strategies
There are 461 students and 20 teachers taking buses on a trip to a museum. Each bus can seat a maximum of 52. What is the *least* number of buses needed for the trip?	Write: $\frac{461+20}{52} \geq b$ Solve $\frac{486}{52} \geq b$ $9.25 \geq b$ A fraction/decimal answer does not make sense because you cannot order a part of a bus. Only an integer answer will work. The lowest integer value in the solution set is 10, so 10 buses will be needed for the trip.	Ignore your real life experience with field trips and buses, like how big or small are the students and teachers, or if student attendance will be influenced by how interesting the museum sounds.

Geometry Based Inequalities

Typical Problem in English	Mathematical Translation	Hints and Strategies
The length of a rectangle is 15 and its width is *w*. The perimeter of the rectangle is, *at most*, 50. Write and solve an inequality to find the longest possible width.	The formula for the perimeter of a rectangle is $2l + 2w = P$. Substitute information from the context into this formula and write: $2(15) + 2w \leq 50$ Then, solve for *w*. $2(15) + 2w \leq 50$ $30 + 2w \leq 50$ $2w \leq 20$ $w \leq 10$ The longest possible width is 10 feet.	Use a formula and substitute information from the problem into the formula.

DEVELOPING ESSENTIAL SKILLS

A swimmer plans to swim *at least* 100 laps during a 6-day period. During this period, the swimmer will increase the number of laps completed each day by one lap. What is the *least* number of laps the swimmer must complete on the first day?

Write the left expression and inequality sign as follows:

$100 \geq$

Let d represent the number of laps the swimmer must complete on the 1st day.
Let d+1 represent the number of laps the swimmer must complete on the 2nd day.
Let d+2 represent the number of laps the swimmer must complete on the 3rd day.
Let d+3 represent the number of laps the swimmer must complete on the 4th day.
Let d+4 represent the number of laps the swimmer must complete on the 5th day.
Let d+5 represent the number of laps the swimmer must complete on the 6th day.
Let 6d+15 represent the number of laps the swimmer must complete in total. This is the right expression.

Complete the inequality

$$100 \geq 6d + 15$$

Solve the inequality

$$100 \geq 6d + 15$$

$$85 \geq 6d$$

$$\frac{85}{6} \geq d$$

$$14.\overline{6} \geq d$$

A swimmer cannot swim a fraction of a lap, so round up to the next integer.
The swimmer must complete 15 laps on the first day.

REGENTS EXAM QUESTIONS (through June 2018)

A.CED.A.1 A.CED.C.3: Modeling Linear Inequalities

151) Connor wants to attend the town carnival. The price of admission to the carnival is \$4.50, and each ride costs an additional 79 cents. If he can spend at most \$16.00 at the carnival, which inequality can be used to solve for r, the number of rides Connor can go on, and what is the maximum number of rides he can go on?

1) $0.79 + 4.50r \leq 16.00$; 3 rides
2) $0.79 + 4.50r \leq 16.00$; 4 rides
3) $4.50 + 0.79r \leq 16.00$; 14 rides
4) $4.50 + 0.79r \leq 16.00$; 15 rides

152) Natasha is planning a school celebration and wants to have live music and food for everyone who attends. She has found a band that will charge her \$750 and a caterer who will provide snacks and drinks for \$2.25 per person. If her goal is to keep the average cost per person between \$2.75 and \$3.25, how many people, p, must attend?

1) $225 < p < 325$
2) $325 < p < 750$
3) $500 < p < 1000$
4) $750 < p < 1500$

153) The cost of a pack of chewing gum in a vending machine is \$0.75. The cost of a bottle of juice in the same machine is \$1.25. Julia has \$22.00 to spend on chewing gum and bottles of juice for her team and she must buy seven packs of chewing gum. If b represents the number of bottles of juice, which inequality represents the maximum number of bottles she can buy?

1) $0.75b + 1.25(7) \geq 22$
2) $0.75b + 1.25(7) \leq 22$
3) $0.75(7) + 1.25b \geq 22$
4) $0.75(7) + 1.25b \leq 22$

154) The acidity in a swimming pool is considered normal if the average of three pH readings, p, is defined such that $7.0 < p < 7.8$. If the first two readings are 7.2 and 7.6, which value for the third reading will result in an overall rating of normal?

1) 6.2
2) 7.3
3) 8.6
4) 8.8

155) David has two jobs. He earns \$8 per hour babysitting his neighbor's children and he earns \$11 per hour working at the coffee shop. Write an inequality to represent the number of hours, x, babysitting and the number of hours, y, working at the coffee shop that David will need to work to earn a minimum of \$200. David worked 15 hours at the coffee shop. Use the inequality to find the number of full hours he must babysit to reach his goal of \$200.

156) Joy wants to buy strawberries and raspberries to bring to a party. Strawberries cost \$1.60 per pound and raspberries cost \$1.75 per pound. If she only has \$10 to spend on berries, which inequality represents the situation where she buys x pounds of strawberries and y pounds of raspberries?

1) $1.60x + 1.75y \le 10$
2) $1.60x + 1.75y \ge 10$
3) $1.75x + 1.60y \le 10$
4) $1.75x + 1.60y \ge 10$

SOLUTIONS

151) ANS: 3
Strategy: Write and solve an inequality that relates total costs to how much money Connor has.

STEP 1. Write the inequality:
The price of admission comes first and is \$4.50. Write +4.50
Each ride (r) costs an additional 0.79. Write +0.79r
Total costs can be expressed as: $4.50 + 0.79r$
$4.50 + 0.79r$ must be less than or equal to the \$16 Connor has.
Write: $4.50 + 0.79r \le 16.00$

STEP 2: Solve the inequality.

Notes	Left Expression	Sign	Right Expression
Given	$4.50 + 0.79r$	$\le$	16.00
Subtract 4.50 from both expressions	$0.79r$	$\le$	11.50
Divide both expressions by 0.79	r	$\le$	$\frac{11.50}{.79}$
Simplify	r	$\le$	14.55696203
Interpret	r	$\le$	14 rides

The correct answer choice is c: $4.50 + 0.79r \le 16.00$; 14 rides

DIMS? Does It Make Sense? Yes. Admissions costs \$4.50 and 14 rides cost $14 \times .79 = \$11.06$. After 14 rides, Connor will only have 45 cents left, which is not enough to go on another ride.

$$\$16 - (\$4.50 + \$11.05)$$

$$\$16 - (\$15.55)$$

$$\$0.45$$

PTS: 2 NAT: A.CED.A.1 TOP: Modeling Linear Inequalities

152) ANS: 4
Strategy:

STEP1. Use the definition of average cost.

$$\text{Average Cost} = \frac{\text{total costs}}{\text{number of persons sharing the cost}}$$

Total costs for the band and the caterer are: $\$750 + \$2.25p$

If the average cost is \$3.25, the formula is $\$3.25 = \frac{\$750 + \$2.25p}{p}$

Solve for p

$\$3.25p = \$750 + \$2.25p$

$p = 750$

If the average cost is \$2.75, the formula is $\$2.75 = \frac{\$750 + \$2.25p}{p}$

Solve for p

$\$2.75p = \$750 + \$2.25p$

$.50p = 750$

$p = 1500$

DIMS? Does It Make Sense? Yes. If 750 people attend, the average cost is \$2.25 per person. If 1500 people attend, the average cost is \$3.25 per person. For any number of people between 750 and 1500, the average cost per person will be between \$2.25 and \$3.25.

PTS: 2 NAT: A.CED.A.3 TOP: Modeling Linear Inequalities

153) ANS: 4

Strategy: Examine the answer choices and eliminate wrong answers.

STEP 1. Eliminate answer choices *a* and *c* because both of them have greater than or equal signs. Julia must spend less than she has, not more.

STEP 2. Choose between answer choices *b* and *d*. Answer choice *d* is correct because the term $0.75(7)$ means that Julia must buy 7 packs of chewing gum @ \$0.75 per pack. Answer choice b is incorrect because the term $1.25(7)$ means that Julia will buy 7 bottles of juice.

DIMS? Does It Make Sense? Yes. Answer choice d shows in the first term that Julia will buy 7 packs of gum and the total of the entire expression must be equal to or less than \$22.00.

PTS: 2 NAT: A.CED.A.3 TOP: Modeling Linear Inequalities

154) ANS: 2

Step 1. Recognize that the problem is asking you to identify one pH reading that will result in an average of three readings that is greater than or equal to 7.0 and less than or equal to 7.8.
Step 2. Use algebraic notation to represent the average of three pH readings, then find the answer that gives an average within the required interval.
Step 3.

$$pH_{(average)} = \frac{pH_1 + pH_2 + pH_3}{3}$$

$$pH_{(average)} = \frac{7.2 + 7.6 + pH_3}{3}$$

$$pH_{(average)} = \frac{14.8 + pH_3}{3}$$

Choice a) $pH_{(average)} = \frac{14.8 + pH_3}{3} = \frac{14.8 + 6.2}{3} = \frac{21}{3} = 7$. This average is not in the required interval, so choice a) is not a correct answer.

Choice b) $pH_{(average)} = \frac{14.8 + pH_3}{3} = \frac{14.8 + 7.3}{3} = \frac{22.1}{3} = 7.3\bar{6} \approx 7.4$. This average is in the required interval, so choice b) is a correct answer.

Choice c) $pH_{(average)} = \frac{14.8 + pH_3}{3} = \frac{14.8 + 8.6}{3} = \frac{23.4}{3} = 7.8$. This average is not in the required interval, so choice c) is not a correct answer.

Choice d) $pH_{(average)} = \frac{14.8 + pH_3}{3} = \frac{14.8 + 8.8}{3} = \frac{23.6}{3} = 7.8\bar{6} \approx 7.9$. This average is not in the required interval, so choice d) is not a correct answer.

Step 4. Does it make sense? Yes.

$$7.0 < p < 7.8$$

$$7.0 < 7.4 < 7.8$$

$$7.0 < \text{choice b} < 7.8$$

PTS: 2 NAT: A.CED.A.1 TOP: Modeling Linear Inequalities

155) ANS:
David must babysit five full hours to reach his goal of $200.

Strategy: Write an inequality to represent David’s income from both jobs, then use it to solve the problem, then interpret the solution.

STEP 1. Write the inequality.
Let x represent the number of hours that David babysits.
Let y represent the number of hours that David works at the coffee shop.
Write: $8x + 11y \geq 200$

STEP 2. Substitute 15 for y and solve for x.

$$8x + 11y \geq 200$$

$$8x + 11(15) \geq 200$$

$$8x + 165 \geq 200$$

$$8x \geq 200 - 165$$

$$8x \geq 35$$

$$x \geq \frac{35}{8}$$

$$x \geq 4.375$$

STEP 3. Interpret the solution.
The problem asks for the number of full hours, so the solution, $x \geq 4.375$, must be rounded up to 5 full hours.

DIMS? Does It Make Sense? Yes. If David works 15 hours at the coffee shop and 5 hours at the library, he will earn more than 200

$$8(5) + 11(15) \geq 200$$

$$40 + 165 \geq 200$$

$$205 \geq 200$$

What does not make sense is why David earns $8 per hour babysitting and Edith, in the previous problem, only earns $4 per hour.

PTS: 4 NAT: A.CED.A.3 TOP: Modeling Linear Inequalities

156) ANS: 1

Strategy: There are three terms in each answer choice - sort the information in the problem to write mathematical terms, then eliminate wrong answers.

Given the Words	Write
Strawberries cost $1.60 per pound.... *x* pounds of strawberries	$1.60x$
raspberries cost $1.75 per pound.... *y* pounds of raspberries	$1.75y$
she only has $10 to spend	≤ 10

Combine all three terms and the inequality sign to write: $1.60x + 1.75y \leq 10$

PTS: 2 NAT: A.CED.A.1 TOP: Modeling Linear Inequalities

F – Inequalities, Lesson 4, Graphing Linear Inequalities (r. 2018)

INEQUALITIES
Graphing Linear Inequalities

Common Core Standard	Next Generation Standard
A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	**AI-A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. **Note: Graphing linear equations is a fluency recommendation for Algebra I. Students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity; as well as modeling linear phenomena (including modeling using systems of linear inequalities in two variables).**

LEARNING OBJECTIVES

Students will be able to:

1) Graph a single inequality involving two variables on a coordinate plane.
    a. Determine if the boundary line is a solid line or a dashed line.
    b. Determine if the solution set is shaded above or below the boundary line.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

boundary line
dashed line
linear inequality

shading
solid line
solution set

testing a solution

BIG IDEAS

A linear inequality describes a region of the coordinate plane that has a **boundary line**. Every point in the region is a **solution of the inequality**.

The **solution set** of a linear inequality includes all ordered pairs that make the inequality true. The graph of an inequality represents the solution set.

Graphing a Linear Inequality

Step One. Change the inequality sign to an equal sign and graph the boundary line in the same manner that you would graph a linear equation.

- When the inequality sign **contains** an equality bar beneath it, use a solid line for the boundary. Any point (ordered pair) on the boundary line is part of the solution set.
- When the inequality sign **does not contain** an equality bar beneath it, use a dashed line for the boundary. Any point (ordered pair) on the boundary line is *not* part of the solution set.

Step Two. Restore the inequality sign and test a point to see which side of the boundary line the solution is on. The point (0,0) is a good point to test since it simplifies any multiplication. However, if the boundary line passes through the point (0,0), another point not on the boundary line must be selected for testing.

- If the test point makes the inequality true, shade the side of the boundary line that includes the test point.
- If the test point makes the inequality not true, shade the side of the boundary line does not include the test point.

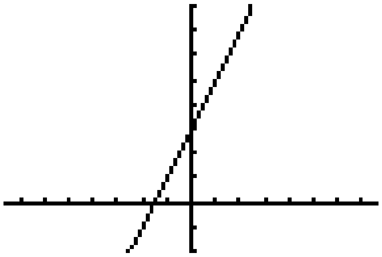
NOTE: If the dependent variable is isolated in the left expression of the inequality, a simplified way to determine which side of the line to shade is as follows:

- If the inequality sign contains >, shade *above* the boundary line.
  - Examples: $y > x$ and $y \geq x$ are shaded *above* the boundary line.
- If the inequality sign contains <, shade *below* the boundary line.
  - Examples: $y < x$ and $y \leq x$ are shaded *below* the boundary line.

Example Graph $y < 2x + 3$

First, change the inequality sign an equal sign and graph the line: $y = 2x + 3$. This is the boundary line of the solution. Since there is no equality line beneath the inequality symbol, use a dashed line for the boundary.

NOTE: A graphing calculator can be used if the inequality has the dependent variable isolated as the in the left expression of the inequality



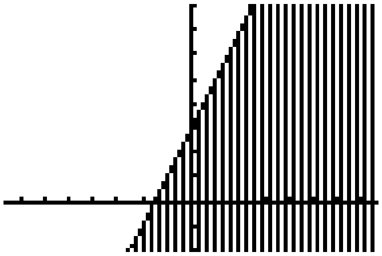
Next, **test a point** to see which side of the boundary line the solution is on. Try (0,0), since it makes the multiplication easy, but remember that any point will do.

$y < 2x + 3$

$0 < 2(0) + 3$

$0 < 3$ True, so the solution of the inequality is the region that contains the point $(0,0)$.

Therefore, we shade the side of the boundary line that contains the point (0,0).



Note: Most graphing calculators do not have the ability to distinguish between solid and dashed lines on a graph of an inequality.

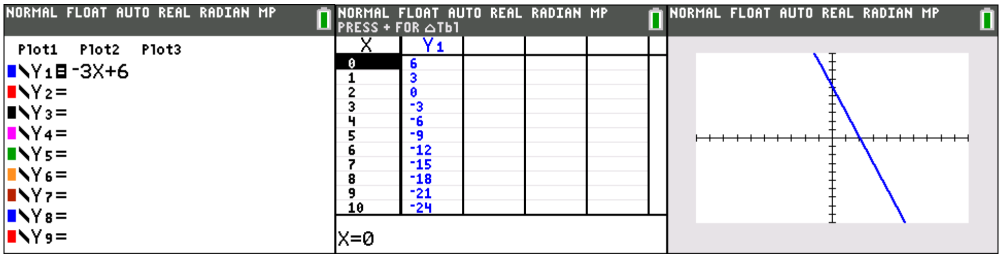
DEVELOPING ESSENTIAL SKILLS

Graph the inequality $3x + 2y \le y + 6$ and determine if point with coordinates (3,8) is in the solution set.

STEP 1. Isolate the dependent variable in the left expression of the inequality.

$$3x + 2y \le y + 6$$
$$3x + y \le 6$$
$$y \le -3x + 6$$

STEP 2. Input the transformed inequality in a graphing computer and use the table and graph views to plot the boundary line.



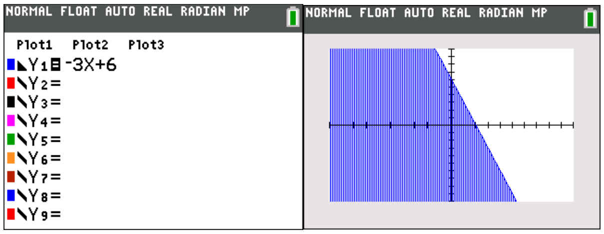
Y1 ◼ -3X+6
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
Y8=
Y9=

X	Y1
0	6
1	3
2	0
3	-3
4	-6
5	-9
6	-12
7	-15
8	-18
9	-21
10	-24

X=0

Since the inequality $\le$ sign contains an equal bar, the boundary line is a solid line and any points on the boundary line are included in the solution set.

STEP 3. Since the dependent variable is isolated in the left expression, and the inequality sign includes <, shade the area *below* the boundary line. (NOTE: The graphing calculator can be set to show < or > inequalities.)



```
NORMAL FLOAT AUTO REAL RADIAN MP
 Plot1  Plot2  Plot3
■◣Y1■-3X+6
■\Y2=
■\Y3=
■\Y4=
■\Y5=
■\Y6=
■\Y7=
■\Y8=
■\Y9=
```

STEP 4. Inspect the graph to determine if the point (3,8) is included in the solution set. It is not.

STEP 5. Do a check to see if the point (3,8) makes the original inequality true.

$$3x+2y\le y+6$$

$$3(3)+2(8)\le(8)+6$$

$$9+16\le 14$$

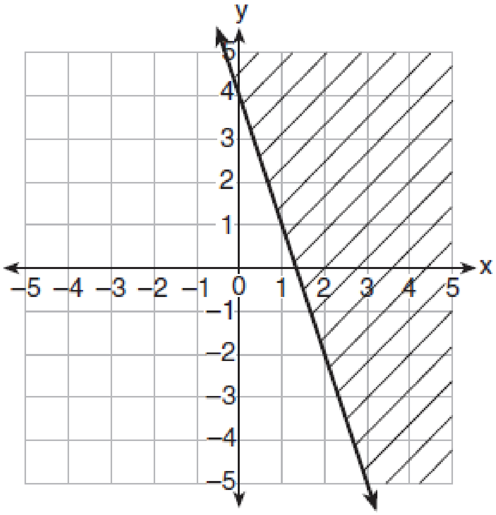
$$25\le 14 \text{ not true}$$

Since the inequality is not true for the point (3,8), the point is not in the solution set.

<u>REGENTS EXAM QUESTIONS (through June 2018)</u>

A.REI.D.12: Graphing Linear Inequalities

157) Which inequality is represented in the graph below?



1) $y\ge -3x+4$

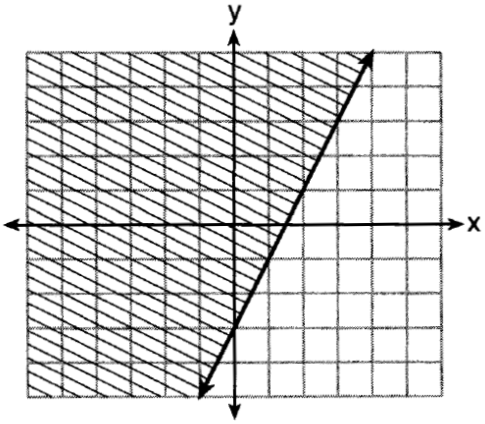
2) $y\le -3x+4$

3) $y\ge -4x-3$

4) $y\le -4x-3$

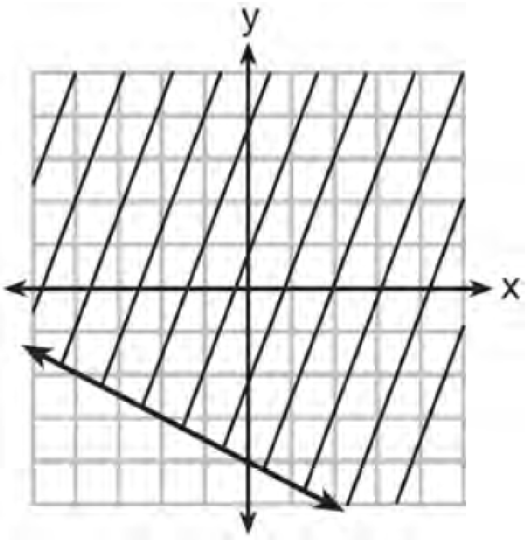
158) On the set of axes below, graph the inequality $2x+y>1$.

159) Which inequality is represented by the graph below?



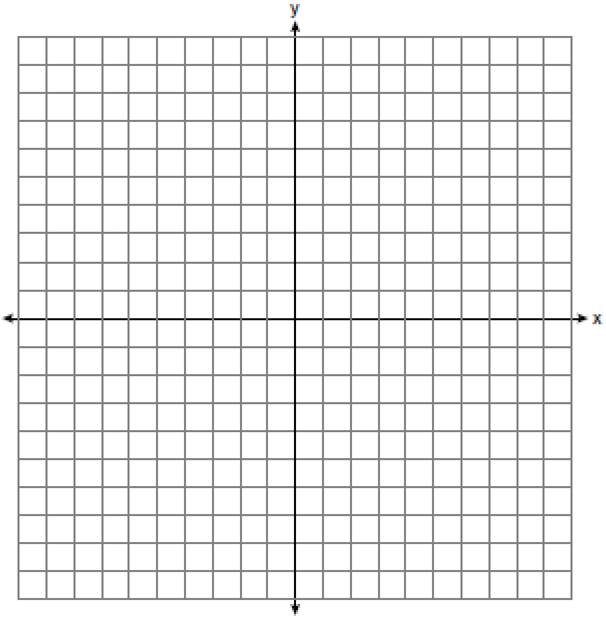
1) $y \leq 2x - 3$
2) $y \geq 2x - 3$
3) $y \leq -3x + 2$
4) $y \geq -3x + 2$

160) Shawn incorrectly graphed the inequality $-x - 2y \geq 8$ as shown below:

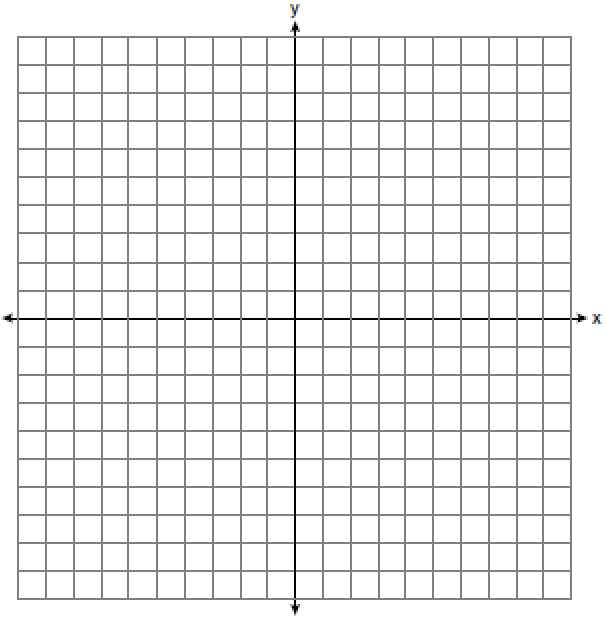


Explain Shawn's mistake.

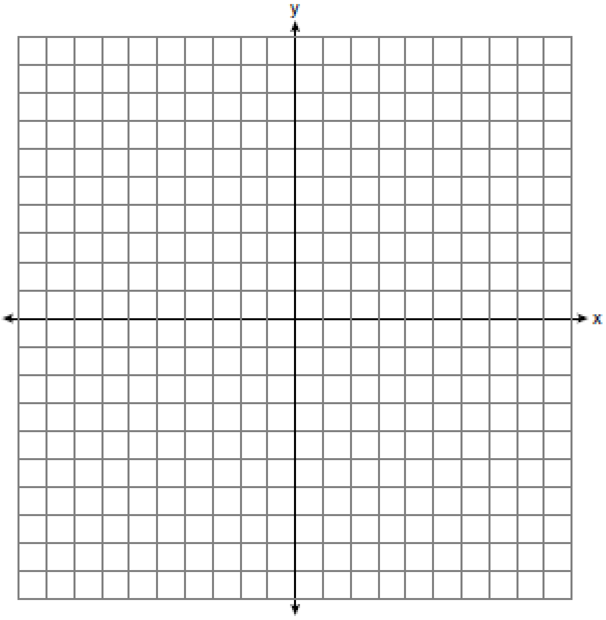
Graph the inequality correctly on the set of axis below.



161) Graph the inequality $y > 2x - 5$ on the set of axes below. State the coordinates of a point in its solution.



162) Graph the inequality $y + 4 < -2(x - 4)$ on the set of axes below.



SOLUTIONS

157) ANS: 1

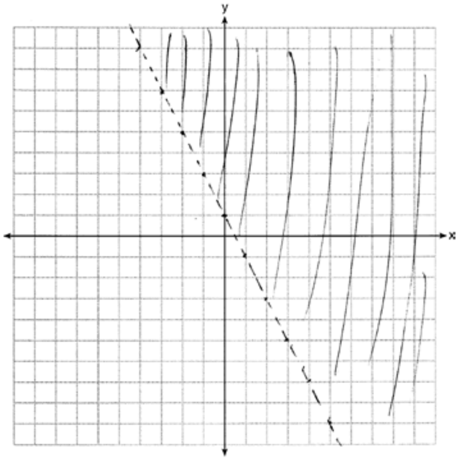
Strategy: Use the slope intercept form of a line, $y = mx + b$, to construct the inequality from the graph.

The line passes though points (0,4) and (1,1), so the slope is $m - \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-4}{1-0} = \frac{-3}{1} = -3$. The y-intercept is 4.

The equation of the boundary line is $y = -3x + 4$, so eliminate choices *c* and *d.*
The shading is above the line, so eliminate choice *b.*
The inequality is $y \geq -3x + 4$, so answer choice *a* is correct.

PTS: 2 NAT: A.REI.D.12 TOP: Linear Inequalities

158) ANS:



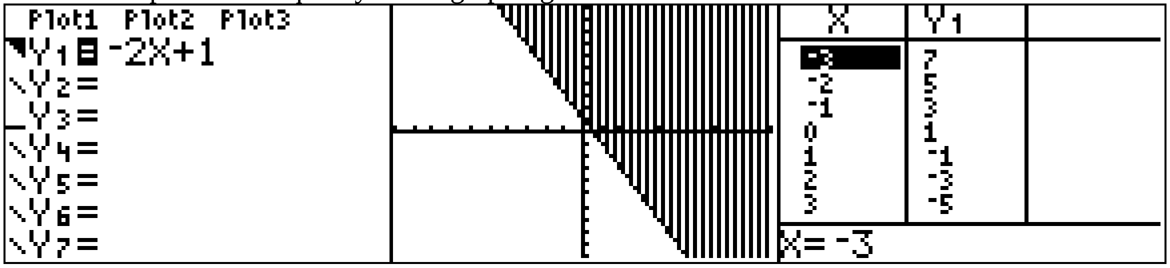
Strategy: Transpose the inequality, put it in a graphing calculator, then use the table and graph views to create the graph on paper.

STEP 1. Transpose the inequality for input into a graphing calculator.

$$2x + y > 1$$

$$y > -2x + 1$$

STEP 2. Inpout the inequality into a graphing calculator.



Plot1 Plot2 Plot3
Y1 ⊟ -2X+1
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

X	Y1
-3	7
-2	5
-1	3
0	1
1	-1
2	-3
3	-5

X= -3

STEP 3. Use information from the graph and table views to create the graph on paper. Be sure to make the line dotted.

PTS: 2 NAT: A.REI.D.12 TOP: Graphing Linear Inequalities

159) ANS: 2 PTS: 2 NAT: A.REI.D.12 TOP: Graphing Linear Inequalities

160) ANS:

Shawn’s mistake was he shaded the wrong side of the boundary line.

$$-x - 2y \geq 8$$

$$-x - 8 \geq 2y$$

$$\frac{-x}{2} - 4 \geq y$$

$$y \leq \frac{-x}{2} - 4$$

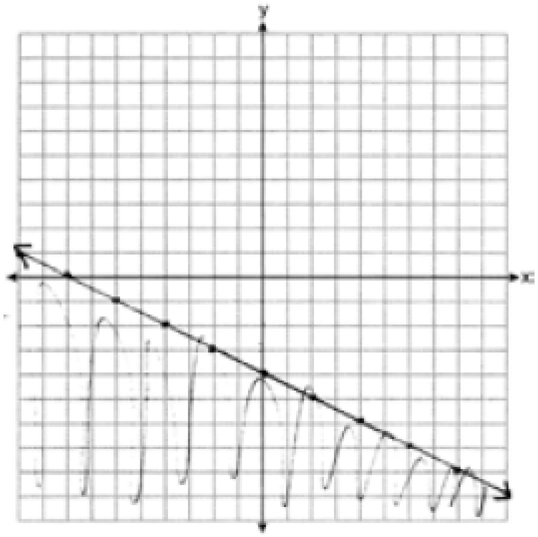
$$y = mx + b$$

Shawn’s y-intercept is correct. $b = -4$

Shawn’s slope is correct. $m = -\frac{1}{2}$

Shawn correctly graphed a solid boundary line. $\geq$

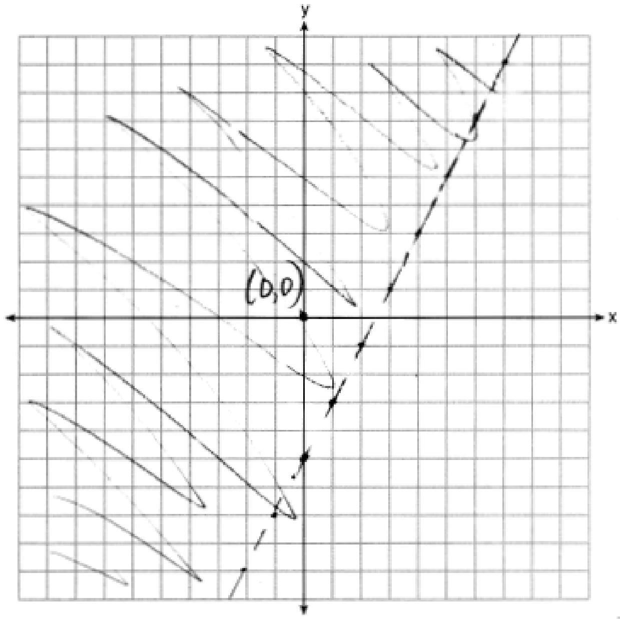
Shawn’s mistake was he shaded the wrong side of the boundary line.



PTS: 4 NAT: A.REI.D.12

161) ANS:

Strategy: Use the slope intercept form of the inequality to plot the y-intercept at -5, then use the slope of $\frac{2}{1}$ to find another point on the boundary line. Plot the boundary line as a dashed. Shade the area above the boundary line. Select any number in the shaded area.

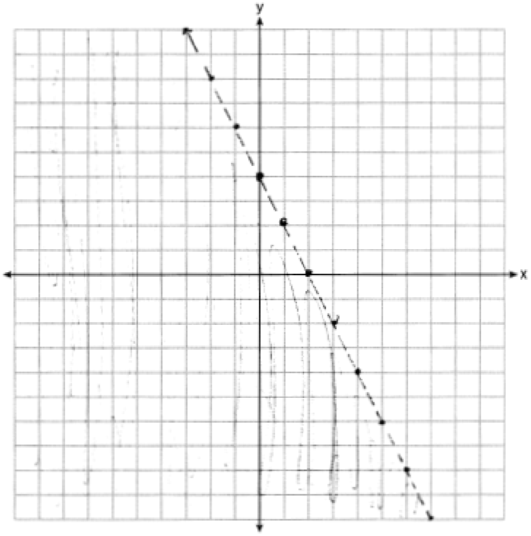


Check (0,0) in the inequality as follows:

$$y > 2x - 5$$
$$0 > 2(0) - 5$$
$$0 > -5 \text{ True}$$

PTS: 2 NAT: A.REI.D.12 TOP: Graphing Linear Inequalities

162) ANS:



$y < -2x + 4$

PTS: 2 NAT: A.REI.D.12 TOP: Graphing Linear Inequalities

G – Absolute Value, Lesson 1, Graphing Absolute Value Functions (r. 2018)

ABSOLUTE VALUE

Graphing Absolute Value Functions

Common Core Standards	**Next Generation Standards**
F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ **F-BF.B.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, ~~$f(kx)$~~, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing ~~even and odd~~ functions from their graphs and algebraic expressions for them. ~~PARCC: Identifying the effect on the graph of replacing $f(x)$ by $f(x) +k$, $kf(x)$, and $f(x+k)$ for specific values of k (both positive and negative) is limited to linear and quadratic functions. Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. Tasks do not involve recognizing even and odd functions.~~	**AI-F.IF.7** Graph functions and show key features of the graph by hand and by using technology where appropriate. ★ (Shared standard with Algebra II) **AI-F.BF.3a** Using $f(x) + k$, $k\,f(x)$, and $f(x + k)$: i) identify the effect on the graph when replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, and $f(x + k)$ for specific values of k (both positive and negative); ii) find the value of k given the graphs; iii) write a new function using the value of k; and iv) use technology to experiment with cases and explore the effects on the graph. (Shared standard with Algebra II) **Note: Tasks are limited to linear, quadratic, square root, and absolute value functions; and exponential functions of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).**

LEARNING OBJECTIVES

Students will be able to:

1) Solve and graph absolute value functions with the aid of technology.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

absolute value
absolute value function

catalog (of graphing calculator)
plus or minus (±) sign

ray
transformation

BIG IDEAS

The **absolute value** of a number is defined as the number's distance from zero on a number line. Distance is always positive, so the absolute value of a number is always positive. For example, $|-3| = 3$, $|3| = 3$, $|-x| = x$.

NOTE: $-|x| = -x$ $-|-x| = -x$ Pay attention to whether a negative sign is inside or outside the absolute value parentheses.

An **absolute value function** is a function that contains an absolute value term or expression. Examples are $|x|$ and $|x+1|$.

How to Solve an Absolute Value Function

STEP 1: Isolate the absolute value expression.
STEP 2: Remove absolute value signs and add ± to opposite expression.
STEP 3: Eliminate the ± sign by writing two new equations.
STEP 4. Solve both equations.

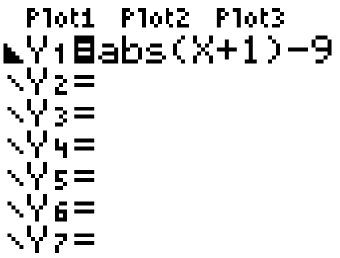
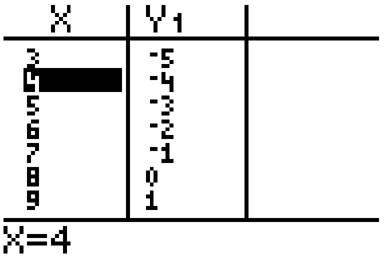
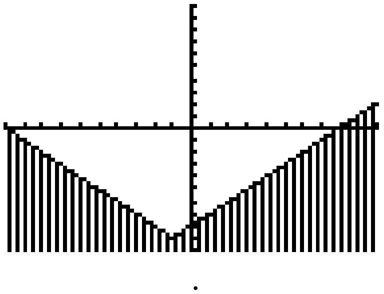
Examples:

Solve for x. $4 = \|x\| - 2$	Solve for x. $4 = \|x+3\| - 2$
STEP 1: Isolate the absolute value expression. $6 = \|x\|$	STEP 1: Isolate the absolute value expression. $6 = \|x+3\|$
STEP 2: Remove absolute value signs and add ± to opposite expression. $\pm 6 = x$	STEP 2: Remove absolute value signs and add ± to opposite expression. $\pm 6 = x + 3$
STEP 3: Eliminate the ± sign by writing two new equations. $+Eq \quad +6 = x$ $-Eq \quad -6 = x$	STEP 3: Eliminate the ± sign by writing two new equations. $+Eq \quad +6 = x + 3$ $-Eq \quad -6 = x + 3$
STEP 4. Solve both equations. (*Unnecessary in this example.*)	STEP 4. Solve both equations. $+Eq \quad +6 = x + 3$ $3 = x$ $-Eq \quad -6 = x + 3$ $-9 = x$

Using a Graphing Calculator with Absolute Value Functions:

Absolute value functions may be input in a graphing calculator by moving all terms to the right expression of the function and setting the left expression to zero. The inequality is then entered into the graphing calculator's y-editor using the ABS function, which is found in the calculator's catalog. Once input, the graph and table views of the function may be inspected.

Example: Given: $|x+1| - 3 > 6$

STEP 1. Transform the absolute value function so that the left expression equals zero, as shown below. $|x+1|-3>6$ $|x+1|-9>0$ $0<|x+1|-9$	STEP 2. Input the function. NOTE: The abs entry is found in the graphing calculator's catalog.  Plot1 Plot2 Plot3 Y1=abs(X+1)-9 Y2= Y3= Y4= Y5= Y6= Y7= NOTE: Since this example is an inequality, pay attention to the inequality sign on the far left of the calculator's input screen.	STEP 3. Inspect the table and graph views of the function.  X Y1 3 -5 4 -4 5 -3 6 -2 7 -1 8 0 9 1 X=4 

Graphing an Absolute Value Function

STEP 1. Input the absolute value function in the graphing calculator.

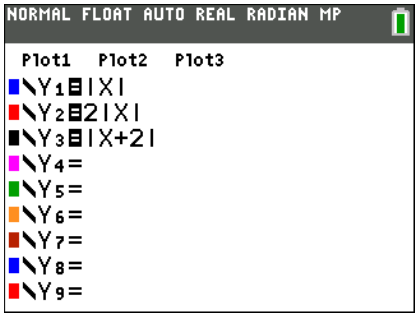
STEP 2. Inspect the graph and table views.

STEP 3. Plot three points: 1) the vertex; 2) a second point for the line to the left of the vertex; and 3) a third point for the line to the right of the vertex.

STEP 4. Draw rays from the vertex through the two points.

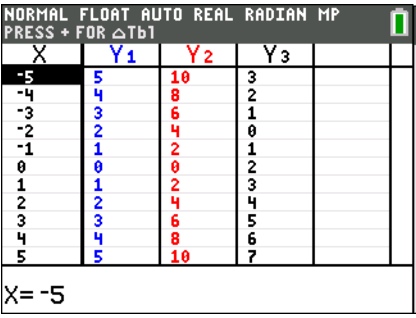
DEVELOPING ESSENTIAL SKILLS

Use technology to explain how what happens to the graph of $f(x)=|x|$ under the transformations $g(x)=2|x|$ and $h(x)=|x+2|$.



NORMAL FLOAT AUTO REAL RADIAN MP

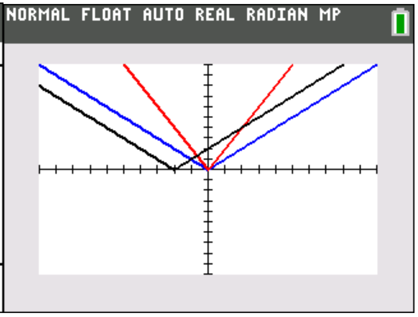
Plot1 Plot2 Plot3
Y1=|X|
Y2=2|X|
Y3=|X+2|
Y4=
Y5=
Y6=
Y7=
Y8=
Y9=



NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl

X	Y1	Y2	Y3
-5	5	10	3
-4	4	8	2
-3	3	6	1
-2	2	4	0
-1	1	2	1
0	0	0	2
1	1	2	3
2	2	4	4
3	3	6	5
4	4	8	6
5	5	10	7

X= -5

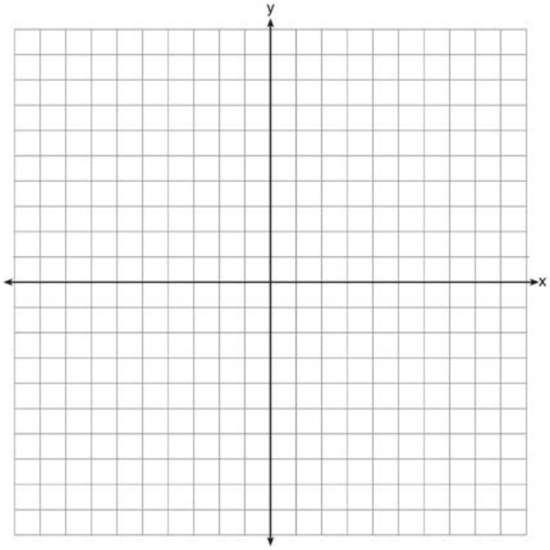


$g(x)=2|x|$ gets narrower with the vertex at the same point.

$h(x)=|x+2|$ shifts two units to the left.

F.IF.C.7b, F.BF.B.3: Graphing Absolute Value Functions

163) On the set of axes below, graph the function $y = |x + 1|$.

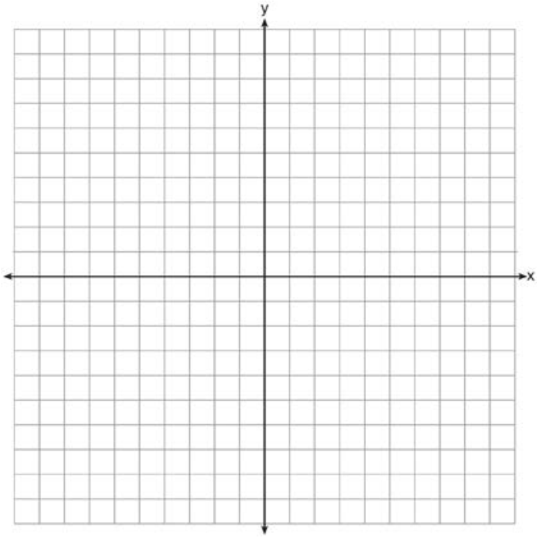


State the range of the function. State the domain over which the function is increasing.

164) What is the *minimum* value of the function $y = |x + 3| - 2$?

1) -2	3) 3
2) 2	4) -3

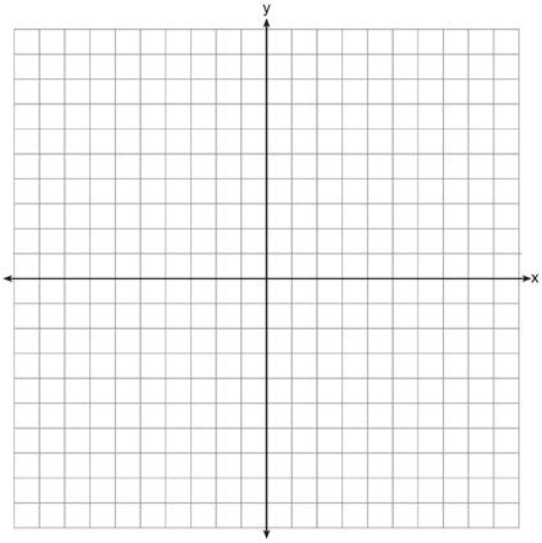
165) Graph the function $y = |x - 3|$ on the set of axes below.



Explain how the graph of $y = |x - 3|$ has changed from the related graph $y = |x|$.

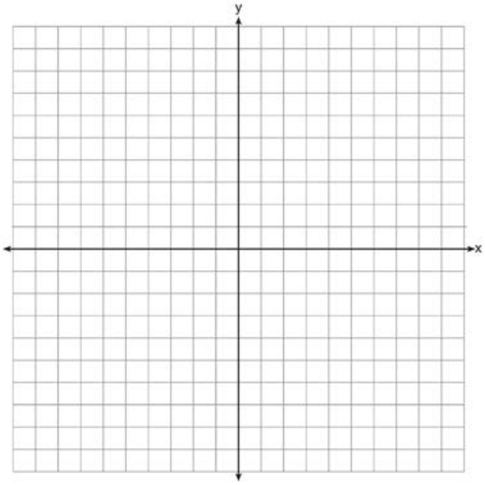
166) Describe the effect that each transformation below has on the function $f(x) = |x|$, where $a > 0$.
$g(x) = |x - a|$
$h(x) = |x| - a$

167) On the axes below, graph $f(x) = |3x|$.



If $g(x) = f(x) - 2$, how is the graph of $f(x)$ translated to form the graph of $g(x)$?
If $h(x) = f(x - 4)$, how is the graph of $f(x)$ translated to form the graph of $h(x)$?

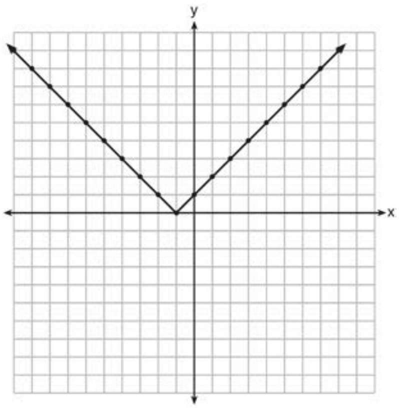
168) On the axes below, graph $f(x) = |3x|$.



If $g(x) = f(x) - 2$, how is the graph of $f(x)$ translated to form the graph of $g(x)$?
If $h(x) = f(x - 4)$, how is the graph of $f(x)$ translated to form the graph of $h(x)$?

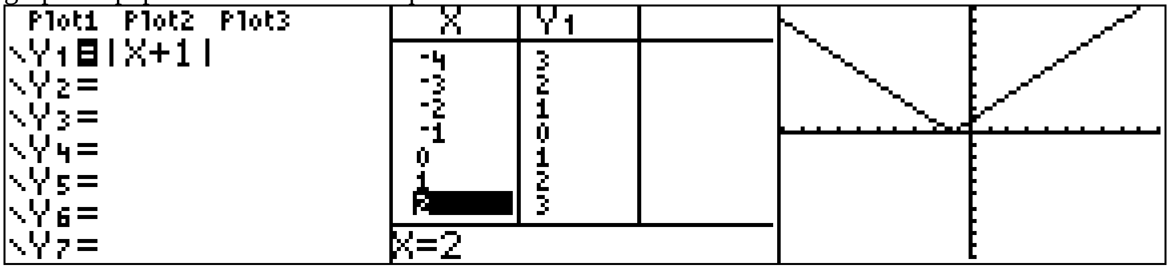
SOLUTIONS

163) ANS:



The range is $y \geq 0$.
The function is increasing for $x > -1$.

Strategy: Input the function in a graphing calculator and use the table and graph views to complete the graph on paper and to answer the questions.



Plot1 Plot2 Plot3
\Y1=|X+1|
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=

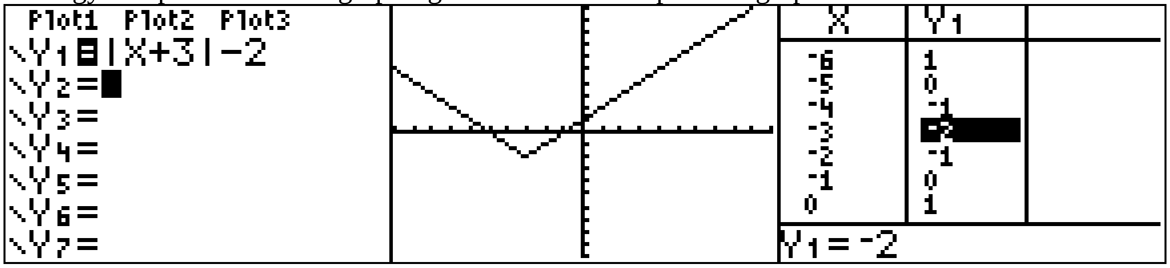
X	Y1
-4	3
-3	2
-2	1
-1	0
0	1
1	2
2	3

X=2

PTS: 4 NAT: F.IF.C.7 TOP: Graphing Absolute Value Functions

164) ANS: 1

Strategy: Input function in graphing calculator and inspect the graph and table views to find answer.



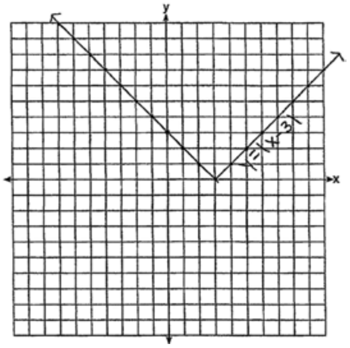
Plot1 Plot2 Plot3
\Y1=|X+3|-2
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=

X	Y1
-6	1
-5	0
-4	-1
-3	-2
-2	-1
-1	0
0	1

Y1=-2

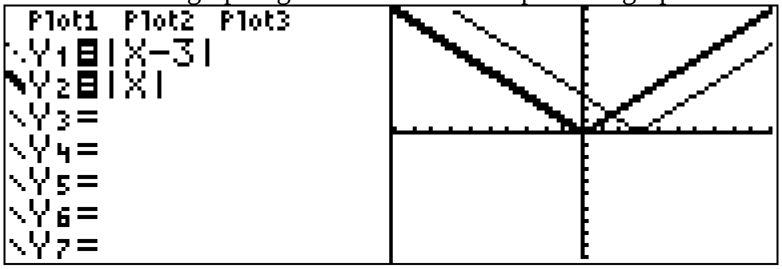
PTS: 2 NAT: F.IF.C.7 TOP: Graphing Absolute Value Functions

165) ANS:



The graph has shifted three units to the right.

Strategy: Input both functions in a graphing calculator and compare the graphs.



```
Plot1 Plot2 Plot3
\Y1=|X-3|
\Y2=|X|
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

PTS: 2 NAT: F.BF.B.3 TOP: Transformations with Functions and Relations

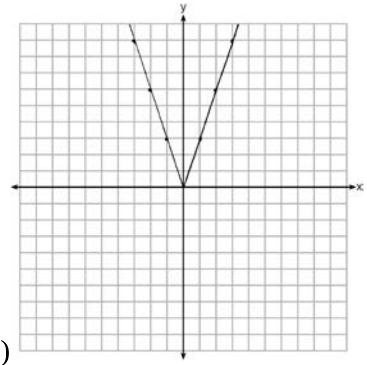
166) ANS:

$g(x) = |x - a|$ moves $f(x)$ "a" units to the right.

$h(x) = |x| - a$ moves $f(x)$ down by "a" units.

PTS: 2 NAT: F.BF.B.3 TOP: Graphing Absolute Value Functions

167) ANS:

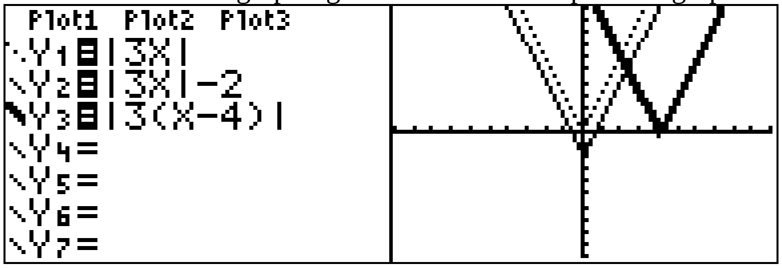


a)

b) If $g(x) = f(x) - 2$, the graph of $f(x)$ is translated 2 down to form the graph of $g(x)$.

c) If $h(x) = f(x - 4)$, the graph of $f(x)$ translated 4 right to form the graph of $h(x)$.

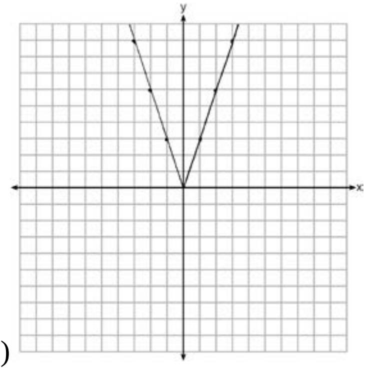
Strategy: Input the three functions in a graphing calculator and compare the graphs.



```
Plot1 Plot2 Plot3
\Y1=|3X|
\Y2=|3X|-2
\Y3=|3(X-4)|
\Y4=
\Y5=
\Y6=
\Y7=
```

PTS: 4 NAT: F.BF.B.3 TOP: Transformations with Functions and Relations

168) ANS:

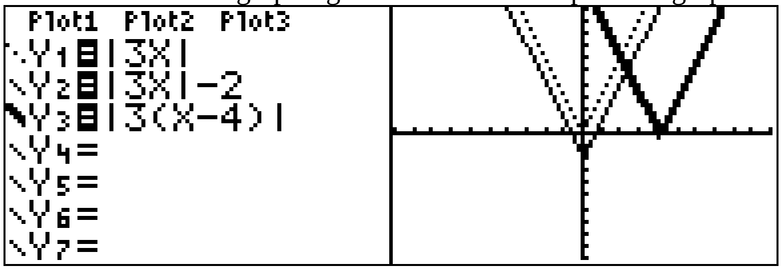


a)

b) If $g(x) = f(x) - 2$, the graph of $f(x)$ is translated 2 down to form the graph of $g(x)$.

c) If $h(x) = f(x - 4)$, the graph of $f(x)$ translated 4 right to form the graph of $h(x)$.

Strategy: Input the three functions in a graphing calculator and compare the graphs.



```
 Plot1  Plot2  Plot3
\Y1 =|3X|
\Y2 =|3X|-2
\Y3 =|3(X-4)|
\Y4 =
\Y5 =
\Y6 =
\Y7 =
```

PTS: 4 NAT: F.BF.B.3 TOP: Transformations with Functions and Relations

H – Quadratics, Lesson 1, Solving Quadratics (r. 2018)

QUADRATICS

Solving Quadratics

Common Core Standards	**Next Generation Standards**
A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	**AI-A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (Shared standard with Algebra II)
A-REI.B.4a Solve quadratic equations in one variable. NYSED: Solutions may include simplifying radicals.	**AI-A.REI.4** Solve quadratic equations in one variable. Note: Solutions may include simplifying radicals.

NOTE: This lesson is in four parts and typically requires four or more days to complete.

LEARNING OBJECTIVES

Students will be able to:

1) Transform a quadratic equation into standard form and identify the values of a, b, and c.
2) Convert factors of quadratics to solutions.
3) Convert solutions of quadratics to factors.
4) Solve quadratics using the quadratic formula.
5) Solve quadratics using the completing the square method.
6) Solve quadratics using the factoring by grouping method.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

box method of factoring
completing the square
constant
factoring by grouping
factors
forms of a quadratic
linear term
multiplication property of zero
quadratic equation
quadratic formula
quadratic term
roots
solutions
standard form of a quadratic
x-axis intercepts
zeros

Part 1 – Overview of Quadratics

BIG IDEAS

The **standard form** of a quadratic is: $ax^2 + bx + c = 0$.

- ax^2 is the quadratic term
- bx is the linear term
- c is the constant term

Note: If the quadratic terms is removed, the remaining terms are a linear equation.

The definition of a **quadratic equation** is: an equation of the second degree.
Examples of quadratics in different **forms**:

Forms	Examples
standard form	$6x^2 + 11x - 35 = 0$ $2x^2 - 4x - 2 = 0$ $-4x^2 - 7x + 12 = 0$ $20x^2 - 15x - 10 = 0$ $x^2 - x - 3 = 0$ $5x^2 - 2x - 9 = 0$ $3x^2 + 4x + 2 = 0$ $-x^2 + 6x + 18 = 0$
without the *bx* term (the linear term)	$2x^2 - 64 = 0$ $x^2 - 16 = 0$ $9x^2 + 49 = 0$ $-2x^2 - 4 = 0$ $4x^2 + 81 = 0$ $-x^2 - 9 = 0$ $3x^2 - 36 = 0$ $6x^2 + 144 = 0$
without the *c* term (the constant term)	$x^2 - 7x = 0$ $2x^2 + 8x = 0$ $-x^2 - 9x = 0$ $x^2 + 2x = 0$ $-6x^2 - 3x = 0$ $-5x^2 + x = 0$ $-12x^2 + 13x = 0$ $11x^2 - 27x = 0$

factored forms	$(x+2)(x-3)=0$ $(x+1)(x+6)=0$ $(x-6)(x+1)=0$ $(x-5)(x+3)=0$ $(x-5)(x+2)=0$ $(x-4)(x+2)=0$ $(2x+3)(3x-2)=0$ $-3(x-4)(2x+3)=0$
other forms	$x(x-2)=4$ $x(2x+3)=12$ $3x(x+8)=-2$ $5x^2=9-x$ $-6x^2=-2+x$ $x^2=27x-14$ $x^2+2x=1$ $4x^2-7x=15$ $-8x^2+3x=-100$ $25x+6=99x^2$

(Source: your dictionary.com)

Multiplication Property of Zero: The **multiplication property of zero** says that if the product of two numbers or expressions is zero, then one or both of the numbers or expressions must equal zero. More simply, if $x \cdot y = 0$, then either $x = 0$ or $y = 0$, or, both x and y equal zero.

Example: The quadratic equation $(x+2)(x-4)=0$ has two factors: $(x+2)$ and $(x-4)$. The multiplication property of zero says that one or both of these factors must equal zero, because the product of these two factors is zero. Therefore, write two equations, as follows:

Eq #1 $(x+2)=0$ Therefore, $x=-2$

Eq #2 $(x-4)=0$ Therefore, $x=4$

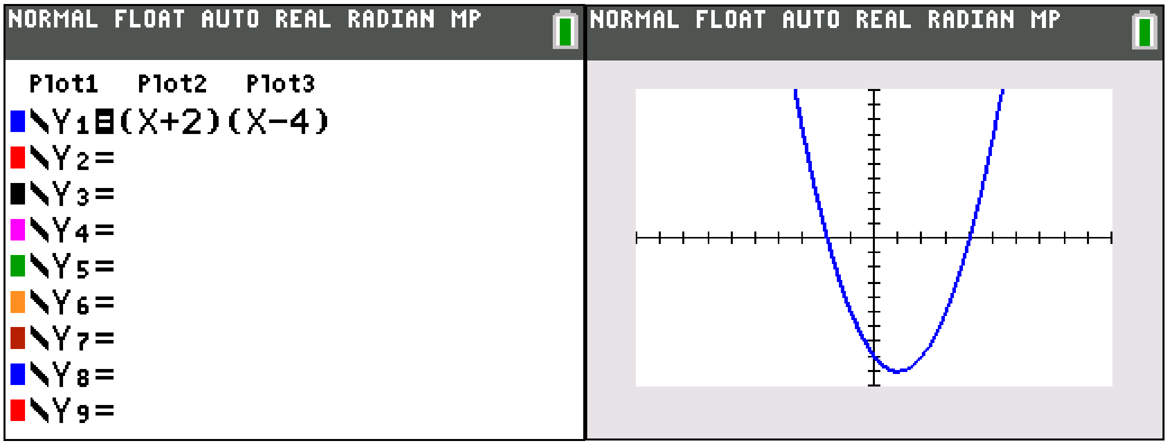
By the multiplication property of zero, $x=\{-2,+4\}$.

Zeros: A **zero** of a quadratic equation is a **solution** or **root** of the equation. The words **zero**, **solution**, and **root** all mean the same thing. The zeros of a quadratic equation are the value(s) of x when $y=0$. A quadratic equation can have one, two, or no zeros. There are four general strategies for finding the zeros of a quadratic equation:

1) Solve the quadratic equation using the quadratic formula.
2) Solve the quadratic equation using the completing the square method.
3) Solve the quadratic equation using the factoring by grouping method.
4) Input the quadratic equation into a graphing calculator and find the x-axis intercepts.

x-axis intercepts: The zeros of a quadratic can be found by inspecting the graph view of the equation. In graph form, the zeros of a quadratic equation are the <u>x-values of the coordinates of the **x-axis intercepts**</u> of the graph of the equation. The graph of a quadratic equation is called a parabola and can intercept the x-axis in one, two, or no places.

Example: Find the x-axis intercepts of the quadratic equation $(x+2)(x-4)=0$ by inspecting the x-axis intercepts of its graph.



```
Plot1  Plot2  Plot3
Y1=(X+2)(X-4)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
Y8=
Y9=
```

The coordinates of the x-axis intercepts are are $(-2,0)$ and $(4,0)$. These x-axis intercepts show that the values of x when y=0 are -2 and 4, so the solutions of the quadratic equation are $x=\{-2,+4\}$.

<u>The Difference Between Zeros and Factors</u>

Factor: A <u>**facto**</u>r is:

1) a whole number that is a <u>**divisor**</u> of another number, or
2) an algebraic expression that is a <u>**divisor**</u> of another algebraic expression.

Examples:

- 1, 2, 3, 4, 6, and 12 all divide the number 12,
  so 1, 2, 3, 4, 6, and 12 are all factors of 12.
- $(x-3)$ and $(x+2)$ will divide the trinomial expression x^2-x-6,
  so $(x-3)$ and $(x+2)$ are both factors of the x^2-x-6.

<u>Start with Factors and Find Zeros</u>

Remember that the <u>**factors**</u> of an expression are *related to* the <u>**zeros**</u> of the expression by the <u>**multiplication property of zero**</u>. Thus, if you know the <u>**factors**</u>, it is easy to find the <u>**zeros**</u>.

Example: If the factors of the quadratic equation $2x^2+5x+6=0$ are $(2x+2)$ and $(x+3)$, then by the multiplication property of zero: either $(2x+2)=0$, or $(x+3)=0$, or both equal zero. Solving each equation for *x* results in the zeros of the equation, as follows:

$$\begin{aligned}(2x+2)&=0\\2x&=-2\\x&=-1\end{aligned}\qquad\qquad\begin{aligned}(x+3)&=0\\x&=-3\end{aligned}$$

Start with Zeros and Find Factors

If you know the **zeros** of an expression, you can work backwards using the **multiplication property of zero** to find the **factors** of the expression. For example, if you inspect the graph of an equation and find that it has **x-intercepts** at $(3,0)$ and $(-2,0)$, then you know that the solutions are $x=3$ and $x=-2$. You can use these two equations to find the factors of the quadratic expression, as follows:

$$\begin{aligned}x&=3\\(x-3)&=0\end{aligned}$$

$$\begin{aligned}x&=-2\\(x+2)&=0\end{aligned}$$

The factors of a quadratic equation with zeros of 3 and -2 are $(x-3)$ and $(x+2)$.

With practice, you can probably move back and forth between the **zeros** of an expression and the **factors** of an expression with ease.

Part 1 – Overview of Quadratics

DEVELOPING ESSENTIAL SKILLS

Convert the following quadratic equations to standard form and identify the values of a, b, and c:

$x(x-2)=4$	$x^2-2x-4=0$	a= 1 , b=-2 , c=-4
$x(2x+3)=12$	$2x^2+6x-12=0$	a= 2, b=6 , c= -12
$3x(x+8)=-2$	$3x^2+24x+2=0$	a= 3 , b= 24, c= 2
$5x^2=9-x$	$5x^2+x-9=0$	a= 5 , b= 1, c= –9
$-6x^2=-2+x$	$-6x^2-x+2=0$	a= -6, b= -1, c= 2
$x^2=27x-14$	$x^2-27x+14=0$	a= 1 , b=-27 , c= 14
$x^2+2x=1$	$x^2+2x-1=0$	a= 1 , b=2 , c=-1
$4x^2-7x=15$	$4x^2-7x-15=0$	a= 4 , b=-7 , c=-1
$-8x^2+3x=-100$	$-8x^2+3x+100=0$	a= -8 , b=3 , c=100
$25x+6=99x^2$	$-99x^2+25x+6=0$	a= -99 , b=25 , c=6
$2x^2=64$	$2x^2-64=0$	a=2 , b= 0, c=
$0=-16+x^2$	$x^2-16=0$	a=1 , b= 0, c= -16
$49=-9x^2$	$9x^2+49=0$	a= 9 , b=0 , c=49
$x^2=7x$	$x^2-7x=0$	a= 1 , b=-7 , c=0
$2x^2=-+8x$	$2x^2+8x=0$	a= 2 , b=8 , c= 0
$0=-9x-x^2$	$-x^2-9x=0$	a=-1 , b=-9 , c= 0

Find the zeros of the following quadratic equations:

a. $(x+2)(x-3)=0$	a. $x=\{-2,3\}$
b. $(x+1)(x+6)=0$	b. $x=\{-6,1\}$
c. $(x-6)(x+1)=0$	c. $x=\{-1,6\}$
d. $(x-5)(x+3)=0$	d. $x=\{3,5\}$
e. $(x-5)(x+2)=0$	e. $x=\{2,5\}$
f. $(x-4)(x+2)=0$	f. $x=\{-2,4\}$
g. $(2x+3)(3x-2)=0$	g. $x=\left\{-\frac{3}{2},\frac{2}{3}\right\}$
h. $-3(x-4)(2x+3)=0$	h. $x=\left\{-\frac{3}{2},4\right\}$

Part 2 – The Quadratic Formula

The **quadratic formula** is: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Quadratic Formula Song

SOLVING QUADRATIC EQUATIONS STRATEGY #1: Use the Quadratic Formula

Start with any quadratic equation in the form of $ax^2 + bx + c = 0$	$x^2 + 2x - 24 = 0$ The right expression *must* be zero.
Identify the values of a, b, and c.	$a = 1$, $b = 2$, and $c = -24$
Substitute the values of a, b, and c into the quadratic formula, which is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \dfrac{-(2) \pm \sqrt{(2)^2 - 4(1)(-24)c}}{2(1)}$
Solve for x	$x = \dfrac{-(2) \pm \sqrt{100}}{2}$ $x = \dfrac{-(2) \pm 10}{2}$ $x = \dfrac{-(2) + 10}{2} \Rightarrow x = \dfrac{8}{2} \Rightarrow x = 4$ $x = \dfrac{-(2) - 10}{2} \Rightarrow x = \dfrac{-12}{2} = -6$

The quadratic formula can be used to solve any quadratic equation.

Part 2 – The Quadratic Formula

DEVELOPING ESSENTIAL SKILLS

Solve the following quadratic equations using the quadratic formula. Leave answers in simplest radical form.

$x^2 - x - 3 = 0$	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ a = 1, b = -1, c = -3 $x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)}$ $x = \dfrac{1 \pm \sqrt{1+12}}{2}$ $x = \dfrac{1 \pm \sqrt{13}}{2}$

$20x^2 - 15x - 10 = 0$	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ a = 20, b = -15, c = -10 $x=\frac{-(-15)\pm\sqrt{(-15)^2-4(20)(-10)}}{2(20)}$ $x=\frac{15\pm\sqrt{225+800}}{40}$ $x=\frac{15\pm\sqrt{1025}}{40}$ $x=\frac{15\pm\sqrt{25}\times\sqrt{41}}{40}$ $x=\frac{15\pm5\sqrt{41}}{40}$ $x=\frac{3\pm\sqrt{41}}{8}$
$2x^2 - 4x - 2 = 0$	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ a=2, b=-4,c=-2 $x=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(-2)}}{2(2)}$ $x=\frac{4\pm\sqrt{16+16}}{4}$ $x=\frac{4\pm\sqrt{32}}{4}$ $x=\frac{4\pm\sqrt{16}\times\sqrt{2}}{4}$ $x=\frac{4\pm4\sqrt{2}}{4}$ $x=1\pm\sqrt{2}$

$6x^2 + 11x = 35$	$6x^2 + 11x -35 = 0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ a = 6, b = 11, c =-35 $x=\frac{-(11)\pm\sqrt{(11)^2-4(6)(-35)}}{2(6)}$ $x=\frac{-11\pm\sqrt{121+840}}{12}$ $x=\frac{-11\pm\sqrt{961}}{12}$ $x=\frac{-11\pm 31}{12}$ $x=\frac{20}{12}$ and $x=\frac{-42}{12}$ $x=\left\{\frac{5}{3},-\frac{7}{2}\right\}$
$-\ 7x\ +12 = 4x^2$	$-4x^2 - 7x +12 = 0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ a = -4, b = -7, c = 12 $x=\frac{-(-7)\pm\sqrt{(-7)^2-4(-4)(12)}}{2(-4)}$ $x=\frac{7\pm\sqrt{49+192}}{-8}$ $x=\frac{7\pm\sqrt{241}}{-8}$

Part 3 – The Box Method of Factoring

$$\begin{array}{c|c|c|} & gcf & gcf \\ \hline gcf & ax^2 & mx \\ \hline gcf & nx & c \\ \hline \end{array}$$

The Box Method
for
Factoring a Trinomial

$$ax^2 + bx + c = 0$$

$$bx = mx + nx$$

INSTRUCTIONS	EXAMPLE
STEP 1 Start with a factorable quadratic in standard form: $ax^2 + bx + c = 0$ and a 2-row by 2-column table.	Solve by factoring: $6x^2 - x - 12 = 0$
STEP 2 Copy the quadratic term into the upper left box and the constant term into the lower right box.	$\begin{array}{\|c\|c\|}\hline 6x^2 & \\ \hline & -12 \\ \hline\end{array}$
STEP 3 Multiply the quadratic term by the constant term and write the product to the right of the table.	$\begin{array}{\|c\|c\|}\hline 6x^2 & \\ \hline & -12 \\ \hline\end{array}$ $6x^2 \times -12 = \boxed{-72x^2}$
STEP 4 Factor the product from STEP 3 until you obtain two factors that *sum* to the linear term (*bx*).	$1x \times -72x$ $-1x \times 72x$ $2x \times -36x$ $-2x \times 36x$ $3x \times -24x$ $-3x \times 24x$ $4x \times -18x$ $-4x \times 18x$ $6x \times -12x$ $-6x \times 12x$ $8x \times -9x$ These two factors sum to *bx* $-8x \times 9x$

STEP 5 Write one of the two factors found in STEP 4 in the upper right box and the other in the lower left box. Order does not matter.	$\begin{array}{\|c\|c\|}\hline 6x^2 & -9x \\ \hline 8x & -12 \\ \hline\end{array}$
STEP 6 Find the greatest common factor of each row and each column and record these factors to the left of each row and above each column. Give each factor the same plus or minus value as the nearest term in a box. NOTE: If all four of the greatest common factors share a common factor, reduce each factor by the common factor and add the common factor as a third factor. Eg. $(3x-9)(3x-15) \Rightarrow 3(x-3)(x-5)$	$\begin{array}{c\|c\|c\|} & 2x & -3 \\ \hline 3x & 6x^2 & -9x \\ \hline 4 & 8x & -12 \\ \hline\end{array}$
STEP 7 Write the expressions above and beside the box as binomial factors of the original trinomial.	$(2x-3)(3x+4)=0$
STEP 8 Check to see that the factored quadratic is the same as the original quadratic.	$(2x-3)(3x+4)=0$ $6x^2+8x-9x-12=0$ $6x^2-9x-12=0$ check
STEP 9 Convert the factors to zeros.	$(2x-3)=0$ $2x=3$ $x=\boxed{\frac{3}{2}}$ $(3x+4)=0$ $3x=-4$ $x=\boxed{-\frac{4}{3}}$

Part 3 – The Box Method of Factoring

DEVELOPING ESSENTIAL SKILLS

Solve each quadratic by factoring.

$x^2-2x-8 \ = \ 0$	$\begin{array}{c\|c\|c\|} & x & -4 \\ \hline x & x^2 & -4x \\ \hline 2 & 2x & -8 \\ \hline\end{array}$

<table>
<tr>
<td></td>
<td>$\left(x - 4\right)\left(x + 2\right) = 0$
$\boxed{x=\{-2,4\}}$</td>
</tr>
<tr>
<td>$x^2-3x-10=0$</td>
<td>
<table>
<tr><td></td><td>x</td><td>-5</td></tr>
<tr><td>x</td><td>x^2</td><td>-5x</td></tr>
<tr><td>2</td><td>2x</td><td>-10</td></tr>
</table>
$\left(x - 5\right)\left(x + 2\right) = 0$
$\boxed{x=\{5,-2\}}$</td>
</tr>
<tr>
<td>$x^2-2x-15=0$</td>
<td>
<table>
<tr><td></td><td>x</td><td>-5</td></tr>
<tr><td>x</td><td>x^2</td><td>-5x</td></tr>
<tr><td>3</td><td>3x</td><td>-15</td></tr>
</table>
$\left(x - 5\right)\left(x + 3\right) = 0$
$\boxed{\text{x}=\{-3,5\}}$</td>
</tr>
<tr>
<td>$6x^2+5x-6$
$6x^2-4x+9x-6$
$\left(2x+3\right)\left(3x - 2\right) = 0$</td>
<td>
<table>
<tr><td></td><td>3x</td><td>-2</td></tr>
<tr><td>2x</td><td>$6x^2$</td><td>-4x</td></tr>
<tr><td>3</td><td>9x</td><td>-6</td></tr>
</table>
$\left(2x+3\right)\left(3x - 2\right) = 0$
$\boxed{x=\left\{-\frac{3}{2},\frac{2}{3}\right\}}$</td>
</tr>
<tr>
<td>$10x^2+4x-6=0$
$10x^2-6x+10x-6=0$
$\left(2x + 2\right)\left(5x - 3\right) = 0$</td>
<td>
<table>
<tr><td></td><td>10x</td><td>-6</td></tr>
<tr><td>2x</td><td>$10x^2$</td><td>-6x</td></tr>
<tr><td>2</td><td>10x</td><td>-6</td></tr>
</table>
$\left(10x-6\right)\left(2x+2\right) = 0$
$2\left(5x-3\right)\left(x+1\right) = 0$
$\boxed{\text{x}=\left\{-1,\frac{3}{5}\right\}}$</td>
</tr>
</table>

Part 4 – Completing the Square

SOLVING QUADRATIC EQUATIONS STRATEGY #3: Completing the Square

completing the square algorithm
A process used to change an expression of the form ax^2 +bx +c into a perfect square binomial by adding a suitable constant.

Source: NYSED Mathematics Glossary

PROCEDURE TO FIND THE ZEROS AND EXTREMES OF A QUADRATIC	
Start with any quadratic equation of the general form $ax^2+bx+c=0$	
STEP 1 **Isolate all terms with** x^2 **and** x **on one side of the equation. If** $a\neq 1$**, divide every term in the equation by** a **to get one expression in the form of** x^2+bx	
STEP 2 **Complete the Square by adding** $\left(\frac{b}{2}\right)^2$ **to both sides of the equation.**	
STEP 3 **Factor the side containing** $x^2+bx+\left(\frac{b}{2}\right)^2$ **into a binomial expression of the form** $\left(x+\frac{b}{2}\right)^2$	
STEP 4a **(solving for roots and zeros only)** **Take the square root of both sides of the equation and simplify,**	**STEP 4b** **(solving for maxima and minima only)** **Multiply both sides of the equation by *a*.** **Move all terms to left side of equation.** **Solve the factor in parenthesis for axis of symmety and x-value of the vertex..** **The number not in parentheses is the y-value of the vertes.**

STEPS:	EXAMPLE A	EXAMPLE B
Start with any quadratic equation of the general form $ax^2+bx+c=n$	$x^2+2x+3=4$	$5x^2+2x+3=4$

STEP 1) **Isolate all terms with x^2 and x on one side of the equation.** **If $a \neq 1$, divide every term in the equation by a to get one expression in the form of $x^2 + bx$**	$x^2 + 2x = 1$	$5x^2 + 2x = 1$ $\frac{5x^2}{5} + \frac{2x}{5} = \frac{1}{5}$ $x^2 + \frac{2}{5}x = \frac{1}{5}$
STEP 2) **Complete the Square by adding $\left(\frac{b}{2}\right)^2$ to both sides of the equation.**	$b = 2,\ \frac{b}{2} = \frac{2}{2} = 1,\ \left(\frac{b}{2}\right)^2 = (1)^2$ $x^2 + 2x + (1)^2 = 1 + (1)^2$ $x^2 + 2x + (1)^2 = 2$	$b = \frac{2}{5},\ \frac{b}{2} = \frac{1}{5},\ \left(\frac{b}{2}\right)^2 = \left(\frac{1}{5}\right)^2$ $x^2 + \frac{2}{5}x + \left(\frac{1}{5}\right)^2 = \frac{1}{5} + \left(\frac{1}{5}\right)^2$
STEP 3) **Factor the side containing $x^2 + bx + \left(\frac{b}{2}\right)^2$ into a binomial expression of the form $\left(x + \frac{b}{2}\right)^2$**	$(x+1)^2 = 2$	$\left(x + \frac{1}{5}\right)^2 = \frac{1}{5} + \left(\frac{1}{5}\right)^2$ $\left(x + \frac{1}{5}\right)^2 = \frac{5}{25} + \frac{1}{25}$ $\left(x + \frac{1}{5}\right)^2 = \frac{6}{25}$
STEP 4a) **Take the square roots of both sides of the equation and simplify.**	$\sqrt{(x+1)^2} = \sqrt{2}$ $x + 1 = \pm\sqrt{2}$ $x = \boxed{-1 \pm \sqrt{2}}$	$\sqrt{\left(x + \frac{1}{5}\right)^2} = \sqrt{\frac{6}{25}}$ $x + \frac{1}{5} = \pm\frac{\sqrt{6}}{5}$ $x = -\frac{1}{5} \pm \frac{\sqrt{6}}{5} = \boxed{\frac{1 \pm \sqrt{6}}{5}}$

STEP 4b **Multiply both sides of the equation by *a*. Move all terms to left side of equation. Solve the factor in parenthesis for axis of symmety and x-value of the vertex. The number not in parentheses is the y-value of the vertes.**	$1(x+1)^2 = 1(2)$ $(x+1)^2 = 2$ $(x+1)^2 - 2 = 0$ vertex form. -1 is the axis of symmetry -2 is the y value of the vertex The vertex is at $(-1,-2)$ $(x+1)^2 = 2$	$5\left(x+\frac{1}{5}\right)^2 = 5\left(\frac{6}{25}\right)$ $5\left(x+\frac{1}{5}\right)^2 = \frac{6}{5}$ $5\left(x+\frac{1}{5}\right)^2 - \frac{6}{5} = 0$ vertex form. $-\frac{1}{5}$ is the axis of symmetry $-\frac{6}{5}$ is the y value of the vertex The vertex is at $\left(-\frac{1}{5},-\frac{6}{5}\right)$

Part 4 – Completing the Square

DEVELOPING ESSENTIAL SKILLS

Solve the following quadratic equations by completing the square.

$x^2 - x - 3 = 0$	$x^2 - x - 3 = 0$ $x^2 - x = 3$ $x^2 - x + \left(\frac{1}{2}\right)^2 = 3 + \left(\frac{1}{2}\right)^2$ $\left(x - \frac{1}{2}\right)^2 = 3 + \frac{1}{4}$ $x - \frac{1}{2} = \pm\sqrt{\frac{13}{4}}$ $x = \frac{1}{2} \pm \frac{\sqrt{13}}{2}$ $x = \frac{1 \pm \sqrt{13}}{2}$

$20x^2 - 15x - 10 = 0$	$20x^2 - 15x - 10 = 0$ $20x^2 - 15x = 10$ $\frac{20x^2}{20} - \frac{15x}{20} = \frac{10}{20}$ $x^2 - \frac{3x}{4} = \frac{1}{2}$ $x^2 - \frac{3x}{4} + \left(-\frac{3}{8}\right)^2 = \frac{1}{2} + \left(-\frac{3}{8}\right)^2$ $\left(x - \frac{3}{8}\right)^2 = \frac{32}{64} + \frac{9}{64}$ $\left(x - \frac{3}{8}\right)^2 = \frac{41}{64}$ $x - \frac{3}{8} = \pm\sqrt{\frac{41}{64}}$ $x = \frac{3}{8} \pm \frac{\sqrt{41}}{8}$ $x = \frac{3 + \sqrt{41}}{8}$
$2x^2 - 4x - 2 = 0$	$2x^2 - 4x - 2 = 0$ $2x^2 - 4x = 2$ $\frac{2x^2}{2} - \frac{4x}{2} = \frac{2}{2}$ $x^2 - 2x = 1$ $x^2 - 2x + \left(-\frac{2}{2}\right)^2 = 1 + \left(-\frac{2}{2}\right)^2$ $(x - 1)^2 = 1 + 1$ $x - 1 = \pm\sqrt{2}$ $x = 1 \pm \sqrt{2}$

$6x^2 + 11x = 35$	$6x^2+11x -35 = 0$ $6x^2+11x = 35$ $\frac{6x^2}{6}+\frac{11x}{6}=\frac{35}{6}$ $x^2+\frac{11x}{6}=\frac{35}{6}$ $x^2+\frac{11x}{6}+\left(\frac{11}{12}\right)^2=\frac{35}{6}+\left(\frac{11}{12}\right)^2$ $\left(x+\frac{11}{12}\right)^2=\frac{35}{6}+\frac{121}{144}$ $\left(x+\frac{11}{12}\right)^2=\frac{840}{144}+\frac{121}{144}$ $\left(x+\frac{11}{12}\right)^2=\frac{961}{144}$ $x+\frac{11}{12}=\pm\sqrt{\frac{961}{144}}$ $x+\frac{11}{12}=\pm\frac{31}{12}$ $x=-\frac{11}{12}\pm\frac{31}{12}$ $x=\frac{42}{12}$ and $\frac{-20}{12}$ $x=\frac{7}{2}$ and $-\frac{5}{3}$ $x=\left\{\frac{5}{3},-\frac{7}{2}\right\}$

$-\ 7x\ +12\ =\ 4\mathrm{x}^2$	$-4x^2\text{-}7x\text{=-}12$ $\frac{-4x^2}{-4}\text{-}\frac{7x}{-4}=\frac{\text{-}12}{-4}$ $x^2+\frac{7}{4}x=3$ $x^2+\frac{7}{4}x+\left(\frac{7}{8}\right)^2=3+\left(\frac{7}{8}\right)^2$ $x^2+\frac{7}{4}x+\left(\frac{7}{8}\right)^2=\frac{192}{64}+\frac{49}{64}$ $\left(x+\frac{7}{8}\right)^2=\frac{241}{64}$ $x+\frac{7}{8}=\pm\frac{\sqrt{241}}{8}$ $x=\frac{7}{8}\pm\frac{\sqrt{241}}{8}$ $x=\frac{7\pm\sqrt{241}}{8}$

A.APR.B.3, A.REI.B.4: Solving Quadratics

169) Solve $8m^2 + 20m = 12$ for *m* by factoring.

170) Keith determines the zeros of the function $f(x)$ to be -6 and 5. What could be Keith's function?

1) $f(x) = (x+5)(x+6)$	3) $f(x) = (x-5)(x+6)$
2) $f(x) = (x+5)(x-6)$	4) $f(x) = (x-5)(x-6)$

171) In the equation $x^2 + 10x + 24 = (x+a)(x+b)$, *b* is an integer. Find algebraically *all* possible values of *b*.

172) Which equation has the same solutions as $2x^2 + x - 3 = 0$

1) $(2x-1)(x+3) = 0$	3) $(2x-3)(x+1) = 0$
2) $(2x+1)(x-3) = 0$	4) $(2x+3)(x-1) = 0$

173) The zeros of the function $f(x) = 3x^2 - 3x - 6$ are

1) -1 and -2	3) 1 and 2
2) 1 and -2	4) -1 and 2

174) The zeros of the function $f(x) = 2x^2 - 4x - 6$ are

1) 3 and -1	3) -3 and 1
2) 3 and 1	4) -3 and -1

175) Janice is asked to solve $0 = 64x^2 + 16x - 3$. She begins the problem by writing the following steps:

Line 1 $0 = 64x^2 + 16x - 3$
Line 2 $0 = B^2 + 2B - 3$
Line 3 $0 = (B+3)(B-1)$

Use Janice’s procedure to solve the equation for x.

Explain the method Janice used to solve the quadratic equation.

176) What is the solution set of the equation $(x-2)(x-a) = 0$?

1) -2 and *a*	3) 2 and *a*
2) -2 and -*a*	4) 2 and -*a*

177) The function $r(x)$ is defined by the expression $x^2 + 3x - 18$. Use factoring to determine the zeros of $r(x)$. Explain what the zeros represent on the graph of $r(x)$.

178) If the quadratic formula is used to find the roots of the equation $x^2 - 6x - 19 = 0$, the correct roots are

1) $3 \pm 2\sqrt{7}$	3) $3 \pm 4\sqrt{14}$
2) $-3 \pm 2\sqrt{7}$	4) $-3 \pm 4\sqrt{14}$

179) Which equation has the same solution as $x^2 - 6x - 12 = 0$?

1) $(x+3)^2 = 21$	3) $(x+3)^2 = 3$
2) $(x-3)^2 = 21$	4) $(x-3)^2 = 3$

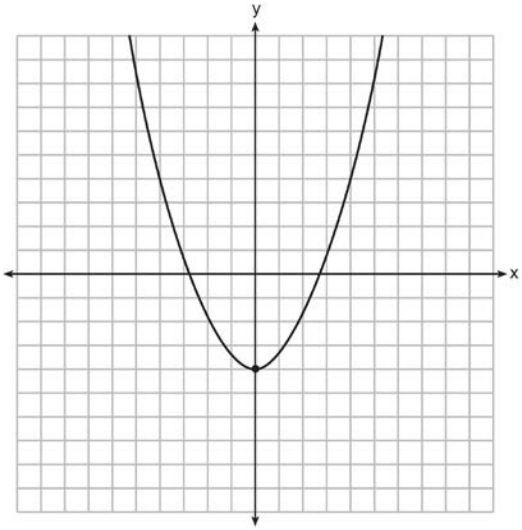
180) What are the roots of the equation $x^2 + 4x - 16 = 0$?

1) $2 \pm 2\sqrt{5}$
3) $2 \pm 4\sqrt{5}$
2) $-2 \pm 2\sqrt{5}$
4) $-2 \pm 4\sqrt{5}$

181) Write an equation that defines $m(x)$ as a trinomial where $m(x) = (3x - 1)(3 - x) + 4x^2 + 19$. Solve for *x* when $m(x) = 0$.

182) If $4x^2 - 100 = 0$, the roots of the equation are
1) -25 and 25
3) -5 and 5
2) -25, only
4) -5, only

183) Ryker is given the graph of the function $y = \frac{1}{2}x^2 - 4$. He wants to find the zeros of the function, but is unable to read them exactly from the graph.



Find the zeros in simplest radical form.

184) A student was given the equation $x^2 + 6x - 13 = 0$ to solve by completing the square. The first step that was written is shown below.

$$x^2 + 6x = 13$$

The next step in the student's process was $x^2 + 6x + c = 13 + c$. State the value of *c* that creates a perfect square trinomial. Explain how the value of *c* is determined.

185) Which equation has the same solutions as $x^2 + 6x - 7 = 0$?
1) $(x + 3)^2 = 2$
3) $(x - 3)^2 = 16$
2) $(x - 3)^2 = 2$
4) $(x + 3)^2 = 16$

186) Solve the equation $4x^2 - 12x = 7$ algebraically for *x*.

187) When directed to solve a quadratic equation by completing the square, Sam arrived at the equation $\left(x - \frac{5}{2}\right)^2 = \frac{13}{4}$. Which equation could have been the original equation given to Sam?
1) $x^2 + 5x + 7 = 0$
3) $x^2 - 5x + 7 = 0$

2) $x^2+5x+3=0$ 4) $x^2-5x+3=0$

188) A student is asked to solve the equation $4(3x-1)^2-17=83$. The student's solution to the problem starts as $4(3x-1)^2=100$

$(3x-1)^2=25$

A correct next step in the solution of the problem is

1) $3x-1=\pm 5$
2) $3x-1=\pm 25$
3) $9x^2-1=25$
4) $9x^2-6x+1=5$

189) What are the solutions to the equation $x^2-8x=10$?

1) $4\pm\sqrt{10}$
2) $4\pm\sqrt{26}$
3) $-4\pm\sqrt{10}$
4) $-4\pm\sqrt{26}$

190) The solution of the equation $(x+3)^2=7$ is

1) $3\pm\sqrt{7}$
2) $7\pm\sqrt{3}$
3) $-3\pm\sqrt{7}$
4) $-7\pm\sqrt{3}$

191) When solving the equation $x^2-8x-7=0$ by completing the square, which equation is a step in the process?

1) $(x-4)^2=9$
2) $(x-4)^2=23$
3) $(x-8)^2=9$
4) $(x-8)^2=23$

192) Solve the equation for *y*: $(y-3)^2=4y-12$

193) Fred's teacher gave the class the quadratic function $f(x)=4x^2+16x+9$.

a) State two different methods Fred could use to solve the equation $f(x)=0$.

b) Using one of the methods stated in part *a,* solve $f(x)=0$ for *x,* to the *nearest tenth.*

194) What is the solution of the equation $2(x+2)^2-4=28$?

1) 6, only
2) 2, only
3) 2 and -6
4) 6 and -2

195) Amy solved the equation $2x^2+5x-42=0$. She stated that the solutions to the equation were $\frac{7}{2}$ and -6. Do you agree with Amy's solutions? Explain why or why not.

196) The height, *H*, in feet, of an object dropped from the top of a building after *t* seconds is given by $H(t)=-16t^2+144$. How many feet did the object fall between one and two seconds after it was dropped? Determine, algebraically, how many seconds it will take for the object to reach the ground.

197) What are the solutions to the equation $3x^2+10x=8$?

1) $\frac{2}{3}$ and -4
2) $-\frac{2}{3}$ and 4
3) $\frac{4}{3}$ and -2
4) $-\frac{4}{3}$ and 2

198) Find the zeros of $f(x) = (x-3)^2 - 49$, algebraically.

199) Which value of x is a solution to the equation $13 - 36x^2 = -12$?

1) $\frac{36}{25}$
2) $\frac{25}{36}$
3) $-\frac{6}{5}$
4) $-\frac{5}{6}$

200) The method of completing the square was used to solve the equation $2x^2 - 12x + 6 = 0$. Which equation is a correct step when using this method?

1) $(x-3)^2 = 6$
2) $(x-3)^2 = -6$
3) $(x-3)^2 = 3$
4) $(x-3)^2 = -3$

201) What are the solutions to the equation $x^2 - 8x = 24$?

1) $x = 4 \pm 2\sqrt{10}$
2) $x = -4 \pm 2\sqrt{10}$
3) $x = 4 \pm 2\sqrt{2}$
4) $x = -4 \pm 2\sqrt{2}$

202) Solve the equation $x^2 - 6x = 15$ by completing the square.

203) What are the solutions to the equation $3(x-4)^2 = 27$?

1) 1 and 7
2) -1 and -7
3) $4 \pm \sqrt{24}$
4) $-4 \pm \sqrt{24}$

204) The quadratic equation $x^2 - 6x = 12$ is rewritten in the form $(x+p)^2 = q$, where q is a constant. What is the value of p?

1) -12
2) -9
3) -3
4) 9

205) Solve for x to the *nearest tenth*: $x^2 + x - 5 = 0$.

SOLUTIONS

169) ANS:

$m = \frac{1}{2}$ and $m = -3$

Strategy: Factor by grouping.

$8m^2 + 20m = 12$

$8m^2 + 20m - 12 = 0$

$|ac| = 96$

The factors of 96 are:

1 and 96

2 and 48

3 and 32

4 and 24 (use these)

$8m^2 + 24m - 4m - 12 = 0$

$\left(8m^2 + 24m\right) - (4m + 12) = 0$

$8m(m + 3) - 4(m + 3) = 0$

$(8m - 4)(m + 3) = 0$

Use the multiplication property of zero to solve for m.

$8m - 4 = 0$ $8m = 4$ $m = \frac{4}{8}$ $m = \frac{1}{2}$	$m + 3 = 0$ $m = -3$

PTS: 2 NAT: A.SSE.B.3 TOP: Solving Quadratics

170) ANS: 3

Strategy: Convert the zeros to factors.

If the zeros of $f(x)$ are -6 and 5, then the factors of $f(x)$ are $(x + 6)$ and $(x - 5)$.
Therefore, the function can be written as $f(x) = (x + 6)(x - 5)$.
The correct answer choice is *c*.

PTS: 2 NAT: A.SSE.B.3 TOP: Solving Quadratics

171) ANS:
6 and 4

Strategy: Factor the trinomial $x^2 + 10x + 24$ into two binomials.

$x^2 + 10x + 24$

$(x+\_\_\_)(x+\_\_\_)$

The factors of 24 are:

1 and 24

2 and 12

3 and 8

4 and 6 (use these)

$(x+4)(x+6)$

Possible values for *a* and *c* are 4 and 6.

PTS: 2 NAT: A.SSE.B.3 TOP: Solving Quadratics

172) ANS: 4

Strategy 1: Factor by grouping.

$2x^2 + x - 3 = 0$

$|ac| = 6$

Factors of 6 are

1 and 6

2 and 3 (use these)

$2x^2 + 3x - 2x - 3 = 0$

$(2x^2 + 3x) - (2x + 3) = 0$

$x(2x - 3) - 1(2x + 3) = 0$

$(x - 1)(2x + 3) = 0$

Answer choice *d* is correct

Strategy 2: Work backwards by using the disrtibutive property to expand all answer choices and match the expanded trinomials to the function $2x^2 + x - 3 = 0$.

a. $(2x-1)(x+3)=0$ $2x^2+6x-x-3$ $2x^2+5x-3$ (Wrong Choice)	c. $(2x-3)(x+1)=0$ $2x^2+2x-3x-3$ $2x^2-x-3$ (Wrong Choice)
b. $(2x+1)(x-3)=0$ $2x^2-6x+x-3=0$ $2x^2-5x-3=0$ (Wrong Choice)	d. $(2x+3)(x-1)=0$ $2x^2-2x+3x-3=0$ $2x^2+x-3=0$ (Correct Choice)

PTS: 2 NAT: A.SSE.B.3 TOP: Solving Quadratics

173) ANS: 4

Strategy 1. Factor, then use the multiplication property of zero to find zeros.

$$3x^2 - 3x - 6 = 0$$

$$3(x^2 - x - 2) = 0$$

$$3(x - 2)(x + 1) = 0$$

$$x = 2, -1$$

Strategy 2. Use the quadratic formula.

$a = 3, b = -3,$ and $c = -6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-6)}}{2(3)}$$

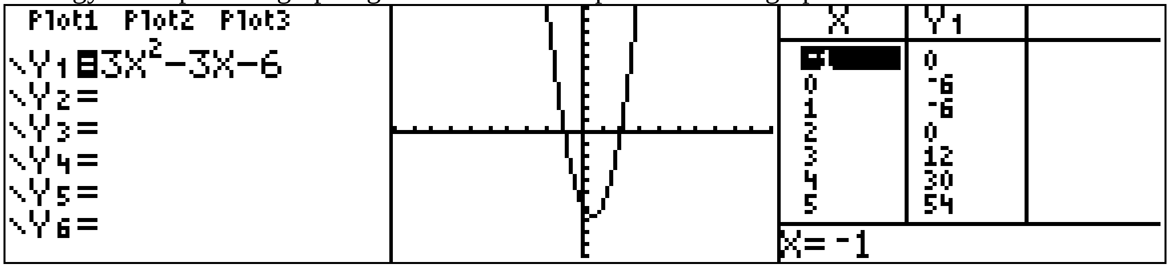
$$x = \frac{3 \pm \sqrt{9 + 72}}{6}$$

$$x = \frac{3 \pm \sqrt{81}}{6}$$

$$x = \frac{3 \pm 9}{6}$$

$$x = \frac{12}{6} = 2 \text{ and } x = \frac{-6}{6} = -1$$

Strategy 3. Input into graphing calculator and inspect table and graph.



```
 Plot1  Plot2  Plot3
\Y1=3X²-3X-6
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

X	Y1	
-1	0	
0	-6	
1	-6	
2	0	
3	12	
4	30	
5	54	

X= -1

PTS: 2 NAT: A.SSE.B.3 TOP: Solving Quadratics

174) ANS: 1

Strategy #1: Solve by factoring:

$$f(x) = 2x^2 - 4x - 6$$

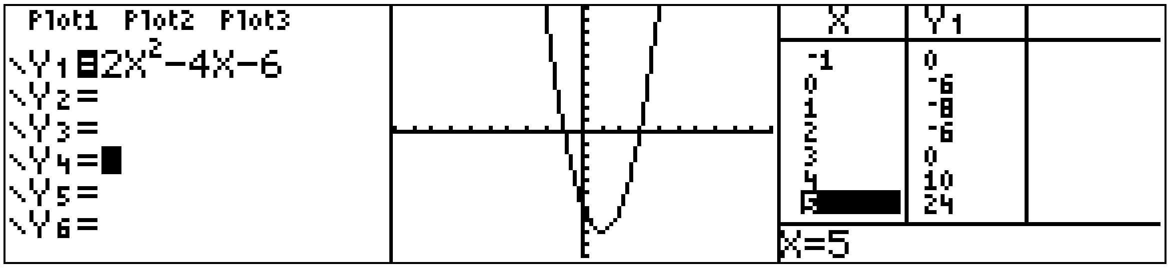
$$0 = 2x^2 - 4x - 6$$

$$0 = 2(x^2 - 2x - 3)$$

$$0 = 2(x - 3)(x + 1)$$

$$x = 3 \textit{ and } x = -1$$

Strategy #2: Solve by inputing equation into graphing calculator, the use the graph and table views to identify the zeros of the function.



Plot1 Plot2 Plot3

$Y_1 = 2X^2 - 4X - 6$

$Y_2 =$

$Y_3 =$

$Y_4 =$

$Y_5 =$

$Y_6 =$

X	Y1
-1	0
0	-6
1	-8
2	-6
3	0
4	10
5	24

X=5

The graph and table views show the zeros to be at -1 and 3.

PTS: 2 NAT: A.SSE.B.3 TOP: Solving Quadratics

KEY: zeros of polynomials

175) ANS:

Use Janice's procedure to solve for X.

Line 4 $B = -3$ and $B = 1$

Line 5 Therefore:

$8x = -3$ and $8x = 1$

$x = -\frac{3}{8}$ $x = \frac{1}{8}$

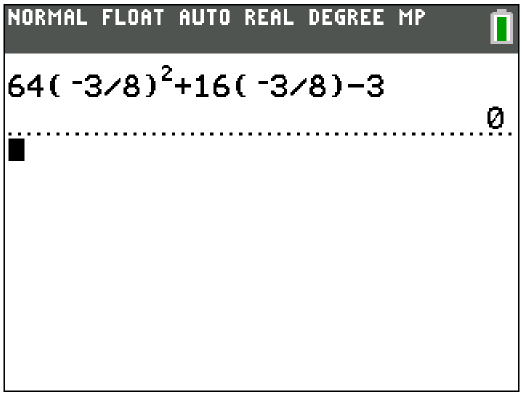
Explain the method Janice used to solve the quadratic formula.

Janice made the problem easier by substituting B for *8x*, then solving for B. After solving for B, she reversed her substitution and solved for *x*.

Check:

$x = -\frac{3}{8}$

$0 = 64x^2 + 16x - 3$



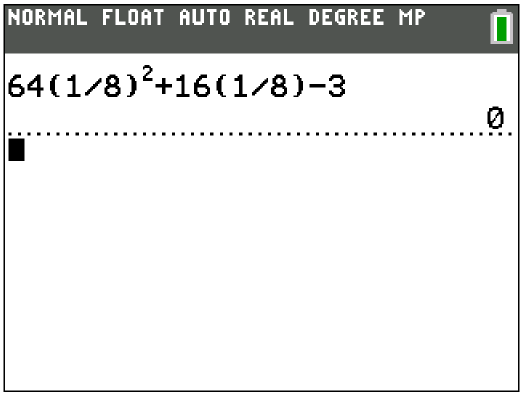
NORMAL FLOAT AUTO REAL DEGREE MP

$64(^-3/8)^2+16(^-3/8)-3$

0

$x = \frac{1}{8}$

$0 = 64x^2 + 16x - 3$



NORMAL FLOAT AUTO REAL DEGREE MP

$64(1/8)^2+16(1/8)-3$

0

PTS: 4 NAT: A.SSE.B.3a

176) ANS: 3

The solution set of a quadratic equation includes all values of x when y equals zero. In the equation $(x-2)(x-a) = 0$, the value of y is zero and $(x-2)$ and $(x-a)$ are factors whose product is zero.

The multiplication property of zero says, if the product of two factors is zero, then one or both of the factors must be zero.

Therefore, we can write: $x-2=0$ and $x-a=0$.

$x=2$ $x=a$

PTS: 2 NAT: A.SSE.B.3 TOP: Solving Quadratics

177) ANS:

$x=\{-6,3\}$

Factor $x^2+3x-18$ as follows:

$$x^2+3x-18=0$$

$$(x+6)(x-3)=0$$

Then, use the multiplication property of zero to find the zeros, as follows:

$$x+6=0 \quad \text{and} \quad x-3=0$$

$$x=-6 \qquad\qquad x=3$$

The zeros of a function are the *x*-values when $y=0$. On a graph, the zeros are the values of *x* at the *x*-axis intercepts.

PTS: 4 NAT: A.SSE.B.3 TOP: Solving Quadratics

178) ANS: 1

Strategy: Use the quadratic equation to solve $x^2-6x-19=0$, where $a=1$, $b=-6$, and $c=-19$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(-19)}}{2(1)}$$

$$x=\frac{6\pm\sqrt{112}}{2}$$

$$x=\frac{6\pm\sqrt{16}\cdot\sqrt{7}}{2}$$

$$x=\frac{6\pm4\sqrt{7}}{2}$$

$$x=3\pm2\sqrt{7}$$

Answer choice *a* is correct.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: quadratic formula

179) ANS: 2

Strategy: Use the distributive property to expand each answer choice, the compare the expanded trinomial to the given equation $x^2-6x-12=0$. Equivalent equations expressed in different terms will have the same solutions.

a.	c.

$(x+3)^2 = 21$ $(x+3)(x+3) = 21$ $x^2 + 6x + 9 = 21$ $x^2 + 6x - 12 = 0$ (Wrong Choice)	$(x+3)^2 = 3$ $(x+3)(x+3) = 3$ $x^2 + 6x + 9 = 3$ $x^2 + 6x + 6 = 0$ (Wrong Choice)
b. $(x-3)^2 = 21$ $(x-3)(x-3) = 21$ $x^2 - 6x + 9 = 21$ $x^2 - 6x - 12 = 0$ (Correct Choice)	d. $(x-3)^2 = 3$ $(x-3)(x-3) = 3$ $x^2 - 6x + 9 = 3$ $x^2 - 6x + 6 = 0$ (Wrong Choice)

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

180) ANS: 2

Strategy 1: Use the quadratic equation to solve $x^2 + 4x - 16 = 0$, where $a = 1$, $b = 4$, and $c = -16$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-16)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{80}}{2}$$

$$x = \frac{-4 \pm \sqrt{16}\sqrt{5}}{2}$$

$$x = \frac{-4 \pm 4\sqrt{5}}{2}$$

$$x = -2 \pm 2\sqrt{5}$$

Answer choice *b* is correct.

Strategy 2: Solve by completing the square:

$$x^2 + 4x - 16 = 0$$

$$x^2 + 4x = 16$$

$$(x+2)^2 = 16 + 2^2$$

$$(x+2)^2 = 20$$

$$\sqrt{(x+2)^2} = \sqrt{20}$$

$$x + 2 = \pm 2\sqrt{5}$$

$$x = -2 \pm 2\sqrt{5}$$

Answer choice *b* is correct.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: quadratic formula

181) ANS:
$x = -8$ and $x = -2$

Strategy: Transform the expression $(3x-1)(3-x) + 4x^2 + 19$ to a trinomial, then set the expression equal to 0 and solve it.

STEP 1. Transform $(3x-1)(3-x) + 4x^2 + 19$ into a trinomial.

$$(3x-1)(3-x) + 4x^2 + 19$$

$$9x - 3x^2 - 3 + x + 4x^2 + 19$$

$$x^2 + 10x + 16$$

STEP 2. Set the trinomial expression equal to 0 and solve.

$$x^2 + 10x + 16 = 0$$

$$(x+8)(x+2) = 0$$

$$x = -8 \text{ and} - 2$$

PTS: 4 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: factoring

182) ANS: 3
Strategy: Solve using root operations.

$$4x^2 - 100 = 0$$

$$4x^2 = 100$$

$$x^2 = 25$$

$$\sqrt{x^2} = \sqrt{25}$$

$$x = \pm 5$$

Answer choice *c* is correct.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: taking square roots

183) ANS:
$x = \pm 2\sqrt{2}$

Strategy: Use root operations to solve for x in the equeation $y = \frac{1}{2}x^2 - 4$.

$$\frac{1}{2}x^2 - 4 = 0$$
$$x^2 - 8 = 0$$
$$x^2 = 8$$
$$\sqrt{x^2} = \sqrt{8}$$
$$x = \pm\sqrt{8}$$
$$x = \pm\sqrt{4}\sqrt{2}$$
$$x = \pm 2\sqrt{2}$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: taking square roots

184) ANS:

The value of c that creates a perfect square trinomial is $\left(\frac{6}{2}\right)^2$, which is equal to 9.

The value of c is determined by taking half the value of *b*, when $a = 1$, and squaring it. In this problem, $b = 6$, so $\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

185) ANS: 4
Strategy: Use the distributive property to expand each answer choice, the compare the expanded trinomial to the given equation $x^2 + 6x - 7 = 0$. Equivalent equations expressed in different terms will have the same solutions.

a. $(x+3)^2 = 2$ $(x+3)(x+3) = 2$ $x^2 + 6x + 9 = 2$ $x^2 + 6x + 7 = 0$ (Wrong Choice)	c. $(x-3)^2 = 16$ $(x-3)(x-3) = 16$ $x^2 - 6x + 9 = 16$ $x^2 - 6x - 7 = 0$ (Wrong Choice)
b. $(x-3)^2 = 2$ $(x-3)(x-3) = 2$ $x^2 - 6x + 9 = 2$ $x^2 - 6x + 7 = 0$ (Wrong Choice)	d. $(x+3)^2 = 16$ $(x+3)(x+3) = 16$ $x^2 + 6x + 9 = 16$ $x^2 + 6x - 7 = 0$ (Correct Choice)

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

186) ANS:

Strategy 1: Solve using factoring by grouping.

$$4x^2 - 12x = 7$$

$$4x^2 - 12x - 7 = 0$$

$$|ac| = 28$$

The factors of 28 are

1 and 28

2 and 14 (use these)

$$4x^2 - 14x + 2x - 7 = 0$$

$$\left(4x^2 - 14x\right) + (2x - 7) = 0$$

$$2x(2x - 7) + 1(2x - 7) = 0$$

$$(2x + 1)(2x - 7) = 0$$

$$x = -\frac{1}{2}$$

$$x = \frac{7}{2}$$

Strategy 2: Solve by completing the square.

$$4x^2 - 12x = 7$$

$$\frac{4x^2}{4} - \frac{12x}{4} = \frac{7}{4}$$

$$x^2 - 3x = \frac{7}{4}$$

$$x^2 - 3x + \left(\frac{-3}{2}\right)^2 = \frac{7}{4} + \left(\frac{-3}{2}\right)^2$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{7}{4} + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{16}{4}$$

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{4}$$

$$x - \frac{3}{2} = \pm 2$$

$$x = \frac{3}{2} \pm 2$$

$$x = -\frac{1}{2} \text{ and } \frac{7}{2}$$

Strategy 3. Solve using the quadratic formula, where $a = 4$, $b = -12$, and $c = -7$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(-7)}}{2(4)}$$

$$x = \frac{12 \pm \sqrt{144 + 112}}{8}$$

$$x = \frac{12 \pm \sqrt{256}}{8}$$

$$x = \frac{12 \pm 16}{8}$$

$$x = \frac{3 \pm 4}{2}$$

$$x = -\frac{1}{2} \text{ and } \frac{7}{2}$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: factoring

187) ANS: 4

Strategy: Assume that Sam’s equation is correct, then expand the square in his equation and simplify.

$$x^2 - 5x + 3 = 0$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{13}{4}$$

$$\left(x - \frac{5}{2}\right)\left(x - \frac{5}{2}\right) = \frac{13}{4}$$

$$x^2 - 5x + \frac{25}{4} = \frac{13}{4}$$

$$x^2 - 5x = \frac{13}{4} - \frac{25}{4}$$

$$x^2 - 5x = \frac{-12}{4}$$

$$x^2 - 5x = -3$$

$$x^2 - 5x + 3 = 0$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: completing the square

188) ANS: 1

Strategy: The next step should be to take the square roots of both expressions.

$$(3x - 1)^2 = 25$$

$$\sqrt{(3x - 1)^2} = \sqrt{25}$$

$$3x - 1 = \pm 5$$

The correct answer choice is *a*.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: completing the square

189) ANS: 2

$$x^2 - 8x = 10$$

$$x^2 - 8x + (4)^2 = 10 + (4)^2$$

$$(x-4)^2 = 10 + 16$$

$$(x-4)^2 = 26$$

$$\sqrt{(x-4)^2} = \sqrt{26}$$

$$x - 4 = \pm\sqrt{26}$$

$$x = 4 \pm \sqrt{26}$$

$$(x-4)^2 = 26$$

$$x - 4 = \pm\sqrt{26}$$

$$x = 4 \pm \sqrt{26}$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

190) ANS: 3

Strategy 1: Solve using root operations.

$$(x+3)^2 = 7$$

$$\sqrt{(x+3)^2} = \sqrt{7}$$

$$x + 3 = \pm\sqrt{7}$$

$$x = -3 \pm \sqrt{7}$$

Strategy 2. Solve using the quadratic equation.

$$(x+3)^2 = 7$$

$$x^2 + 6x + 9 = 7$$

$$x^2 + 6x + 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1,\ b = 6,\ c = 2$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 8}}{2}$$

$$x = \frac{-6 \pm \sqrt{28}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{7}}{2}$$

$$x = -3 \pm \sqrt{7}$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

191) ANS: 2

$$x^2 - 8x - 7 = 0$$

$$x^2 - 8x = 7$$

$$x^2 - 8x + (-4)^2 = 7 + (-4)^2$$

$$x^2 - 8x + 16 = 7 + 16$$

$$(x-4)^2 = 23$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

192) ANS:
The solutions are $y = 3$ and $y = 7$.

$$(y-3)^2 = 4y-12$$

$$y^2-6y+9 = 4y-12$$

$$y^2-10y+21 = 0$$

$$(y-7)(y-3) = 0$$

$$y-7 = 0$$

$$y = 7$$

$$y-3 = 0$$

$$y = 3$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: factoring

193) ANS:

a) Quadratic formula and completing the square.

b) -0.7 and -3.3

Complete the Square Method	Quadratic Formula Method
	$f(x) = 4x^2+16x+9$ a=4, b=16, c=9 $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $x = \frac{-16 \pm \sqrt{(16)^2-4(4)(9)}}{2(4)}$ $x = \frac{-16 \pm \sqrt{112}}{8}$ $x = \frac{-16+\sqrt{112}}{8} = \frac{-5.416}{8} = -.677 = -0.7$ $x = \frac{-16-\sqrt{112}}{8} = \frac{-26.583}{8} = -3.322 = -3.3$

$$f(x) = 4x^2 + 16x + 9$$

$$4x^2 + 16x + 9 = 0$$

$$4x^2 + 16x = -9$$

$$\frac{4x^2}{4} + \frac{16x}{4} = \frac{-9}{4}$$

$$x^2 + 4x = -\frac{9}{4}$$

$$x^2 + 4x + (2)^2 = -\frac{9}{4} + (2)^2$$

$$(x+2)^2 = -\frac{9}{4} + 4$$

$$(x+2)^2 = -\frac{9}{4} + \frac{16}{4}$$

$$(x+2)^2 = \frac{7}{4}$$

$$x + 2 = \pm\sqrt{\frac{7}{4}}$$

$$x + 2 = \pm\frac{\sqrt{7}}{2}$$

$$x = -2 \pm \frac{\sqrt{7}}{2}$$

$$x = -2 + \frac{\sqrt{7}}{2} = -0.677 = -0.7$$

$$x = -2 - \frac{\sqrt{7}}{2} = -3.322 = -3.3$$

PTS: 1 NAT: A.REI.A.1

194) ANS: 3

Step 1. Understand that solving the equation means isolating the value of x.

Step 2. Strategy. Isolate x.

Step 3. Execution of strategy.

$$2(x+2)^2-4=28$$
$$2(x+2)^2=28+4$$
$$2(x+2)^2=32$$
$$\frac{2(x+2)^2}{2}=\frac{32}{2}$$
$$(x+2)^2=16$$
$$x+2=\sqrt{16}$$
$$x+2=\pm 4$$
$$x=-2\pm 4$$
$$x=2$$
$$x=-6$$

Step 4. Does it make sense? Yes. The values 2 and -6 satisfy the equation $2(x+2)^2-4=28$.

x=2	x=-6
$2(x+2)^2-4=28$	$2(x+2)^2-4=28$
$2(2+2)^2-4=28$	$2(-6+2)^2-4=28$
$2(4)^2-4=28$	$2(-4)^2-4=28$
$2(16)-4=28$	$2(16)-4=28$
$32-4=28$	$32-4=28$
$28=28$	$28=28$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: taking square roots

195) ANS:
Yes. I agree with Amy’s solution. I get the same solutions by using the quadratic formula.

$$2x^2 + 5x - 42 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-42)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{25 + 336}}{4}$$

$$x = \frac{-5 \pm \sqrt{361}}{4}$$

$$x = \frac{-5 \pm 19}{4}$$

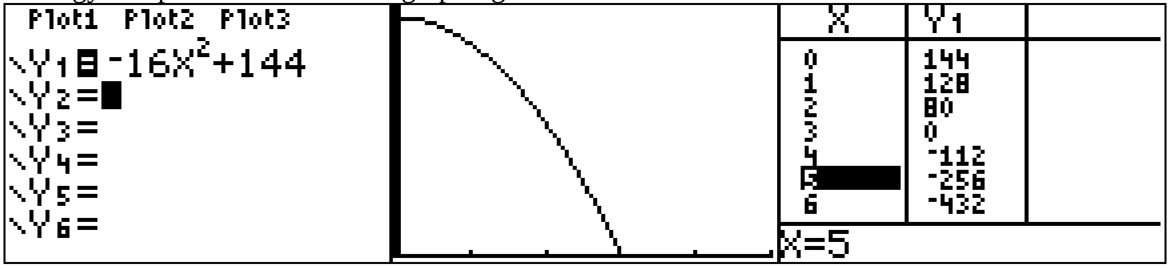
$$x = \frac{14}{4} = \frac{7}{2}$$

$$x = \frac{-24}{4} = -6$$

NOTE: Acceptable explanations could also be made by: 1) substituting Amy's solutions into the original equation and showing that both solutions make the equation balance; 2) solving the quadratic by completing the square and getting Amy's solutions; or 3) solving the quadratic by factoring and getting Amy's solutions.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: factoring NOT: NYSED classifies this as A.REI.A

196) ANS:
How many feet did the object fall between one and two seconds after it was dropped?
Strategy: Input the function in a graphing calculator.



```
Plot1 Plot2 Plot3
\Y1=-16X²+144
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

```
X    Y1
0    144
1    128
2    80
3    0
4    -112
5    -256
6    -432
X=5
```

After one second, the object is 128 feet above the ground.
After two seconds, the object is 80 feet above the ground.
The object fell $128 - 80 = 48$ feet between one and two seconds after it was dropped.

Determine algebraically how many seonds it will take for the object to reach the ground.

$$H(t) = -16t^2 + 144$$
$$0 = -16t^2 + 144$$
$$16t^2 = 144$$
$$t^2 = \frac{144}{16}$$
$$t^2 = 9$$
$$t = 3$$

The object will hit the ground after 3 seconds.

PTS: 4 NAT: A.SSE.B.3 TOP: Solving Quadratics

197) ANS: 1

$$3x^2 + 10x = 8$$
$$3x^2 + 10x - 8 = 0$$
$$a = 3 \quad b = 10 \quad c = -8$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-10 \pm \sqrt{10^2 - 4(3)(-8)}}{2(3)}$$
$$x = \frac{-10 \pm \sqrt{196}}{6}$$
$$x = \frac{-10 \pm 14}{6}$$
$$x = \frac{4}{6} = \frac{2}{3} \text{ and } x = \frac{-24}{6} = -4$$

PTS: 2 NAT: A.REI.B.4

198) ANS:

$\{10, -4\}$

$$f(x) = (x-3)^2 - 49$$
$$0 = (x-3)^2 - 49$$
$$49 = (x-3)^2$$
$$\pm 7 = x - 3$$
$$3 \pm 7 = x$$
$$x = 10 \text{ and } x = -4$$

PTS: 2 NAT: A.REI.B.4

199) ANS: 4

Given	$13 - 36x^2$	=	-12

Add (12)	+12	=	+12
Simplify	$25 - 36x^2$	=	0
Add ($36x^2$)	$+36x^2$	=	$+36x^2$
Simplify	25	=	$+36x^2$
Divide (36)	$\frac{25}{36}$	=	$\frac{36x^2}{36}$
Simplify	$\frac{25}{36}$	=	x^2
Square Root	$\pm\frac{5}{6}$	=	x

The only correct answer choice is $-\frac{5}{6}$.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: taking square roots

200) ANS: 1

Given	$2x^2 - 12x + 6$	=	0
Divide by 2	$\frac{2x^2 - 12x + 6}{2}$	=	$\frac{0}{2}$
Simplify	$x^2 - 6x + 3$	=	0
Subtract 3	-3	=	-3
Simplify	$x^2 - 6x$	=	-3
Complete the Square	$x^2 - 6x + \left(\frac{-6}{2}\right)^2$	=	$-3 + \left(\frac{-6}{2}\right)^2$
Simplify	$x^2 - 6x + (-3)^2$	=	$-3 + (-3)^2$
Factor and Simplify	$(x-3)^2$	=	-3 + 9
Simplify	$(x-3)^2$	=	6

$$2(x^2 - 6x + 3) = 0$$

$$x^2 - 6x = -3$$

$$x^2 - 6x + 9 = -3 + 9$$

$$(x-3)^2 = 6$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

201) ANS: 1

Strategy 1: Use the quadratic equation to solve $x^2 - 8x = 24$, where $a = 1$, $b = -8$, and $c = -24$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-24)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{160}}{2}$$

$$x = \frac{8 \pm \sqrt{16}\sqrt{10}}{2}$$

$$x = \frac{8 \pm 4\sqrt{10}}{2}$$

$$x = 4 \pm 2\sqrt{10}$$

Answer choice *a* is correct.

Strategy 2. Solve by completing the square.

$$x^2 - 8x = 24$$

$$(x-4)^2 = 24 + (-4)^2$$

$$(x-4)^2 = 24 + 16$$

$$(x-4)^2 = 40$$

$$\sqrt{(x-4)^2} = \sqrt{40}$$

$$x - 4 = \pm 2\sqrt{10}$$

$$x = 4 \pm 2\sqrt{10}$$

Answer choice *a* is correct.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

202) ANS:

$$x^2 - 6x = 15$$

$$x^2 - 6x + \left(\frac{-6}{2}\right)^2 = 15 + \left(\frac{-6}{2}\right)^2$$

$$x^2 - 6x + (-3)^2 = 15 + (-3)^2$$

$$(x-3)^2 = 15 + 9$$

$$(x-3)^2 = 24$$

$$\sqrt{(x-3)^2} = \sqrt{24}$$

$$x - 3 = \pm\sqrt{24}$$

$$x = 3 \pm \sqrt{24}$$

$$x = 3 \pm \sqrt{4}\sqrt{6}$$

$$x = 3 \pm 2\sqrt{6} \quad \text{Answer}$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

203) ANS: 1

$$3(x-4)^2 = 27$$

$$\frac{3(x-4)^2}{3} = \frac{27}{3}$$

$$(x-4)^2 = 9$$

$$\sqrt{(x-4)^2} = \sqrt{9}$$

$$x - 4 = \pm 3$$

$$x = 1, 7$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: taking square roots

204) ANS: 3

Strategy: Rewrite $x^2 - 6x = 12$ in the form of $(x+p)^2 = q$ and find the value of p.

Notes	Left Epression	Sign	Right Expression
Given	$x^2 - 6x$	=	12
Complete the Square	$x^2 - 6x + (-3)^2$	=	$12 + (-3)^2$
Exponents and Parentheses	$x^2 - 6x + 9$	=	$12 + 9$
Factor left expression and simplify right expression	$(x-3)^2$	=	21
Compare to form given in the question.	$(x+p)^2$	=	q

$$p = -3$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

205) ANS:
Answer: -2.8, 1.8

Strategy: Use the quadratic formula

STEP 1. Identify the values of a, b, and c in $x^2 + x - 5 = 0$.

$$a = 1$$

$$b = 1$$

$$c = -5$$

STEP 2. Substitute these values in the quadratic formula and solve.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 + 20}}{2}$$

$$x = \frac{-1 \pm \sqrt{21}}{2}$$

$$x = \frac{-1 \pm 4.58}{2}$$

$$x = \frac{-1 + 4.58}{2} \qquad x = \frac{-1 - 4.58}{2}$$

$$x = \frac{4.58}{2} \qquad x = \frac{-5.58}{2}$$

$$x = 1.79 \approx 1.8 \qquad x = -2.79 \approx -2.8$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: quadratic formula

H – Quadratics, Lesson 2, Using the Discriminant (r. 2018)

QUADRATICS
Using the Discriminant

Common Core Standard	**Next Generation Standard**
A-REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a* + *bi*, *a* - *bi* for real numbers *a* and *b*. PARCC: Tasks do not require students to write solutions for ic equations that have roots with non-zero imaginary parts. r, tasks can require the student to recognize cases in which a ic equation has no real solutions.	**AI-A.REI.4b** Solve quadratic equations by: i) inspection, ii) taking square roots, iii) factoring, iv) completing the square, v) the quadratic formula, and **vi) graphing.** **Recognize when the process yields no real solutions.** (Shared standard with Algebra II) Notes: • **Solutions may include simplifying radicals or writing solutions in simplest radical form.** • An example for inspection would be $x^2 = 49$, where a student should know that the solutions would include 7 and -7. • **When utilizing the quadratic formula, there are no coefficient limits.** • **The discriminant is a sufficient way to recognize when the process yields no real solutions.**

LEARNING OBJECTIVES

Students will be able to:

1) Identify the number and characteristics of solutions to quadratic equations based on analysis of the discriminant.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

discriminant
real solutions
imaginary solutions
standard form of a quadratic
solution
zero
root
x-axis intercept

BIG IDEAS

Standard Form of a Quadratic: $ax^2 + bx + c = 0$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

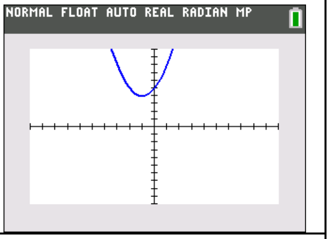
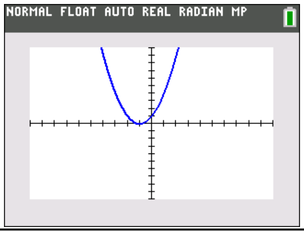
Discriminant = $b^2 - 4ac$

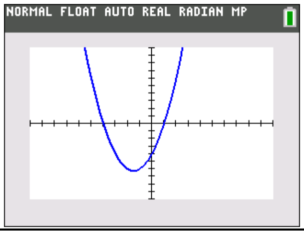
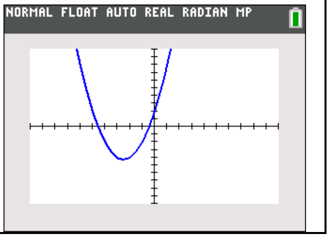
Analyzing the Discriminant

The discriminant can be used to determine the number of and type of solutions to a quadratic equation.

Every quadratic can have zero, one, or two solutions.

Solutions can be real or imaginary numbers.

If the Value of the Discriminant Is:	Characteristics and Number of Solutions of the Quadratic Equation Are:	Examples
Negative $0 > b^2 - 4ac$	If the value of the discriminant is negative, then there will be two imaginary number solutions and no x-axis intercepts.	$y = x^2 + 2x + 5$ $b^2 - 4ac = 2^2 - 4(1)(5)$ $b^2 - 4ac = -16$ 
Zero $0 = b^2 - 4ac$	If the value of the discriminant is zero, then there will be one real solution and the graph will touch the x-axis at one and only one point.	$y = x^2 + 2x + 1$ $b^2 - 4ac = 2^2 - 4(1)(1)$ $b^2 - 4ac = 0$ 

Positive Perfect Square $b^2 - 4ac > 0$	If the value of the discriminant is a positive perfect square, then there will be two integer solutions and two x-axis intercepts.	$y = x^2 + 3x - 4$ $b^2 - 4ac = 3^2 - 4(1)(-4)$ $b^2 - 4ac = 25$ 
Positive Not a Perfect Square $b^2 - 4ac > 0$	If the value of the discriminant is positive, but not a perfect square, then there will be two real number solutions and two x-axis intercepts.	$y = x^2 + 5x + 2$ $b^2 - 4ac = 5^2 - 4(1)(2)$ $b^2 - 4ac = 17$ 

DEVELOPING ESSENTIAL SKILLS

Determine the number and characteristics of the following quadratic equations by analyzing the discriminant.

1. $-3n^2 + 4n + 6 = 6$
2. $-p^2 + 4p - 7 = -3$
3. $-x^2 + 5x - 3 = -3$
4. $3v^2 + 3v + 2 = 2$
5. $6v^2 - 2v + 6 = 4$
6. $p^2 - 4p - 1 = -5$
7. $-3x^2 - 2x + 4 = 4$
8. $2x^2 + 4x + 11 = 5$
9. $6x^2 + 6x + 3 = 3$
10. $3a^2 - a - 4 = -2$

Answers

1. 16; two real solutions
2. 0; one real solution
3. 25; two real solutions
4. 9; two real solutions
5. -44; two imaginary solutions
6. 0; one real solution
7. 4; two real solutions
8. -32; two imaginary solutions
9. 36; two real solutions
10. 25; two real solutions

A.REI.B.4: Using the Discriminant

206) How many real solutions does the equation $x^2 - 2x + 5 = 0$ have? Justify your answer.

207) How many real-number solutions does $4x^2 + 2x + 5 = 0$ have?
1) one
2) two
3) zero
4) infinitely many

SOLUTIONS

206) ANS:
No Real Solutions

Strategy 1. Evaluate the discriminant $b^2 - 4ac$ for $a = 1$, $b = -2$, and $c = 5$.

$b^2 - 4ac$

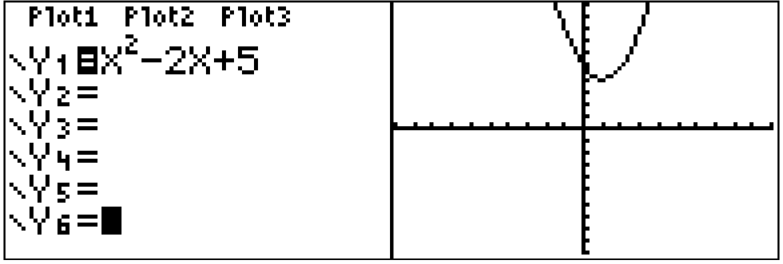
$(-2)^2 - 4(1)(5)$

$4 - 20$

-16

Because the value of the discriminant is negative, there are no real solutions.

Strategy 2.
Input the equation in a graphing calculator and count the x-intercepts.



\Y1 = X²−2X+5
\Y2 =
\Y3 =
\Y4 =
\Y5 =
\Y6 =

The graph does not intercept the x-axis, so there are no real solutions.

Strategy 3
Solve the quadratic to see how many real solutions there are.

$$x^2 - 2x + 5 = 0$$

$$x^2 - 2x = -5$$

$$(x-1)^2 = -5 + (-1)^2$$

$$(x-1)^2 = -5 + 1$$

$$(x-1)^2 = -4$$

$$x - 1 = \sqrt{-4}$$

$$x - 1 = \pm 2i$$

$$x = 1 \pm 2i$$

Both solutions involve imaginary numbers, so there are no real solutions.

PTS: 2 NAT: A.REI.B.4 TOP: Using the Discriminant

207) ANS: 3

Strategy: Use the discriminant, which is $b^2 - 4ac$.

If the discriminant is > 0, then the quadratic has two real-number solutions.
If the discriminant is = 0, then the quadratic has one real-numer solution.
If the discriminant is < 0, the the quadratic has zero real-number solutions.

STEP 1. Identify the values of *a, b*, and *c* in the quadratic equation $4x^2 + 2x + 5 = 0$.

$$a = 4$$

$$b = 2$$

$$c = 5$$

STEP 2. Substitute the values into $b^2 - 4ac$ and evaluate.

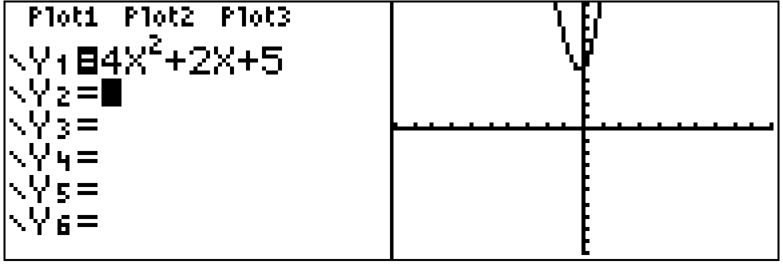
$$b^2 - 4ac$$

$$(2)^2 - 4(4)(5)$$

$$4 - 80$$

$$-76$$

The quadratic has zero real-number solutions.
CHECK by inputting the quadratic equation in a graphing calculator and looking at the graph view.



Plot1 Plot2 Plot3
\Y1=4X²+2X+5
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=

The number of solutions is equal to the number of x-axis intercepts. In this case, the parabola opens upward and does not cross the x-axis, which means it has zero real-numer solutions.

PTS: 2 NAT: A.REI.B.4 TOP: Using the Discriminant
KEY: AI

H – Quadratics, Lesson 3, Modeling Quadratics (r. 2018)

QUADRATICS
Modeling Quadratics

Common Core Standard	Next Generation Standard
A-CED.1 Create equations and inequalities in one variable ~~and use them to solve problems~~. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* PARCC: Tasks are limited to linear, quadratic, or exponential equations with integer exponents.	**AI-A.CED.1** Create equations and inequalities in one variable to **represent a real-world context**. (Shared standard with Algebra II) Notes: • **This is strictly the development of the model (equation/inequality).** • **Limit equations to linear, quadratic, and exponentials of the form** $f(x) = a(b)^x$ **where** $a > 0$ **and** $b > 0$ ($b \neq 1$). • **Work with geometric sequences may involve an exponential equation/formula of the form** $a_n = ar^{n-1}$**, where a is the first term and r is the common ratio.** • **Inequalities are limited to linear inequalities.** • Algebra I tasks do not involve compound inequalities.

NOTE: This lesson is related to Expressions and Equations, Lesson 4, Modeling Linear Equations

LEARNING OBJECTIVES

Students will be able to:

model quadratic equations that reflect real-world contexts, including:
- a. product of consecutive integer contexts,
- b. product of ages contexts, and
- c. squared number contexts.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

consecutive integers

consecutive odd integers

consecutive even integers

BIG IDEAS

General Approach

The general approach to modelling quadratics is:

1. Read and understand the entire problem.
2. Underline key words, focusing on variables, operations, and equalities or inequalities.
3. Convert the key words to mathematical notation (consider meaningful variable names other than x and y).
4. Write the final expression or equation.
5. Check the final expression or equation for reasonableness.

Product of Consecutive Integer Problems: The key to solving *product* of consecutive integer problems is also defining the variables.

Typical Problem in Context	Mathematical Translation	Hints and Strategies
Find three consecutive positive even integers such that the product of the second and third integers is twenty more than ten times the first integer.	6, 8, 10. Three consecutive even integers are x, $x+2$ and $x+4$. $(x+2)(x+4)=10x+20$ $x^2+6x+8=10x+20$ $x^2-4x-12=0$ $(x-6)(x+2)=0$ $x=6$	For consecutive integer problems, define your variables as x, x + 1, etc. For consecutive *even or odd* integer problems, define your variables as x, x +2, etc.

Product of Ages Problems: The key to solving *product* of consecutive integer problems is also defining the variables.

Typical Problem in Context	Mathematical Translation	Hints and Strategies
Brian is 3 years older than Doug. The product of their ages is 40. How old is Doug?	Let d represent Doug's age. Let d+3 represent Brian's age. Let $d(d+3)=40$ represent the product of their ages. Solve for *d*. $d(d+3)=40$ $d^2+3d=40$ $d^2+3d-40=0$ $(d+8)(d-5)=0$ $d=\{-8,5\}$ Reject -8 because age cannot be negative. Doug is 5 years old.	Define your variables carefully.

Squared Number Problems:

Typical Problem in Context	Mathematical Translation	Hints and Strategies

When 36 is subtracted from the square of a number, the result is five times the number. What is the positive solution?	9 Let the square of a number be represented by x^2 Let five times the number be represented by $5x$ Write: $x^2 - 36 = 5x$ $x^2 - 5x - 36 = 0$ $(x-9)(x+4) = 0$ $x = \{-4, 9\}$ The problem says to select the positive solution.	Underline key words.

DEVELOPING ESSENTIAL SKILLS

1) Find three consecutive positive even integers such that the product of the second and third integers is twenty more than ten times the first integer. [Only an algebraic solution can receive full credit.]

2) When 36 is subtracted from the square of a number, the result is five times the number. What is the positive solution?
   1) 9
   2) 6
   3) 3
   4) 4

3) Noj is 5 years older than Jacob. The product of their ages is 84. How old is Noj?

4) The square of a positive number is 24 more than 5 times the number. What is the value of the number?

5) Find three consecutive odd integers such that the product of the first and the second exceeds the third by 8.

6) Three brothers have ages that are consecutive even integers. The product of the first and third boys' ages is 20 more than twice the second boy's age. Find the age of *each* of the three boys.

7) Tamara has two sisters. One of the sisters is 7 years older than Tamara. The other sister is 3 years younger than Tamara. The product of Tamara's sisters' ages is 24. How old is Tamara?

Answers

1) 6, 8, 10.
   Let x represent the first integer.
   Let x+2 represent the second integer.
   Let x+4 represent the third integer
   Write

$$(x+2)(x+4)=10x+20$$
$$x^2+6x+8=10x+20$$
$$x^2-4x-12=0$$
$$(x-6)(x+2)=0$$
$$x=\{-2,6\}$$

Reject the negative integer solution for x because the problem calls for a positive integer solution.

2) Let x^2 represent the square of a number.
Let 5x represent five times the number.
Write:

$$x^2-36=5x$$
$$x^2-5x-36=0$$
$$(x-9)(x+4)=0$$
$$x=\{-4,9\}$$

The positive solution is 9.

3) Let N represent Noj’s age.
Let N-5 represent Jacob’s age.
Write:

$$N(N-5)=84$$
$$N^2-5N-84=0$$
$$(N-12)(N+7)=0$$
$$N=\{-7,12\}$$

Reject the negative solution because age cannot be negative.
Noj is 12 years old.

4) Let x^2 represent the square of a number.
Let 5x represent 5 times the number.
Write:

$$x^2=24+5x$$
$$x^2-5x-24=0$$
$$(x-8)(x+3)=0$$
$$x=\{-3,8\}$$

Reject the negative solution
The number is 8.

5) Let x represent the first odd integer.
Let x+2 represent the second consecutive odd integer.
Let x+4 represent the third consecutive odd integer.
Write:

$$x(x+2)-(x+4)=8$$
$$x^2+2x-x-4=8$$
$$x^2+x-12=0$$
$$(x+4)(x-3)=0$$
$$x=\{-4,3\}$$

Reject the even integer solution.
The three consecutive odd integers are 3, 5, and 7

6) Let x represent the age of the first brother.
Let x+2 represent the age of the second brother.
Let x+4 represent the age of the third brother.
Write:

$$x(x+4)=20+2(x+2)$$
$$x^2+4x=20+2x+4$$
$$x^2-2x=24$$
$$x^2-2x-24=0$$
$$(x+6)(x-4)=0$$
$$x=\{-6,4\}$$

Reject the negative solution because age cannot be negative.
The ages of the three brothers are 4, 6, and 8.

7) Let x represent Tamara's age.
Let x+7 represent the age of Tamara's older sister.
Let x-3 represent the age of Tamara's younger sister.
Write:

$$(x+7)(x-3)=24$$
$$x^2+7x-3x-21=24$$
$$x^2+4x-21=24$$
$$x^2+4x-45=0$$
$$(x+9)(x-5)=0$$
$$x=\{-9,5\}$$

Reject the negative solution.
Tamara's age is 5.

REGENTS EXAM QUESTIONS (through June 2018)

A.CED.A.1: Modeling Quadratics

208) Sam and Jeremy have ages that are consecutive odd integers. The product of their ages is 783. Which equation could be used to find Jeremy's age, *j*, if he is the younger man?

1) $j^2 + 2 = 783$

2) $j^2 - 2 = 783$

3) $j^2 + 2j = 783$

4) $j^2 - 2j = 783$

209) Abigail's and Gina's ages are consecutive integers. Abigail is younger than Gina and Gina's age is represented by *x*. If the difference of the square of Gina's age and eight times Abigail's age is 17, which equation could be used to find Gina's age?

1) $(x+1)^2 - 8x = 17$

2) $(x-1)^2 - 8x = 17$

3) $x^2 - 8(x+1) = 17$

4) $x^2 - 8(x-1) = 17$

SOLUTIONS

208) ANS: 3
Strategy: Deconstruct the problem to find the information needed to write the equation.

Let *j* represent Jeremy’s age. The last sentence says *j* represents Jeremy’s age.

Let $\left(j+2\right)$ represent Sam’s age. The problem tells us that Sam and Jeremy have ages that are **consecutive odd integers**. The consecutive odd integers that could be ages are $\{1, 3, 5, 7, 9, \ldots\ldots\}$ and each odd integer is 2 more that the odd integer before it. Thus, if Jeremy is 2 years younger than Sam, as the problem says, then Sam’s age can be represented as $\left(j+2\right)$.

The second sentence says, “The product of their ages is 783.” Product is the result of multiplication, so we can write $j\left(j+2\right) = 783$. Since this is not an answer choice, we must manipulate the equation:

$$j\left(j+2\right) = 783$$

$$j^2 + 2j = 783$$

Our equation is now identical to answer choice *c*, which is the correct answer.

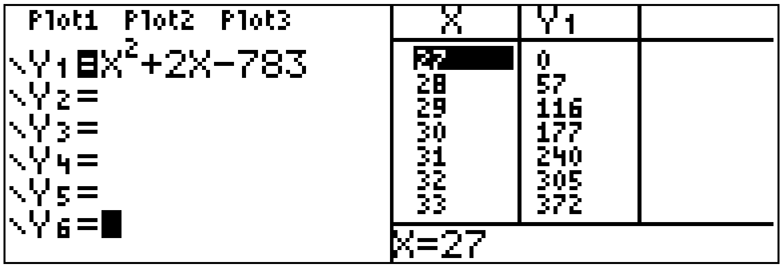
DIMS? Does It Make Sense? Yes. Jeremy is 27 and Sam is 29. The product of their ages is $27 \times 29 = 783$.
In order to input this into a graphing calculator, the equation must be transformed as follows:

$$j^2 + 2j = 783$$

$$j^2 + 2j - 783 = 0$$

$$0 = j^2 + 2j - 783$$

$$y_1 = x^2 + 2x - 783$$



```
 Plot1  Plot2  Plot3
\Y1=X²+2X-783
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

X	Y1	
27	0	
28	57	
29	116	
30	177	
31	240	
32	305	
33	372	

X=27

PTS: 2 NAT: A.CED.A.1 TOP: Modeling Quadratics

209) ANS: 4

If Gina’s age is x, then Abigail’s age is $x - 1$.
The square of Gina’s age is represented by x^2.
Eight times Abigail's age can be represented as $8(x - 1)$.
The difference of the square of Gina's age and eight times Abigail's age is 17 can be represented as $x^2 - 8(x - 1) = 17$.
Check by solving for x, as follows:

$$x^2 - 8(x - 1) = 17$$

$$x^2 - 8x + 8 = 17$$

$$x^2 - 8x - 9 = 0$$

$$(x - 9)(x + 1) = 0$$

$$x = 9$$

Gina is 9 years old and Abigail is 8 years old.

PTS: 2 NAT: A.CED.A.1 TOP: Modeling Quadratics

H – Quadratics, Lesson 4, Geometric Applications of Quadratics (r. 2018)

QUADRATICS
Geometric Applications of Quadratics

Common Core Standard	**Next Generation Standard**
A-CED.1 Create equations and inequalities in one variable ~~and use them to solve problems~~. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* PARCC: Tasks are limited to linear, quadratic, or exponential equations with integer exponents.	**AI-A.CED.1** Create equations and inequalities in one variable to **represent a real-world context**. (Shared standard with Algebra II) Notes: • **This is strictly the development of the model (equation/inequality).** • **Limit equations to linear, quadratic, and exponentials of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).** • **Work with geometric sequences may involve an exponential equation/formula of the form $a_n = ar^{n-1}$, where a is the first term and r is the common ratio.** • **Inequalities are limited to linear inequalities.** • Algebra I tasks do not involve compound inequalities.

LEARNING OBJECTIVES

Students will be able to:

1) model quadratic equations that reflect real-world contexts involving the area and dimensions of two-dimensional geometric figures.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

area area formulas length width

BIG IDEAS

Geometric Area Problems: Quadratics are frequently used to model problems involving geometric area. The keys to solving geometric area problems are to use a geometric area formula and draw a sketch to represent the problem.

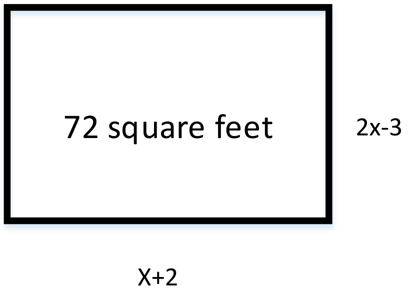
Typical Problem in Context	Mathematical Translation	Hints and Strategies
The area of the rectangular playground enclosure at South School is 500 square meters. The length of the playground is 5 meters longer than the width. Find the dimensions of the playground, in meters.	width = 20 and length = 25 $A=lw$ Let A=500 Let width = w Let length = w+5 Write: $A = lw$ $500 = (w+5)w$ $500 = w^2 + 5w$ $0 = w^2 + 5w - 500$ $0 = (w+25)(w-20)$ $w = \{-25, 20\}$ Reject -25 as a solution because width cannot be negative.	Start with a formula. Define variables. Substitute known information into the formula.

Sketching a Diagram Can Help to Understand and Solve a Problem

The general strategy for solving problems that involve geometric applications of quadratics is to substitute terms with a common variable for length and width in common area formulas. Drawing a picture can also help.

For example: A rectangular garden has length of x+2 and width of 2x-3, and the area of the garden is 72 square feet. What are the dimensions of the garden?

Start by drawing a picture to help understand the problem.



Then, use the formula for finding the area of a rectangle, which is:

$$A = lw$$

Substitute information about the length and width of the garden into the area formula for a rectangle, then write:

$$A = lw$$
$$72 = (x+2)(2x-3)$$

$$A = (x+2)(2x-3)$$

The area of the garden is 72 square feet, so we can write:

$$72 = (x+2)(2x-3)$$

Solve for x, then for x+2 and 2x-3. The length is 8 feet and the width is 9 feet.

DEVELOPING ESSENTIAL SKILLS

1) A contractor needs 54 square feet of brick to construct a rectangular walkway. The length of the walkway is 15 feet more than the width. Write an equation that could be used to determine the dimensions of the walkway. Solve this equation to find the length and width, in feet, of the walkway.

2) A rectangle has an area of 24 square units. The width is 5 units less than the length. What is the length, in units, of the rectangle?

3) Jack is building a rectangular dog pen that he wishes to enclose. The width of the pen is 2 yards less than the length. If the area of the dog pen is 15 square yards, how many yards of fencing would he need to completely enclose the pen?

4) A rectangular park is three blocks longer than it is wide. The area of the park is 40 square blocks. If *w* represents the width, write an equation in terms of *w* for the area of the park. Find the length and the width of the park.

5) What is the length of one side of the square whose perimeter has the same numerical value as its area?

Answers

1) The formula for the area of a rectangle is $A = lw$
Let 54 represent A.
Let w represent the width of the rectangle.
Let w+15 represent the length of the rectangle.
Write:

$$A = lw$$
$$54 = (w+15)w$$
$$54 = w^2 + 15w$$
$$0 = w^2 + 15w - 54$$
$$0 = (w+18)(w-3)$$
$$w = \{-18, 3\}$$

Reject the negative solution.
The width of the sidewalk is 3 feet.
The length of the sidewalk is 18 feet.

2) The formula for the area of a rectangle is $A = lw$
Let 24 represent A.
Let *l* represent the length of the rectangle.

Let l-5 represent the width of the rectangle.
Write:

$$24 = l(l-5)$$
$$24 = l^2 - 5l$$
$$0 = l^2 - 5l - 24$$
$$0 = (l-8)(l+3)$$
$$l = \{-3, 8\}$$

Reject the negative solution.
The length of the rectangle is 8 units.

3) The formula for the area of a rectangle is $A = lw$
Let 15 represent A.
Let l represent the length of the rectangle.
Let l-2 represent the width of the rectangle.
Write:

$$A = lw$$
$$15 = l(l-2)$$
$$15 = l^2 - 2l$$
$$0 = l^2 - 2l - 15$$
$$0 = (l-5)(l+3)$$
$$l = \{-3, 5\}$$

Reject the negative solution.
If the length is 5, the width is 3.
The formula for the perimeter of a rectangle is $P = 2l + 2w$, so the length of fence needed is

$$P = 2l + 2w$$
$$P = 2(5) + 2(3)$$
$$P = 16$$

16 yards of fencing are needed.

4) The formula for the area of a rectangle is $A = lw$
The units in this problem are blocks.
Let 40 represent A.
Let w represent the width of the rectangle.
Let w+3 represent the length of the rectangle.
Write:

$$40 = w(w+3)$$
$$40 = w^2 + 3w$$
$$0 = w^2 + 3w - 40$$
$$0 = (w+8)(w-5)$$
$$w = \{-8, 5\}$$

Reject the negative solution.
The park is 5 blocks wide and 8 blocks long.

5) The formula for the area of a square is $A = s^2$
The formula for the perimeter of a square is $P = 4s$
Write:

$$4s = s^2$$
$$0 = s^2 - 4s$$
$$0 = s(s-4)$$
$$s = \{0, 4\}$$

Reject the zero solution.
The length of one side of the square is 4 units.

REGENTS EXAM QUESTIONS (through June 2018)

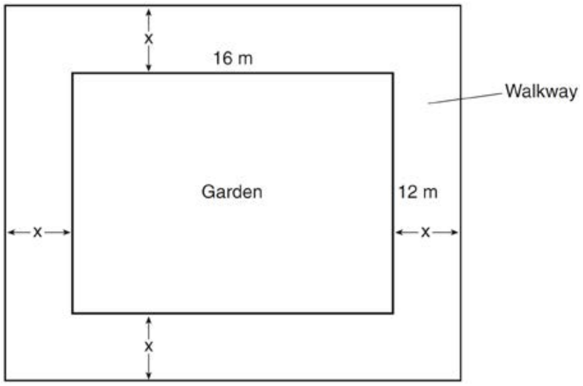
A.CED.A.1: Geometric Applications of Quadratics

210) The length of the shortest side of a right triangle is 8 inches. The lengths of the other two sides are represented by consecutive odd integers. Which equation could be used to find the lengths of the other sides of the triangle?

1) $8^2 + (x+1) = x^2$
2) $x^2 + 8^2 = (x+1)^2$
3) $8^2 + (x+2) = x^2$
4) $x^2 + 8^2 = (x+2)^2$

211) New Clarendon Park is undergoing renovations to its gardens. One garden that was originally a square is being adjusted so that one side is doubled in length, while the other side is decreased by three meters. The new rectangular garden will have an area that is 25% more than the original square garden. Write an equation that could be used to determine the length of a side of the original square garden. Explain how your equation models the situation. Determine the area, in square meters, of the new rectangular garden.

212) A rectangular garden measuring 12 meters by 16 meters is to have a walkway installed around it with a width of x meters, as shown in the diagram below. Together, the walkway and the garden have an area of 396 square meters.



Write an equation that can be used to find x, the width of the walkway. Describe how your equation models the situation. Determine and state the width of the walkway, in meters.

213) A school is building a rectangular soccer field that has an area of 6000 square yards. The soccer field must be 40 yards longer than its width. Determine algebraically the dimensions of the soccer field, in yards.

214) A landscaper is creating a rectangular flower bed such that the width is half of the length. The area of the flower bed is 34 square feet. Write and solve an equation to determine the width of the flower bed, to the *nearest tenth of a foot*.

215) A rectangular picture measures 6 inches by 8 inches. Simon wants to build a wooden frame for the picture so that the framed picture takes up a maximum area of 100 square inches on his wall. The pieces of wood that he uses to build the frame all have the same width. Write an equation or inequality that could be used to determine the maximum width of the pieces of wood for the frame Simon could create. Explain how your equation or inequality models the situation. Solve the equation or inequality to determine the maximum width of the pieces of wood used for the frame to the *nearest tenth of an inch*.

216) Joe has a rectangular patio that measures 10 feet by 12 feet. He wants to increase the area by 50% and plans to increase each dimension by equal lengths, *x*. Which equation could be used to determine *x*?

1) $(10+x)(12+x)=120$
2) $(10+x)(12+x)=180$
3) $(15+x)(18+x)=180$
4) $(15)(18)=120+x^2$

217) A contractor has 48 meters of fencing that he is going to use as the perimeter of a rectangular garden. The length of one side of the garden is represented by *x*, and the area of the garden is 108 square meters. Determine, algebraically, the dimensions of the garden in meters.

SOLUTIONS

210) ANS: 4

Strategy: Use the Pythagorean Theorem, the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

$$a^2+b^2=c^2$$

The shortest side must be one of the legs, since the longest side is always the hypotenuse.
Substitute 8 for a in the equation.

$$a^2+b^2=c^2$$

$$8^2+b^2=c^2$$

The lengths of sides b and c are consecutive odd integers. Let x represent the smaller odd integer and let $(x+2)$ represent the larger consecutive odd integer. Side c must be represented by $(x+2)$ because side c represents the hypotenuse, which is always the longest side of a right triangle. Therefore, side b is represented by x and side c is represented by $(x+2)$. Substitute these values into the equation.

$$8^2+b^2=c^2$$

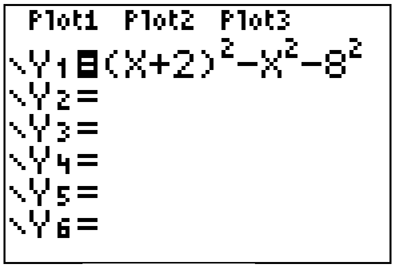
$$8^2+x^2=(x+2)^2$$

By using the commutative property to rearrange the two terms in the right expression, we obtain the same equation as answer choice d.

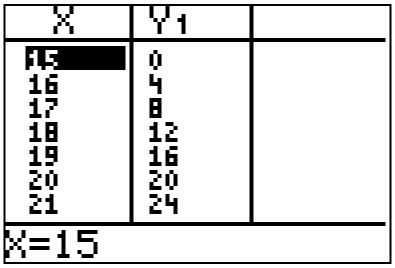
$$8^2+x^2=(x+2)^2$$

$$x^2+8^2=(x+2)^2$$

DIMS? Does It Make Sense? Yes. Transorm the equation for input into a graphing calculator as follows: $0=(x+2)^2-x^2-8^2$ and we find that the other two sides of the right triangle are 15 and 17.



```
Plot1  Plot2  Plot3
\Y1■(X+2)²-X²-8²
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```



X	Y1
15	0
16	4
17	8
18	12
19	16
20	20
21	24

X=15

By the Pythagorean Theorem, $8^2 + 15^2 = 17^2$

$$64 + 225 = 289$$

$$289 = 289$$

Everything checks!

PTS: 2 NAT: A.CED.A.1 TOP: Geometric Applications of Quadratics

211) ANS:

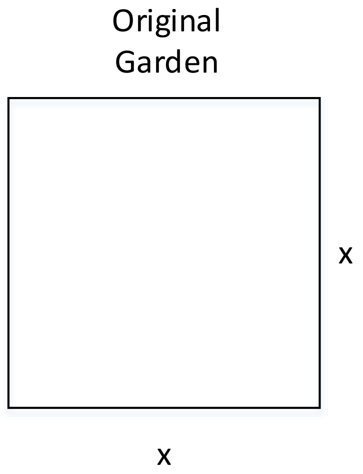
a) $1.25x^2 = (2x)(x-3)$

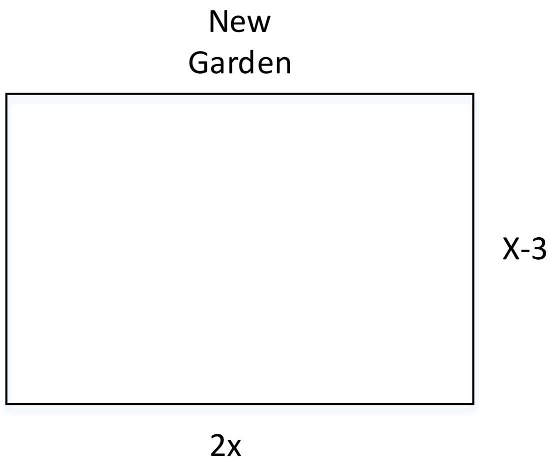
b) Because the original garden is a square, x^2 represents the original area, $x-3$ represents the side decreased by 3 meters, $2x$ represents the doubled side, and $1.25x^2$ represents the new garden with an area 25% larger.

c) The length of a side of the original square garden was 8 meters.
The area of the new rectangular garden is 80 square meters.

Strategy: Draw two pictures: one picture of the garden as it was in the past and one picture of the garden as it will be in the future. Then, write and solve an equation to determine the length of a side of the original garden.

STEP 1. Draw 2 pictures.





Area of original garden is x^2. Area of new garden is $1.25x^2$.

STEP 2: Use the area formula, $A = \text{length} \times \text{width}$, to write an equation for the area of the new garden.

$$A = \text{length} \times \text{width}$$

$$1.25x^2 = (2x)(x-3)$$

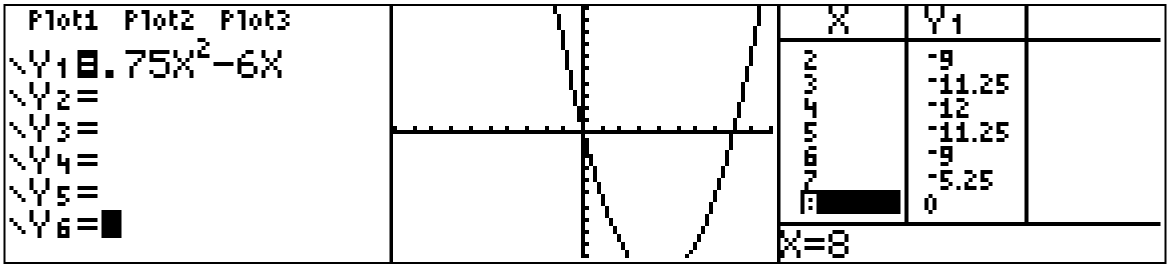
STEP 3: Transform the equation for input into a graphing calculator and solve.

$$1.25x^2=(2x)(x-3)$$

$$1.25x^2=2x^2-6x$$

$$0=2x^2-1.25x^2-6x$$

$$0=0.75x^2-6x$$



```
Plot1  Plot2  Plot3
\Y1=.75X²-6X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

X	Y1
2	-9
3	-11.25
4	-12
5	-11.25
6	-9
7	-5.25
8	0

X=8

The length on a side of the original square garden was 8 meters.

The area of the new garden is $1.25(8)^2 = 1.25(64) = 80$ square meters.

DIMS? Does It Make Sense? Yes. The dimensions of the original square garden are 8 meters on each side and the area was 64 square meters. The dimensions of the new rectangular garden are 16 meters length and 5 meters width. The new garden will have area of 80 meters. The area of the new garden is 1.25 times the area of the original garden.

PTS: 6 NAT: A.CED.A.1 TOP: Geometric Applications of Quadratics

212) ANS:

a) $396 = (16+2x)(12+2x)$.

b) The length, $16+2x$, and the width, $12+2x$, are multiplied and set equal to the area.

c) The width of the walkway is 3 meters.

Strategy: Use the picture, the area formula ($Area = length \times width$), and information from the problem to write an equation, then solve the equation.

STEP 1. Use the area formula, the picture, and information from the problem to write an equation.

$$Area = length \times width$$

$$396 = (16+2x)(12+2x)$$

STEP 2. Solve the equation.

$$396 = (16+2x)(12+2x)$$

$$396 = (16\times 12) + (16\times 2x) + (2x\times 12) + (2x\times 2x)$$

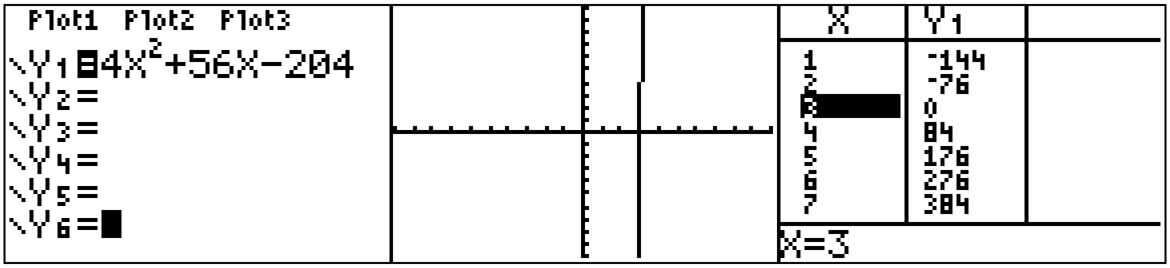
$$396 = 192 + 32x + 24x + 4x^2$$

$$396 = 192 + 56x + 4x^2$$

$$396 = 4x^2 + 56x + 192$$

$$0 = 4x^2 + 56x + 192 - 396$$

$$0 = 4x^2 + 56x - 204$$



```
Plot1 Plot2 Plot3
\Y1=4X²+56X-204
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

```
X     Y1
1     -144
2     -76
3     0
4     84
5     176
6     276
7     384
X=3
```

The width of the walkway is 3 meters.

DIMS? Does It Make Sense? Yes. The garden plus walkway is $16+2(3)=22$ meters long and $12+2(3)=18$ meters wide. $Area=22\times 18=396$, which fits the information in the problem.

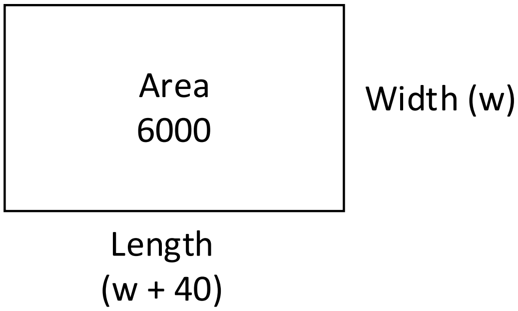
PTS: 4 NAT: A.CED.A.1 TOP: Geometric Applications of Quadratics

213) ANS:

The soccer field is 60 yards wide and 100 yards long.

Strategy: Draw and label a picture, then use the picture to write and solve an equation based on the area formula: $\text{Area} = \text{length} \times \text{width}$

STEP 1: Draw and label a picture.



STEP 2: Write and solve an equation based on the area formula: $\text{Area} = \text{width} \times \text{length}$

$$
\begin{aligned}
6000 &= w(w+40)\\
6000 &= w^2+40w\\
0 &= w^2+40w-6000\\
0 &= (w+100)(w-60)\\
w &= -100 \ \text{ reject - distance should be positive}\\
w &= 60\\
w+40 &= 100
\end{aligned}
$$

DIMS? Does It Make Sense? Yes. If the width of the soccer field is 60 yards and the length of the soccer field is 100 yards, then the area of the soccer field will be 6,000 square yards, as required by the problem.

PTS: 4 NAT: A.CED.A.1 TOP: Geometric Applications of Quadratics

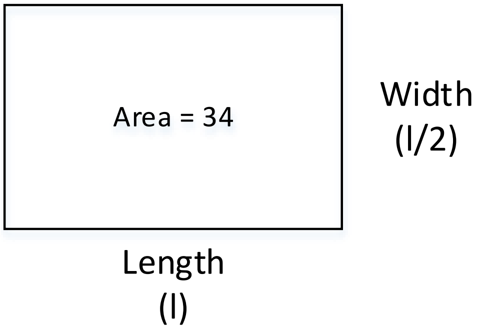
214) ANS:

a) Equation$34 = l\left(\frac{1}{2}l\right)$

b) The width of the flower bed is approximately 4.1 feet.

Strategy: Draw a picture, then write and solve an equation based on the area formula, $\text{Area} = \text{length} \times \text{width}.$

STEP 1. Draw a picture.



STEP 2: Write and solve an equation based on the area formula.

$$\text{Area} = \text{length} \times \text{width}.$$

$$34 = l\left(\frac{l}{2}\right)$$

$$34 = \frac{l^2}{2}$$

$$68 = l^2$$

$$\sqrt{68} = \sqrt{l^2}$$

$$8.2 \approx l$$

$$4.1 \approx w$$

PTS: 2 NAT: A.CED.A.1 TOP: Geometric Applications of Quadratics

215) ANS:

The maximum width of the frame should be 1.5 inches.

Strategy: Write an inequality, then solve it.

STEP 1: Write the inequality.

The picture is 6 inches by 8 inches. The area of the picture is (6×8) square inches.

The width of the frame is an unknown variable represented by x.

Two widths of the frame (2x) must be added to the length and width of the picture. Therefore, the area of the picture with frame is $(6 + 2x)(8 + 2x)$ square inches

The area of the picture with frame, $(6 + 2x)(8 + 2x)$ square inches, must be less than or equal $(\leq)$ to 100.

Write $(6 + 2x)(8 + 2x) \leq 100$

STEP 2: Solve the inequality.

Notes	Left Expression	Sign	Right Expression
Given	$(6+2x)(8+2x)$	$\le$	100
Use Distributive Property to Clear Parentheses	$48+12x+16x+4x^2$	$\le$	100
Commutative Property	$4x^2+12x+16x+48$	$\le$	100
Combine Like Terms	$4x^2+28x+48$	$\le$	100
Subtract 100 from both expressions	$4x^2+28x-52$	$\le$	0

Use the Quadratic Formula: a=4, b=28, c=-52

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-28\pm\sqrt{28^2-4(4)(-52)}}{2(4)}$$

$$x=\frac{-28\pm\sqrt{1616}}{8}$$

$$x=\frac{-28\pm\sqrt{1616}}{8}$$

$$x=\frac{-28\pm 40.1995}{8}$$

$$x=\frac{-28+40.1995}{8}$$

$$x=\frac{12.1995}{8}$$

$$x=1.5\text{ inches}$$

DIMS? Does It Make Sense? Yes. If the frame is 1.5 inches wide, then the total picture with frame will be

$$(6+2\times 1.5)(8+2\times 1.5)$$

$$(9)(11)$$

$$99\text{ square inches}$$

PTS: 6 NAT: A.CED.A.1 TOP: Geometric Applications of Quadratics

216) ANS: 2

Strategy: STEP 1. First, determine the area of the current rectangular patio and increase its size by 50%, which will be the size of the new patio. STEP 2. Then, increase each dimension of the current rectangular patio by x, as follows:

STEP 1.

$$\text{Area} = \text{length} \times \text{width}$$

$$\text{Current Patio}$$

$$A = 10 \times 12$$

$$A = 120$$

$$\text{New Patio}$$

$$A = 120 \times 150\%$$

$$A = 120 \times 1.5$$

$$A = 180$$

The new patio will have an area of 180 square feet. Eliminate choice (a).
STEP 2.

$$(10 + x)(12 + x) = 180$$

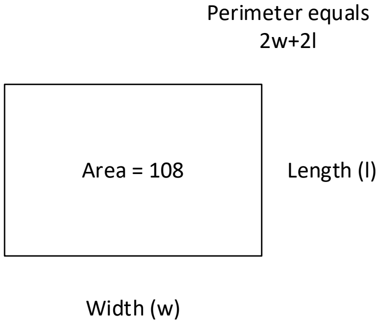
Choose answer b.

PTS: 2 NAT: A.CED.A.1 TOP: Geometric Applications of Quadratics

217) ANS:
The garden is a rectangle that measures 18 meters by 6 meters.

Strategy: . Solve as a system of two equations, because the question requires solving for two variables: length and width.

STEP 1. Draw a picture that illustrates the information in the problem.



STEP 2. Using the picture, write two equations using length and area formulas for rectangles. Let *l* represent the unknown *length* of the garden and let *w* represent the unknown *width* of the garden.

The first equation, Eq_1, is based on the formula for the perimeter of a rectangle, which is $P = 2l + 2w$.

The second equation, Eq_2, is based on the area formula for rectangles, which is $A = lw$

$$Eq_1 \qquad 48 = 2l + 2w$$

$$Eq_2 \qquad lw = 108$$

STEP 2. Isolate the length variable Eq_2

$$lw = 108$$

$$Eq_{2_a} \qquad l = \frac{108}{w}$$

STEP 3. Solve Eq_1 and Eq_{2_a} as a system using substitution, as follows:

$$Eq_1 \qquad 48 = 2l + 2w$$

$$Eq_{2_a} \qquad l = \frac{108}{w}$$

$$48 = 2\left(\frac{108}{w}\right) + 2w$$

$$48w = 2(108) + 2w^2$$

$$2w^2 - 48w + 216 = 0$$

$$2w^2 - 48w = -216$$

$$w^2 - 24w = -108$$

$$w^2 - 24w + (-12)^2 = -108 + (-12)^2$$

$$(w - 12)^2 = -108 + 144$$

$$(w - 12)^2 = 36$$

$$w = \pm 6$$

The garden is 6 meters wide. The length of the garden can be found using $Eq_{2_a} \qquad l = \frac{108}{w}$.

$$Eq_{2_a} \qquad l = \frac{108}{w}$$

$$l = \frac{108}{6}$$

$$l = 18$$

PTS: 4 NAT: A.CED.A.1 TOP: Geometric Applications of Quadratics

H – Quadratics, Lesson 5, Vertex Form of a Quadratic (r. 2018)

QUADRATICS

Vertex Form of a Quadratic

Common Core Standards	**Next Generation Standards**
F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	**AI-F.IF.8** Write a function in different but equivalent forms to reveal and explain different properties of the function. (Shared standard with Algebra II)
F-IF.C.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	**AI-F.IF.8a For a quadratic function, use an algebraic process** to find zeros, maxima, minima, and symmetry of the graph, and interpret these in terms of context. **Note: Algebraic processes include but not limited to factoring, completing the square, use of the quadratic formula, and the use of the axis of symmetry.**

LEARNING OBJECTIVES

Students will be able to:

1) Transform quadratics equations to and between standard, factored, and vertex forms of a quadratic.
2) Identify the zeros, maxima, minima, and axis-of symmetry of parabolas.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

axis of symmetry
completing the square
maxima
minima
parabola
standard form of a parabola
turning point
vertex
vertex form of a quadratic
x-axis intercepts
zeros

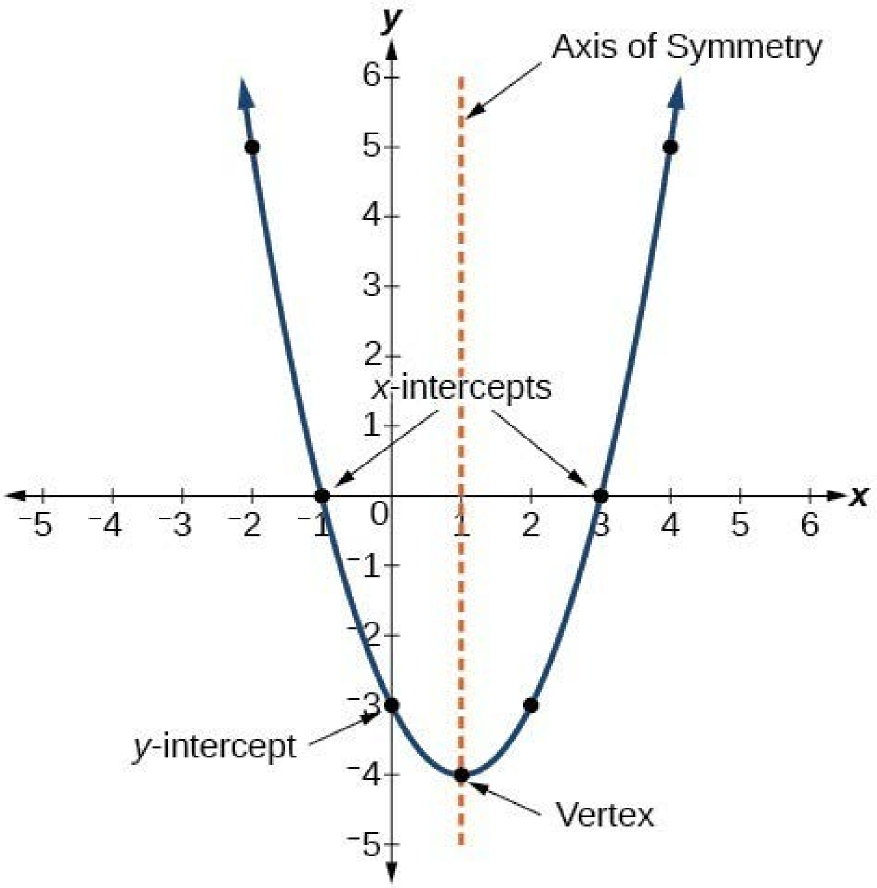
BIG IDEAS

The graph of a quadratic is called a **parabola**, and a parabola has several characteristics, including:

1) **vertex**, also known as the turning point. The vertex is the highest or lowest point on a parabola and is usually expressed as a coordinate pair.
2) **maxima** is the y-value of the turning point when the graph opens downward.
3) **minima** is the y-value of the turning point when the graph opens upward.
4) **axis of symmetry** is a vertical line that passes through the vertex of a parabola and divides the parabola into two symmetrical halves. It is sometimes called the line of reflection.
5) **zeros**, also known as roots or solutions, are the x-values of the coordinates of the **x-axis intercepts**.

There are three general forms of a quadratic equation:

1) standard form, given by $ax^2 - bx - c = 0$, where ax^2 is the quadratic term, bx is the linear term, and c is the constant. A positive value of a indicates the parabola opens upwards and a negative value of a indicates the parabola opens downward. As the value of a approaches zero, the appearance of the parabola approaches the appearance of a horizontal line.
2) vertex form, given by $a(x-h)^2 + k = 0$, where (h,k) is the vertex of the parabola and $x = h$ is the axis of symmetry.
3) factored form, given by $a\left(x-r\right)\left(x-s\right)$, where r and s are solutions.



In standard form, the function rule for this parabola is:

$$x^2 - 2x - 3 = 0$$

This form reveals that the parabola opens upward and has a y-intercept of 3.

In vertex form, the function rule for this parabola is:

$$\left(x-1\right)^2 - 4 = 0$$

The form reveals the vertex is at (1,-4) and the axis of symmetry is x=1.

In factored form, the function rule for this parabola is

$$(x-3)(x+1) = 0$$

This form reveals the solutions are $x = \left\{-1, 3\right\}$

The ability to transform quadratic equations between standard, vertex, and quadratic forms is useful for identifying the characteristics of their graphs.

DEVELOPING ESSENTIAL SKILLS

Complete the following table.

Standard Form	**Vertex Form**	**Factored Form**	**Vertex and Axis of Symmetry**	**Solutions**
$x^2-10x+21=0$				
	$(x-1)^2-4=0$			
		$(x-2)(x+4)=0$		
		$3(x-5)(x-3)=0$		
$x^2+4x-5=0$				

Answers

Standard Form	**Vertex Form**	**Factored Form**	**Vertex and Axis of Symmetry**	**Solutions**
$x^2-10x+21=0$	$(x-5)^2-4=0$	$(x-7)(x-3)=0$	$(5,-4)$ $x=5$	$x=\{3,7\}$
$x^2-2x-3=0$	$(x-1)^2-4=0$	$(x-3)(x+1)=0$	$(1,-4)$ $x=1$	$x=\{-1,3\}$
$x^2+2x-8=0$	$(x+1)^2-9=0$	$(x-2)(x+4)=0$	$(-1,-9)$ $x=-1$	$x=\{-4,2\}$
$3x^2-24x+45=0$	$3(x-4)^2-3=0$	$3(x-5)(x-3)=0$	$(4,-3)$ $x=4$	$x=\{3,5\}$
$x^2+4x-5=0$	$(x+2)^2-9=0$	$(x+5)(x-1)=0$	$(-2,-9)$ $x=-2$	$x=\{-5,1\}$

REGENTS EXAM QUESTIONS (through June 2018)

F.IF.C.8: Vertex Form of a Quadratic

218) a) Given the function $f(x)=-x^2+8x+9$, state whether the vertex represents a maximum or minimum point for the function. Explain your answer.
b) Rewrite *f(x)* in vertex form by completing the square.

219) If Lylah completes the square for $f(x) = x^2 - 12x + 7$ in order to find the minimum, she must write $f(x)$ in the general form $f(x) = (x - a)^2 + b$. What is the value of a for $f(x)$?

1) 6
2) -6
3) 12
4) -12

220) In the function $f(x) = (x - 2)^2 + 4$, the minimum value occurs when x is

1) -2
2) 2
3) -4
4) 4

221) Which equation is equivalent to $y - 34 = x(x - 12)$?

1) $y = (x - 17)(x + 2)$
2) $y = (x - 17)(x - 2)$
3) $y = (x - 6)^2 + 2$
4) $y = (x - 6)^2 - 2$

222) Which equation and ordered pair represent the correct vertex form and vertex for $j(x) = x^2 - 12x + 7$?

1) $j(x) = (x - 6)^2 + 43,\ (6, 43)$
2) $j(x) = (x - 6)^2 + 43,\ (-6, 43)$
3) $j(x) = (x - 6)^2 - 29,\ (6, -29)$
4) $j(x) = (x - 6)^2 - 29,\ (-6, -29)$

223) The function $f(x) = 3x^2 + 12x + 11$ can be written in vertex form as

1) $f(x) = (3x + 6)^2 - 25$
2) $f(x) = 3(x + 6)^2 - 25$
3) $f(x) = 3(x + 2)^2 - 1$
4) $f(x) = 3(x + 2)^2 + 7$

SOLUTIONS

218) ANS:

a) The vertex represents a maximum since $a < 0$.

b) $f(x) = -(x - 4)^2 + 25$

$$\left.\begin{aligned} f(x) &= -x^2+8x+9 \\ -x^2+8x+9 &= 0 \end{aligned}\right\}\text{(set } f(x) \text{ to 0)}$$

$$\left.\begin{aligned} -x^2+8x &= -9 \\ \frac{-x^2}{-1}+\frac{8x}{-1} &= \frac{-9}{-1} \\ x^2-8x &= 9 \end{aligned}\right\}\text{(isolate both variables with 1 as coefficient of leading variable)}$$

$$\left.\begin{aligned} \text{x}^2-8x+(-4)^2 &= 9+(-4)^2 \\ (x-4)^2 &= 9+16 \\ (x-4)^2 &= 25 \end{aligned}\right\}\text{(complete the square)}$$

$$\left.\begin{aligned} -1(x-4)^2 &= -1(25) \\ -1(x-4)^2+25 &= 0 \end{aligned}\right\}\text{(multiply by a)}$$

The vertex is at (4,25), but this information is not required by the problem.

PTS: 4 NAT: F.IF.C.8 TOP: Graphing Quadratic Functions

219) ANS: 1

Strategy: Transform $f(x) = x^2 - 12x + 7$ into the form of $f(x) = (x-a)^2 + b$ and find the value of a.

$$x^2 - 12x + 7 = f(x)$$

$$x^2 - 12x + 7 = 0$$

$$x^2 - 12x = -7$$

$$x^2 - 12x + \left(\frac{-12}{2}\right)^2 = -7 + \left(\frac{-12}{2}\right)^2$$

$$x^2 - 12x + (-6)^2 = -7 + (-6)^2$$

$$(x-6)^2 = -7 + 36$$

$$(x-6)^2 = +29$$

$$(x-6)^2 - 29 = 0$$

$$f(x) = (x-6)^2 - 29$$

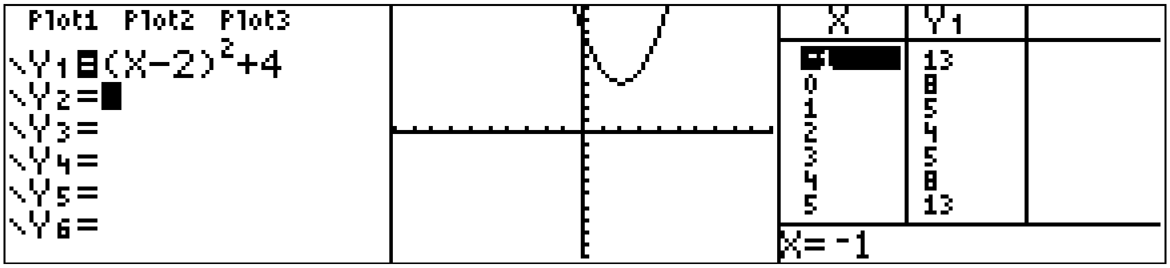
If $-a = -6$, then $a = 6$.

PTS: 2 NAT: A.SSE.B.3 TOP: Solving Quadratics
KEY: completing the square

220) ANS: 2

Strategy #1. Recognize that the function $f(x) = (x - 2)^2 + 4$ is expressed in vertex form, and that the vertex is located at $\left(2, 4\right)$. Accordingly, the minimum value of $f(x)$ occurs when $x = 2$.

Strategy #2: Input the function rule in a graphing calculator, then examine the graph and table views to determine the vertex. The problem wants to know the x value of the when $f(x)$ is at its minimum.



```
 Plot1  Plot2  Plot3
\Y1=(X-2)²+4
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

X	Y1
-1	13
0	8
1	5
2	4
3	5
4	8
5	13

X=-1

The minimum value of $f(x) = 4$ when x is equal to 2.

Strategy #3: Substitute each value of x into the equation and determine the minimum value of $f(x)$.

$f(x) = (x - 2)^2 + 4$

$f(-2) = (-2 - 2)^2 + 4$

$f(-2) = (-4)^2 + 4$

$f(-2) = 16 + 4$

$f(-2) = 20$

$f(2) = (2 - 2)^2 + 4$

$f(2) = (0)^2 + 4$

$f(2) = 4$

$f(-4) = (-4 - 2)^2 + 4$

$f(-4) = (-6)^2 + 4$

$f(-4) = 36 + 4$

$f(-4) = 40$

$f(4) = (4 - 2)^2 + 4$

$f(4) = (2)^2 + 4$

$f(4) = 4 + 4$

$f(4) = 8$

PTS: 2 NAT: A.SSE.B.3 TOP: Vertex Form of a Quadratic

NOT: NYSED classifies this as A.SSE.3

221) ANS: 4

Strategy: Simplify the equation $y - 34 = x(x - 12)$.

$$y - 34 = x(x - 12)$$

$$y - 34 = x^2 - 12x$$

$$y = x^2 - 12x + 34$$

$$y = x^2 - 12x + 36 - 2$$

$$y = (x - 6)^2 - 2$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics

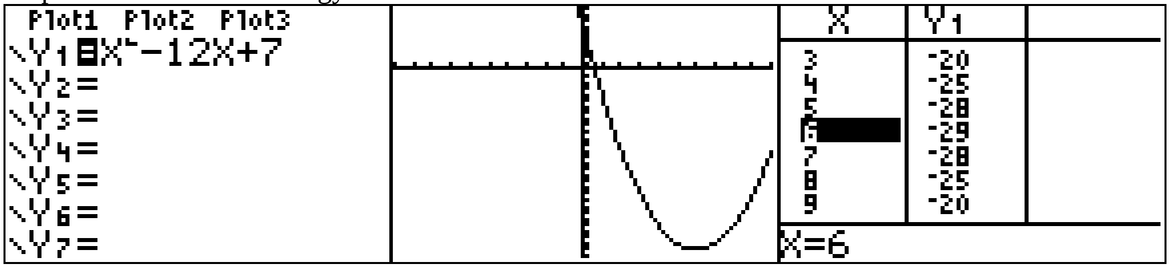
KEY: completing the square

222) ANS: 3

Step 1. Understand from the answer choices that the problem wants us to choose the answer that is equivalent to $j(x) = x^2 - 12x + 7$.

Step 2. Strategy: Input $j(x) = x^2 - 12x + 7$ in a graphing calulator and inspect the table and graph views of the function, then eliminate wrong answers.

Step 3. Execute the strategy.

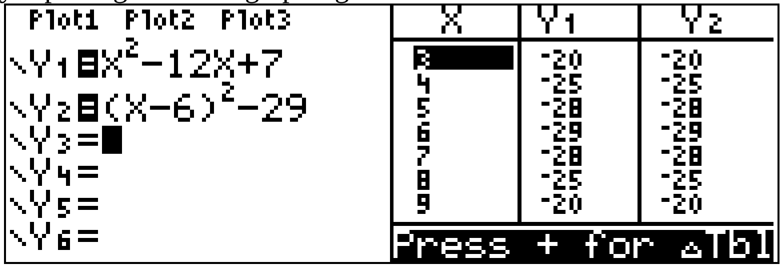


```
 Plot1  Plot2  Plot3
\Y1=X²-12X+7
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
 X      Y1
 3      -20
 4      -25
 5      -28
 6      -29
 7      -28
 8      -25
 9      -20
X=6
```

Choice c) is correct because it is the only answer choice that shows the vertex at (6, -29).

Step 4. Does it make sense? Yes. You can see that $j(x) = x^2 - 12x + 7$ and $j(x) = (x - 6)^2 - 29,\ (6, -29)$ are the same funtion by inputting both in a graphing calculator.



```
 Plot1  Plot2  Plot3
\Y1=X²-12X+7
\Y2=(X-6)²-29
\Y3=
\Y4=
\Y5=
\Y6=
```

```
 X      Y1      Y2
 3      -20     -20
 4      -25     -25
 5      -28     -28
 6      -29     -29
 7      -28     -28
 8      -25     -25
 9      -20     -20
Press + for ΔTbl
```

PTS: 2 NAT: F.IF.C.8 TOP: Vertex Form of a Quadratic

223) ANS: 3

$$3x^2 + 12x + 11$$

$$3x^2 + 12x = -11$$

$$x^2 + 4x = \frac{-11}{3}$$

$$x^2 + 4x + \left(\frac{4}{2}\right)^2 = \frac{-11}{3} + \left(\frac{4}{2}\right)^2$$

$$(x+2)^2 = \frac{-11}{3} + 4$$

$$(x+2)^2 = \frac{1}{3}$$

$$3(x+2)^2 = 1$$

$$3(x+2)^2 - 1 = 0$$

PTS: 2 NAT: A.SSE.B.3b TOP: Families of Functions

QUADRATICS

Graphing Quadratic Functions

Common Core Standard	Next Generation Standard
F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* PARCC: Tasks have a real-world context. Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piece-wise defined functions (including step functions and absolute value functions) and exponential functions with domains in the integers.	**AI-F.IF.4** For a function that models a relationship between two quantities: i) interpret key features of graphs and tables in terms of the quantities; and ii) sketch graphs showing key features given a verbal description of the relationship. (Shared standard with Algebra II) Notes: • Algebra I key features include the following: intercepts, **zeros**; intervals where the function is increasing, decreasing, positive, or negative; maxima, minima; and symmetries. • Tasks have a real-world context and are limited to the following functions: linear, quadratic, square root, piecewise defined (including step and absolute value), and **exponential functions of the form** $f(x) = a(b)^x$ **where** $a > 0$ **and** $b > 0$ ($b \neq 1$).
F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	**AI-F.IF.7** Graph functions and show key features of the graph by hand and by using technology where appropriate. (Shared standard with Algebra II)

LEARNING OBJECTIVES

Students will be able to:

1) Create a table of values from a function rule.
2) Sketch a graph from a table of values.
3) Identify and interpret in context key features of graphs, including: intercepts, zeros; intervals where the function is increasing, decreasing, positive, or negative; maxima, minima; and symmetries.

Overview of Lesson

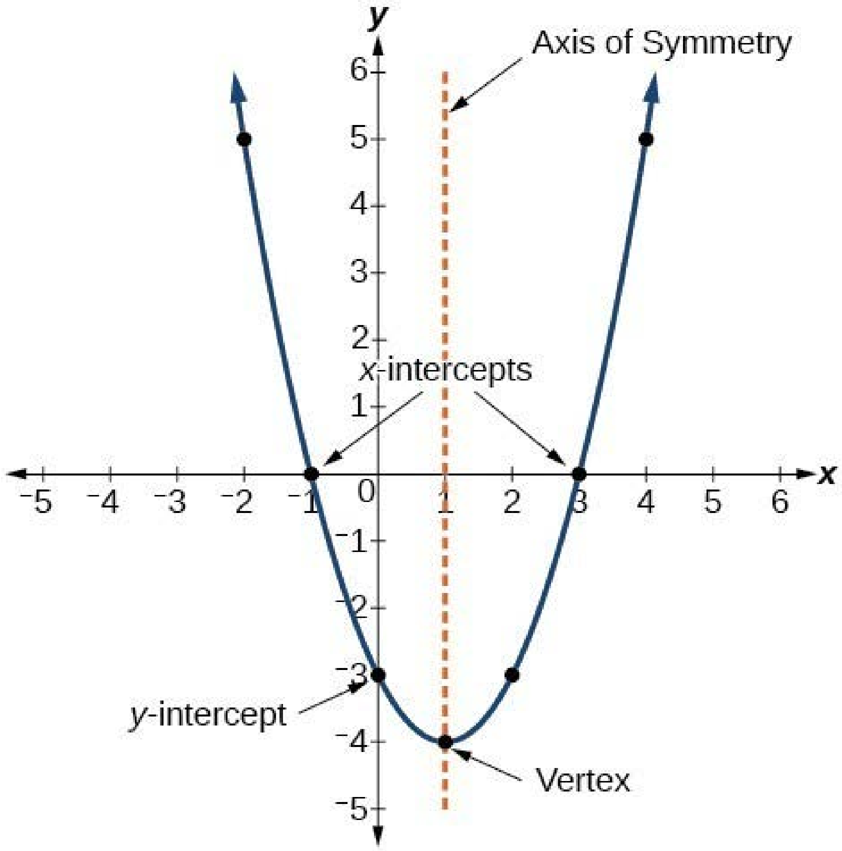
Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

axis of symmetry
decreasing intervals
increasing intervals
intercepts
maxima
minima
negative slope
no slope
positive slope
symmetries
turning point
vertex
zeros

BIG IDEAS

Identify key features of graphs:



axis of symmetry: the axis of symmetry is the vertical line whose equation is $x = 1$
decreasing intervals: the function is decreasing over the interval $x < 0$
increasing intervals: the function is increasing over the interval $0 < x$
intercepts: the x-axis intercepts are -1 and 3, the y-axis intercept is -3
maxima: there is no maxima
minima: the minima is -4
negative slope: the slope is negative on the left side of the axis of symmetry
no slope: the function has no slope at the vertex (1,-4)
positive slope: the slope is positive on the right side of the axis of symmetry
symmetries: the graph left of the axis of symmetry is symmetrical to the graph right of the axis of symmetry
turning point: the turning point is (1,-4)
vertex: the vertex is at (1,-4)
zeros: the zeros are $x = \{-1, 3\}$, these are also the roots and solutions of the function.

Sketching Graphs of Quadrilaterals

STEP 1 Create a table of values.

- Use a graphing calculator whenever possible.
- Include the vertex in the table.

STEP 2 Select coordinate pairs that are easy to plot on a Cartesian plane.

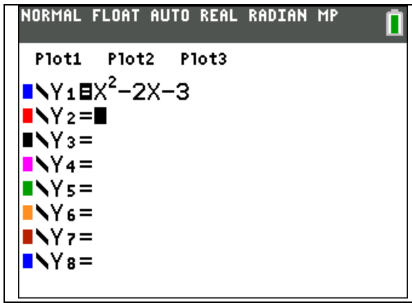
- Start with the vertex.
- Select integer values when possible.
- Select an equal number of points on either side of the vertex
- 5 coordinate pairs are usually enough.

STEP 3 Connect the plotted points.

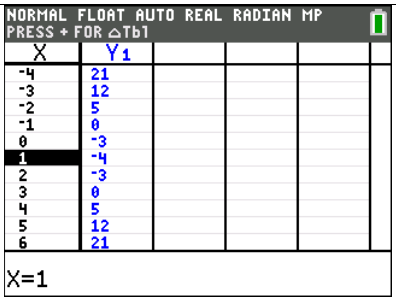
- Use the graph view of the function as a model for your sketch.
- Label the graph.
  - Include the function rule and at least three coordinates of plot points, and/or
  - Show the function rule and construct a table of values of plotted points on the page with the graph.

Example

Use technology to show three views of the following function: x^2-2x-3



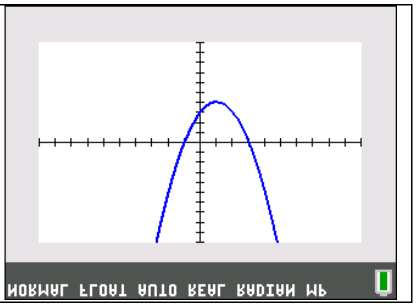
```
NORMAL FLOAT AUTO REAL RADIAN MP
 Plot1  Plot2  Plot3
■\Y1=X²-2X-3
■\Y2=■
■\Y3=
■\Y4=
■\Y5=
■\Y6=
■\Y7=
■\Y8=
```



```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl
```

X	Y1
-4	21
-3	12
-2	5
-1	0
0	-3
1	-4
2	-3
3	0
4	5
5	12
6	21

X=1



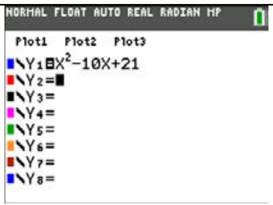
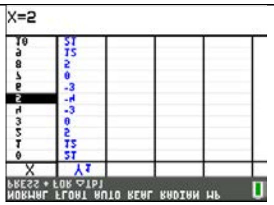
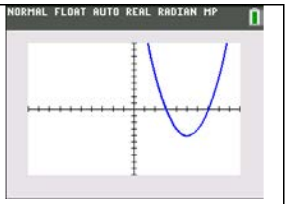
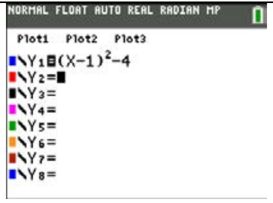
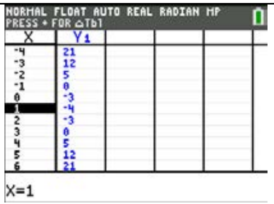
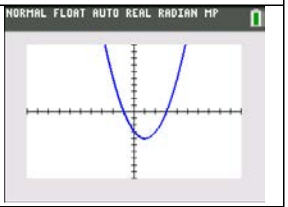
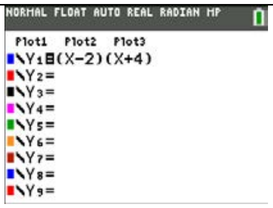
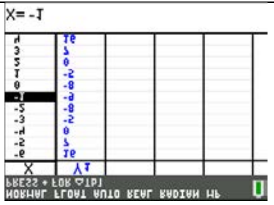
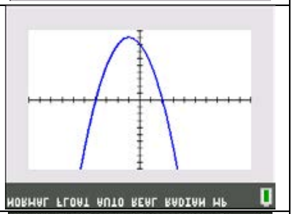
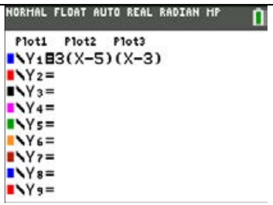
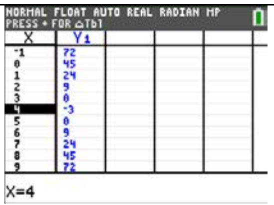
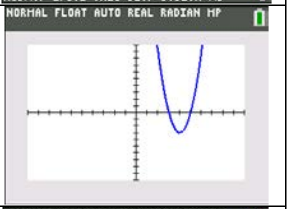
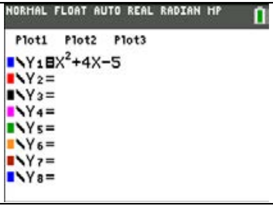
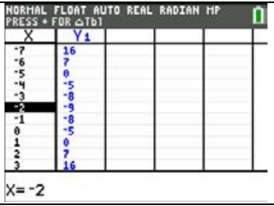
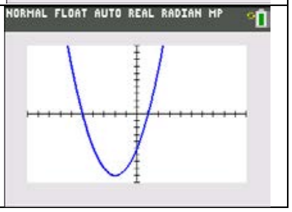
DEVELOPING ESSENTIAL SKILLS

Sketch graphs of each of the following functions using technology and graph paper:

$x^2-10x+21=0$
$(x-1)^2-4=0$
$(x-2)(x+4)=0$
$3(x-5)(x-3)=0$
$x^2+4x-5=0$

Answers

Note: Students may submit answers on graph paper.

$x^2-10x+21=0$	 NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 $Y_1=X^2-10X+21$ $Y_2=$ $Y_3=$ $Y_4=$ $Y_5=$ $Y_6=$ $Y_7=$ $Y_8=$	 X=2	
$(x-1)^2-4=0$	 NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 $Y_1=(X-1)^2-4$ $Y_2=$ $Y_3=$ $Y_4=$ $Y_5=$ $Y_6=$ $Y_7=$ $Y_8=$	 NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR △Tbl X: -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6 Y_1: 21, 12, 5, 0, -3, -4, -3, 0, 5, 12, 21 X=1	
$(x-2)(x+4)=0$	 NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 $Y_1=(X-2)(X+4)$ $Y_2=$ $Y_3=$ $Y_4=$ $Y_5=$ $Y_6=$ $Y_7=$ $Y_8=$ $Y_9=$		
$3(x-5)(x-3)=0$	 NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 $Y_1=3(X-5)(X-3)$ $Y_2=$ $Y_3=$ $Y_4=$ $Y_5=$ $Y_6=$ $Y_7=$ $Y_8=$ $Y_9=$	 NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR △Tbl X: -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 Y_1: 72, 45, 24, 9, 0, -3, 0, 9, 24, 45, 72 X=4	
$x^2+4x-5=0$	 NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 $Y_1=X^2+4X-5$ $Y_2=$ $Y_3=$ $Y_4=$ $Y_5=$ $Y_6=$ $Y_7=$ $Y_8=$	 NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR △Tbl X: -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3 Y_1: 16, 7, 0, -5, -8, -9, -8, -5, 0, 7, 16 X= -2	

REGENTS EXAM QUESTIONS (through June 2018)

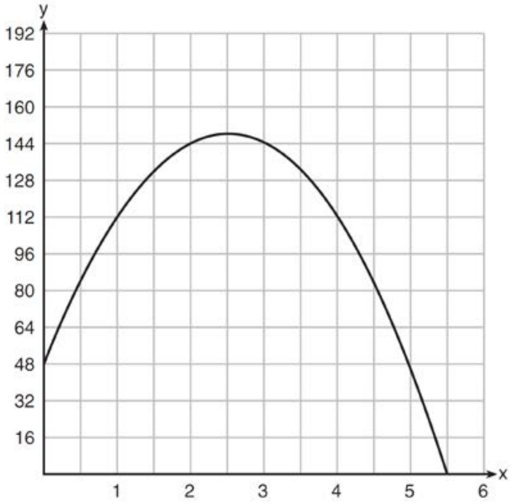
F.IF.B.4, F.IF.C.7: Graphing Quadratic Functions

224) Let $h(t)=-16t^2+64t+80$ represent the height of an object above the ground after t seconds. Determine the number of seconds it takes to achieve its maximum height. Justify your answer.

State the time interval, in seconds, during which the height of the object *decreases*. Explain your reasoning.

225) A toy rocket is launched from the ground straight upward. The height of the rocket above the ground, in feet, is given by the equation $h(t)=-16t^2+64t$, where t is the time in seconds. Determine the domain for this function in the given context. Explain your reasoning.

226) A ball is thrown into the air from the edge of a 48-foot-high cliff so that it eventually lands on the ground. The graph below shows the height, y, of the ball from the ground after x seconds.



For which interval is the ball's height always *decreasing*?

1) $0 \le x \le 2.5$
2) $0 < x < 5.5$
3) $2.5 < x < 5.5$
4) $x \ge 2$

227) Morgan throws a ball up into the air. The height of the ball above the ground, in feet, is modeled by the function $h(t) = -16t^2 + 24t$, where t represents the time, in seconds, since the ball was thrown. What is the appropriate domain for this situation?

1) $0 \le t \le 1.5$
2) $0 \le t \le 9$
3) $0 \le h(t) \le 1.5$
4) $0 \le h(t) \le 9$

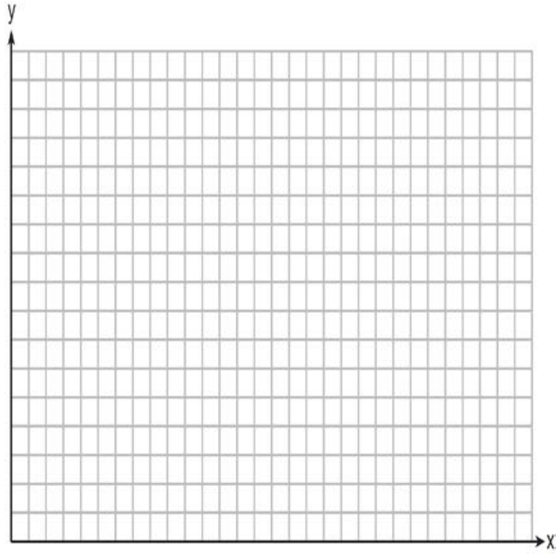
228) The height of a rocket, at selected times, is shown in the table below.

Time (sec)	0	1	2	3	4	5	6	7
Height (ft)	180	260	308	324	308	260	180	68

Based on these data, which statement is *not* a valid conclusion?

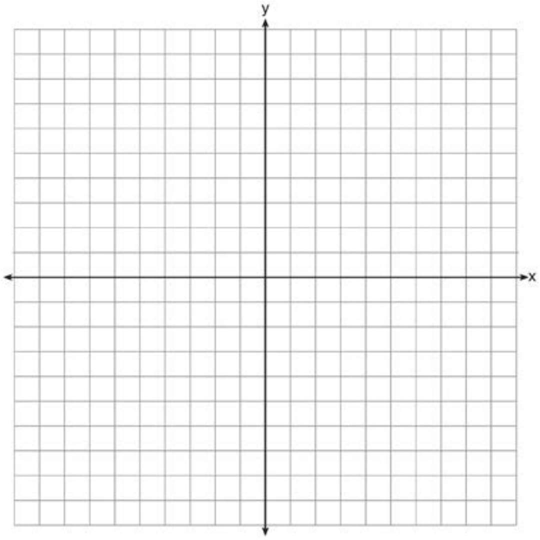
1) The rocket was launched from a height of 180 feet.
2) The maximum height of the rocket occurred 3 seconds after launch.
3) The rocket was in the air approximately 6 seconds before hitting the ground.
4) The rocket was above 300 feet for approximately 2 seconds.

229) A football player attempts to kick a football over a goal post. The path of the football can be modeled by the function $h(x) = -\frac{1}{225}x^2 + \frac{2}{3}x$, where x is the horizontal distance from the kick, and $h(x)$ is the height of the football above the ground, when both are measured in feet. On the set of axes below, graph the function $y = h(x)$ over the interval $0 \le x \le 150$.



Determine the vertex of $y = h(x)$. Interpret the meaning of this vertex in the context of the problem. The goal post is 10 feet high and 45 yards away from the kick. Will the ball be high enough to pass over the goal post? Justify your answer.

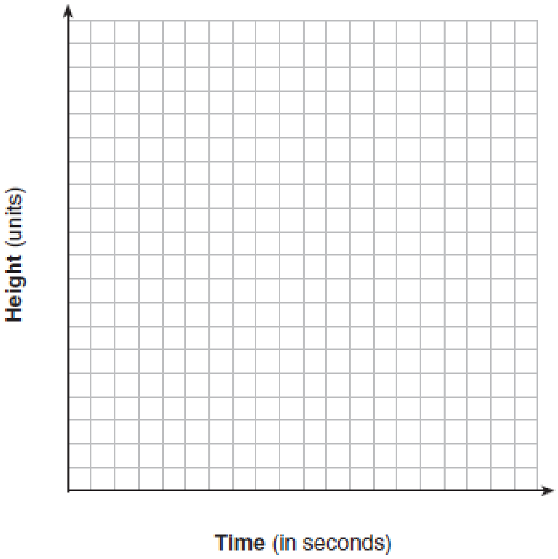
230) On the set of axes below, draw the graph of $y = x^2 - 4x - 1$.



State the equation of the axis of symmetry.

231) If the zeros of a quadratic function, *F*, are -3 and 5, what is the equation of the axis of symmetry of *F*? Justify your answer.

232) Alex launched a ball into the air. The height of the ball can be represented by the equation $h = -8t^2 + 40t + 5$, where *h* is the height, in units, and *t* is the time, in seconds, after the ball was launched. Graph the equation from $t = 0$ to $t = 5$ seconds.



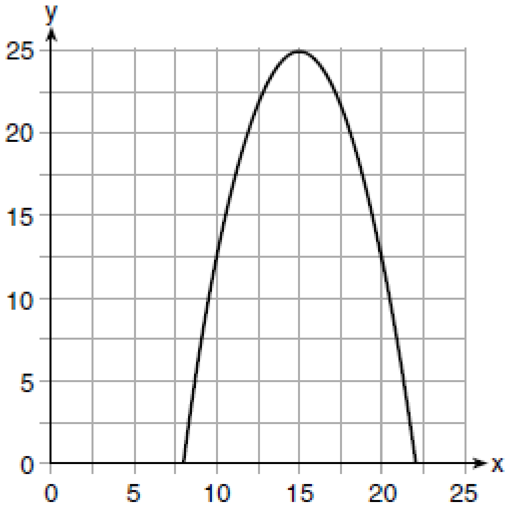
State the coordinates of the vertex and explain its meaning in the context of the problem.

233) An Air Force pilot is flying at a cruising altitude of 9000 feet and is forced to eject from her aircraft. The function $h(t) = -16t^2 + 128t + 9000$ models the height, in feet, of the pilot above the ground, where t is the time, in seconds, after she is ejected from the aircraft. Determine and state the vertex of $h(t)$. Explain what the second coordinate of the vertex represents in the context of the problem. After the pilot was ejected, what is the maximum number of feet she was above the aircraft's cruising altitude? Justify your answer.

234) The expression $-4.9t^2 + 50t + 2$ represents the height, in meters, of a toy rocket t seconds after launch. The initial height of the rocket, in meters, is

1) 0	3) 4.9
2) 2	4) 50

235) The graph of a quadratic function is shown below.



An equation that represents the function could be

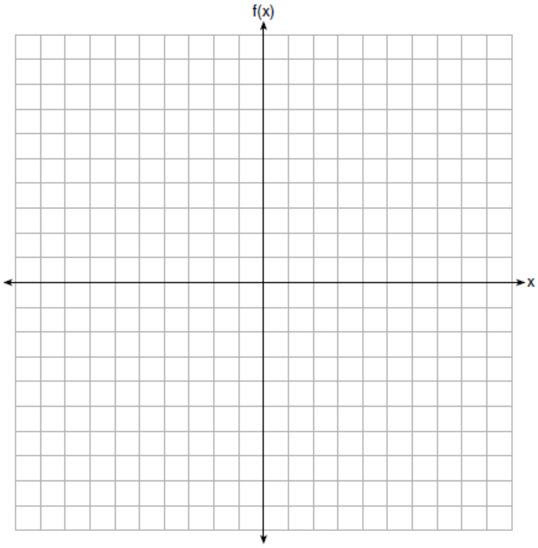
1) $q(x) = \frac{1}{2}(x + 15)^2 - 25$

2) $q(x) = -\frac{1}{2}(x + 15)^2 - 25$

3) $q(x) = \frac{1}{2}(x - 15)^2 + 25$

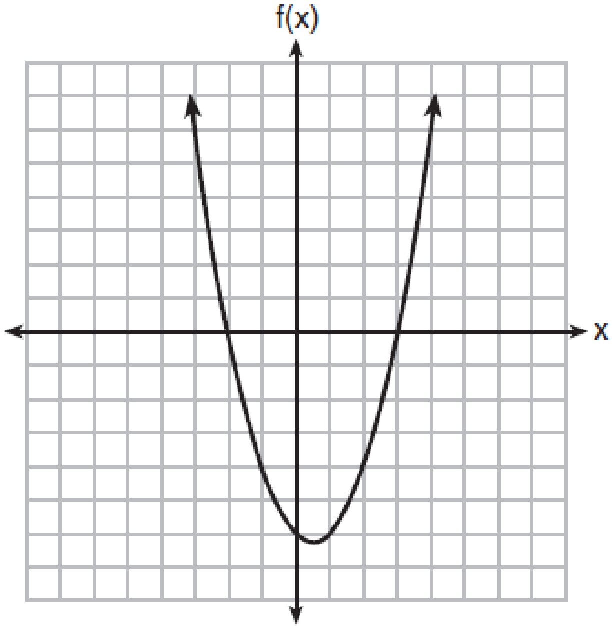
4) $q(x) = -\frac{1}{2}(x - 15)^2 + 25$

236) Graph the function $f(x) = -x^2 - 6x$ on the set of axes below.



State the coordinates of the vertex of the graph.

237) The graph of the function $f(x) = ax^2 + bx + c$ is given below.



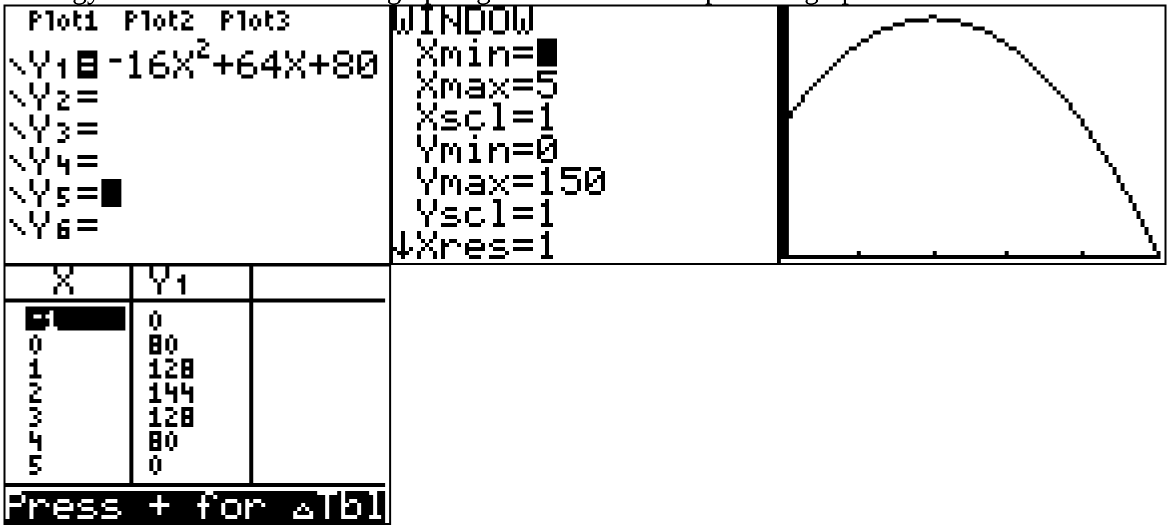
Could the factors of $f(x)$ be $(x + 2)$ and $(x - 3)$? Based on the graph, explain why or why *not*.

238) When an apple is dropped from a tower 256 feet high, the function $h(t) = -16t^2 + 256$ models the height of the apple, in feet, after t seconds. Determine, algebraically, the number of seconds it takes the apple to hit the ground.

SOLUTIONS

224) ANS:
The object reaches its maximum height at 2 seconds.
The height of the object decreases between 2 seconds and 5 seconds.

Strategy: Inut the function in a graphing calculator and inspect the graph and table of values.



```
Plot1  Plot2  Plot3
\Y1 = -16X²+64X+80
\Y2 =
\Y3 =
\Y4 =
\Y5 =
\Y6 =
```

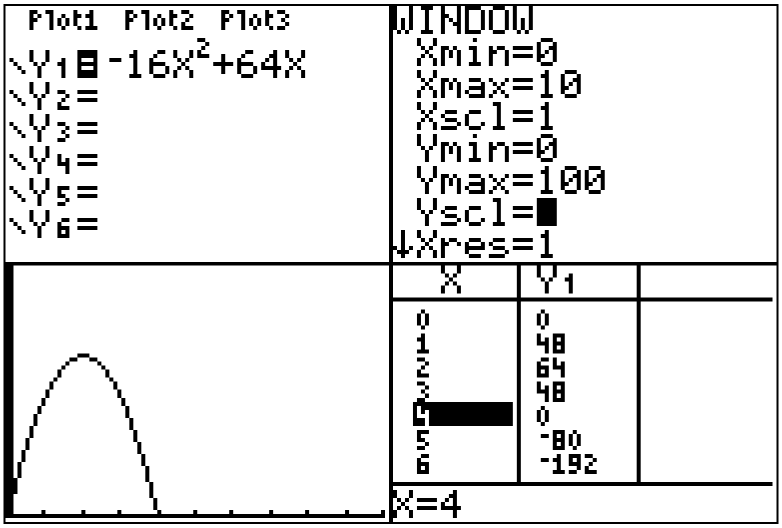
```
WINDOW
 Xmin=
 Xmax=5
 Xscl=1
 Ymin=0
 Ymax=150
 Yscl=1
↓Xres=1
```

X	Y1
-1	0
0	80
1	128
2	144
3	128
4	80
5	0

Press + for ΔTbl

PTS: 4 NAT: F.IF.B.4 TOP: Graphing Quadratic Functions

225) ANS:
The rocket launches at $t = 0$ and lands at $t = 4$, so the domain of the function is $0 \le x \le 4$.
Strategy: Input the function into a graphing calculator and determine the flight of the rocket using the graph and table views of the function.



```
Plot1  Plot2  Plot3
\Y1 = -16X²+64X
\Y2 =
\Y3 =
\Y4 =
\Y5 =
\Y6 =
```

```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=100
 Yscl=
↓Xres=1
```

X	Y1
0	0
1	48
2	64
3	48
4	0
5	-80
6	-192

X=4

The toy rocket is in the air between 0 and 4 seconds, so the domain of the function is $0 \le x \le 4$.

PTS: 2 NAT: F.IF.B.4 TOP: Graphing Quadratic Functions

226) ANS: 3

Strategy: Identify the domain of x that corresponds to a negative slope (decreasing height) in the function, then eliminate wrong answers.

STEP 1. The axis of symmetry for the parabola is $x = 2.5$ and the graph has a negative slope after $x = 2.5$ all the way to $x = 5.5$, meaning that the height of the ball is decreasing over this interval.

STEP 2. Eliminate wrong answers.
Answer choice a can be eliminated because the the slope of the graph increases over the interval $0 \le x \le 2.5$.
Answer choice b can be eliminated because the the slope of the graph both increases and decreases over the interval $0 \le x \le 2.5$.
Answer choice c is the correct choice, because it shows the domain of x where the graph has a negative slope.
Answer choice d can be eliminated because the the slope of the graph increases from $x \ge 2$ until $x = 2.5$.

PTS: 2 NAT: F.IF.B.4 TOP: Graphing Quadratic Functions

227) ANS: 1

$$h(t) = -16t^2 + 24t$$
$$0 = -16t^2 + 24t$$
$$0 = -8t(2t - 3)$$

$$-8t = 0 \quad \text{and} \quad 2t - 3 = 0$$
$$t = \frac{0}{-8} \qquad 2t = 3$$
$$t = 0 \qquad t = \frac{3}{2}$$

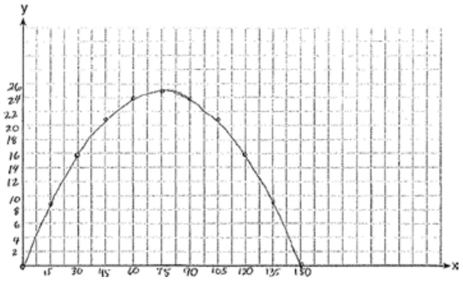
The appropriate domain for this function is $0 \le t \le 1.5$.

PTS: 2 NAT: F.IF.B.4 TOP: Graphing Quadratic Functions
KEY: context

228) ANS: 3
Strategy: Eliminate wrong answers.
Eliminate “The rocket was launched from a height of 180 feet” because the ordered pair (0, 180) indicates this statement is true.
Eliminate “The maximum height of the rocket occurred 3 seconds after launch” because the ordered pair (3,324) is the vertex of the parabola modeled by the table.
Select “The rocket was in the air approximately 6 seconds before hitting the ground” because the ordered pair (7, 68) shows that the rocket had not yet hit the ground after 7 seconds.
Eliminate ‘The rocket was above 300 feet for approximately 2 seconds” because the rocket was above 300 feet during the 2 second interval between the ordered pairs (2, 308) and (4, 308).

PTS: 2 NAT: F.IF.B.4

229) ANS:

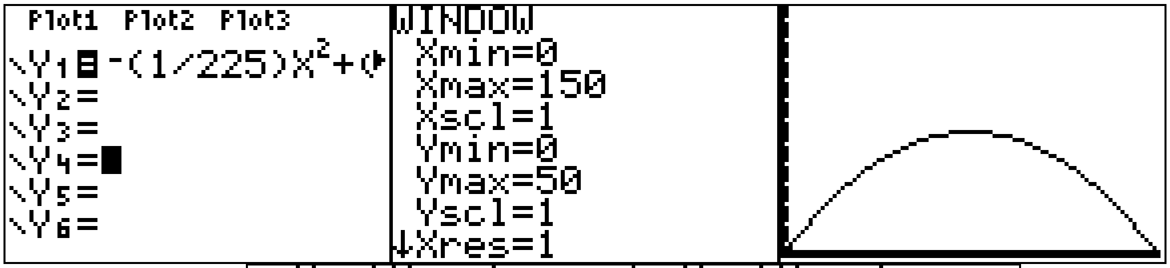
a) 

b) The vertex is at $(75, 25)$. This means that the ball will reach it highest (25 feet) when the horizontal distance is 75 feet.

c) No, the ball will not clear the goal post because it will be less than 10 feet high.

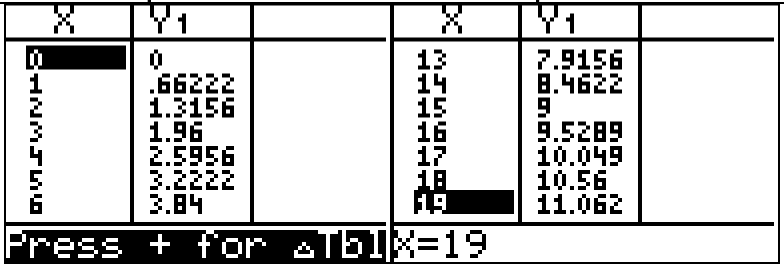
Strategy: Input the equation into a graphing calculator and use the table and graph views to complete the graph on paper, then find the vertex and determine if the ball will pass over the goal post.

STEP 1. Input $h(x) = -\frac{1}{225}x^2 + \frac{2}{3}x$ into a graphing calculator. Set the window to reflect the interval $0 \le x \le 150$ and estimate the height to be approximately $\frac{1}{3}$ the domain of *x*.



```
 Plot1  Plot2  Plot3
\Y1■-(1/225)X²+(
\Y2=
\Y3=
\Y4=█
\Y5=
\Y6=
```

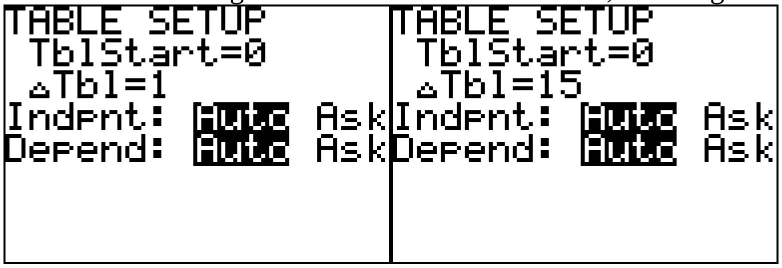
```
WINDOW
 Xmin=0
 Xmax=150
 Xscl=1
 Ymin=0
 Ymax=50
 Yscl=1
↓Xres=1
```



```
X      Y1
0      0
1      .66222
2      1.3156
3      1.96
4      2.5956
5      3.2222
6      3.84
Press + for △Tbl
```

```
X      Y1
13     7.9156
14     8.4622
15     9
16     9.5289
17     10.049
18     10.56
19     11.062
X=19
```

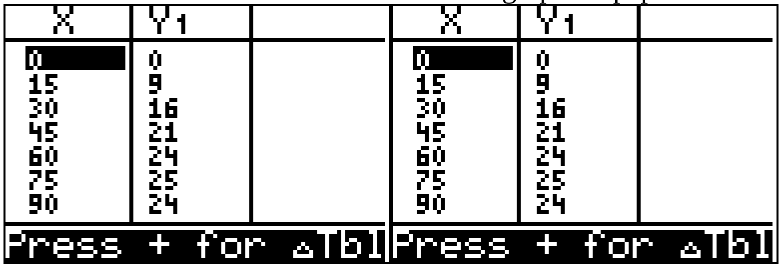
Observe that the table of values has integer solutions at 15 unit intervals, so change the △Tbl to 15.



```
TABLE SETUP
 TblStart=0
 △Tbl=1
Indpnt: Auto  Ask
Depend: Auto  Ask
```

```
TABLE SETUP
 TblStart=0
 △Tbl=15
Indpnt: Auto  Ask
Depend: Auto  Ask
```

The change in △Tbl results in a table of values that is easier to graph on paper.

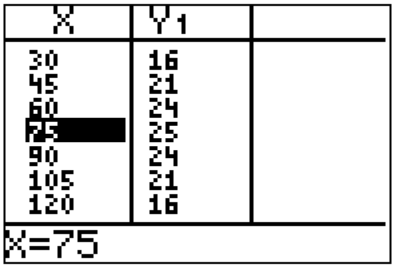


```
X      Y1
0      0
15     9
30     16
45     21
60     24
75     25
90     24
Press + for △Tbl
```

```
X      Y1
0      0
15     9
30     16
45     21
60     24
75     25
90     24
Press + for △Tbl
```

Use the graph view and the table of values to complete the graph on paper.

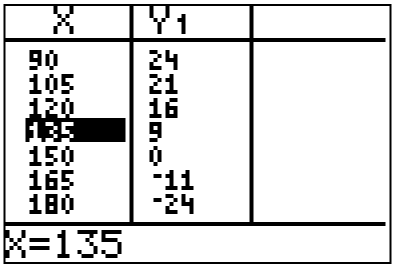
STEP 2. Use the table of values to find the vertex. The vertex is located at $\left(75, 25\right)$.



X	Y1
30	16
45	21
60	24
75	25
90	24
105	21
120	16

X=75

STEP 3. Convert 45 yards to 135 feet and determine if the the ball will be 10 feet or higher when $x = 135$
.



X	Y1
90	24
105	21
120	16
135	9
150	0
165	-11
180	-24

X=135

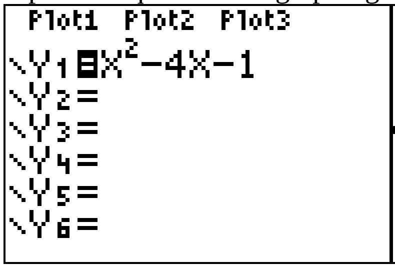
or $y = -\frac{1}{225}(135)^2 + \frac{2}{3}(135) = -81 + 90 = 9$

The ball will be 9 feet above the ground and will not go over the 10 feet high goal post.

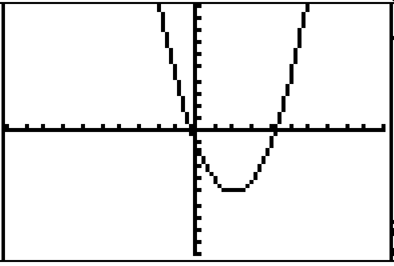
PTS: 6 NAT: F.IF.B.4 TOP: Graphing Quadratic Functions

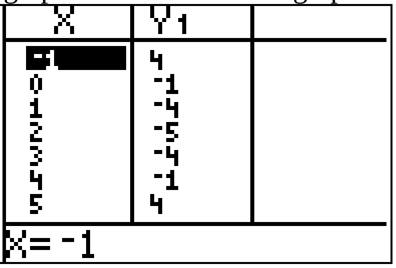
230) ANS:

Input the equation in a graphing calculator, then use the table and graph views to draw the graph.



Plot1 Plot2 Plot3
$\backslash Y_1 = X^2 - 4X - 1$
$\backslash Y_2 =$
$\backslash Y_3 =$
$\backslash Y_4 =$
$\backslash Y_5 =$
$\backslash Y_6 =$

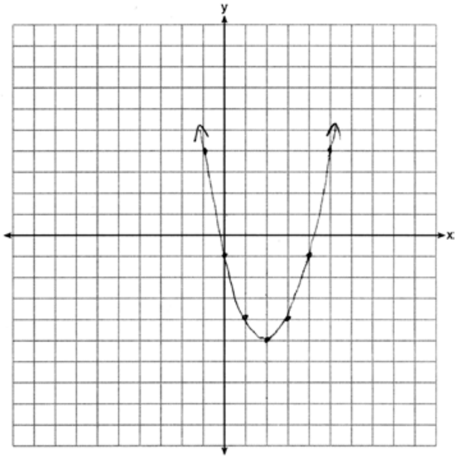




X	Y1
-1	4
0	-1
1	-4
2	-5
3	-4
4	-1
5	4

X= -1

The axis of symmetry is $x = 2$



The equation for the axis of symmetry can also be found using the formula $x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$

PTS: 2 NAT: F.IF.B.4 TOP: Graphing Quadratic Functions
NOT: NYSED classifies this as A.REI.D

231) ANS:

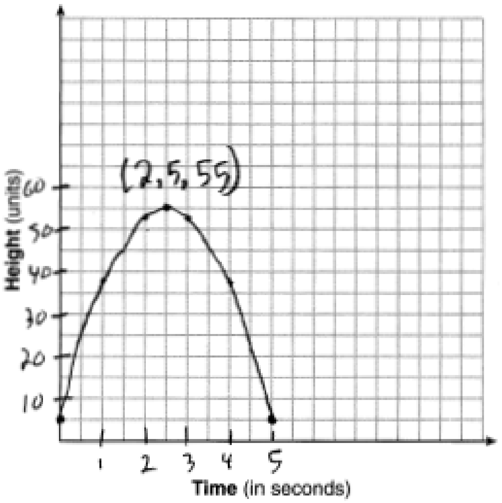
The equation of the axis of symmetry is $x = 1$.

The axis of symmetry is the vertical line that is midway between the zeros of a quadratic.

$$\frac{-3+5}{2} = 1$$

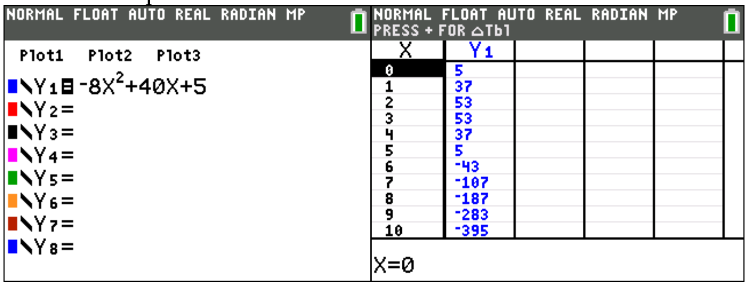
PTS: 2 NAT: F.IF.B.4 TOP: Graphing Quadratic Functions
KEY: no context

232) ANS:



The ball reaches a maximum height of 55 units at 2.5 seconds.

Strategy: Input the equation in a graphing calculator, then use the table of values to plot the graph and answer the questions.



NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3
$Y_1 = -8X^2+40X+5$
$Y_2 =$
$Y_3 =$
$Y_4 =$
$Y_5 =$
$Y_6 =$
$Y_7 =$
$Y_8 =$

NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1
0	5
1	37
2	53
3	53
4	37
5	5
6	-43
7	-107
8	-187
9	-283
10	-395

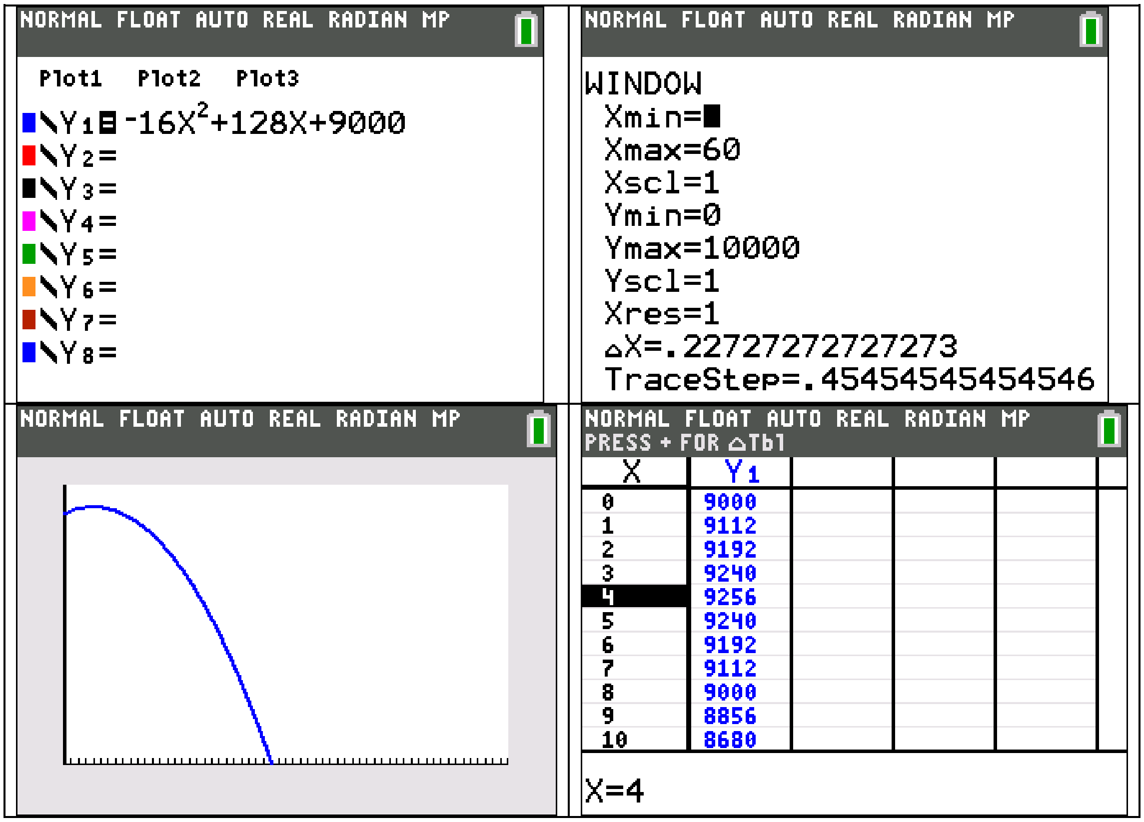
X=0

PTS: 4 NAT: F.IF.B.4 TOP: Graphing Quadratic Functions

233) ANS:

The vertex occurs at (4, 9256). This means that 4 seconds after the pilot ejects from the plane, she is 9,256 feet above the ground.

After being ejected at a crusing altitude of 9,000 feet, the maximum number of feet she was above cruising altitude was $9256 - 9000 = 256$ feet.



Y1=-16X²+128X+9000
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
Y8=

WINDOW
Xmin=0
Xmax=60
Xscl=1
Ymin=0
Ymax=10000
Yscl=1
Xres=1
△X=.2272727272727273
TraceStep=.4545454545454546

X	Y1
0	9000
1	9112
2	9192
3	9240
4	9256
5	9240
6	9192
7	9112
8	9000
9	8856
10	8680

X=4

PTS: 4 NAT: F.IF.B.4 TOP: Graphing Quadratic Functions
KEY: context

234) ANS: 2

The initial height of the rocket is when $t = 0$.

$$-4.9t^2 + 50t + 2$$

$$-4.9(0)^2 + 50(0) + 2$$

$$0 + 0 + 2$$

$$2$$

PTS: 2 NAT: F.IF.B.4 TOP: Graphing Quadratic Functions
KEY: context

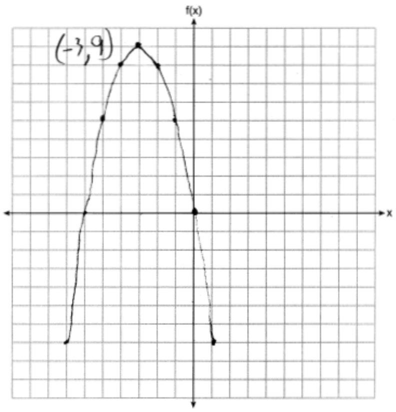
235) ANS: 4

The graph shows the vertex to be at (15, 25).

Each of the answer choices is in vertex form: $y = a(x-h)^2 + k$, where h is the x-coordinate of the vertex and k is the y-coorindate of the vertex. Stubstituting the x and y coordinatges of the vertex into $y = a(x-h)^2 + k$ results in $y = a(x-15)^2 + 25$. Since the parabola opens downward, the value of a must be negative. The correct answer is $q(x) = -\frac{1}{2}(x-15)^2 + 25$.

PTS: 2 NAT: F.IF.B.4 TOP: Graphing Quadratic Functions
KEY: no context

236) ANS:



x	y
0	0
-1	5
-2	8
-3	9
-4	8
-5	5
-6	0
HINT:	Use graphing calculator!

The coordinates of the vertex are (-3, 9)

PTS: 2 NAT: F.IF.B.4 TOP: Graphing Quadratic Functions
KEY: no context

237) ANS:
Yes, because the factors of a function and the zeros of a function are related through the multiplication property of zero. The multiplication property of zero states that the product of any number and zero is zero. This also means that if the product of two numbers is zero, then one or both of the factors must be zero.

If the factors of $f(x)$ are $(x+2)$ and $(x-3)$, then the function rule is $f(x)=(x+2)(x-3)$ and the zeros of the function will occur when $f(x)=0$. By the multiplication property of zero, when $(x+2)(x-3)=0$, either $(x+2)$, $(x-3)$, or both $(x+2)$ and $(x-3)$, must equal zero.

If $(x+2)=0$, then $x=-2$, which is shown as an x-intercept on the graph.
If $(x-3)=0$, then $x=3$, which is also shown as an x-intercept on the graph.

PTS: 2 NAT: F.IF.C.7 TOP: Graphing Quadratic Functions

238) ANS:
Answer: 4
Strategy: The apple will be on the ground when its height is zero, so evaluate the function $h(t)=-16t^2+256$ for $h(t)=0$.

$$h(t)=-16t^2+256$$
$$0=-16t^2+256$$
$$16t^2=256$$
$$t^2=16$$
$$t=4$$

Check:

$$h(4)=-16(4)^2+256$$
$$h(4)=-16(16)+256$$
$$h(4)=-256+256$$
$$h(4)=0$$

PTS: 2 NAT: F.IF.C.7 TOP: Graphing Quadratic Functions

I – Systems, Lesson 1, Solving Linear Systems (r. 2018)

SYSTEMS

Solving Linear Systems

Common Core Standards	Next Generation Standard
A-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	**STANDARD REMOVED**
A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. PARCC: Tasks have a real-world context. Tasks have hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).	**AI-A.REI.6a** Solve systems of linear equations in two variables **both algebraically and graphically.** **Note: Algebraic methods include both elimination and substitution**.

LEARNING OBJECTIVES

Students will be able to:

1) Solve systems of linear equation by graphing.
2) Solve systems of linear equations using the substitution method.
3) Solve systems of linear equations using the elimination method.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

context view
distinct
elimination method
equation rule view
graph view
non-distinct
point of intersection
solution to a system of equations
substitution method
system of equations
table view

BIG IDEAS

Facts About Systems of Linear Equations

1. A **system of linear equations** is a collection of two or more linear equations that have the same set of variables.
2. A **solution of a system of linear equations** is the set of values that simultaneously satisfy each and every linear equation in the system. Systems of linear equations can be grouped into three categories according to the number of solutions they have.
   a) **Infinitely Many Solutions**: A system of linear equations has infinitely many solutions when the equations represent the same line on a graph.
   b) **No Solutions**: A system of linear equations has no solutions when the equations represent parallel lines on a graph.
   c) **One Solution**: A system of linear equations has one and only solution when the equations represent distinct, non-parallel lines on a graph.
3. For a system of linear equations to have one solution, the number of distinct linear equations in the system must correspond to the number of variables in the system. For example, two variables require two distinct linear equations, three variables require three distinct linear equations, etc.

Distinct vs Non-Distinct Equations

Two equations are distinct if they describe different mathematical relationships between the variables. For example $y = 2x$ and $y = 3x$ describe different mathematical relationships between the variables x and y.

Two equations are non-distinct if they describe the same mathematical relationships between the variables. For example $y = 2x$ and $2y = 4x$ and $3y = 6x$ all describe the same mathematical relationships between the variables x and y, which is the idea that the value of y is two times the value of x. When linear equations are non-distinct, their graphs and tables of values will be identical.

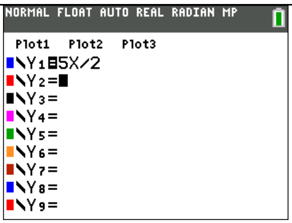
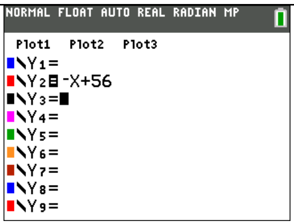
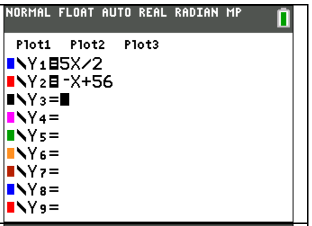
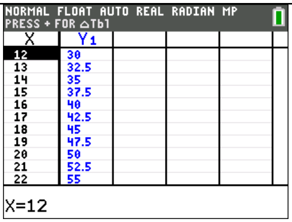
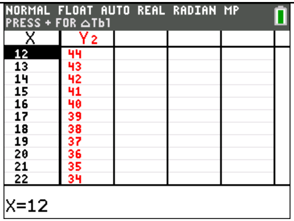
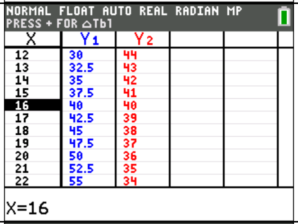
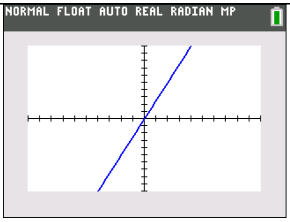
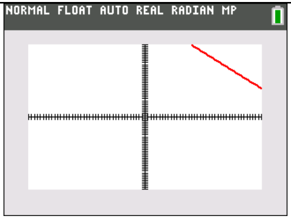
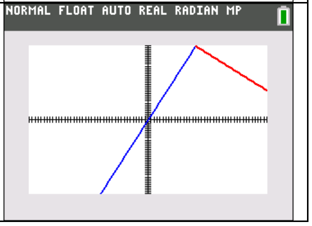
Views of Linear Equations vs Views of Systems of Linear Equations

Linear equations can be expressed in four different ways, called views. These views are:

1) an equation (or function rule) view;
2) a table view;
3) a graph view; and
4) a context view.

Systems of linear equations can be expressed using the same four views. With systems of linear equations, however, each of the four views shows two or more equations simultaneously, and it becomes important to know which values are associated with each equation. Color is used in the following examples to help distinguish between equations.

	Single Linear Equation	Single Linear Equation	System of Linear Equations
Context View	Two numbers are in the ratio 2:5.	If 6 is subtracted from the sum of two numbers, the result is 50.	Two numbers are in the ratio 2:5. If 6 is subtracted from their sum, the result is 50. What is the larger number?
Equation View	$\frac{x}{y} = \frac{2}{5}$	$(x + y) - 6 = 50$	$\begin{cases} \frac{x}{y} = \frac{2}{5} \\ (x + y) - 6 = 50 \end{cases}$

Equation View (Calculator Input)	 NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 $Y_1=5X/2$ $Y_2=$ $Y_3=$ $Y_4=$ $Y_5=$ $Y_6=$ $Y_7=$ $Y_8=$ $Y_9=$	 NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 $Y_1=$ $Y_2=-X+56$ $Y_3=$ $Y_4=$ $Y_5=$ $Y_6=$ $Y_7=$ $Y_8=$ $Y_9=$	 NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 $Y_1=5X/2$ $Y_2=-X+56$ $Y_3=$ $Y_4=$ $Y_5=$ $Y_6=$ $Y_7=$ $Y_8=$ $Y_9=$
Table View *Note that the table view for the system of linear equations has only one column for the x values.*	 NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR △Tbl X Y_1 12 30 13 32.5 14 35 15 37.5 16 40 17 42.5 18 45 19 47.5 20 50 21 52.5 22 55 X=12	 NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR △Tbl X Y_2 12 44 13 43 14 42 15 41 16 40 17 39 18 38 19 37 20 36 21 35 22 34 X=12	 NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR △Tbl X Y_1 Y_2 12 30 44 13 32.5 43 14 35 42 15 37.5 41 16 40 40 17 42.5 39 18 45 38 19 47.5 37 20 50 36 21 52.5 35 22 55 34 X=16
Graph View	 NORMAL FLOAT AUTO REAL RADIAN MP	 NORMAL FLOAT AUTO REAL RADIAN MP	 NORMAL FLOAT AUTO REAL RADIAN MP

Solutions to systems of equations

The solution to a system of linear equation is ordered pair of values that satisfies each equation in the system simultaneously (at the same time).

- In the **function rule view**, the solution is the ordered pair of values that makes each equation balance.

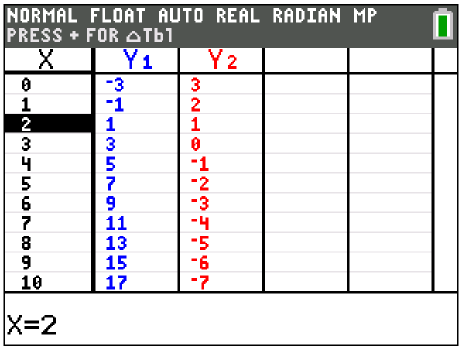
  EXAMPLE: The system $\begin{cases} 2x-y=3 \\ x+y=3 \end{cases}$ has a common solution of $(2,1)$.

  When the values $x=2$ and $y=1$ are inputs, both equations balance, as shown below:

$$\begin{aligned} &(2,1) \\ 2x-y&=3 \\ 2(2)-(1)&=3 \\ 4-1&=3 \\ 3&=3 \quad \textit{check} \end{aligned}$$

$$\begin{aligned} &(2,1) \\ x+y&=3 \\ (2)+(1)&=3 \\ 3&=3 \quad \textit{check} \end{aligned}$$

- In the **table view**, the solution is the ordered pair of values that are the same for both equations.

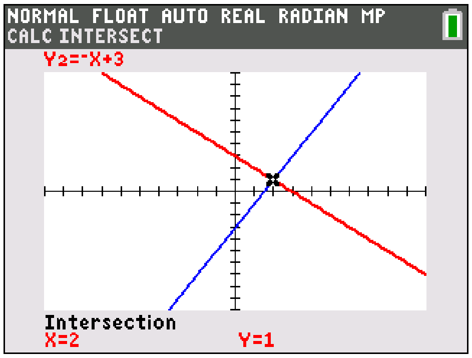


NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y_1	Y_2
0	-3	3
1	-1	2
2	1	1
3	3	0
4	5	-1
5	7	-2
6	9	-3
7	11	-4
8	13	-5
9	15	-6
10	17	-7

X=2

- In the **graph view**, the solutions are the coordinates of the point of intersection.


NORMAL FLOAT AUTO REAL RADIAN MP
CALC INTERSECT
Y₂=⁻X+3
Intersection
X=2
Y=1

Solution Strategies

Elimination Method – an Algebraic Strategy

Overview of Strategy: Eliminate one variable by addition or subtraction, then solve for the remaining variable, then the second variable.

STEPS	EXAMPLE
STEP 1 Read and understand the problem.	Solve the following system of equations by elimination. $\begin{cases}4M+3C=12\\5C+6M=19\end{cases}$
STEP 2 Line up the like terms in columns	$3C+4M=12$ $5C+6M=19$
STEP 3 Multiply each equation by the leading coefficient of the other equation, which will result in both equations having the same leading coefficient.	$5(3C+4M=12)\Rightarrow 15C+20M=60$ $3(5C+6M=19)\Rightarrow 15C+18M=57$
STEP 4 Add or subtract the like terms in the two equations to form a third equation, in which the leading coefficient is zero.	$15C+20M=60$ *subtract* $\underline{15C+18M=57}$ $0C+\ 2M=\ 3$
STEP 5 Solve the new equation for the first variable.	$0C+\ 2M=\ 3$ $2M=3$ $M=\boxed{\frac{3}{2}}$
STEP 6 Input the value found in STEP 5 into either of the original equations and solve for the second variable.	$4M+3C=12$ $4\left(\frac{3}{2}\right)+3C=12$ $\frac{12}{2}+3C=12$ $6+3C=12$ $3C=6$ $C=\boxed{2}$
STEP 7 Check your solutions in both equations.	$4M+3C=12$ $4\left(\frac{3}{2}\right)+3(2)=12$ $\frac{12}{2}+6=12$ $6+6=12$ $12=12$ *check* $5C+6M=19$ $5(2)+6\left(\frac{3}{2}\right)=19$ $10+\frac{18}{2}=19$ $10+9=19$ $19=19$ *check*

Substitution Method – an Algebraic Strategy

Overview of Strategy: Isolate one variable in either equation, then substitute its equivalent expression into the other equation. This results in a new equation with only one variable. Solve for the first variable, then use the value of the first variable in either equation to solve for the second variable.

STEPS	EXAMPLE
STEP 1 Read and understand the problem.	Solve the following system of equations by substitution. $\begin{cases} 4M+3C=12 \\ 5C+6M=19 \end{cases}$
STEP 2 Isolate one variable from one equation.	$4M+3C=12$ $4M=12-3C$ $M=3-\frac{3}{4}C$
STEP 3 Substitute the isolated value into the other equation.	$5C+6M=19$ $5C+6\left(3-\frac{3}{4}C\right)=19$
STEP 4 Solve the new equation with one variable.	$5C+6\left(3-\frac{3}{4}C\right)=19$ $5C+18-\frac{18}{4}C=19$ $20C+72-18C=76$ $2C=4$ $C=\boxed{2}$
STEP 5 Input the value found in STEP 4 into either of the original equations and solve for the second variable.	$4M+3C=12$ $4M+3(2)=12$ $4M=6$ $M=\boxed{\frac{3}{2}}$
STEP 6 Check your solutions in both equations.	$\begin{array}{ll} 4M+3C=12 & 5C+6M=19 \\ 4\left(\frac{3}{2}\right)+3(2)=12 & 5(2)+6\left(\frac{3}{2}\right)=19 \\ \frac{12}{2}+6=12 & 10+\frac{18}{2}=19 \\ 6+6=12 & 10+9=19 \\ 12=12 \ \textit{check} & 19=19 \ \textit{check} \end{array}$

DEVELOPING ESSENTIAL SKILLS

Solve each of the following systems by two algebraic methods: 1) by elimination; and 2) by substitution.

1. $\begin{cases} 4x+2y=16 \\ 3x+3y=15 \end{cases}$

2. $\begin{cases} 3x+y=7 \\ 2x+2y=6 \end{cases}$

3. $\begin{cases} 2x+3y=80 \\ 4x+2y=80 \end{cases}$

4. $\begin{cases} 5a+4b=65 \\ 4a+3b=50 \end{cases}$

5. $\begin{cases} 2m+4j=28 \\ 3m+2j=30 \end{cases}$

Answers

1. $\begin{cases} 4x+2y=16 \\ 3x+3y=15 \end{cases}$

Elimination

$$\begin{array}{ll} Eq.\#1 & 4x+2y=16 \\ Eq.\#2 & 3x+3y=15 \end{array}$$

$$\begin{array}{ll} Eq.\#1 & 3(4x+2y=16) \rightarrow 12x+6y=48 \\ Eq.\#2 & 4(3x+3y=15) \rightarrow 12x+12y=60 \end{array}$$

$$\begin{array}{ll} Eq.\#1b & 12x+6y=48 \\ Eq.\#2b & 12x+12y=60 \\ \hline & 0x+\ 6y=12 \end{array}$$

$$\begin{aligned} 0x+\ 6y&=12 \\ 6y&=12 \\ y&=\boxed{2} \end{aligned}$$

$$\begin{aligned} &Eq.\#1 \\ 4x+2y&=16 \\ 4x+2(2)&=16 \\ 4x+4&=16 \\ 4x&=12 \\ x&=\boxed{3} \end{aligned}$$

Substitution

$$3x+3y=15$$
$$x=-y+5$$
$$4x+2y=16$$
$$4(-y+5)+2y=16$$
$$-4y+20+2y=16$$
$$-2y=-4$$
$$y=\boxed{2}$$
$$3x+3y=15$$
$$3x+3(2)=15$$
$$3x+6=15$$
$$3x=9$$
$$x=\boxed{3}$$

2. $\begin{cases} 3x+y=7 \\ 2x+2y=6 \end{cases}$

Elimination

$$3x+\quad y=7$$
$$2x+2y=6$$

$$2\left(3x+y=7\right)\ \rightarrow 6x+2y=14$$
$$3\left(2x+2y=6\right)\rightarrow 6x+6y=18$$

$$0x+4y=4$$

$$y=\boxed{1}$$

$$3x+y=7$$
$$3x+1=7$$
$$3x=6$$
$$x=\boxed{2}$$

Substitution

$$3x + y = 7$$
$$y = -3x + 7$$
$$2x + 2y = 6$$
$$2x + 2(-3x + 7) = 6$$
$$2x - 6x + 14 = 6$$
$$-4x = -8$$
$$x = \boxed{2}$$
$$3x + y = 7$$
$$3(2) + y = 7$$
$$y = \boxed{1}$$

3. $\begin{cases} 2x + 3y = 80 \\ 4x + 2y = 80 \end{cases}$

Elimination

$$2x + 3y = 80$$
$$4x + 2y = 80$$

$$4(2x + 3y = 80) \rightarrow 8x + 12y = 320$$
$$2(4x + 2y = 80) \rightarrow 8x + 4y = 160$$

$$0x + 8y = 160$$

$$y = \boxed{20}$$

$$2x + 3(y) = 80$$
$$2x + 3(20) = 80$$
$$2x + 60 = 80$$
$$2x = 20$$
$$x = \boxed{10}$$

Substitution

$$4x+2y=80$$
$$y=-2x+40$$
$$2x+3y=80$$
$$2x+3(-2x+40)=80$$
$$2x-6x+120=80$$
$$-4x=-40$$
$$x=\boxed{10}$$
$$4(10)+2y=80$$
$$40+2y=80$$
$$2y=40$$
$$y=\boxed{20}$$

4. $\begin{cases} 5a+4b=65 \\ 4a+3b=50 \end{cases}$

Elimination

$$5a+4b=65$$
$$4a+3b=50$$

$$4(5a+4b=65) \to 20a+16b=260$$
$$5(4a+3b)=50 \to 20a+15b=250$$

$$0a+1b=10$$

$$\boxed{b=10}$$

$$4a+3b=50$$
$$4a+3(10)=50$$
$$4a+30=50$$
$$4a=20$$
$$a=\boxed{5}$$

Substitution

$$5a+4b=65$$

$$a=\frac{-4}{5}b+13$$

$$4a+3b=50$$

$$4\left(\frac{-4}{5}b+13\right)+3b=50$$

$$\frac{-16}{5}b+52+3b=50$$

$$-16b+260+15b=250$$

$$-b=-10$$

$$b=\boxed{10}$$

$$5a+4b=65$$

$$5a+4(10)=65$$

$$5a+40=65$$

$$5a=25$$

$$a=\boxed{5}$$

5. $\begin{cases} 2m+4j=28 \\ 3m+2j=30 \end{cases}$

Elimination

$$2m+4j=28$$

$$3m+2j=30$$

$$3(2m+4j=28)\rightarrow 6m+12j=84$$

$$2(3m+2j=30)\rightarrow 6m+4j=60$$

$$0m+8j=24$$

$$j=\boxed{3}$$

$$2m+4(j)=28$$

$$2m+4(3)=28$$

$$2m+12=28$$

$$2m=16$$

$$m=\boxed{8}$$

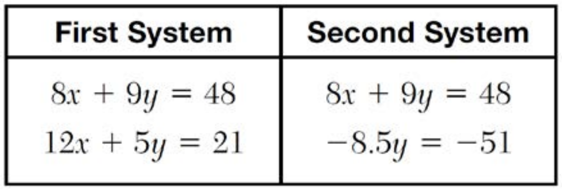
Substitution

$$2m+4j=28$$
$$m=-2j+14$$
$$3m+2j=30$$
$$3(-2j+14)+2j=30$$
$$-6j+42+2j=30$$
$$-4j=-12$$
$$j=\boxed{3}$$
$$2m+4j=28$$
$$2m+4(3)=28$$
$$2m+12=28$$
$$2m=16$$
$$m=\boxed{8}$$

REGENTS EXAM QUESTIONS (through June 2018)

A.REI.C.5, A.REI.C.6: Solving Linear Systems

239) Albert says that the two systems of equations shown below have the same solutions.



First System	Second System
$8x + 9y = 48$ $12x + 5y = 21$	$8x + 9y = 48$ $-8.5y = -51$

Determine and state whether you agree with Albert. Justify your answer.

240) Which system of equations has the same solution as the system below?

$$2x+2y=16$$
$$3x-y=4$$

1) $2x+2y=16$
$6x-2y=4$

2) $2x+2y=16$
$6x-2y=8$

3) $x+y=16$
$3x-y=4$

4) $6x+6y=48$
$6x+2y=8$

241) Which pair of equations could *not* be used to solve the following equations for x and y?

$$4x+2y=22$$
$$-2x+2y=-8$$

1) $4x+2y=22$
$2x-2y=8$

3) $12x+6y=66$
$6x-6y=24$

2) $4x + 2y = 22$
$-4x + 4y = -16$

4) $8x + 4y = 44$
$-8x + 8y = -8$

242) A system of equations is given below.

$$x + 2y = 5$$
$$2x + y = 4$$

Which system of equations does *not* have the same solution?

1) $3x + 6y = 15$
$2x + y = 4$

2) $4x + 8y = 20$
$2x + y = 4$

3) $x + 2y = 5$
$6x + 3y = 12$

4) $x + 2y = 5$
$4x + 2y = 12$

243) A system of equations is shown below.

Equation *A*: $5x + 9y = 12$
Equation *B*: $4x - 3y = 8$

Which method eliminates one of the variables?

1) Multiply equation *A* by $-\frac{1}{3}$ and add the result to equation *B*.
2) Multiply equation *B* by 3 and add the result to equation *A*.
3) Multiply equation *A* by 2 and equation *B* by -6 and add the results together.
4) Multiply equation *B* by 5 and equation *A* by 4 and add the results together.

244) Which system of equations does *not* have the same solution as the system below?

$$4x + 3y = 10$$
$$-6x - 5y = -16$$

1) $-12x - 9y = -30$
$12x + 10y = 32$

2) $20x + 15y = 50$
$-18x - 15y = -48$

3) $24x + 18y = 60$
$-24x - 20y = -64$

4) $40x + 30y = 100$
$36x + 30y = -96$

245) Guy and Jim work at a furniture store. Guy is paid \$185 per week plus 3% of his total sales in dollars, *x*, which can be represented by $g(x) = 185 + 0.03x$. Jim is paid \$275 per week plus 2.5% of his total sales in dollars, *x*, which can be represented by $f(x) = 275 + 0.025x$. Determine the value of *x*, in dollars, that will make their weekly pay the same.

246) In attempting to solve the system of equations $y = 3x - 2$ and $6x - 2y = 4$, John graphed the two equations on his graphing calculator. Because he saw only one line, John wrote that the answer to the system is the empty set. Is he correct? Explain your answer.

247) What is the solution to the system of equations below?

$$y = 2x + 8$$
$$3(-2x + y) = 12$$

1) no solution
2) infinite solutions
3) $(-1, 6)$
4) $\left(\frac{1}{2}, 9\right)$

248) The line represented by the equation $4y + 2x = 33.6$ shares a solution point with the line represented by the table below.

x	y
–5	3.2
–2	3.8
2	4.6
4	5
11	6.4

The solution for this system is

1) $(-14.0, -1.4)$
2) $(-6.8, 5.0)$
3) $(1.9, 4.6)$
4) $(6.0, 5.4)$

SOLUTIONS

239) ANS:

Albert is correct. Both systems have the same solution $\left(\frac{-3}{4}, 6\right)$.

Strategy: Solve one system of equations, then test the solution in the second system of equations.

STEP 1. Solve the first system of equations.

$$Eq.\ 1\quad 8x + 9y = 48$$

$$Eq.\ 2\quad 12x + 5y = 21$$

Multiply Eq. 1 by 3 and Multiply Eq. 2 by 2.

Then solve for the first variable

$$24x + 27y = 144$$

$$\underline{24x + 10y = 42}$$

$$17y = 102$$

$$y = 6$$

Solve for the second variable.

$$8x + 9(6) = 48$$

$$8x = -6$$

$$x = -\frac{3}{4}$$

The solution is $\left(\frac{-3}{4}, 6\right)$

STEP 2: Test the second system of equations using the same solution set.

$8x+9y=48$ $8\left(\frac{-3}{4}\right)+9(6)=48$ $-6+54=48$ $48=48$	$-8.5y=-51$ $-8.5(6)=-51$ $-51=-51$

DIMS? Does It Make Sense? Yes. The solution $\left(\frac{-3}{4},6\right)$ makes both equations balance.

PTS: 4 NAT: A.REI.C.5 TOP: Solving Linear Systems

240) ANS: 2

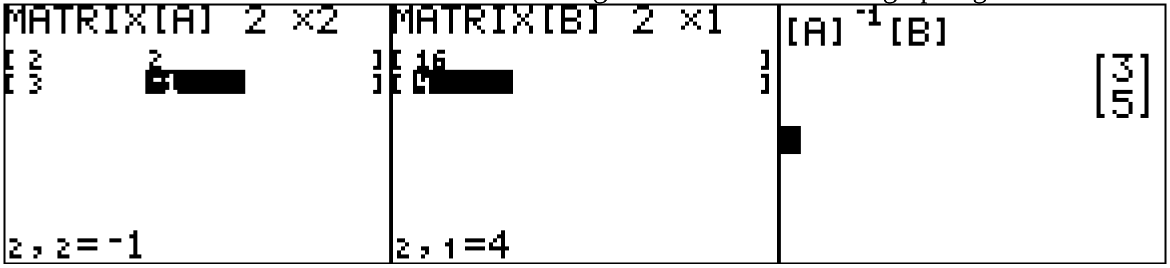
Strategy: Find equivalent forms of the system and eliminate wrong answers.

STEP 1. Eliminate answer choices *c* and *d* because the first equation in each system is not a multiple of any equation in the original system.

STEP 2. Eliminate answer choice *a* because $6x-2y=4$ is not a multiple of $3x-y=4$.

Choose answer choice *b* as the only remaining choice.

DIMS? Does It Make Sense? Yes. Check using the matrix feature of a graphing calculator.

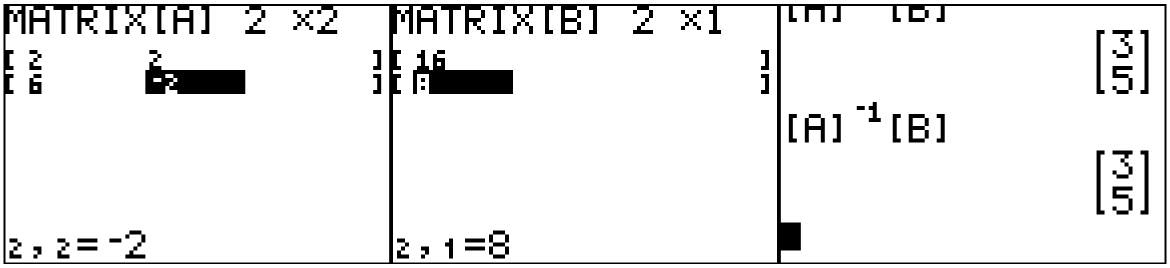


```
MATRIX[A] 2 ×2
[ 2        2    ]
[ 3        -1   ]

2,2=-1
```

```
MATRIX[B] 2 ×1
[ 16  ]
[ 4   ]

2,1=4
```

```
[A]-1[B]
                [[3]
                 [5]]
```

The solution set $(3,5)$ also works for the system in answer choice *b*.



```
MATRIX[A] 2 ×2
[ 2        2    ]
[ 6        -2   ]

2,2=-2
```

```
MATRIX[B] 2 ×1
[ 16  ]
[ 8   ]

2,1=8
```

```
                [[3]
                 [5]]
[A]-1[B]
                [[3]
                 [5]]
```

PTS: 2 NAT: A.REI.C.5 TOP: Solving Linear Systems

241) ANS: 4

Strategy: Eliminate wrong answers by deciding which systems of equations are made of multiples of the original system of equations and which system is made of equations that are not multiples of the orginal system of equations.

Choice (a) is a multiple of the original system of equations.

$$\begin{pmatrix} 4x+2y=22 \\ 2x-2y=8 \end{pmatrix} = \begin{pmatrix} 1(4x+2y=22) \\ -1(-2x+2y=-8) \end{pmatrix}$$

Choice (b) is a multiple of the original system of equations.

$$\begin{bmatrix} 4x+2y=22 \\ -4x+4y=-16 \end{bmatrix} = \begin{bmatrix} 1\left(4x+2y=22\right) \\ 2\left(-2x+2y=-8\right) \end{bmatrix}$$

Choice (c) is a multiple of the original system of equations.

$$\begin{bmatrix} 12x+6y=66 \\ 6x-6y=24 \end{bmatrix} = \begin{bmatrix} 3\left(4x+2y=22\right) \\ -3\left(-2x+2y=-8\right) \end{bmatrix}$$

Choice (d) is ***not*** a multiple of the original system of equations.

$$\begin{bmatrix} 8x+4y=44 \\ -8x+8y=-8 \end{bmatrix} \neq \begin{bmatrix} 8x+4y=44 \\ -8x+8y=-8 \end{bmatrix}$$

PTS: 2 NAT: A.REI.C.5 TOP: Solving Linear Systems

242) ANS: 4

Strategy: Determine which equations in the answer choices describe the same relationshops between variables as the equations in the problem. If one equation is a multiple of another equation, both equation describe the same relationship between variables and both equations will have the same solutions.

Eliminate $\begin{matrix} 3x+6y=15 \\ 2x+y=4 \end{matrix}$ because $3x+6y=15 \Rightarrow 3\left(x+2y=5\right)$

Eliminate $\begin{matrix} 4x+8y=20 \\ 2x+y=4 \end{matrix}$ because $4x+8y=20 \Rightarrow 4\left(x+2y=5\right)$

Eliminate $\begin{matrix} x+2y=5 \\ 6x+3y=12 \end{matrix}$ because $6x+3y=12 \Rightarrow 3\left(2x+y=4\right)$

Choose $\begin{matrix} x+2y=5 \\ 4x+2y=12 \end{matrix}$ because $4x+2y=12$ is not a multiple of $2x+y=4$

PTS: 2 NAT: A.REI.C.5 TOP: Other Systems

243) ANS: 2

STEP 1: Multiply equation B by 3

$$\begin{aligned} &Eq.A \quad 5x+9y=12 \\ &Eq.B \quad 3(4x-3y=8) \\ &Eq.B_2 \quad 1x-9y=24 \end{aligned}$$

STEP 2: Add Eq.A and Eq.B$_2$

$$\begin{aligned} &Eq.A \quad 5x+9y=12 \\ &\underline{Eq.B_1 \quad 1x-9y=24} \\ &\qquad\quad 6x \qquad = 36 \end{aligned}$$

Note that the y variable is eliminated.

PTS: 2 NAT: A.REI.C.5 TOP: Solving Linear Systems

244) ANS: 4

Strategy: Determine which systems are multiples of the original system.

$4x+3y=10$ $-6x-5y=-16$	$times-3=$ $times-2=$	$-12x-9y=-30$ $12x+10y=32$
$4x+3y=10$ $-6x-5y=-16$	$times\ 5=$ $times\ 3=$	$20x+15y=50$ $-18x-15y=-48$
$4x+3y=10$ $-6x-5y=-16$	$times\ 6=$ $times\ 4=$	$24x+18y=60$ $-24x-20y=-64$
$4x+3y=10$ $-6x-5y=-16$	$times\ 10=$ $times\ ????\neq$	$40x+30y=100$ $36x+30y=-96$ NOTE: The second equation is not a multiple of $-6x-5y=-16$

$36x+30y=96$

PTS: 2 NAT: A.REI.C.5 TOP: Solving Linear Systems

245) ANS:
$18,000

Strategy: Set both function equal to one another and solve for *x*.

STEP 1. Set both functions equal to one another.

$$g(x)=185+0.03x$$
$$f(x)=275+0.025x$$
$$185+0.03x=275+0.025x$$
$$0.03x-0.025x=275-185$$
$$0.005x=90$$
$$x=18,000$$

PTS: 2 NAT: A.REI.C.6 TOP: Solving Linear Systems

246) ANS:
No. There are infinite solutions.

The equations $y=3x-2$ and $6x-2y=4$ describe identical relationships between the variables x and y. When $6x-2y=4$ is transformed to sloped intercept format ($y=mx+b$), the result is $y=3x-2$. Therefore, this systems consists of two identical relationships between variables, and every solution to $y=3x-2$ solves both equations. Thus, there are infinite solutions.

Given (Eq. #2)	$6x-2y$	=	4
Divide (2)	$\frac{6x-2y}{2}$	=	$\frac{4}{2}$
Simplify	$3x-y$	=	2
Subtract (3x)	-3x	=	-3x
Simplify	-y	=	-3x+2

Multiply (-1)	y	=	3x-2

PTS: 2 NAT: A.REI.C.6 TOP: Solving Linear Systems
KEY: substitution

247) ANS: 1
Use substitution to solve.

$$y = 2x + 8$$

$$3(-2x + y) = 12$$

$$3[-2x + (2x + 8)] = 12$$

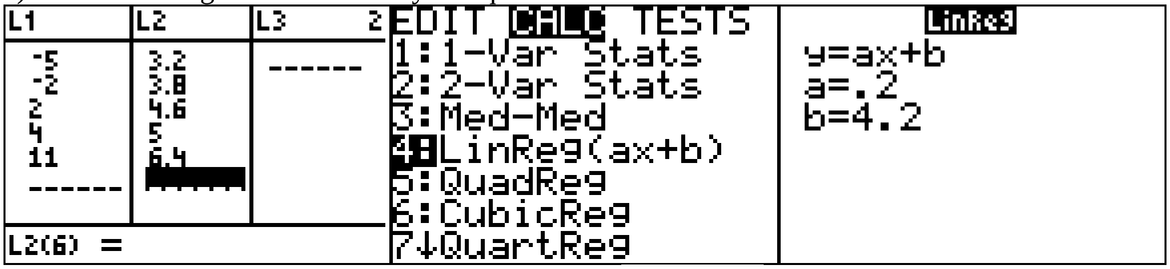
$$3[8] = 12$$

$$24 \neq 12$$

There is no solution to this system of equations.

PTS: 2 NAT: A.REI.C.6 TOP: Solving Linear Systems
KEY: substitution

248) ANS: 4
Step 1. Understand that this question is asking for the coordinates of the intersection of two different lines: the first line is represented by the equation $4y + 2x = 33.6$ and the second line is represented by the table.
Step 2. Strategy: a) Identify the function rule for the data in the table; b) transform $4y + 2x = 33.6$ into $y = mx + b$ format; and c) input both equations into a graphing calculator to find their intersection.
Step 3. Execution of strategy:
a) Use linear regression to identify an equation for the table.



L1	L2	L3 2
-5	3.2	------
-2	3.8	
2	4.6	
4	5	
11	6.4	

L2(6) =

EDIT CALC TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7↓QuartReg

LinReg
y=ax+b
a=.2
b=4.2

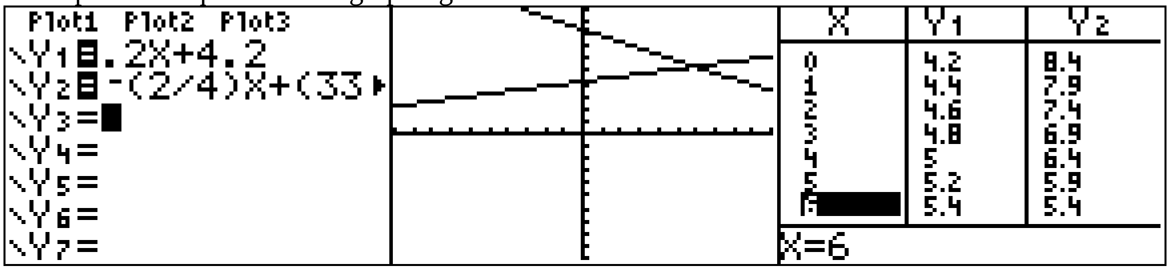
The table values can be represented by the equation $y = .2x + 4.2$
b) Transform $4y + 2x = 33.6$ into $y = mx + b$ format.

$$4y + 2x = 33.6$$

$$4y = -2x + 33.6$$

$$y = -\frac{2}{4}x + \frac{33.6}{4}$$

cI Input both equations in a graphing calculator.



Plot1 Plot2 Plot3
\Y1=.2X+4.2
\Y2=-(2/4)X+(33▸
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=

X	Y1	Y2
0	4.2	8.4
1	4.4	7.9
2	4.6	7.4
3	4.8	6.9
4	5	6.4
5	5.2	5.9
6	5.4	5.4

X=6

The lines intersect at (6, 5.4). Choice d) is the correct answer.

PTS: 2 NAT: A.REI.C.6 TOP: Solving Linear Systems

SYSTEMS

Modeling Linear Systems

Common Core Standard	Next Generation Standard
A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	AI-A.CED.2 Create equations and linear inequalities in two variables to represent a real-world context. Notes: • This is strictly the development of the model (equation/inequality). • Limit equations to linear, quadratic, and exponentials of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ $(b \neq 1)$.

NOTE: This lesson is related to Expressions and Equations, Lesson 4, Modeling Linear Equations.

LEARNING OBJECTIVES

Students will be able to:

1) Create function rules for systems of linear equations from real-world contexts.
2) Solve problems involving systems of equations based on real-world contexts.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

system of equations

defining variables

key words

BIG IDEAS

General Approach

The general approach is as follows:

1. Read and understand the entire problem.
2. Underline key words, focusing on variables, operations, and equalities or inequalities.
3. Convert the key words to mathematical notation (consider meaningful variable names other than x and y).
4. Write two or more equations with the same variables.
5. Check the final system of linear equations for reasonableness.

<table>
<tr><th>Example</th><th>Equations</th><th>Check</th></tr>
<tr><td>Jack bought 3 slices of cheese pizza and 4 slices of mushroom pizza for a total cost of $12.50.
Grace bought 3 slices of cheese pizza and 2 slices of mushroom pizza for a total cost of $8.50.
What is the cost of one slice of mushroom pizza?</td><td>Equation #1.
$3C+4M=12.50$
Equation #2.
$3C+2M=8.50$</td><td rowspan="2">$3(\cancel{C}1.50)+4(\cancel{M}2.00)=12.50$
$4.50+8.00=12.50$
$12.50=12.50$
$3(\cancel{C}1.50)+2(\cancel{M}2.00)=8.50$
$4.50+4.00=8.50$
$8.50=8.50$</td></tr>
<tr><td colspan="2">Variables:
Let C represent the cost of one slice of cheeses pizza.
Let M represent the cost of one slice of mushroom pizza.</td></tr>
<tr><td colspan="3">Solution:
$Eq.\#1 \quad 3C+4M=12.50$
$\underline{Eq.\#2 \quad 3C+2M=8.50}$
Subtract Eq.#2 from Eq.#1
$Eq.\#3 \quad 0C+2M=4.00$
Solve Eq.#3
$M=2.00$
$\boxed{\text{The cost of one slice of mushroom pizza is \$2.00}}$
Replace M in Eq#2 with 2.00
$Eq.\#2 \quad 3C+2(\cancel{M}2.00)=8.50$
$3C+4.00=8.50$
$3C=8.50-4.00$
$3C=4.50$
$C=\frac{4.50}{3}=1.50$</td></tr>
</table>

DEVELOPING ESSENTIAL SKILLS

Write and solve a system of linear equations for each real-world context below:

Problem 1

Tanisha and Rachel had lunch at the mall.

Tanisha ordered three slices of pizza and two colas. Tanisha’s bill was $6.00.

Rachel ordered two slices of pizza and three colas. Rachel’s bill was $5.25.

What was the price of one slice of pizza? What was the price of one cola?

Problem 2

When Tony received his weekly allowance, he decided to purchase candy bars for all his friends.

Tony bought three Milk Chocolate bars and four Creamy Nougat bars, which cost a total of $4.25 without tax.

Then he realized this candy would not be enough for all his friends, so he returned to the store and bought an additional six Milk Chocolate bars and four Creamy Nougat bars, which cost a total of $6.50 without tax.

How much did *each* type of candy bar cost?

Problem 3

Alexandra purchases two doughnuts and three cookies at a doughnut shop and is charged $3.30.

Briana purchases five doughnuts and two cookies at the same shop for $4.95.

All the doughnuts have the same price and all the cookies have the same price. Find the cost of one doughnut and find the cost of one cookie.

Problem 4

Ramón rented a sprayer and a generator.

On his first job, he used each piece of equipment for 6 hours at a total cost of $90.

On his second job, he used the sprayer for 4 hours and the generator for 8 hours at a total cost of $100.

What was the hourly cost of *each* piece of equipment?

Problem 5

The cost of 3 markers and 2 pencils is $1.80.

The cost of 4 markers and 6 pencils is $2.90.

What is the cost of each item? Include appropriate units in your answer.

Answers

<table>
<tr><th>Problem 1 080233a</th><th>Equations</th><th>Check</th></tr>
<tr><td>Tanisha and Rachel had lunch at the mall.
Tanisha ordered three slices of pizza and two colas. Tanisha’s bill was $6.00.
Rachel ordered two slices of pizza and three colas. Rachel’s bill was $5.25.
What was the price of one slice of pizza? What was the price of one cola?</td><td>Equation #1.
$3P + 2C = 6.00$
Equation #2.
$2P + 3C = 5.25$</td><td rowspan="2">Equation #1.
$3\left(\cancel{P}1.50\right) + 2\left(\cancel{C}.75\right) = 6.00$
$4.50 + 1.50 = 6.00$
$6.00 = 6.00$
Equation #2.
$2\left(\cancel{P}1.50\right) + 3\left(\cancel{C}.75\right) = 5.25$
$3.00 + 2.25 = 5.25$
$5.25 = 5.25$</td></tr>
<tr><td colspan="2">Variables:
Let P represent the cost of a slice of pizza.
Let C represent the cost of a cola.</td></tr>
</table>

Solution

$$Eq.\#1 \qquad 3P + 2C = 6.00$$
$$Eq.\#2 \qquad 2P + 3C = 5.25$$

Multiply Eq.#1 by 2

Multiply Eq.#2 by 3

$$Eq\#1 \qquad 6P + 4C = 12.00$$
$$Eq.\#2 \qquad 6P + 9C = 15.75$$

Subtract Eq.#1 from Eq.#2

$$\text{Eq.\#3} \qquad 5C = 3.75$$

$$\boxed{C = \frac{3.75}{5} = .75}$$

Substitute .75 for C in Eq.#1

$$3P + 2\left(\cancel{C}.75\right) = 6.00$$
$$3P + 1.50 = 6.00$$
$$3P = 4.50$$
$$\boxed{P = 1.50}$$

Substitute .75 for C in Eq.#1

$$3P + 2\left(\cancel{C}.75\right) = 6.00$$
$$3P + 1.50 = 6.00$$
$$3P = 4.50$$
$$\boxed{P = 1.50}$$

Problem 2 010232a	**Equations**	**Check**
When Tony received his weekly allowance, he decided to purchase candy bars for all his friends. Tony bought three Milk Chocolate bars and four Creamy Nougat bars, which cost a total of $4.25 without tax. Then he realized this candy would not be enough for all his friends, so he returned to the store and bought an additional six Milk Chocolate bars and four Creamy Nougat bars, which cost a total of $6.50 without tax. How much did *each* type of candy bar cost?	Equation #1. $3M+4C=4.25$ Equation #2. $6M+4C=6.50$	Equation #1. $3(\cancel{M}.75)+4(\cancel{C}.50)=4.25$ $2.25+2.00=4.25$ $4.25=4.25$ Equation #2. $6(\cancel{M}.75)+4(\cancel{C}.50)=6.50$ $4.50+2.00=6.50$ $6.50=6.50$

Variables:

Let C represent the cost of a Creamy Nougat bar.

Let M represent the cost of a Milk Chocolate bar.

Solution

$Eq.\#1 \qquad 3M+4C=4.25$

$Eq.\#2 \qquad 6M+4C=6.50$

Subtract Eq.#1 from Eq.#2

Eq.#3 3M+0C=2.25

$$\boxed{\text{M}=\frac{2.25}{3}=.75}$$

Substitute .75 for M in Eq.#1

$3(\cancel{M}.75)+4C=4.25$

$2.25+4C=4.25$

$4C=2.00$

$$\boxed{C=\frac{2.00}{4}=.50}$$

<table>
<tr><td>Problem 3 010332a</td><td>Equations</td><td>Check</td></tr>
<tr><td>Alexandra purchases two doughnuts and three cookies at a doughnut shop and is charged $3.30.
Briana purchases five doughnuts and two cookies at the same shop for $4.95.
All the doughnuts have the same price and all the cookies have the same price. Find the cost of one doughnut and find the cost of one cookie.</td><td>Equation #1.
$2D+3C=3.30$
Equation #2.
$5D+2C=4.95$</td><td rowspan="2">Equation #1.
$2(\cancel{D}.75)+3(\cancel{C}.60)=3.30$
$1.50+1.80=3.30$
$3.30=3.30$
Equation #2.
$5(\cancel{D}.75)+2(\cancel{C}.60)=4.95$
$3.75+1.20=4.95$
$4.95=4.95$</td></tr>
<tr><td colspan="2">Variables:
Let D represent the cost of a donut.
Let C represent the cost of a cookie.</td></tr>
<tr><td colspan="3">Solution
$Eq.\#1 \quad 2D+3C=3.30$
$Eq.\#2 \quad 5D+2C=4.95$
Multiply Eq.#1 by 5
Multiply Eq.#2 by 2
$Eq\#1 \quad 10D+15C=16.50$
$Eq.\#2 \quad 10D+\ \ 4C=\ \ 9.90$
Subtract Eq.#2 from Eq.#1
Eq.#3 0D+11C = 6.60
$\boxed{C=\frac{6.60}{11}=.60}$
Substitute .60 for C in Eq.#1
$10D+15(\cancel{C}.60)=16.50$
$10D+9=16.50$
$10D=7.50$
$\boxed{D=\frac{7.50}{10}=.75}$</td></tr>
</table>

Problem 4 060133a

Equations

Check

Ramón rented a sprayer and a generator.

On his first job, he used each piece of equipment for 6 hours at a total cost of $90.

On his second job, he used the sprayer for 4 hours and the generator for 8 hours at a total cost of $100.

What was the hourly cost of *each* piece of equipment?

Equation #1.

$$6S+6G=90$$

Equation #2.

$$4S+8G=100$$

Equation #1.

$$6\left(\cancel{S}5\right)+6\left(\cancel{G}10\right)=90$$

$$30+60=90$$

$$90=90$$

Equation #2.

$$4\left(\cancel{S}5\right)+8\left(\cancel{G}10\right)=100$$

$$20+80=100$$

$$100=100$$

Variables:

Let S represent the hourly cost of a sprayer.

Let G represent the hourly cost of a generator.

Solution

$$Eq.\#1 \qquad 6S+6G=90$$

$$Eq.\#2 \qquad 4S+8G=100$$

Multiply Eq.#1 by 4

Multiply Eq.#2 by 6

$$Eq\#1 \qquad 24S+24G=360$$

$$Eq.\#2 \qquad 24S+48G=600$$

Subtract Eq.#1 from Eq.#2

Eq.#3 $\quad$ $0S+24G=240$

$$\boxed{G=\frac{240}{24}=10}$$

Substitute 10 for G in Eq.#1

$$6S+6\left(\cancel{G}10\right)=90$$

$$6S+60=90$$

$$6S=30$$

$$\boxed{S=\frac{30}{6}=5}$$

<table>
<tr><td>Problem 5 080837ia</td><td>Equations</td><td>Check</td></tr>
<tr><td>The cost of 3 markers and 2
pencils is $1.80.
The cost of 4 markers and 6 pencils is $2.90.
What is the cost of each item? Include appropriate units in your answer.</td><td>Equation #1.
$3M+2P=1.80$
Equation #2.
$4M+6P=2.90$</td><td rowspan="2">Equation #1.
$3(\cancel{M}.50)+2(\cancel{P}.15)=1.80$
$1.50+.30=1.80$
$1.80=1.80$
Equation #2.
$4(\cancel{M}.50)+6(\cancel{P}.15)=2.90$
$2.00+.90=2.90$
$2.90=2.90$</td></tr>
<tr><td colspan="2">Variables:
Let M represent the cost of a marker.
Let P represent the cost of a pencil</td></tr>
<tr><td colspan="3">Solution
$Eq.\#1\ \ 3M+2P=1.80$
$Eq.\#2\ \ 4M+6P=2.90$
Multiply Eq.#1 by 4
Multiply Eq.#2 by 3
$Eq.\#1\ \ 12M+8P=7.20$
$Eq.\#2\ \ 12M+18P=8.70$
Subtract Eq.#1 from Eq.#2
$Eq.\#3\ \ 0M+10P=1.50$
$10P=1.50$
$\boxed{P=\frac{1.50}{10}=.15}$
Substitute .15 for P in Eq.#1
$Eq.\#1\ \ 3M+2(\cancel{P}.15)=1.80$
$3M+.30=1.80$
$3M=1.50$
$\boxed{M=\frac{1.50}{3}=.50}$</td></tr>
</table>

REGENTS EXAM QUESTIONS

A.CED.A.2: Modeling Linear Systems

249) An animal shelter spends \$2.35 per day to care for each cat and \$5.50 per day to care for each dog. Pat noticed that the shelter spent \$89.50 caring for cats and dogs on Wednesday. Write an equation to represent the possible numbers of cats and dogs that could have been at the shelter on Wednesday. Pat said that there might have been 8 cats and 14 dogs at the shelter on Wednesday. Are Pat's numbers possible? Use your equation to justify your answer. Later, Pat found a record showing that there were a total of 22 cats and dogs at the shelter on Wednesday. How many cats were at the shelter on Wednesday?

250) During the 2010 season, football player McGee's earnings, *m*, were 0.005 million dollars more than those of his teammate Fitzpatrick's earnings, *f*. The two players earned a total of 3.95 million dollars. Which system of equations could be used to determine the amount each player earned, in millions of dollars?

1) $m + f = 3.95$
   $m + 0.005 = f$
2) $m - 3.95 = f$
   $f + 0.005 = m$
3) $f - 3.95 = m$
   $m + 0.005 = f$
4) $m + f = 3.95$
   $f + 0.005 = m$

251) Jacob and Zachary go to the movie theater and purchase refreshments for their friends. Jacob spends a total of \$18.25 on two bags of popcorn and three drinks. Zachary spends a total of \$27.50 for four bags of popcorn and two drinks. Write a system of equations that can be used to find the price of one bag of popcorn and the price of one drink. Using these equations, determine and state the price of a bag of popcorn and the price of a drink, to the *nearest cent.*

252) Mo's farm stand sold a total of 165 pounds of apples and peaches. She sold apples for \$1.75 per pound and peaches for \$2.50 per pound. If she made \$337.50, how many pounds of peaches did she sell?

1) 11
2) 18
3) 65
4) 100

253) At Bea's Pet Shop, the number of dogs, *d*, is initially five less than twice the number of cats, *c*. If she decides to add three more of each, the ratio of cats to dogs will be $\frac{3}{4}$. Write an equation or system of equations that can be used to find the number of cats and dogs Bea has in her pet shop. Could Bea's Pet Shop initially have 15 cats and 20 dogs? Explain your reasoning. Determine algebraically the number of cats and the number of dogs Bea initially had in her pet shop.

254) Last week, a candle store received \$355.60 for selling 20 candles. Small candles sell for \$10.98 and large candles sell for \$27.98. How many large candles did the store sell?

1) 6
2) 8
3) 10
4) 12

255) The Celluloid Cinema sold 150 tickets to a movie. Some of these were child tickets and the rest were adult tickets. A child ticket cost \$7.75 and an adult ticket cost \$10.25. If the cinema sold \$1470 worth of tickets, which system of equations could be used to determine how many adult tickets, *a*, and how many child tickets, *c*, were sold?

1) $a + c = 150$
   $10.25a + 7.75c = 1470$
3) $a + c = 150$
   $7.75a + 10.25c = 1470$

2) $a + c = 1470$
$10.25a + 7.75c = 150$

4) $a + c = 1470$
$7.75a + 10.25c = 150$

256) For a class picnic, two teachers went to the same store to purchase drinks. One teacher purchased 18 juices boxes and 32 bottles of water, and spent $19.92. The other teacher purchased 14 juice boxes and 26 bottles of water, and spent $15.76.

Write a system of equations to represent the costs of a juice box, *j*, and a bottle of water, *w*.

Kara said that the juice boxes might have cost 52 cents each and that the bottles of water might have cost 33 cents each. Use your system of equations to justify that Kara's prices are *not* possible.

Solve your system of equations to determine the actual cost, in dollars, of each juice box and each bottle of water.

257) Alicia purchased *H* half-gallons of ice cream for $3.50 each and *P* packages of ice cream cones for $2.50 each. She purchased 14 items and spent $43. Which system of equations could be used to determine how many of each item Alicia purchased?

1) $3.50H + 2.50P = 43$
$H + P = 14$

2) $3.50P + 2.50H = 43$
$P + H = 14$

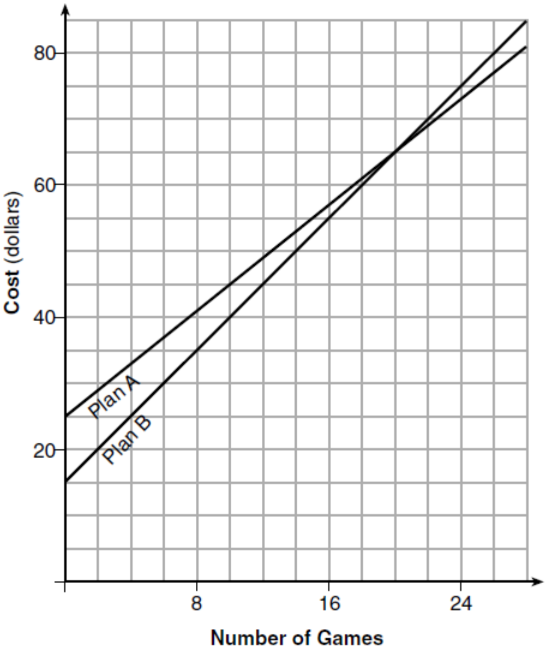
3) $3.50H + 2.50P = 14$
$H + P = 43$

4) $3.50P + 2.50H = 14$
$P + H = 43$

258) Two friends went to a restaurant and ordered one plain pizza and two sodas. Their bill totaled $15.95. Later that day, five friends went to the same restaurant. They ordered three plain pizzas and each person had one soda. Their bill totaled $45.90. Write and solve a system of equations to determine the price of one plain pizza. [Only an algebraic solution can receive full credit.]

259) Ian is borrowing $1000 from his parents to buy a notebook computer. He plans to pay them back at the rate of $60 per month. Ken is borrowing $600 from his parents to purchase a snowboard. He plans to pay his parents back at the rate of $20 per month. Write an equation that can be used to determine after how many months the boys will owe the same amount. Determine algebraically and state in how many months the two boys will owe the same amount. State the amount they will owe at this time. Ian claims that he will have his loan paid off 6 months after he and Ken owe the same amount. Determine and state if Ian is correct. Explain your reasoning.

260) The graph below models the cost of renting video games with a membership in Plan *A* and Plan *B*.



Explain why Plan *B* is the better choice for Dylan if he only has $50 to spend on video games, including a membership fee. Bobby wants to spend $65 on video games, including a membership fee. Which plan should he choose? Explain your answer.

261) Dylan has a bank that sorts coins as they are dropped into it. A panel on the front displays the total number of coins inside as well as the total value of these coins. The panel shows 90 coins with a value of $17.55 inside of the bank. If Dylan only collects dimes and quarters, write a system of equations in two variables or an equation in one variable that could be used to model this situation. Using your equation or system of equations, algebraically determine the number of quarters Dylan has in his bank. Dylan's mom told him that she would replace each one of his dimes with a quarter. If he uses all of his coins, determine if Dylan would then have enough money to buy a game priced at $20.98 if he must also pay an 8% sales tax. Justify your answer.

262) There are two parking garages in Beacon Falls. Garage *A* charges $7.00 to park for the first 2 hours, and each additional hour costs $3.00. Garage *B* charges $3.25 per hour to park. When a person parks for at least 2 hours, write equations to model the cost of parking for a total of x hours in Garage *A* and Garage *B*. Determine algebraically the number of hours when the cost of parking at both garages will be the same.

SOLUTIONS

249)ANS:

a) $2.35c + 5.50d = 89.50$

b) Pat’s numbers are not possible, because the equation does not balance using Pat’s numbers.

c) There were 10 cats in the shelter on Wednesday

Strategy: Use information from the first two sentences to write the equation, then use the equation to see if Pat is correct, then modify the equation for the last part of the question.

STEP 1: Write the equation

Let *c* represent the number of cats in the shelter.

Let *d* represent the number of dogs in the shelter.

$$2.35c + 5.50d = 89.50$$

STEP 2: Use the equation to see if Pat is correct.

$$2.35c + 5.50d = 89.50$$
$$2.35(8) + 5.50(14) \neq 89.50$$
$$18.80 + 77.00 \neq 89.50$$
$$95.80 \neq 89.50$$

STEP 3: Modify the equation to reflect the total number of animals in the shelter.
Let *c* represent the number of cats in the shelter.
Let *(22-c)* represent the number of dogs in the shelter.

$$2.35c + 5.50(22 - c) = 89.50$$
$$2.35c + 121 - 5.50c = 89.50$$
$$-3.15c = -31.50$$
$$c = 10$$

DIMS? Does It Make Sense? Yes. If there were 10 cats in the shelter and 12 dogs, the total costs of caring for the animals would be $89.50.

$$2.35c + 5.50d = 89.50$$
$$2.35(10) + 5.50(12) = 89.50$$
$$23.50 + 66 = 89.50$$
$$89.50 = 89.50$$

PTS: 4 NAT: A.CED.A.2 TOP: Modeling Linear Equations

250) ANS: 4
Strategy: Eliminate wrong answers and choose between the remaining choices..

The problem states that McGee (*m*) and Fitzpatrick's (*f*) combined earning were 3.95 million dollars. This can be represented mathematically as $m + f = 3.95$. Eliminate answer choices b and c because they state that $m - f = 3.95$, which is the difference of their salaries, not the sum.

Choose between answer choices *a* and *d*. Choice a says that Fitzpatrick (*f*) makes more. Choice *d* says that McGee (*m*) makes more. The problem states that McGee (*m*) makes more, so choice *d* is the correct answer.

DIMS? Does It Make Sense? Yes. Solve the system in answer choice D using the substitution method, as follows:

$$\text{Eq.1} \quad m + f = 3.95$$

$$\text{Eq.2} \quad f + 0.005 = m$$

$$\text{Substitute } (f + 0.005) \text{ for } m \text{ in Eq. 1}$$

$$(f + 0.005) + f = 3.95$$

$$2f + 0.005 = 3.95$$

$$2f = 3.95 - 0.005$$

$$2f = 3.945$$

$$f = \frac{3.945}{2}$$

$$f = 1.9725 \text{ million dollars}$$

Fitzpatrick earns \$1,972,500 and McGee earns $\$3,950,000 - \$1,972,500 = \$1,977,500$, which is $\$1,977,500 - \$1,972,500 = \$5,000$ more than Fitzpatrick. \$5,000 is 0.005 million dollars, so everything agrees with the information contained in the problem.

PTS: 2 NAT: A.CED.A.3 TOP: Modeling Linear Systems

251) ANS:

a) $18.25 = 2p + 3d$

$27.50 = 4p + 2d$

b) Drinks cost \$2.25 and popcorn costs \$5.75

Strategy: Write one equation for Jacob and one equation for Zachary, then solve them as a system of equations.

STEP 1: Write 2 equations.

$$\underline{18.25 = \qquad\qquad + 2p \qquad\qquad + 3d}$$

Jacob spends a total of \$18.25 on two bags of popcorn and three drinks

$$18.25 = 2p + 3d$$

$$\underline{27.50 = \qquad\qquad + 4p \qquad\qquad + 2d}$$

Zachary spends a total of \$27.50 for four bags of popcorn and two drinks.

$$27.50 = 4p + 2d$$

STEP 2. Solve both equations as a system of equations.

$$\text{Eq.1}\qquad 18.25 = 2p + 3d$$

$$\text{Eq.2}\qquad 27.50 = 4p + 2d$$

Rewrite both equations

$$\text{Eq.1}\qquad 2p + 3d = 18.25$$

$$\text{Eq.2}\qquad 4p + 2d = 27.50$$

Multiply Eq.1 by 2

$$\text{Eq.1}a\qquad 4p + 6d = 36.50$$

$$\text{Eq.2}\qquad 4p + 2d = 27.50$$

Subtract Eq.2 from Eq1a

$$\text{Eq.3}\qquad 4d = 9.00$$

$$d = \$2.25$$

Substitute 2.25 for d in Eq.1

$$\text{Eq.1}\qquad 18.25 = 2p + 3d$$

$$\text{Eq.1}\qquad 18.25 = 2p + 3(2.25)$$

$$\text{Eq.1}\qquad 18.25 = 2p + 6.75$$

$$\text{Eq.1}\qquad 18.25 - 6.75 = 2p$$

$$\text{Eq.1}\qquad 11.50 = 2p$$

$$\text{Eq.1}\qquad \$5.75 = p$$

Drinks cost \$2.25 and popcorn costs \$5.75

DIMS? Does It Make Sense? Yes. Both equations balance if drinks cost \$2.25 and popcorn costs \$5.75, as shown below:

$$\text{Eq.1}\qquad 18.25 = 2p + 3d$$

$$\text{Eq.2}\qquad 27.50 = 4p + 2d$$

Substitute and Solve

$$\text{Eq.1}\qquad 2(5.75) + 3(2.25) = 18.25$$

$$11.50 + 6.75 = 18.25$$

$$18.25 = 18.25$$

$$\text{Eq.2}\qquad 4(5.75) + 2(2.25) = 27.50$$

$$23.00 + 4.50 = 27.50$$

$$27.50 = 27.50$$

PTS: 2 NAT: A.CED.A.3 TOP: Modeling Linear Systems

252) ANS: 3

Strategy: Write and solve a system of equations to represent the problem.

Let *a* represent the number pounds of apples sold.

Let p represent the number of pounds of peaches sold.

STEP 1. Write a system of equations.
Eq. 1 $a+p=165$
Eq. 2 $\$1.75a+\$2.50p=\$337.50$

STEP 2. Solve the system.

$$Eq.1 \qquad a+p=165$$

$$Eq.2 \qquad 1.75a+2.5p=337.50$$

Multiply Eq. 1 by 1.75

$$Eq.1a \qquad 1.75a+1.75p=1.75(165)$$

Subtract Eq.1a from Eq.2

$$.75p=337.5-1.75(165)$$

$$.75p=48.75$$

$$p=\frac{48.75}{.75}$$

$$p=65$$

DIMS? Does It Make Sense? Yes. If $p=65$, then $a=100$, and these values make both equations balance.

Eq. 1 $a+p=165$	Eq. 2 $\$1.75a+\$2.50p=\$337.50$
$100+65=165$	$\$1.75(100)+\$2.50(65)=\$337.50$
$165=165$	$\$175.00+\$162.50=\$337.50$
	$\$337.50=\$337.50$

PTS: 2 NAT: A.REI.C.6 TOP: Solving Linear Systems

253) ANS:
PART 1: Write an equation or system of equations.
STEP 1. Write an equation from the first sentence to describe the *initial relationship* between the number of dogs and cats.
Let c represent the initial number of cats.
Let d represent the initial number of dogs.

$$d=2c-5$$

STEP 2. Express the current ratio of cats and dogs in the fraction form of $\left(\frac{c}{d}\right)$

STEP 3. Modify the current ratio of cats and dogs to show the addition of three cats and three dogs.

$$\frac{c+3}{d+3}$$

STEP 4. Write a proportion that equates the modified ratio with the fraction $\frac{3}{4}$.

$$\frac{c+3}{d+3}=\frac{3}{4}$$

STEP 5. Write a system of equations using the equation from STEP 1 and the proportion from STEP 4.

$$\boxed{\begin{cases} d = 2c - 5 \\ \dfrac{c+3}{d+3} = \dfrac{3}{4} \end{cases}}$$

PART 2. Answer the question, “Could Bea’s Pet Shop initially have 15 cats and 20 dogs?” and explain your reasoning.

No. The initial relationship between the number of cats and dogs can be expressed mathematically as $d = 2c - 5$. This equation does not balance when $d = 20$ and $c = 15$.

$$d = 2c - 5$$

$$20 \neq 2(15) - 5$$

$$20 \neq 25$$

Part 3. Solve the system of equations to determine the initial number of dogs and cats.

$$\begin{cases} d = 2c - 5 \\ \dfrac{c+3}{d+3} = \dfrac{3}{4} \end{cases}$$

$$\frac{c+3}{(2c-5)+3} = \frac{3}{4}$$

$$\frac{c+3}{2c-2} = \frac{3}{4}$$

$$3(2c-2) = 4(c+3)$$

$$6c - 6 = 4c + 12$$

$$2c = 18$$

$$c = 9$$

$$d = 2c - 5$$

$$d = 2(9) - 5$$

$$d = 18 - 5$$

$$d = 13$$

The initial number of cats is 9 and the initial number of dogs is 13.

PTS: 6 NAT: A.CED.A.3 TOP: Modeling Linear Systems

254) ANS: 2

Strategy: Write and solve a system of equations to represent the problem.

Let *L* represent the number of large candles sold.
Let *S* represent the number of small candles sold.

STEP 1. Write a system of equations.
Eq. 1 $L + S = 20$
Eq. 2 $\$27.98 \times L + \$10.98 \times S = \$355.60$

STEP 2. Solve the system.

$$L + S = 20$$

$$S = 20 - L$$

$$27.98L + 10.98S = 355.60$$

$$\text{Substitute}$$

$$27.98L + 10.98(20 - L) = 355.60$$

$$27.98L + 219.6 - 10.98L = 355.60$$

$$17L = 355.60 - 219.6$$

$$17L = 136$$

$$L = \frac{136}{17}$$

$$L = 8$$

DIMS? Does It Make Sense? Yes. If $L = 8$, then $S = 12$, and these values make both equations balance.

Eq. 1 $L + S = 20$	Eq. 2 $\$27.98 \times L + \$10.98 \times S = \$355.60$
$8 + 12 = 20$	$\$27.98 \times 8 + \$10.98 \times 12 = \$355.60$
$20 = 20$	$\$223.84 + \$131.76 = \$355.60$
	$\$355.60 = \$355.60$

PTS: 2 NAT: A.REI.C.6 TOP: Modeling Linear Systems

255) ANS: 1

Step 1. Recognize this problem as having two variables, a and c.
Step 2. Strategy: Write a system of equations to model the problem.
Step 3. Use information from the first two sentences to write the first equation.

The Celluloid Cinema sold 150 tickets to a movie.
Some of these were child tickets and the rest were adult tickets.

$$a + c = 150$$

Eliminate answer choices b) and d).
Use information from the next two sentences to write the second equation.
A child ticket cost $7.75 and an adult ticket cost $10.25.
If the cinema sold $1470 worth of tickets, ...

$$10.25a + 7.75c = 1470$$

Eliminate choice c). The answer is choice a).

Step 4. Does it make sense? Yes. Answer choice a) shows that the number of adult tickets added to the number of children tickets equals 150, and the income from the adult tickets added to the income from the children tickets equals 1470.

PTS: 2 NAT: A.REI.C. TOP: Modeling Linear Systems

256) ANS:

PART 1: Write a system of equations.
Eq. 1. $18j + 32w = 19.92$
Eq. 2. $14j + 23w = 15.76$

PART 2. Use the system to justify that Kara’s prices are not possible.

Eq. 1. $18j + 32w = 19.92$ Kara's prices work in equation 1.

$$18(0.52) + 32(0.33) = 19.92$$

$$9.36 + 10.56 = 19.92$$

$$19.92 = 19.92$$

Eq. 2. $14j + 23w = 15.76$ Kara's prices do not work with equation 2.

$$14(0.52) + 23(0.33) \neq 15.76$$

$$7.28 + 8.58 \neq 15.76$$

$$15.86 \neq 15.76$$

Kara's prices do not work with both equations, so they do not solve the system of equations.

PART 3 Solve the system of equation to find the price of each juice box and each bottle of water.

Eq. 1. $18j + 32w = 19.92$

Eq. 2. $14j + 23w = 15.76$

Eq. 2a $j = \left(\dfrac{-26w + 15.76}{14}\right)$

Substitute using Eq. 1 and Eq. 2a

$$18\left(\frac{-26w + 15.76}{14}\right) + 32w = 19.92$$

A bottle of water costs 24 cents.

$$(14)18\left(\frac{-26w + 15.76}{14}\right) + (14)32w = (14)19.92$$

$$18(-26w + 15.76) + (14)32w = (14)19.92$$

$$-468w + 283.68 + 448w = 278.88$$

$$-20w = 278.88 - 283.68$$

$$-20w = -4.80$$

$$w = .24$$

Solve for juice.

$$18j + 32w = 19.92$$

A box of juice costs 68 cents.

$$18j + 32(0.24) = 19.92$$

$$18j + 7..68 = 19.92$$

$$j = \frac{19.92 - 7.68}{18}$$

$$j = 0.68$$

PTS: 6 NAT: A.CED.A.2

257) ANS: 1

STEP 1: Define the variables.

Let *H* represent the number of half-gallons of ice cream.

Let *P* represent the number of packages of ice cream cones.

STEP 2: Write two equations:

Eq. #1 $3.50H + 2.50P = 43$

This equation says that \$3.50 times the number of half-gallons of ice cream plus \$2.50 times the number of packages of ice cream cones is \$43.00

Eq.#2 $H + P = 14$

This equation says the the number of half-gallons of ice cream and the number of packages of ice cream cones is 14.

PTS: 2 NAT: A.CED.A.3 TOP: Modeling Linear Systems

258) ANS:

One plain pizza costs \$12.05

Step 1. Define Variables
Let P represent the cost of one plain pizza
Let S represent the cost of one soda.
Step 2. Write 2 equations.
Eq. #1 $P + 2S = 15.95$ (from first and second sentences)
Eq. #2 $3P + 5S = 45.90$ (from third, fourth and fifth sentences)
Step 3. Multiply Eq. #1 times 3
Eq. #1a $3P + 6S = 47.85$
Step 4. Subtract Eq.#2 from Eq.#1a

$$\begin{array}{ll} Eq.\#1a & 3P + 6S = 47.85 \\ \underline{-Eq.\#2} & \underline{3P + 5S = 45.90} \\ & S = 1.95 \end{array}$$

Step 5. Solve for P by substituting 1.95 for S in Eq.#1.

$$P + 2S = 15.95$$
$$P + 2(1.95) = 15.95$$
$$P + 3.90 = 15.95$$
$$P = 12.05$$

Step 6. Check to see that $S = 1.95$ and $P = 12.05$ satisfy both equations.

Eq. #1 $P + 2S = 15.95$

$$12.05 + 2(1.95) = 15.95$$
$$15.95 = 15.95\ check$$

Eq. #2 $3P + 5S = 45.90$

$$3(12.05) + 5(1.95) = 45.90$$
$$45.90 = 45.90\ check$$

PTS: 4 NAT: A.CED.A.3 TOP: Modeling Linear Systems

259) ANS:

Strategy - Part 1:

$$1000 - 60m = 600 - 20m$$

Let m represent the number of months.
Ian’s debt is modeled by $I(m) = 1000 - 60m$
Ken’s debt is modeled by $K(m) = 600 - 20m$
Ian and Ken will owe the same amount when $K(m) = I(m)$, so set both expressions equal, as follows:

$$1000 - 60m = 600 - 20m$$

Strategy - Part 2
Ian and Ken will owe the same amount after 10 months. Both will owe $400.

Given	$1000-60m$	=	$600-20m$
Add (60m)	$+60m$		$+60m$
Simplify	1000	=	$600+40m$
Subtract (600)	-600		-600
Simplify	400	=	$40m$
Divide (40)	$\frac{400}{40}$	=	$\frac{40m}{40}$
Answer	10	=	m

Solve for amount owed after 10 months.
$I(10)=1000-60(10)=400$
$K(10)=600-20(10)=400$

Strategy - Part 3
Ian is wrong. He will still owe his parents $40 after 16 months.
$I(16)=1000-60(16)=40$

PTS: 6 NAT: A.CED.A.3 TOP: Modeling Linear Systems

260) ANS:
When the cost is $50, the graph shows that Plan A purchases a smaller number of games than Plan B.

When $65 is spent, both plans purchase the same amount of games, so it doesn’t matter.

PTS: 2 NAT: A.CED.A.3 TOP: Modeling Linear Systems

261) ANS:
Equation 1. $10d+25q=1755$
Equation 2. $d+q=90$
Dylan has 57 quarters
With all 90 coins being quarters, Dylan would not have enough money to buy the game and pay the sales tax.

Strategy: Let d represent the number of dimes and let q represent the number of quarters. Write two equations: 1) one to represent the total amount of money; and 2) another to represent the total number of coins. Convert all money to cents to avoid working with decimals.
STEP 1. Write two equations.
Equation 1. $10d+25q=1755$
Equation 2. $d+q=90$
STEP 2. Multiply equation 2 by 10, so that the second equation has 10d as a term.
Equation 2b. $10\times\left(d+q=90\right)\Leftrightarrow 10d+10q=900$
STEP 3. Subtract equation 2b from equation 1 and solve for *q*, as follows:

$$\begin{aligned} 10d + 25q &= 1755 \\ \underline{-(10d + 10q = 900)} & \\ 15q &= 855 \\ q &= \frac{855}{15} \\ q &= 57 \end{aligned}$$

The number of quarters Dylan has is 57.

STEP 4. Find out how much money Dylan will have if his mother replaces all the dimes with quarters.

Dylan will still have 90 coins, but they will all be quarters.

$90 \times 25 = 2250$ cents, or $22.50.

STEP 5. Determine how much a $20.95 game costs with 8% sales tax.

$$\begin{array}{r} \$20.96 \\ \times\ 1.08 \\ \hline \$22.65 \end{array}$$

STEP 6. Compare the amount Dylan has from STEP 5 to the amount Dylan needs from STEP 6.

$22.50 < $22.65

Dylan does not have enough money.

PTS: 6 NAT: A.CED.A.3 TOP: Modeling Linear Systems

262) ANS:

Strategy: Write and solve a system of linear equations.

STEP 1.

Let $A(x)$ represent the cost of parking in Garage *A*.

Let $B(x)$ represent the cost of parking in Garage *B*.

Let *x* represent the number of parking hours.

STEP 2.

Write two equations.

$$\text{For all } x \geq 2 \left| \begin{array}{l} A(x) = 7 + 3(x-2) \\ B(x) = 6.50 + 3.25(x-2) \end{array} \right.$$

STEP 3

Let $A(x) = B(x)$ to determine the number of hours when the cost of parking will be the same.

$$\begin{aligned} A(x) &= B(x) \\ 7 + 3(x-2) &= 6.50 + 3.25(x-2) \\ 7 + 3x - 6 &= 6.50 + 3.25x - 6.50 \\ 1 + 3x &= 3.25x \\ 1 &= .25x \\ \frac{1}{.25} &= x \\ 4 &= x \end{aligned}$$

The cost of parking in both garages will be the same for 4 hours.

CHECK

Hours	$A(x)$	$B(x)$
2	$7.00	$6.50

3	$10.00	$9.75
4	$13.00	$13.00

PTS: 4 NAT: A.CED.A.3 TOP: Modeling Linear Systems

SYSTEMS

Graphing Linear Systems

Common Core Standard	Next Generation Standard
A-REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. PARCC: Tasks have a real-world context. Tasks have hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).	**AI-A.REI.6a** Solve systems of linear equations in two variables **both algebraically and graphically.** **Note: Algebraic methods include both elimination and substitution**.

LEARNING OBJECTIVES

Students will be able to:

1) Create function rules, tables of values, and graphs of systems of linear equations from real-world contexts.
2) Use graphs of systems of equations to solve problems involving real-world contexts.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

context view
distinct equation
function rule view
graph view
infinite solutions
no-solution
same equation
simultaneous
solution
system of linear equations
table view

BIG IDEAS

Graphing Method of Solving and System of Linear Equations

Objective: Find the coordinates of the point where the graphs of the equations intersect.

Manually	With Graphing Calculator
STEP #1. Put the equations into slope-intercept form: $y = mx + b$.	STEP #1. Put the equations into slope-intercept form: $y = mx + b$.
STEP #2. Graph both equations on the same coordinate plane.	STEP #2. Input both equations in a graphing calculator.
STEP #3. Identify the coordinates of the point where the two lines intersect. This is the solution to the system of equations.	STEP #3. Use the table and/or graph views to identify the coordinates of the point where the two lines intersect. This is the solution to the system of equations. NOTE: Some calculators also have a *calculate intersection* feature.
STEP #4. Check your solution by substituting it into the original equations. If both equations balance, you have the correct solution.	STEP #4. Check that you have input the equations properly and that both table and graph views show the same solution.

DEVELOPING ESSENTIAL SKILLS

Solve each of the following systems by graphing.

1. $\begin{cases} 4x + 2y = 16 \\ 3x + 3y = 15 \end{cases}$

2. $\begin{cases} 3x + y = 7 \\ 2x + 2y = 6 \end{cases}$

3. $\begin{cases} 2x + 3y = 80 \\ 4x + 2y = 80 \end{cases}$

4. $\begin{cases} 5a + 4b = 65 \\ 4a + 3b = 50 \end{cases}$

5. $\begin{cases} 2m + 4j = 28 \\ 3m + 2j = 30 \end{cases}$

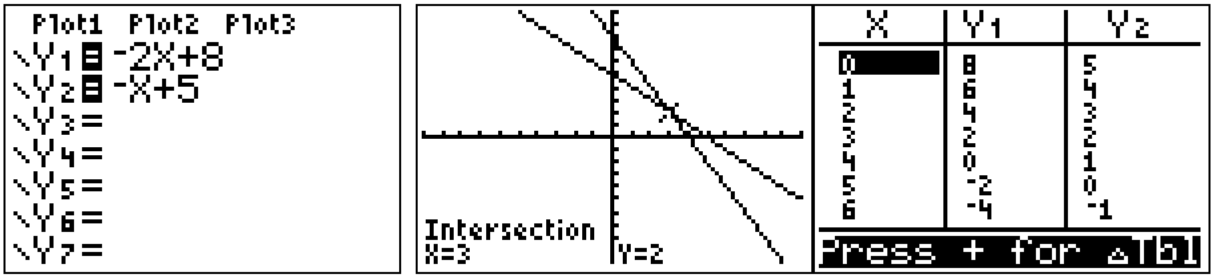
Answers

1. $\begin{cases} 4x + 2y = 16 \\ 3x + 3y = 15 \end{cases}$

Graphing

$$4x + 2y = 16 \rightarrow y = \frac{16 - 4x}{2} \rightarrow y = 8 - 2x \rightarrow y = -2x + 8$$

$$3x + 3y = 15 \rightarrow y = \frac{15 - 3x}{3} \rightarrow y = 5 - x = y = -x + 5$$



```
Plot1 Plot2 Plot3
\Y1=-2X+8
\Y2=-X+5
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
Intersection
X=3          Y=2
```

X	Y1	Y2
0	8	5
1	6	4
2	4	3
3	2	2
4	0	1
5	-2	0
6	-4	-1

Press + for ΔTbl

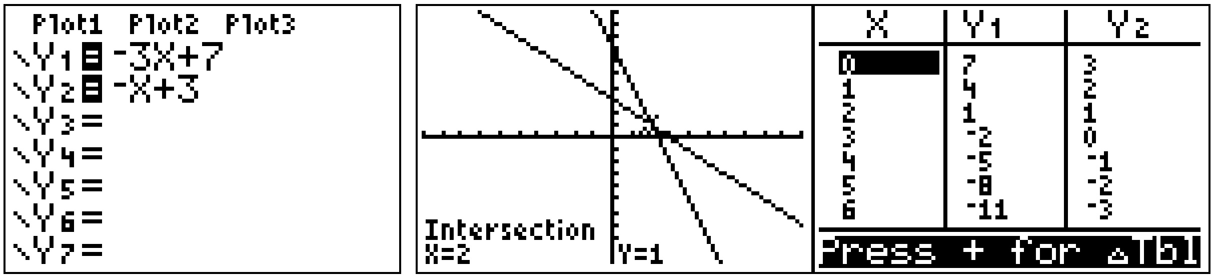
Screenshots from TI 84 Graphing Calculator

2. $\begin{cases} 3x + y = 7 \\ 2x + 2y = 6 \end{cases}$

Graphing

$$3x + y = 7 \rightarrow y = -3x + 7$$

$$2x + 2y = 6 \rightarrow 2y = -2x + 6 \rightarrow y = -x + 3$$



```
Plot1 Plot2 Plot3
\Y1=-3X+7
\Y2=-X+3
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
Intersection
X=2          Y=1
```

X	Y1	Y2
0	7	3
1	4	2
2	1	1
3	-2	0
4	-5	-1
5	-8	-2
6	-11	-3

Press + for ΔTbl

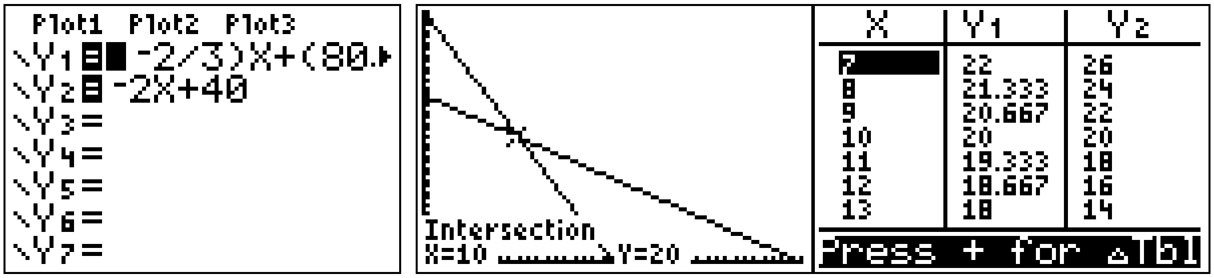
Screenshots from TI 84 Graphing Calculator

3. $\begin{cases} 2x + 3y = 80 \\ 4x + 2y = 80 \end{cases}$

Graphing

$$2x + 3y = 80 \rightarrow 3y = 80 - 2x \rightarrow y = \frac{80 - 2x}{3} \rightarrow y = \frac{-2}{3}x + \frac{80}{3}$$

$$4x + 2y = 80 \rightarrow 2y = 80 - 4x \rightarrow y = \frac{80 - 4x}{2} \rightarrow y = 40 - 2x \rightarrow y = -2x + 40$$



```
Plot1 Plot2 Plot3
\Y1=█-2/3)X+(80.▸
\Y2=-2X+40
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
Intersection
X=10         Y=20
```

X	Y1	Y2
7	22	26
8	21.333	24
9	20.667	22
10	20	20
11	19.333	18
12	18.667	16
13	18	14

Press + for ΔTbl

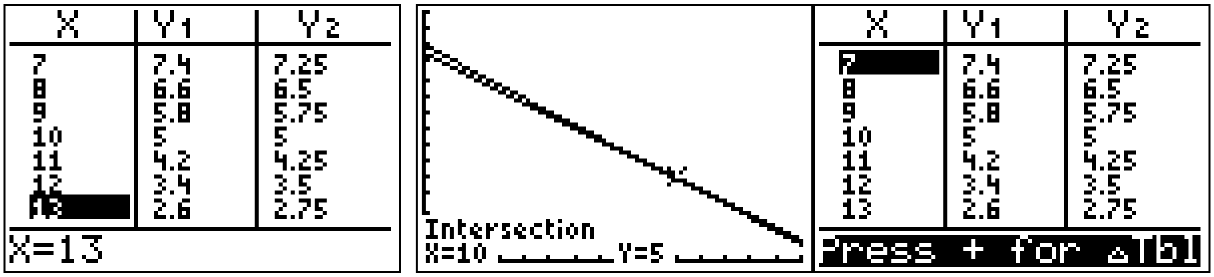
Screenshots from TI 84 Graphing Calculator

4. $\begin{cases} 5a + 4b = 65 \\ 4a + 3b = 50 \end{cases}$

Graphing

$$5a+4b=65 \rightarrow 5a=65-4b \rightarrow a=\frac{65-4b}{5} \rightarrow a=\frac{-4}{5}b+13$$

$$4a+3b=50 \rightarrow 4a=50-3b \rightarrow a=\frac{50-3b}{4} \rightarrow a=\frac{-3}{4}b+\frac{25}{2}$$



X	Y1	Y2
7	7.4	7.25
8	6.6	6.5
9	5.8	5.75
10	5	5
11	4.2	4.25
12	3.4	3.5
13	2.6	2.75

X=13

Intersection
X=10 Y=5

X	Y1	Y2
7	7.4	7.25
8	6.6	6.5
9	5.8	5.75
10	5	5
11	4.2	4.25
12	3.4	3.5
13	2.6	2.75

Press + for ΔTbl

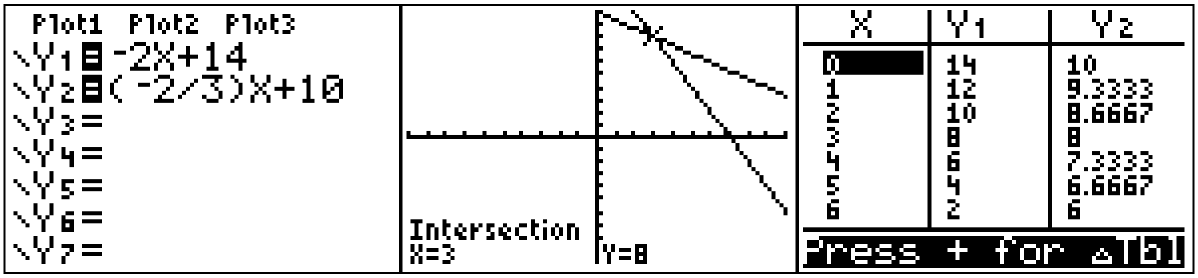
Screenshots from TI 84 Graphing Calculator

5. $\begin{cases} 2m+4j=28 \\ 3m+2j=30 \end{cases}$

Graphing

$$2m+4j=28 \rightarrow 2m=28-4j \rightarrow m=\frac{28-4j}{2} \rightarrow m=-2j+14$$

$$3m+2j=30 \rightarrow 3m=30-2j \rightarrow m=\frac{30-2j}{3} \rightarrow m=\frac{-2}{3}j+10$$



```
Plot1 Plot2 Plot3
\Y1=-2X+14
\Y2=(-2/3)X+10
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Intersection
X=3 Y=8

X	Y1	Y2
0	14	10
1	12	9.3333
2	10	8.6667
3	8	8
4	6	7.3333
5	4	6.6667
6	2	6

Press + for ΔTbl

Screenshots from TI 84 Graphing Calculator

REGENTS EXAM QUESTIONS

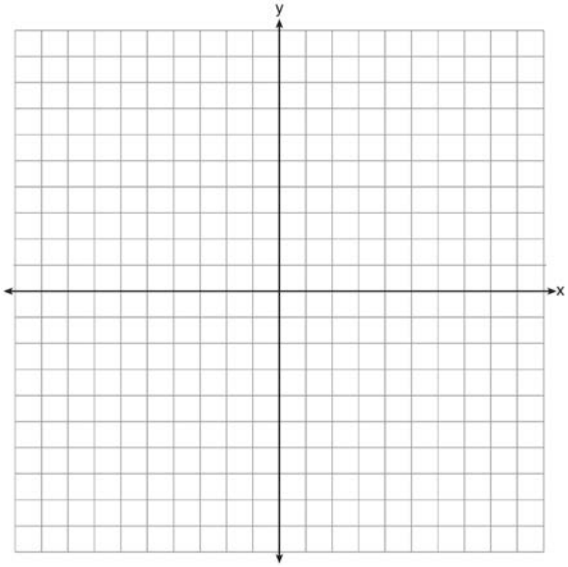
A.REI.C.6: Graphing Linear Systems

263) Next weekend Marnie wants to attend either carnival *A* or carnival *B*. Carnival *A* charges \$6 for admission and an additional \$1.50 per ride. Carnival *B* charges \$2.50 for admission and an additional \$2 per ride.

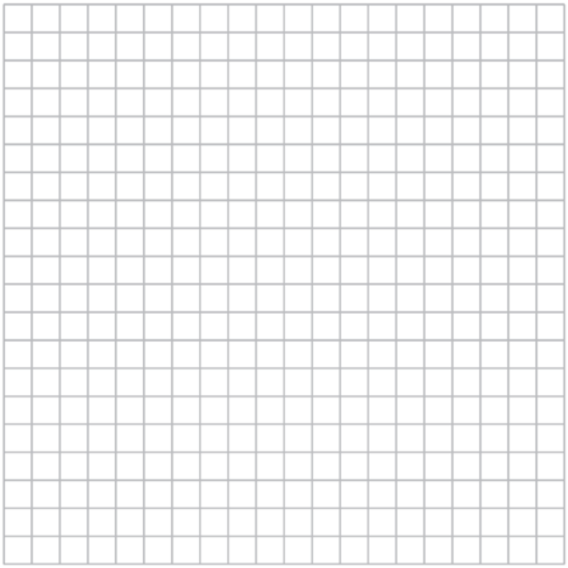
a) In function notation, write $A(x)$ to represent the total cost of attending carnival *A* and going on *x* rides. In function notation, write $B(x)$ to represent the total cost of attending carnival *B* and going on *x* rides.

b) Determine the number of rides Marnie can go on such that the total cost of attending each carnival is the same. [Use of the set of axes below is optional.]

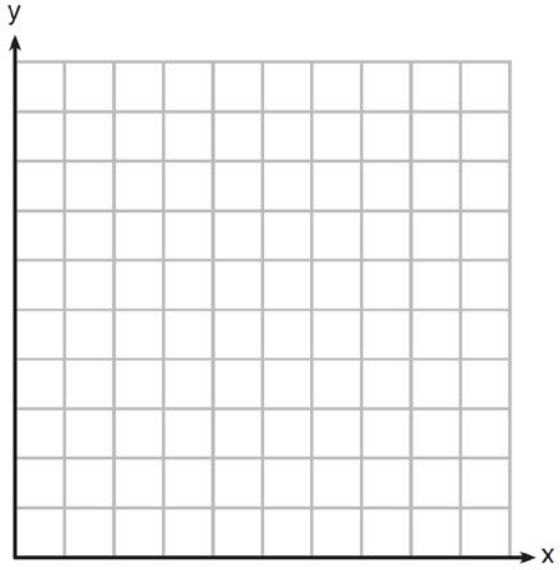
c) Marnie wants to go on five rides. Determine which carnival would have the lower total cost. Justify your answer.



264) A local business was looking to hire a landscaper to work on their property. They narrowed their choices to two companies. Flourish Landscaping Company charges a flat rate of $120 per hour. Green Thumb Landscapers charges $70 per hour plus a $1600 equipment fee. Write a system of equations representing how much each company charges. Determine and state the number of hours that must be worked for the cost of each company to be the same. [The use of the grid below is optional.] If it is estimated to take at least 35 hours to complete the job, which company will be less expensive? Justify your answer.

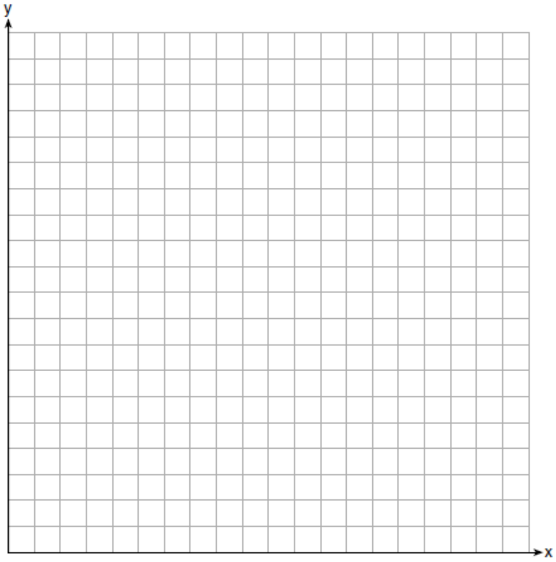


265) Franco and Caryl went to a bakery to buy desserts. Franco bought 3 packages of cupcakes and 2 packages of brownies for $19. Caryl bought 2 packages of cupcakes and 4 packages of brownies for $24. Let *x* equal the price of one package of cupcakes and *y* equal the price of one package of brownies. Write a system of equations that describes the given situation. On the set of axes below, graph the system of equations.



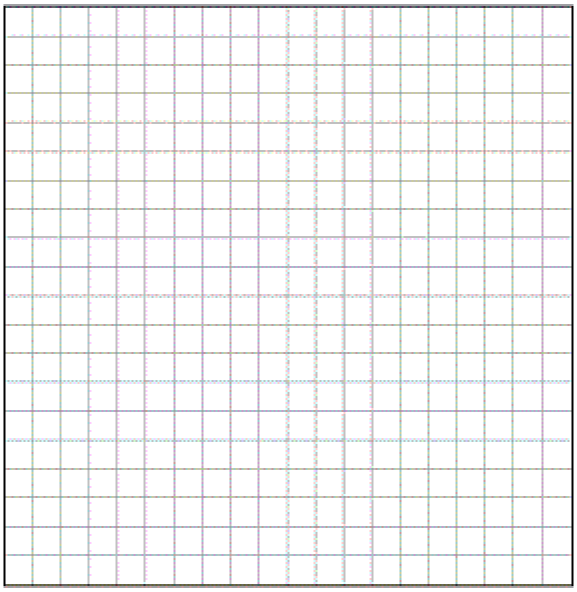
Determine the exact cost of one package of cupcakes and the exact cost of one package of brownies in dollars and cents. Justify your solution.

266) Central High School had five members on their swim team in 2010. Over the next several years, the team increased by an average of 10 members per year. The same school had 35 members in their chorus in 2010. The chorus saw an increase of 5 members per year. Write a system of equations to model this situation, where x represents the number of years since 2010. Graph this system of equations on the set of axes below.



Explain in detail what each coordinate of the point of intersection of these equations means in the context of this problem.

267) Zeke and six of his friends are going to a baseball game. Their combined money totals \$28.50. At the game, hot dogs cost \$1.25 each, hamburgers cost \$2.50 each, and sodas cost \$0.50 each. Each person buys one soda. They spend all \$28.50 on food and soda. Write an equation that can determine the number of hot dogs, x, and hamburgers, y, Zeke and his friends can buy. Graph your equation on the grid below.



Determine how many different combinations, including those combinations containing zero, of hot dogs and hamburgers Zeke and his friends can buy, spending all $28.50. Explain your answer.

268) Rowan has $50 in a savings jar and is putting in $5 every week. Jonah has $10 in his own jar and is putting in $15 every week. Each of them plots his progress on a graph with time on the horizontal axis and amount in the jar on the vertical axis. Which statement about their graphs is true?

1) Rowan's graph has a steeper slope than Jonah's.
2) Rowan's graph always lies above Jonah's.
3) Jonah's graph has a steeper slope than Rowan's.
4) Jonah's graph always lies above Rowan's.

SOLUTIONS

263) ANS:

a) $A(x) = 1.50x + 6$

$B(x) = 2x + 2.50$

b) The total costs are the same if Marnie goes on 7 rides.

c) Carnival *B* has the lower cost for admission and 5 rides. Carnival B costs $12.50 for admission and 5 rides and Carnival A costs $13.50 for admission and 5 rides.

Strategy: Write a system of equations, then input it into a graphing calculator and use it to answer parts *b* and *c* of the problem.

STEP 1. Write a system of equations.

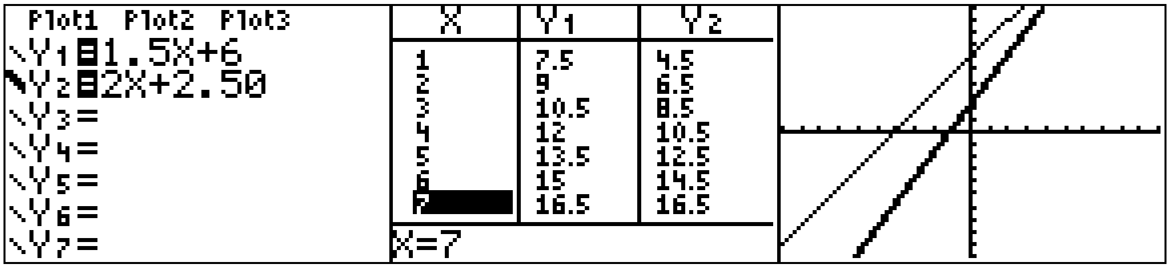
$$A(x) = 1.50x + 6$$

$$B(x) = 2x + 2.50$$

STEP 2. Input the system into a graphing calculator.

Let $A(x) = Y_1$

Let $B(x) = Y_2$



Plot1 Plot2 Plot3
\Y1=1.5X+6
\Y2=2X+2.50
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=

X	Y1	Y2
1	7.5	4.5
2	9	6.5
3	10.5	8.5
4	12	10.5
5	13.5	12.5
6	15	14.5
7	16.5	16.5

X=7

STEP 3. Use the different views of the function to answer parts b and c of the problem.
Part a) The total costs are the same at 7 rides.
Part b) Carnival B costs $12.50 for admission and 5 rides and Carnival A costs $13.50 for admission and 5 rides, so Carnival B has the lower total cost.

PTS: 6 NAT: A.REI.C.6 TOP: Modeling Linear Systems

264) ANS:

a) $F(x) = 120x$

$G(x) = 70x + 1600$

b) The costs will be the same when 32 hours are worked.
c) If the job takes at least 35 hours, Green Thumb Landscapers will be less expensive.

Strategy: Write a system of equations, then set both equations equal to one another and solve for *x*, then answer the questions

STEP 1. Write a system of equations.
Let x represent the number of hours worked.
Let $F(x)$ represent the total costs of Flourish Landscape Company.
Let $G(x)$ represent the total costs of Green Thumb Landscapers.
Write: $F(x) = 120x$

$G(x) = 70x + 1600$

STEP 2. Set both functions equal to one another to find the break even hours..

$$F(x) = 120x$$

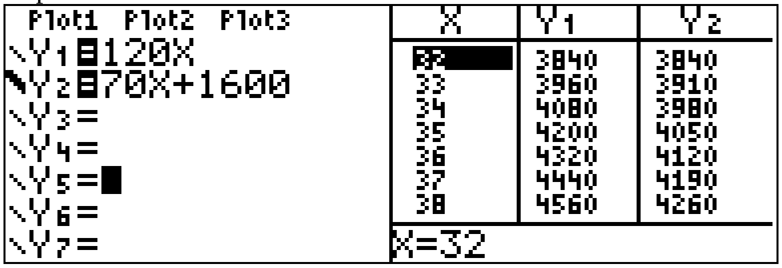
$$G(x) = 70x + 1600$$

$$120x = 70x + 1600$$

$$50x = 1600$$

$$x = 32$$

STEP 3. Input the equations into a graphing calculator to verify the break even amount and determine which company is cheaper for 35 hours or more of work.



Plot1 Plot2 Plot3
\Y1=120X
\Y2=70X+1600
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=

X	Y1	Y2
32	3840	3840
33	3960	3910
34	4080	3980
35	4200	4050
36	4320	4120
37	4440	4190
38	4560	4260

X=32

Green Thumb is less expensive.

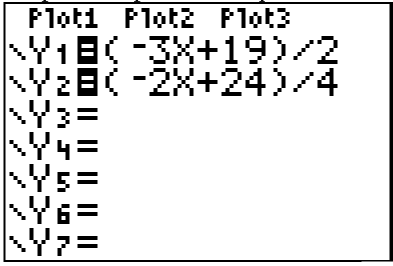
PTS: 6 NAT: A.REI.C.6 TOP: Modeling Linear Systems

265) ANS:

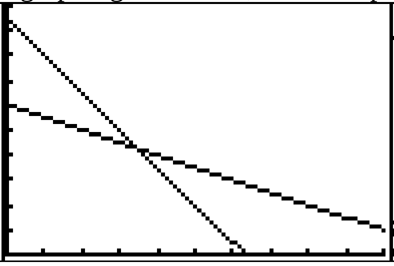
Step 1. Write two equations.

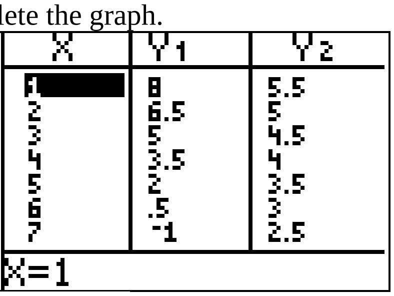
Franco’s Purchase: Franco bought <u>3 packages of cupcakes</u> (3x) and <u>2 packages of brownies</u> (2y) for $19. $3C+2B=19$ $3x+2y=19$ $2y=-3x+19$ $y=\frac{-3x+19}{2}$	Caryl’s Purchase: Caryl bought <u>2 packages of cupcakes</u> (2x) and <u>4 packages of brownies</u> (4y) for $24. $2C+4B=24$ $2x+4y=24$ $4y=-2x+24$ $y=\frac{-2x+24}{4}$

Step 2. Input both equations in a graphing calculator, then complete the graph.



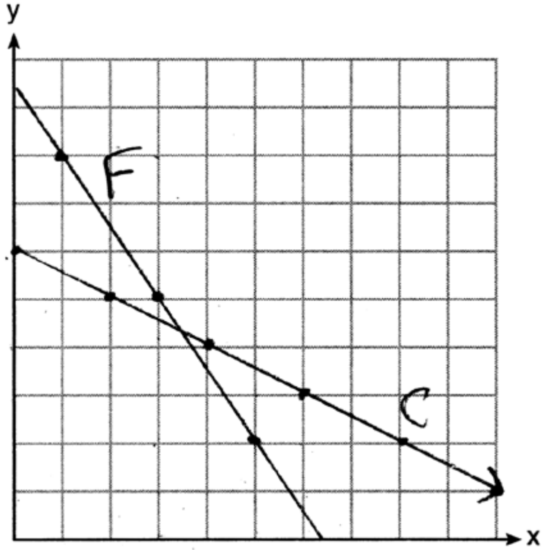
```
 Plot1  Plot2  Plot3
\Y1=(-3X+19)/2
\Y2=(-2X+24)/4
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```





X	Y1	Y2
1	8	5.5
2	6.5	5
3	5	4.5
4	3.5	4
5	2	3.5
6	.5	3
7	-1	2.5

X=1



Determine the <u>exact cost of one package of cupcakes</u> and the <u>exact cost of one package of brownies</u> in dollars and cents. Justify your solution.

Cupcakes	Brownies
$y=\frac{-3x+19}{2}$ $\quad$ $y=\frac{-2x+24}{4}$	$y=\frac{-2x+24}{4}$ $y=\frac{-2(3.5)+24}{4}$ $y=\frac{-7+24}{4}$ $y=\frac{17}{4}$ $y=4.25$

$\frac{-3x+19}{2} = \frac{-2x+24}{4}$ $4(-3x+19) = 2(-2x+24)$ $-12x+76 = -4x+48$ $76-48 = 12x-4x$ $28 = 8x$ $3.5 = x$ A package of cupcakes costs $3.50.	A package of brownies costs $4.25

Check by inserting both values in both equations.

Franco	Caryl
$3C+2B = 19$ $3(3.50)+2(4.25) = 19$ $10.50+8.50 = 19$ $19 = 19$	$2C+4B = 24$ $2(3.50)+4(4.25) = 24$ $7.00+17.00 = 24$ $24 = 24$

PTS: 6 NAT: A.REI.C.6 TOP: Graphing Linear Systems

266) ANS:

Step 1. Create a table of values to model membership in the two clubs, as follows:

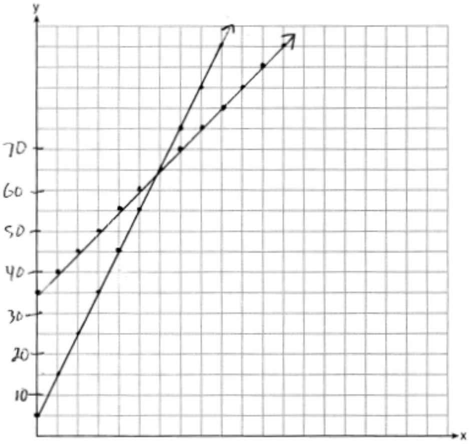
	2010	2011	2012	2013	2014	
x	0	1	2	3	4	
Swim Y_1	5	15	25	35	45	rate of change is a constant: 10 members per year
Chorus Y_2	35	40	45	50	55	rate of change is a constant: 5 members per year

Step 2. Use $y = mx + b$ to write two linear equations to model the data in the table.

$$Y_1 = 10x + 5$$

$$Y_2 = 5x + 35$$

Step 3. Graph the system of equations.



The intersection of these two equations means that in the sixth year, which is 2016, the swim team and chorus will each have 65 members.

PTS: 6 NAT: A.REI.C.6 TOP: Graphing Linear Systems

267) ANS:

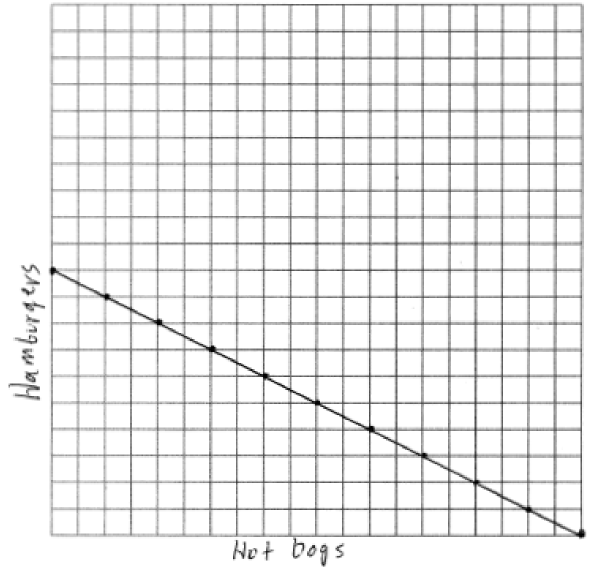
Seven friends have \$28.50. They spend $7 \times 0.50 = 3.50$ on sodas, leaving 25.00 for hot dogs and hamburgers. If hotdogs (x) cost \$1.25 and hamburgers (y) cost \$2.50, the following equation can be used to determine the number of hot dogs and hamburgers the 7 friends can buy.

$$1.25x + 2.5y = 25$$

$$2.5y = 25 - 1.25x$$

$$y = 10 - \frac{1}{2}5x$$

$$y = -\frac{1}{2}x + 10$$



There are 11 combinations, as each dot represents a possible combination.

PTS: 6 NAT: A.REI.C.6 TOP: Graphing Linear Systems

268) ANS: 3

Strategy: Create equations that model Rowan’s and Jonah’s savings plans, then compare the slopes.

STEP 1. Create Equations

$y = ax + b$, where a represents the slope of the line

$R(w) = 50 + 5w$

$R(w) = 5w + 50$ The slope of Rowan's graph is $\frac{5 \text{ rise}}{1 \text{ run}}$, or simply 5.

$J(w) = 10 + 15w$

$J(w) = 15w + 10$ The slope of Jonah's graph is $\frac{15 \text{ rise}}{1 \text{ run}}$, or simply 15.

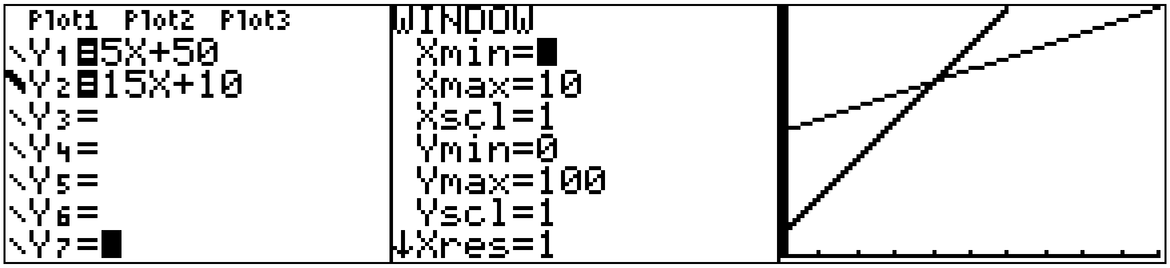
STEP 2. Compare the slopes.

Jonah’s slope is greater than Rowan’s slope because $15 > 5$. Therefore, Jonah’s graph will have a steeper slope.

DIMS? Does It Make Sense? Yes. Input both equations in a graphing calculator, as follows:

$R(w) = 5w + 50$ Transform Rowan's equation to $Y_1 = 5x + 50$ for input.

$J(w) = 15w + 10$ Transform Jonah's equation to $Y_2 = 15w + 10$ for input.



```
Plot1 Plot2 Plot3
\Y1■5X+50
\Y2■15X+10
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=■
```

```
WINDOW
 Xmin=■
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=100
 Yscl=1
↓Xres=1
```

Jonah’s graph, which is bold, is steeper than Rowan’s graph.

PTS: 2 NAT: A.CED.A.2 TOP: Graphing Linear Systems

SYSTEMS

Modeling Systems of Linear Inequalities

CC Standard	**NG Standard**
A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	**AI-A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.

LEARNING OBJECTIVES

Students will be able to:

1) Create a system of linear inequalities from a real-world context.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

see key words below

BIG IDEAS

Modeling systems of linear inequalities is similar to modeling systems of linear equations, except that an inequality sign is used instead of an equal sign.

Key English Words and Their Mathematical Translations

These English Words	Usually Mean	*Examples: English becomes math*
is, are	equals	*the sum of 5 and x is 20* becomes $5 + x = 20$
more than, greater than	inequality $>$	*x is greater than y* becomes $x > y$ *x is more than 5* becomes $x > 5$ *5 is more than x* becomes $5 > x$
greater than or equal to, a minimum of, at least	inequality $\geq$	*x is greater than or equal to y* becomes *the minimum of x is 5* becomes *x is at least 20* becomes
less than	inequality $<$	*x is less than y* becomes *x is less than 5* becomes *5 is less than x* becomes
less than or equal to, a maximum of, not more than	Inequality $\leq$	*X is less than or equal to y* becomes *The maximum of x is 5* becomes *X is not more than* becomes

General Approach

The general approach is as follows:

1. Read and understand the entire problem.
2. Underline key words, focusing on variables, operations, and equalities or inequalities.
3. Convert the key words to mathematical notation (consider meaningful variable names other than x and y).
4. Write two or more inequalities with the same variables.
5. Check the final system of linear inequalities for reasonableness.

Example	***Inequalities***
A high school drama club is putting on their annual theater production. There is a maximum of 800 tickets for the show. The costs of the tickets are \$6 before the day of the show and \$9 on the day of the show. To meet the expenses of the show, the club must sell at least \$5,000 worth of tickets. a) Write a system of inequalities that represent this situation. b) The club sells 440 tickets before the day of the show. Is it possible to sell enough additional tickets on the day of the show to at least meet the expenses of the show? Justify your answer.	*Inequality #1.* $b+d \leq 800$ *Inequality #2.* $6b+9d \geq 5000$

***Variables**:*

Let b represent the number of tickets sold *before* the day of the show.

Let d represent the number of tickets sold the *day* of the show.

Solution Strategy:

Substitute 440 for b in both inequalities.

Inequality #1.

$$b+d \leq 800$$

$$440+d \leq 800$$ They can sell no more than 360 tickets on the day of the show.

$$d \leq 360$$

Inequality #2.

$$\begin{aligned} 6b+9d &\ge 5000 \\ 6(440)+9d &\ge 5000 \\ 2640+9d &\ge 5000 \\ 9d &\ge 5000-2640 \\ 9d &\ge 2360 \\ d &\ge 262.\overline{22} \end{aligned}$$

They need to sell at least 263 tickets on the day of the show.

Yes, it is possible to sell enough additional tickets on the day of the show to meet expenses.

NOTE: Systems of inequalities often have an infinite number of solutions. Graphs are useful to represent the solution sets for such systems. Graphing systems of inequalities is covered in Systems, Lesson 5, Graphing Systems of Linear Inequalities.

DEVELOPING ESSENTIAL SKILLS

Model each context below with a system of inequalities. Define the variables. *Do not solve.*

Contexts	Systems of Inequalities
Nazmun has at least $5,000 in a savings account at the bank. Her savings account balance is more than 5 times greater than her checking account balance.	Let S represent Nazmun's savings account balance. Let C represent Nazmun's checking account balance. Write: $\begin{cases} c \ge 5000 \\ s \ge 5c \end{cases}$
The senior spirit committee is selling food to raise money for the prom. They need to raise at least $500. A deluxe meal with dessert costs $10. A sandwich meal with potato chips costs $5. They have enough food to sell at most 100 meals.	Let *d* represent the number of *deluxe* meals. Let *s* represent the number of *sandwich* meals. Write: $\begin{cases} 10d+5s \ge 500 \\ d+s \le 100 \end{cases}$
Dr. Steve is going to Sal's Diner to buy sandwiches. A small sandwich costs $3.50 and larger hoagie costs $5.00. He needs to buy at least 20 sandwiches, and he can spend no more than $88.	Let s represent the number of *sandwiches.* Let *h* represent the number of *hoagies.* Write: $\begin{cases} 3.5s+5h \le 88 \\ s+h \ge 20 \end{cases}$
The girls soccer team is doing a fundraiser for new soccer uniforms. They need to raise at least $2,000. A local merchant has promised to donate up to 150 plain and deluxe t-shirts to help the team with their fundraiser. Plain t-shirts sell for $8 each and fancy t-shirts sell for $12 each.	Let *p* represent the number of *plain t-shirts.* Let *d* represent the number of *deluxe-shirts.* Write: $\begin{cases} 8p+12d \ge 2000 \\ p+d \le 150 \end{cases}$

Tenzin is working math problems to prepare for the high stakes math exam required for graduation. He wants to work at least 200 math problems before the exam. He estimates that it will take 10 minutes to work a multiple choice problem and 15 minutes to work an open-end problem. He can spend at most 1300 minutes working math problems before the exam. Write a system of inequalities to help Tenzin decide how many multiple choice problems and how many open-end problems he should work before the exam.	Let x represent the number of multiple choice problems. Let y represent the number of open-end problems. Write: $\begin{cases} 10x+15y \le 1300 \\ x+y \ge 200 \end{cases}$

REGENTS EXAM QUESTIONS (through June 2018)

A.CED.A.3: Modeling Systems of Linear Inequalities

269) A high school drama club is putting on their annual theater production. There is a maximum of 800 tickets for the show. The costs of the tickets are $6 before the day of the show and $9 on the day of the show. To meet the expenses of the show, the club must sell at least $5,000 worth of tickets.
a) Write a system of inequalities that represent this situation.
b) The club sells 440 tickets before the day of the show. Is it possible to sell enough additional tickets on the day of the show to at least meet the expenses of the show? Justify your answer.

270) A drama club is selling tickets to the spring musical. The auditorium holds 200 people. Tickets cost $12 at the door and $8.50 if purchased in advance. The drama club has a goal of selling at least $1000 worth of tickets to Saturday's show.

Write a system of intequalities that can be used to model this scenario.

If 50 tickets are sold in advance, what is the minimum number of tickets that must be sold at the door so that the club meets its goal? Justify your answer.

271) The drama club is running a lemonade stand to raise money for its new production. A local grocery store donated cans of lemonade and bottles of water. Cans of lemonade sell for $2 each and bottles of water sell for $1.50 each. The club needs to raise at least $500 to cover the cost of renting costumes. The students can accept a maximum of 360 cans and bottles. Write a system of inequalities that can be used to represent this situation. The club sells 144 cans of lemonade. What is the *least* number of bottles of water that must be sold to cover the cost of renting costumes? Justify your answer.

272) Jordan works for a landscape company during his summer vacation. He is paid $12 per hour for mowing lawns and $14 per hour for planting gardens. He can work a maximum of 40 hours per week, and would like to earn at least $250 this week. If m represents the number of hours mowing lawns and g represents the number of hours planting gardens, which system of inequalities could be used to represent the given conditions?

1) $m+g \le 40$
$12m+14g \ge 250$

3) $m+g \le 40$
$12m+14g \le 250$

2) $m + g \geq 40$
$12m + 14g \leq 250$

4) $m + g \geq 40$
$12m + 14g \geq 250$

SOLUTIONS

269) ANS:

a) $Eq.1 \quad p + d \leq 800$

$Eq.2 \quad \$6p + \$9d \geq 5000$

b) Yes, it is possible. They will need to sell 263 or more tickets on the day of the show. They have 360 tickets left.

Strategy: Write a system of equations, then use it to answer part b.

STEP 1.
Let p represent the number of tickets sold before the day of the show.
Let d represent the number of tickets sold on the day of the show.

Write: $Eq.1 \quad p + d \leq 800$

$Eq.2 \quad \$6p + \$9d \geq \$5000$

STEP 2. Substitute 440 for p in both equations and solve.

$$\begin{aligned} Eq.1 \quad p + d &\leq 800 \\ 440 + d &\leq 800 \\ d &\leq 800 - 440 \\ d &\leq 360 \end{aligned}$$

$$\begin{aligned} Eq.2 \quad \$6p + \$9d &\geq \$5000 \\ \$6(440) + \$9d &\geq \$5000 \\ \$2640 + \$9d &\geq \$5000 \\ \$9d &\geq \$5000 - \$2640 \\ \$9d &\geq \$2360 \\ d &\geq \frac{\$2360}{\$9} \\ d &\geq 262.2 \end{aligned}$$

DIMS? Does It Make Sense? Yes. They could cover their costs by selling 263 tickets and make almost \$9000 over costs if they sell 360 tickets on the day of the show.

PTS: 2 NAT: A.CED.A.3 TOP: Modeling Systems of Linear Inequalities

270) ANS:
Answer: 48 Tickets

PART 1: Write a system of inequalitites.
Let D represent the number of tickets sold at the door.
Let A represent the number of tickets sold in advance.

$$12D + 8.50A \geq 1000$$

$$D + A \leq 200$$

PART 2: Solve for 50 tickets sold in advance.

$$12D + 8.50A \ge 1000$$

$$12D + 8.50(50) \ge 1000$$

$$12D + 425 \ge 1000$$

$$12D \ge 575$$

$$D \ge \frac{575}{12}$$

$$D \ge 47.916$$

The drama club needs to sell at least 48 tickets at the door to meet its goal of making $1000.

PTS: 4 NAT: A.REI.A.2

271) ANS:

STEP 1. Write a system of inequalities.
Let L represent a can of lemondae.
Let W represent a bottle of water.
Write:

Equation 1
$2L + 1.5W \ge 500$
Equation 2
$L + \quad W \le 360$

STEP 2. Use Equation 1 to determine the least amount of W required when L=144.

$$2L + 1.5W \ge 500$$

$$2(144) + 1.5W \ge 500$$

$$288 + 1.5W \ge 500$$

$$1.5W \ge 212$$

$$W \ge \frac{212}{1.5}$$

$$W \ge 141.\overline{33}$$

You cannot sell $.\overline{33}$ bottles of water, so the drama club needs to sell at least

142 bottles
of water

PTS: 4 NAT: A.CED.A.3 TOP: Modeling Systems of Linear Inequalities

272) ANS: 1

Strategy: Translate the words into two inequalities.

Let *m* represent the number of hours mowing.
Let *g* represent the number of hours gardening.

He is paid $12 per hour for mowing lawns and $14 per hour for planting gardens. He can work a maximum of 40 hours per week, and would like to earn at least $250 this week.

Inequality #1 Hours per week.

$$m + g \le 40$$

This inequality says:
the number of hours mowing (m) and the number of hours gardening (g)
must be less than or equal to 40 hours.

Inequality #2 Money earned

$$12m + 14g \ge 250$$

This inequality says:

the money earned mowing (12m) and the money earned gardening (14g)
must be greater than or equal to $250.

SYSTEMS

Graphing Systems of Linear Inequalities

Common Core Standard	Next Generation Standard
A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	**AI-A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. **Note: Graphing linear equations is a fluency recommendation for Algebra I. Students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity; as well as modeling linear phenomena (including modeling using systems of linear inequalities in two variables).**

LEARNING OBJECTIVES

Students will be able to:

1) Graph the solution set of a system of linear inequalities.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

boundary line	shading	solution set
dashed line	solid line	testing a point

BIG IDEAS

A linear inequality describes a region of the coordinate plane that has a **boundary line**. Every point in the region is a **solution of the inequality**.

Two or more linear inequalities together form a **system of linear inequalities**. Note that there are two or more boundary lines in a system of linear inequalities.

A **solution of a system of linear inequalities** makes each inequality in the system true. The graph of a system shows all of its solutions.

Graphing a Linear Inequality

Step One. Change the inequality sign to an equal sign and graph the boundary line in the same manner that you would graph a linear equation.

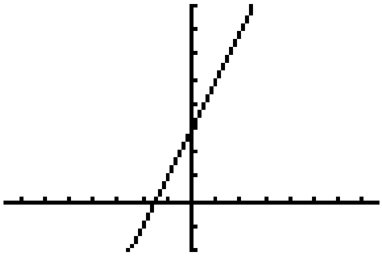
- When the inequality sign **contains** an equality bar beneath it, use a solid line for the boundary.
- When the inequality sign **does not contain** an equality bar beneath it, use a dashed or dotted line for the boundary

Step Two. Restore the inequality sign and test a point to see which side of the boundary line the solution is on. The point (0,0) is a good point to test since it simplifies any multiplication. However, if the boundary line passes through the point (0,0), another point not on the boundary line must be selected for testing.

- If the test point makes the inequality true, shade the side of the boundary line that includes the test point.
- If the test point makes the inequality not true, shade the side of the boundary line does not include the test point.

Example Graph $y < 2x + 3$

First, change the inequality sign an equal sign and graph the line: $y = 2x + 3$. This is the boundary line of the solution. Since there is no equality line beneath the inequality symbol, use a dashed line for the boundary.



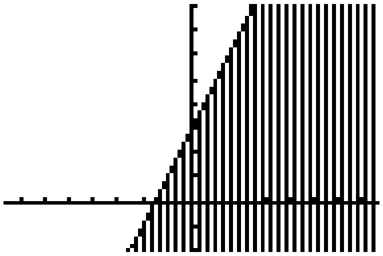
Next, **test a point** to see which side of the boundary line the solution is on. Try (0,0), since it makes the multiplication easy, but remember that any point will do.

$y < 2x + 3$

$0 < 2(0) + 3$

$0 < 3$ True, so the solution of the inequality is the region that contains the point $(0,0)$.

Therefore, we shade the side of the boundary line that contains the point (0,0).



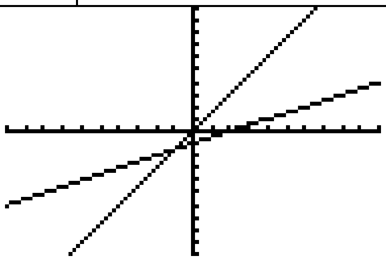
Note: The TI-83+ graphing calculator does not have the ability to distinguish between solid and dashed lines on a graph of an inequality. The less than and greater than symbols are input using the far-left column of symbols that can be accessed through the $\boxed{\text{Y=}}$ feature.

Graphing a System of Linear Inequalities. Systems of linear inequalities are graphed in the same manner as systems of equations are graphed. The solution of the system of inequalities is the region of the coordinate plane that is shaded by both inequalities.

Example: Graph the system: $\begin{array}{l} 4y \ge 6x \\ -3x+6y \le -6 \end{array}$

First, convert both inequalities to slope-intercept form and graph.

$4y \ge 6x$ $\frac{4y}{4} \ge \frac{6x}{4}$ $y \ge \frac{3}{2}x$ m=$\frac{3}{2}$, b=0	$-3x+6y \le -6$ $6y \le -6+3x$ $6y \le 3x-6$ D_2 $\frac{6y}{6} \le \frac{3x}{6} - \frac{6}{6}$ $y \le \frac{1}{2}x - 1$ m=$\frac{1}{2}$, b=-1



Next, test a point in each inequality and shade appropriately.

- Since point (0,0) is on the boundary line of $y \ge \frac{3}{2}x$, select another point, such as (0,1).

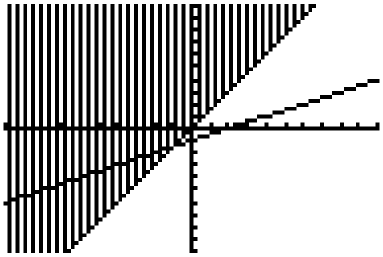
$y \geq \frac{3}{2}x$

Test (0,1)

$1 \geq \frac{3}{2}(0)$

$1 \geq 0$ This is true, so the point (0,1) is in the solution set of this inequality.

Therefore, we shade the side of the boundary line that includes point (0,1).



- Since (0,0) is not on the boundary line of $y \leq \frac{1}{2}x - 1$, we can use (0,0) as our test point, as follows:

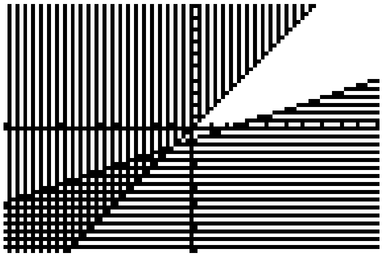
  $y \leq \frac{1}{2}x - 1$

  Test (0,0)

  $0 \leq \frac{1}{2}(0) - 1$

  $0 \leq -1$ This is not true, so the point (0,0) is not in the solution set of this inequality.

  We therefore must shade the side of the boundary line that does not incude the point (0,0).



Note that the system of inequalities divides the coordinate plane into four sections. The solution set for the system of inequalities is the area where the two shaded regions overlap.

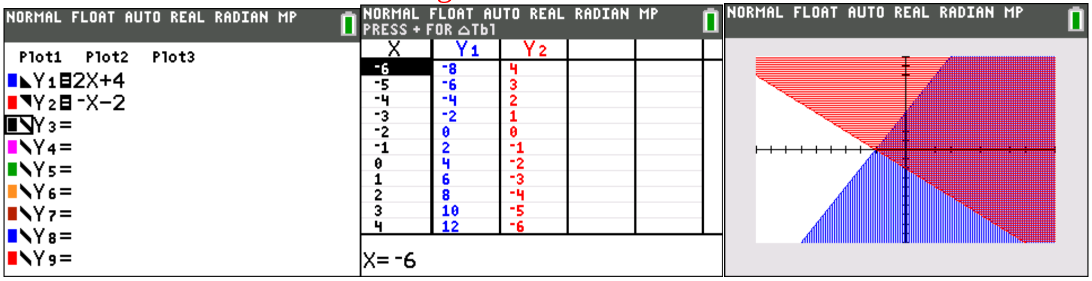
DEVELOPING ESSENTIAL SKILLS

Graph the solution sets of the following systems of inequalities. State if the origin (0,0) *is* or *is not* in the solution set.

1.	2.	3.	4.	5.
$\begin{cases} y \leq 2x + 4 \\ y \geq -x - 2 \end{cases}$	$\begin{cases} y \leq x - 3 \\ y \geq -2x + 2 \end{cases}$	$\begin{cases} y \geq x - 3 \\ y \geq -2x + 2 \end{cases}$	$\begin{cases} y \leq x - 3 \\ y \leq -2x + 2 \end{cases}$	$\begin{cases} y \geq x - 3 \\ y \leq -2x + 2 \end{cases}$

Answers

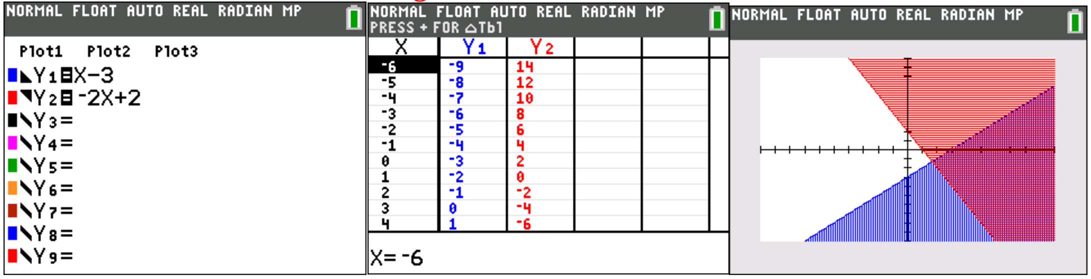
1. The origin is *in* the solution set.



```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
 Y1=2X+4
 Y2=-X-2
 Y3=
 Y4=
 Y5=
 Y6=
 Y7=
 Y8=
 Y9=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl
  X    Y1   Y2
 -6    -8   4
 -5    -6   3
 -4    -4   2
 -3    -2   1
 -2    0    0
 -1    2    -1
 0     4    -2
 1     6    -3
 2     8    -4
 3     10   -5
 4     12   -6
X=-6
```

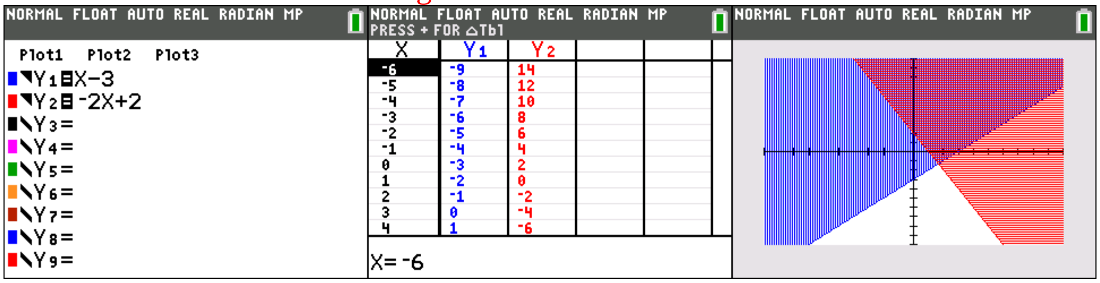
2. The origin is *not in* the solution set.



```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
 Y1=X-3
 Y2=-2X+2
 Y3=
 Y4=
 Y5=
 Y6=
 Y7=
 Y8=
 Y9=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl
  X    Y1   Y2
 -6    -9   14
 -5    -8   12
 -4    -7   10
 -3    -6   8
 -2    -5   6
 -1    -4   4
 0     -3   2
 1     -2   0
 2     -1   -2
 3     0    -4
 4     1    -6
X=-6
```

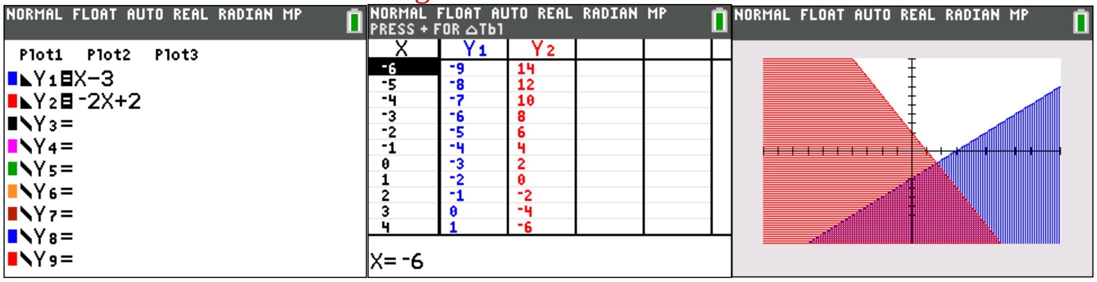
3. The origin is *not in* the solution set.



```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
 Y1=X-3
 Y2=-2X+2
 Y3=
 Y4=
 Y5=
 Y6=
 Y7=
 Y8=
 Y9=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl
  X    Y1   Y2
 -6    -9   14
 -5    -8   12
 -4    -7   10
 -3    -6   8
 -2    -5   6
 -1    -4   4
 0     -3   2
 1     -2   0
 2     -1   -2
 3     0    -4
 4     1    -6
X=-6
```

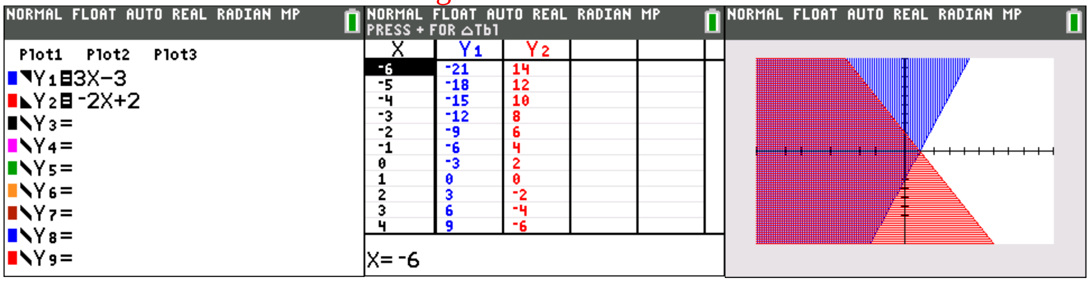
4. The origin is *not in* the solution set.



```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
 Y1=X-3
 Y2=-2X+2
 Y3=
 Y4=
 Y5=
 Y6=
 Y7=
 Y8=
 Y9=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl
  X    Y1   Y2
 -6    -9   14
 -5    -8   12
 -4    -7   10
 -3    -6   8
 -2    -5   6
 -1    -4   4
 0     -3   2
 1     -2   0
 2     -1   -2
 3     0    -4
 4     1    -6
X=-6
```

5. The origin is *in* the solution set.



```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
 Y1=3X-3
 Y2=-2X+2
 Y3=
 Y4=
 Y5=
 Y6=
 Y7=
 Y8=
 Y9=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl
  X    Y1    Y2
 -6    -21   14
 -5    -18   12
 -4    -15   10
 -3    -12   8
 -2    -9    6
 -1    -6    4
 0     -3    2
 1     0     0
 2     3     -2
 3     6     -4
 4     9     -6
X=-6
```

REGENTS EXAM QUESTIONS

A.REI.D.12: Graphing Systems of Linear Inequalities

273)Which ordered pair is *not* in the solution set of $y > -\frac{1}{2}x + 5$ and $y \le 3x - 2$?

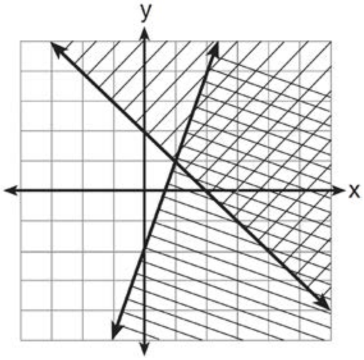
1) $(5,3)$
2) $(4,3)$
3) $(3,4)$
4) $(4,4)$

274) Given: $y + x > 2$

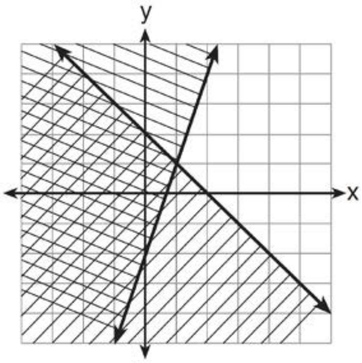
$y \le 3x - 2$

Which graph shows the solution of the given set of inequalities?

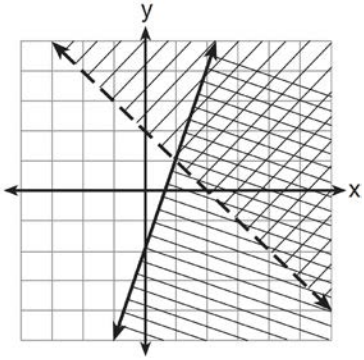
1)



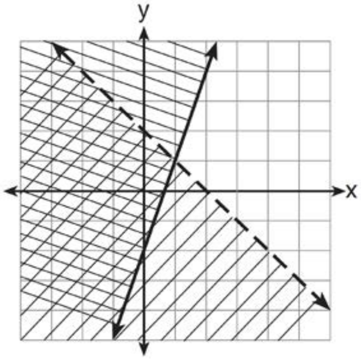
3)



2)



4)



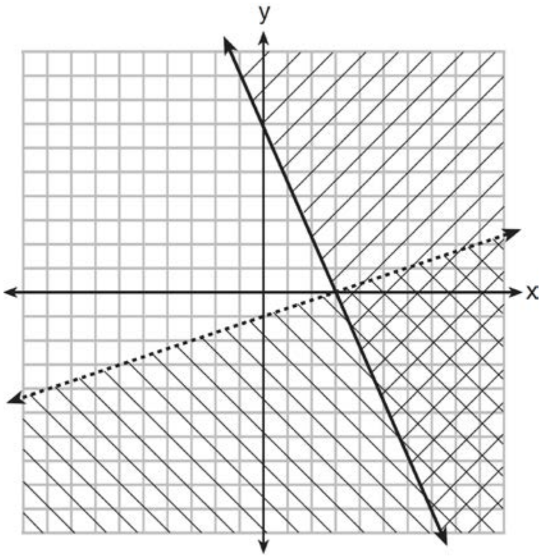
275) Which point is a solution to the system below?

$$2y < -12x + 4$$

$$y < -6x + 4$$

1) $\left(1, \frac{1}{2}\right)$
2) $(0,6)$
3) $\left(-\frac{1}{2}, 5\right)$
4) $(-3,2)$

276) What is one point that lies in the solution set of the system of inequalities graphed below?



1) $(7, 0)$
2) $(3, 0)$
3) $(0, 7)$
4) $(-3, 5)$

277) First consider the system of equations $y = -\frac{1}{2}x + 1$ and $y = x - 5$. Then consider the system of inequalities $y > -\frac{1}{2}x + 1$ and $y < x - 5$. When comparing the number of solutions in each of these systems, which statement is true?

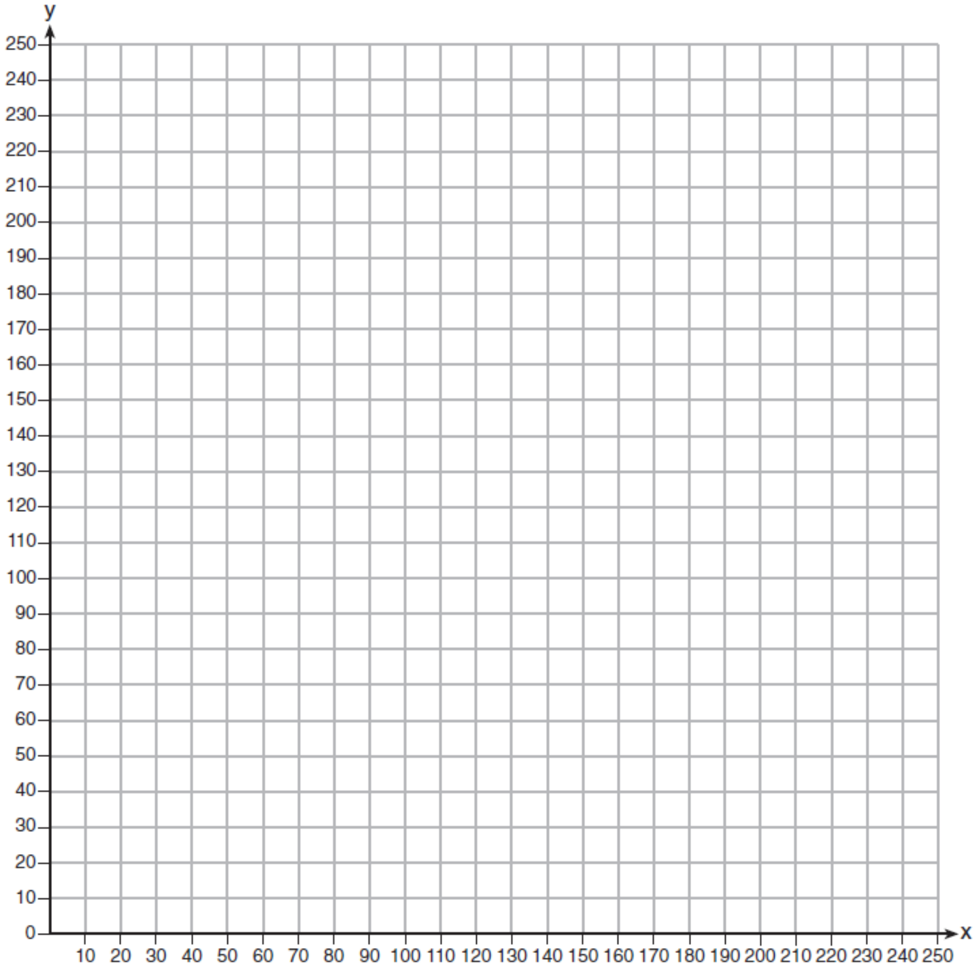
1) Both systems have an infinite number of solutions.
2) The system of equations has more solutions.
3) The system of inequalities has more solutions.
4) Both systems have only one solution.

278) The Reel Good Cinema is conducting a mathematical study. In its theater, there are 200 seats. Adult tickets cost \$12.50 and child tickets cost \$6.25. The cinema's goal is to sell at least \$1500 worth of tickets for the theater.

Write a system of linear inequalities that can be used to find the possible combinations of adult tickets, *x*, and child tickets, *y*, that would satisfy the cinema's goal.

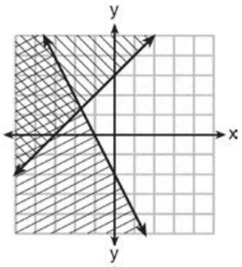
Graph the solution to this system of inequalities on the set of axes below. Label the solution with an *S*.

Marta claims that selling 30 adult tickets and 80 child tickets will result in meeting the cinema's goal. Explain whether she is correct or incorrect, based on the graph drawn.

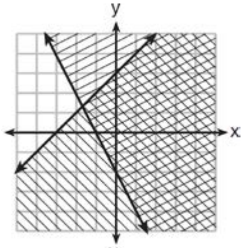


279) Which graph represents the solution of $y \le x + 3$ and $y \ge -2x - 2$?

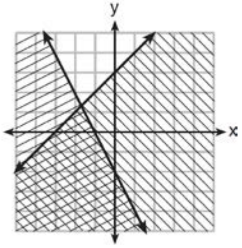
1)



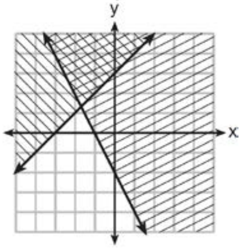
3)



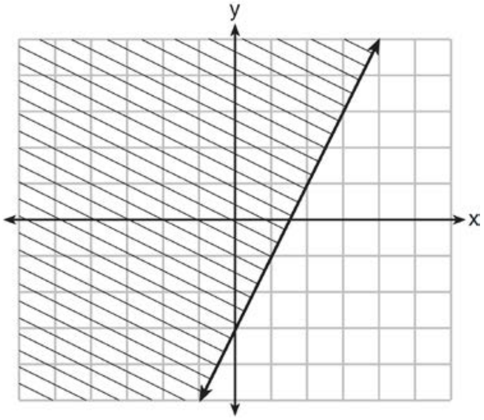
2)



4)

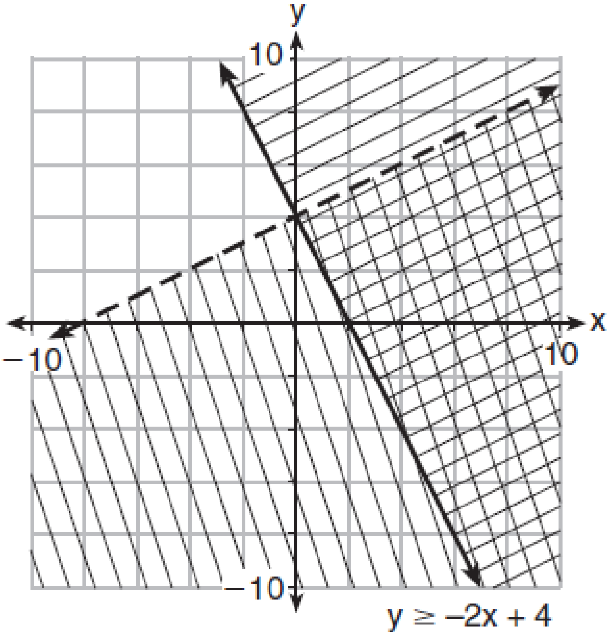


280) The graph of an inequality is shown below.

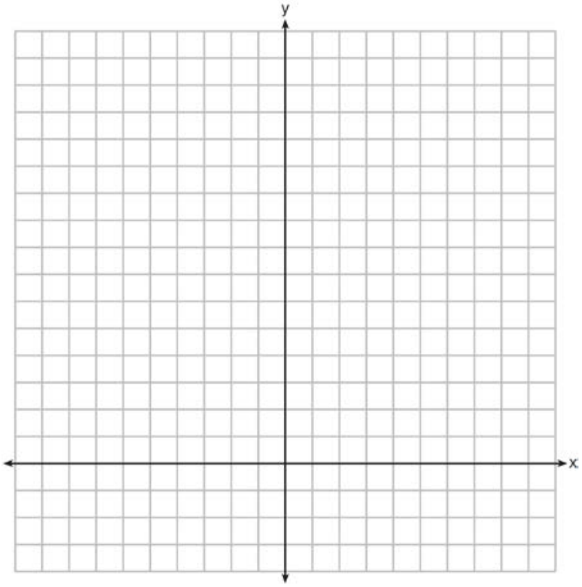


a) Write the inequality represented by the graph.
b) On the same set of axes, graph the inequality $x + 2y < 4$.
c) The two inequalities graphed on the set of axes form a system. Oscar thinks that the point $(2, 1)$ is in the solution set for this system of inequalities. Determine and state whether you agree with Oscar. Explain your reasoning.

281) Determine if the point $(0, 4)$ is a solution to the system of inequalities graphed below. Justify your answer.



282) The sum of two numbers, *x* and *y*, is more than 8. When you double *x* and add it to *y*, the sum is less than 14. Graph the inequalities that represent this scenario on the set of axes below.

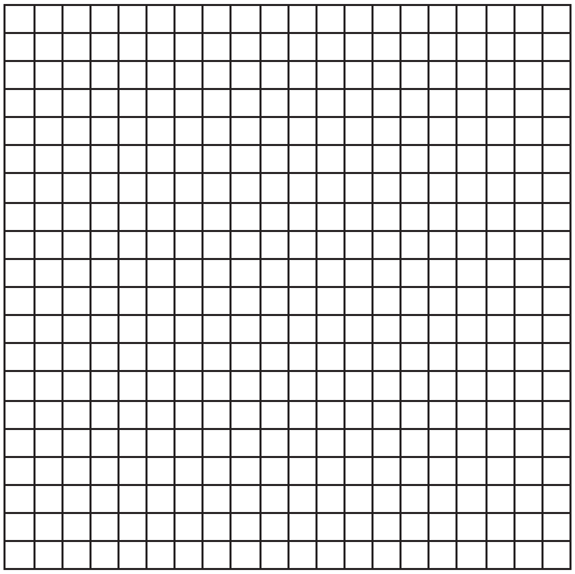


Kai says that the point $(6, 2)$ is a solution to this system. Determine if he is correct and explain your reasoning.

283) Solve the following system of inequalities graphically on the grid below and label the solution *S*.

$$3x + 4y > 20$$

$$x < 3y - 18$$

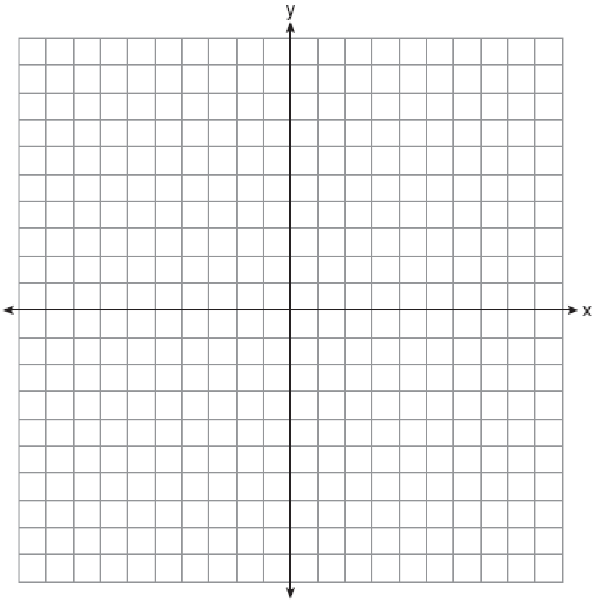


Is the point $(3, 7)$ in the solution set? Explain your answer.

284) On the set of axes below, graph the following system of inequalities:

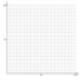
$$2y + 3x \leq 14$$

$$4x - y < 2$$



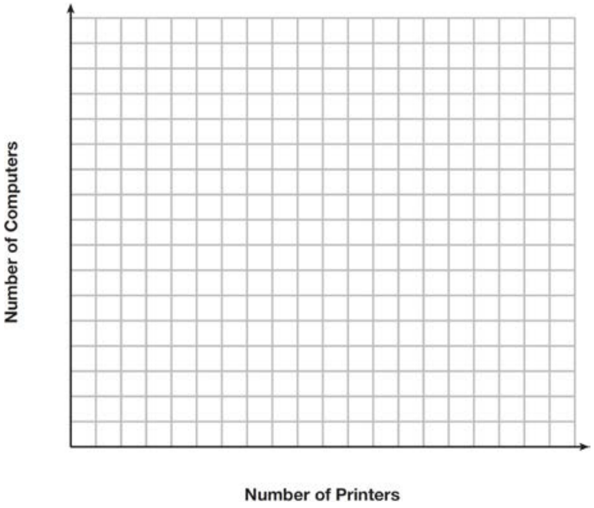
Determine if the point $(1, 2)$ is in the solution set. Explain your answer.

285) Edith babysits for x hours a week after school at a job that pays \$4 an hour. She has accepted a job that pays \$8 an hour as a library assistant working y hours a week. She will work both jobs. She is able to work no more than 15 hours a week, due to school commitments. Edith wants to earn at least \$80 a week, working a combination of both jobs. Write a system of inequalities that can be used to represent the situation. Graph these inequalities on the set of axes below.



Determine and state one combination of hours that will allow Edith to earn *at least* \$80 per week while working *no more than* 15 hours.

286) An on-line electronics store must sell at least \$2500 worth of printers and computers per day. Each printer costs \$50 and each computer costs \$500. The store can ship a maximum of 15 items per day. On the set of axes below, graph a system of inequalities that models these constraints.



Determine a combination of printers and computers that would allow the electronics store to meet all of the constraints. Explain how you obtained your answer.

SOLUTIONS

273) Which ordered pair is *not* in the solution set of $y > -\frac{1}{2}x + 5$ and $y \le 3x - 2$?

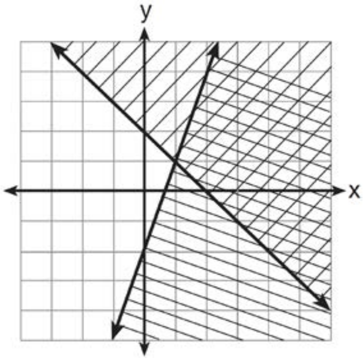
1) $(5, 3)$
3) $(3, 4)$
2) $(4, 3)$
4) $(4, 4)$

274) Given: $y + x > 2$

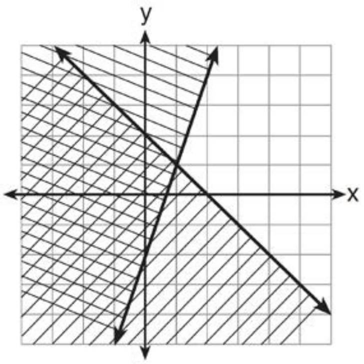
$y \le 3x - 2$

Which graph shows the solution of the given set of inequalities?

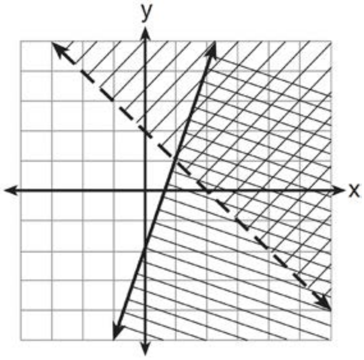
1)



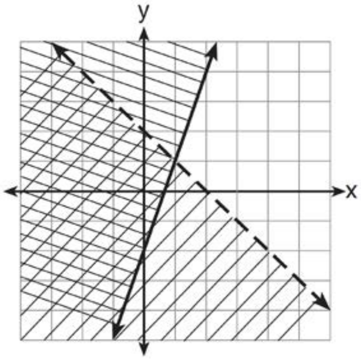
3)



2)



4)



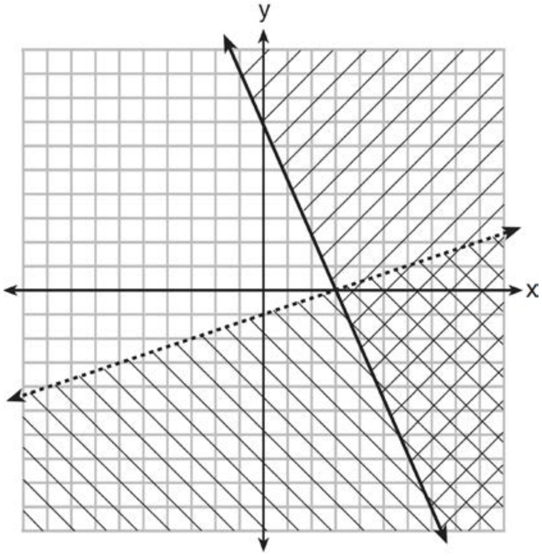
275) Which point is a solution to the system below?

$$2y < -12x + 4$$

$$y < -6x + 4$$

1) $\left(1, \frac{1}{2}\right)$
3) $\left(-\frac{1}{2}, 5\right)$
2) $(0, 6)$
4) $(-3, 2)$

276) What is one point that lies in the solution set of the system of inequalities graphed below?



1) $(7, 0)$
2) $(3, 0)$
3) $(0, 7)$
4) $(-3, 5)$

277) First consider the system of equations $y = -\frac{1}{2}x + 1$ and $y = x - 5$. Then consider the system of inequalities $y > -\frac{1}{2}x + 1$ and $y < x - 5$. When comparing the number of solutions in each of these systems, which statement is true?

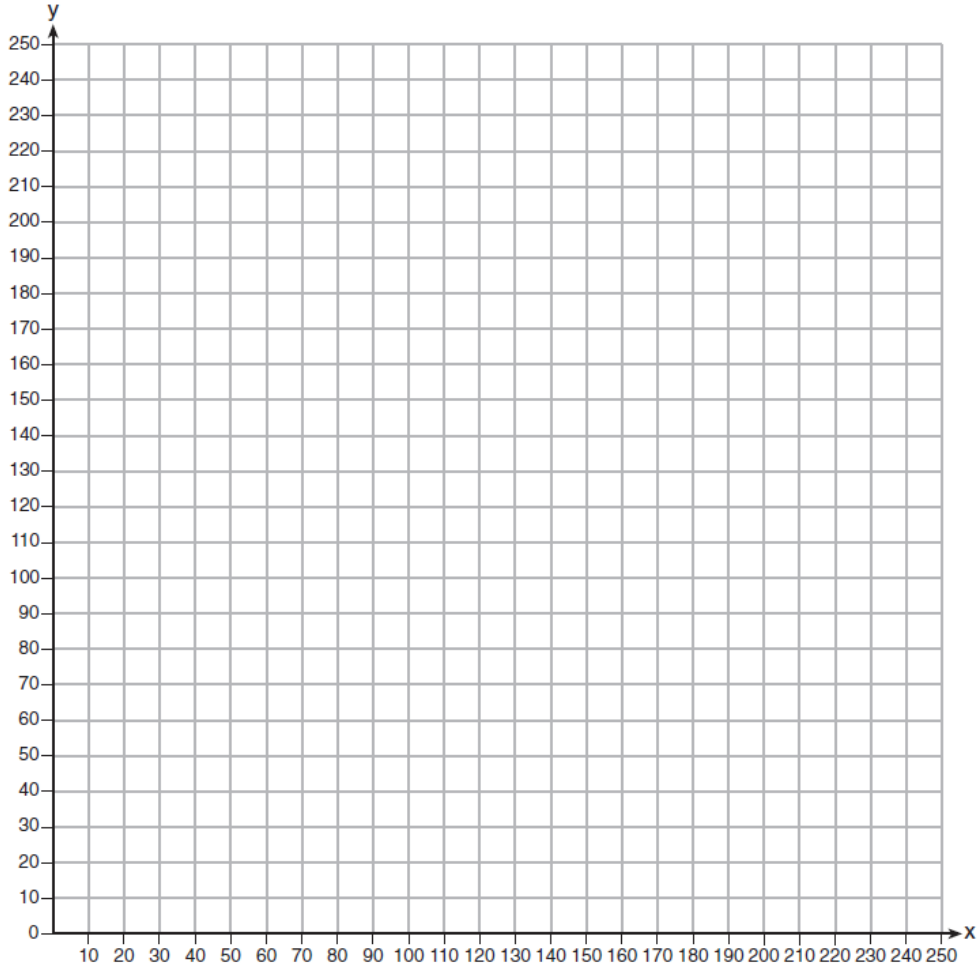
1) Both systems have an infinite number of solutions.
2) The system of equations has more solutions.
3) The system of inequalities has more solutions.
4) Both systems have only one solution.

278) The Reel Good Cinema is conducting a mathematical study. In its theater, there are 200 seats. Adult tickets cost \$12.50 and child tickets cost \$6.25. The cinema's goal is to sell at least \$1500 worth of tickets for the theater.

Write a system of linear inequalities that can be used to find the possible combinations of adult tickets, x, and child tickets, y, that would satisfy the cinema's goal.

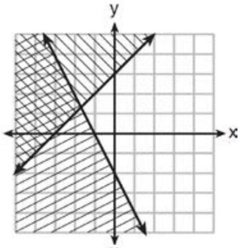
Graph the solution to this system of inequalities on the set of axes below. Label the solution with an *S*.

Marta claims that selling 30 adult tickets and 80 child tickets will result in meeting the cinema's goal. Explain whether she is correct or incorrect, based on the graph drawn.

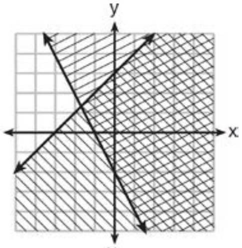


279) Which graph represents the solution of $y \le x + 3$ and $y \ge -2x - 2$?

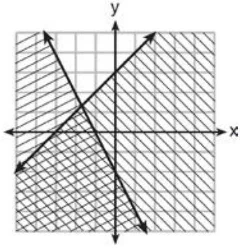
1)



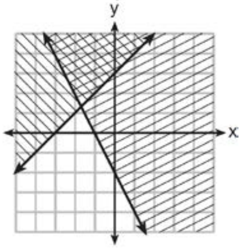
3)



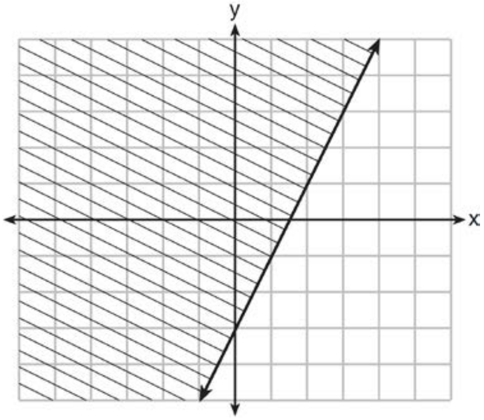
2)



4)

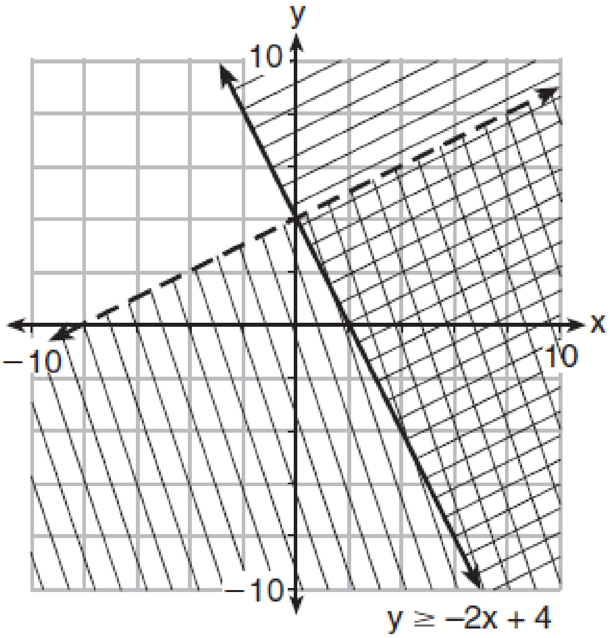


280) The graph of an inequality is shown below.

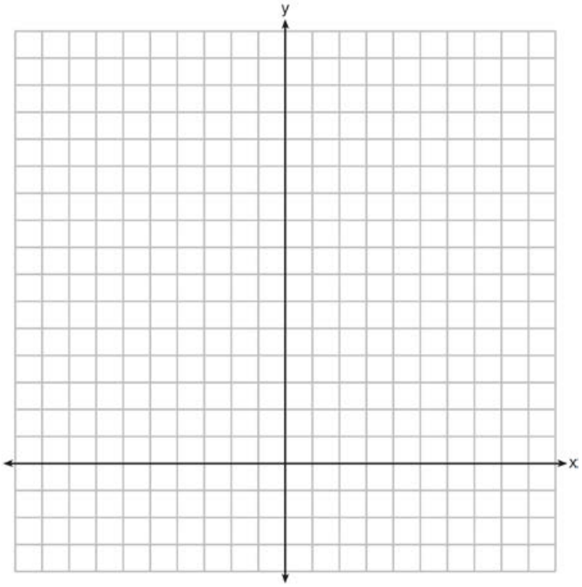


a) Write the inequality represented by the graph.
b) On the same set of axes, graph the inequality $x + 2y < 4$.
c) The two inequalities graphed on the set of axes form a system. Oscar thinks that the point $(2, 1)$ is in the solution set for this system of inequalities. Determine and state whether you agree with Oscar. Explain your reasoning.

281) Determine if the point $(0, 4)$ is a solution to the system of inequalities graphed below. Justify your answer.



282) The sum of two numbers, *x* and *y*, is more than 8. When you double *x* and add it to *y*, the sum is less than 14. Graph the inequalities that represent this scenario on the set of axes below.

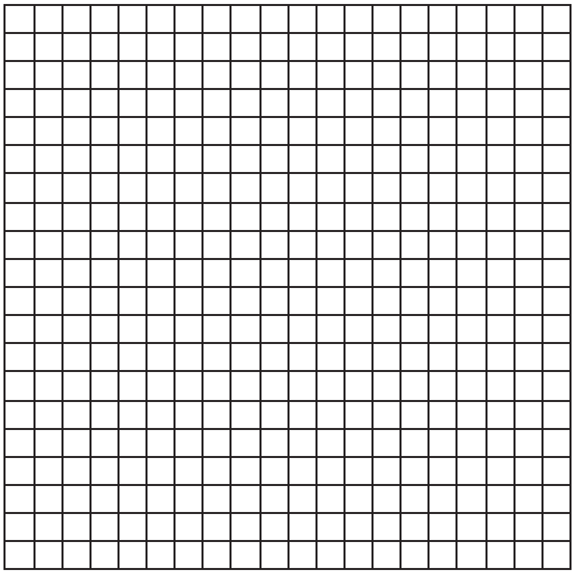


Kai says that the point $(6, 2)$ is a solution to this system. Determine if he is correct and explain your reasoning.

283) Solve the following system of inequalities graphically on the grid below and label the solution *S*.

$$3x + 4y > 20$$

$$x < 3y - 18$$

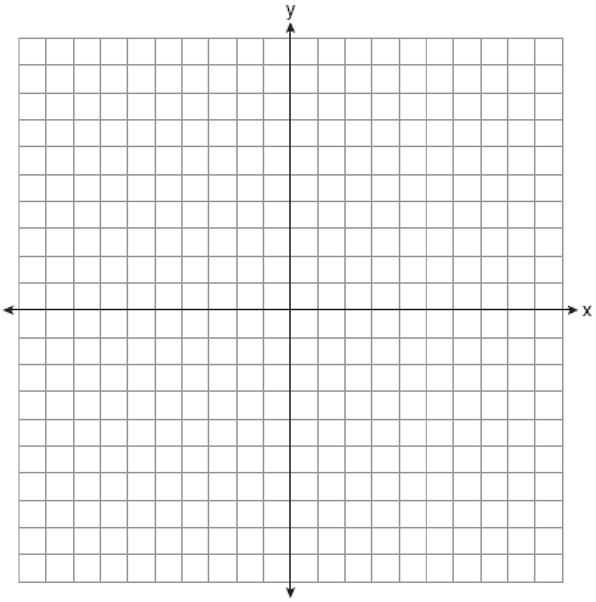


Is the point $(3, 7)$ in the solution set? Explain your answer.

284) On the set of axes below, graph the following system of inequalities:

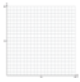
$$2y + 3x \leq 14$$

$$4x - y < 2$$



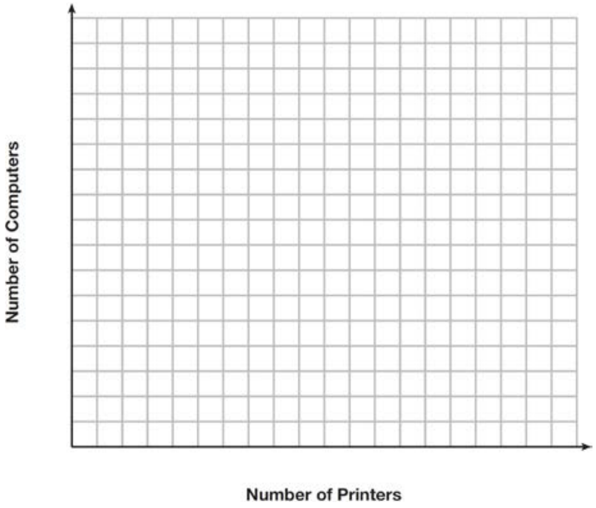
Determine if the point $(1, 2)$ is in the solution set. Explain your answer.

285) Edith babysits for x hours a week after school at a job that pays \$4 an hour. She has accepted a job that pays \$8 an hour as a library assistant working y hours a week. She will work both jobs. She is able to work no more than 15 hours a week, due to school commitments. Edith wants to earn at least \$80 a week, working a combination of both jobs. Write a system of inequalities that can be used to represent the situation. Graph these inequalities on the set of axes below.



Determine and state one combination of hours that will allow Edith to earn *at least* \$80 per week while working *no more than* 15 hours.

286) An on-line electronics store must sell at least \$2500 worth of printers and computers per day. Each printer costs \$50 and each computer costs \$500. The store can ship a maximum of 15 items per day. On the set of axes below, graph a system of inequalities that models these constraints.



Determine a combination of printers and computers that would allow the electronics store to meet all of the constraints.   Explain how you obtained your answer.

I – Systems, Lesson 6, Quadratic-Linear Systems (r. 2018)

SYSTEMS

Quadratic-Linear Systems

Common Core Standard	**Next Generation Standards**
No Current Standard in New York	**AI-A.REI.7a** Solve a system, with rational solutions, consisting of a linear equation and a quadratic equation (parabolas only) in two variables both algebraically and graphically. (Shared standard with Algebra II)
A-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations *y=f(x)* and *y=g(x)* intersect are the solutions of the equation *f(x)=g(x)*; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, ~~or find successive approximations~~. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential~~, and logarithmic functions~~. ~~PARCC: Tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. Finding the solutions approximately is limited to cases where f(x) and g(x) are polynomial functions.~~	**AI-A.REI.11** Given the equations $y = f(x)$ and $y = g(x)$: i) **recognize** that each x-coordinate of the intersection(s) is the solution to the equation $f(x) = g(x)$; ii)find the solutions approximately using technology to graph the functions or make tables of values; and iii) **interpret the solution in context**. (Shared standard with Algebra II) **Notes: Algebra I tasks are limited to cases where *f(x)* and *g(x)* are linear, polynomial, absolute value, and exponential functions of the form** $f(x) = a(b)^x$ **where *a* > 0 and *b* > 0 (*b* ≠ 1).** **Students should be taught to find the solutions approximately by using technology to graph the functions *and* by making tables of values. When solving any problem, students can choose either strategy.**

LEARNING OBJECTIVES

Students will be able to:

1) solve quadratic-linear systems of equations algebraically or by graphing.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

linear equation	quadratic equation	solution

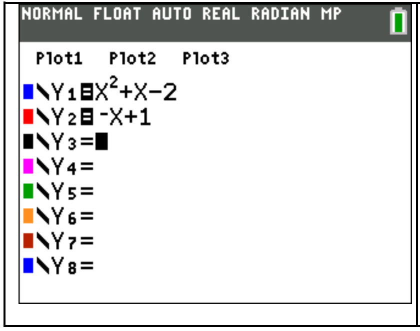
BIG IDEAS

Quadratic-linear systems are solved in the same ways that systems of linear equations and/or systems of linear inequalities are solved, either algebraically or by graphing.

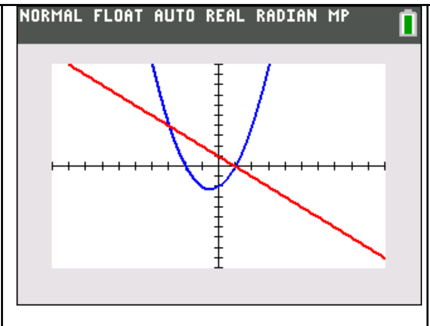
A **solution of a system** of equations makes each equation in the system true. Solutions can be found using three different views of a function. Quadratic linear systems will have:

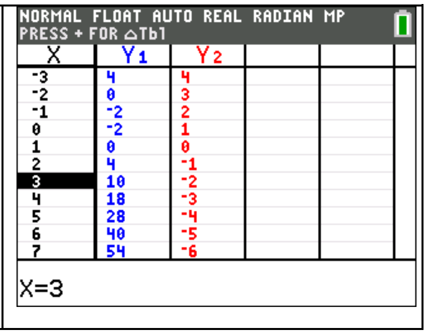
- no solution (the graphs do not intersect),
- one solution (the graphs intersect at one point)
- two solutions (the graphs intersect at two points).

Example: If $y_1 = x^2 + x + 2$ and $y_2 = -x + 1$, then the solution may be found using a graphing calculator, as follows:



```
NORMAL FLOAT AUTO REAL RADIAN MP
 Plot1  Plot2  Plot3
■\Y1=X²+X−2
■\Y2=-X+1
■\Y3=
■\Y4=
■\Y5=
■\Y6=
■\Y7=
■\Y8=
```





```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl
```

X	Y1	Y2
-3	4	4
-2	0	3
-1	-2	2
0	-2	1
1	0	0
2	4	-1
3	10	-2
4	18	-3
5	28	-4
6	40	-5
7	54	-6

X=3

The solutions to this quadratric-linear system are (-3,4) and (1,0).

NOTE: The calculate intersection function of some graphing calculators can be used to identify solutions.

How to Solve a Quadratic Linear System Algebraically

Step 1	Step 2	Step 3	Step 4	Step 4
Isolate the same variable in both equations.	Set the opposite expressions equal to one another.	Solve for the first variable. NOTE: Strategies other than factoring can be used.	Input the solutions from Step 3 into an equation and solve for the second variable.	Write the solutions as ordered pairs.
$y = x^2 + 6x + 3$ $y = 3x + 7$	$x^2 + 6x + 3 = 3x + 7$	$x^2 + 6x + 3 = 3x + 7$ $x^2 + 3x - 4 = 0$ $(x+4)(x-1) = 0$ $x = \{-4, 1\}$	$y = 3x + 7$ $y = 3(-4) + 7$ $y = -5$ $y = 3x + 7$ $y = 3(1) + 7$ $y = 10$	Two solutions: (-4,-5) and (1,10)

DEVELOPING ESSENTIAL SKILLS

Solve the following quadratic-linear systems of equations algebraically and by graphing.

1.	2.	3.	4.	5.
$y=x^2-4x+6$ $y=x+2$	$y=x^2-9x+18$ $y=x+2$	$y=x^2-10x+14$ $y=7x-16$	$y=x^2+5x+4$ $y=x+4$	$y=x^2+8x+16$ $y=x+6$

Answers

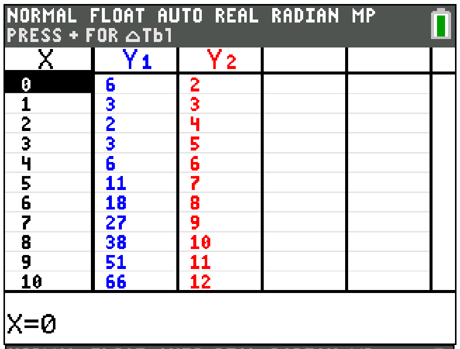
1.

$$y=x^2-4x+6$$
$$y=x+2$$

$$x^2-4x+6=x+2$$
$$x^2-5x+4=0$$
$$(x-4)(x-1)=0$$
$$x=\{1,4\}$$

$$y=x+2$$
$$y=(1)+2=3$$
$$y=(4)+2=6$$

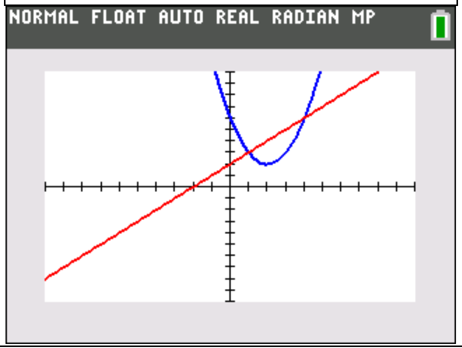
$$\boxed{(1,3)\ \ and\ \ (4,6)}$$



NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1	Y2
0	6	2
1	3	3
2	2	4
3	3	5
4	6	6
5	11	7
6	18	8
7	27	9
8	38	10
9	51	11
10	66	12

X=0



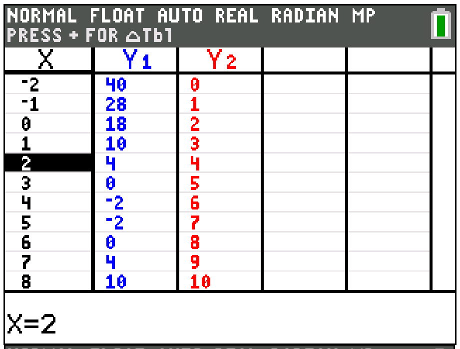
2.

$$y=x^2-9x+18$$
$$y=x+2$$

$$x^2-9x+18=x+2$$
$$x^2-10x+16=0$$
$$(x-8)(x-2)=0$$
$$x=\{2,8\}$$

$$y=x+2$$
$$y=(2)+2=4$$
$$y=(8)+2=10$$

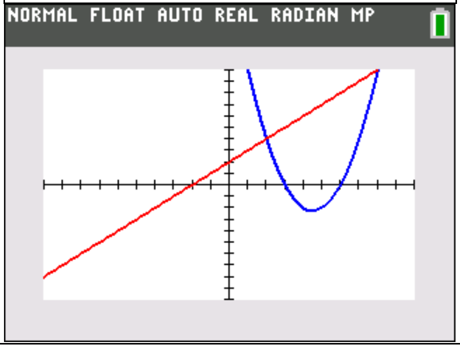
$$\boxed{(2,4)\ \ and\ \ (8,10)}$$



NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1	Y2
-2	40	0
-1	28	1
0	18	2
1	10	3
2	4	4
3	0	5
4	-2	6
5	-2	7
6	0	8
7	4	9
8	10	10

X=2



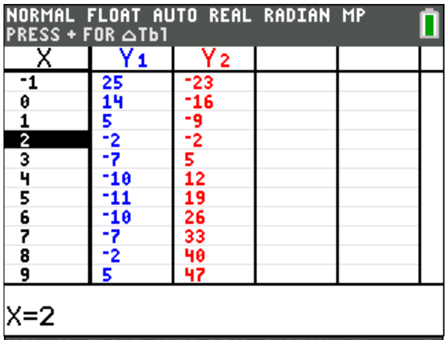
3.

$$y = x^2 - 10x + 14$$
$$y = 7x - 16$$

$$x^2 - 10x + 14 = 7x - 16$$
$$x^2 - 17x + 30 = 0$$
$$(x-15)(x-2) = 0$$
$$x = \{2, 15\}$$

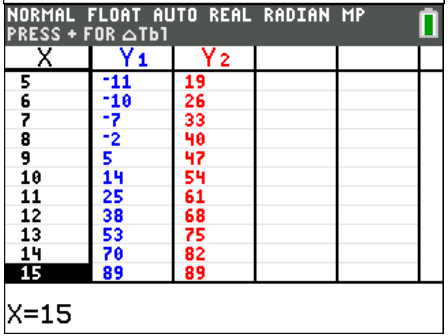
$$y = 7x - 16$$
$$y = 7(2) - 16 = -2$$
$$y = 7(15) - 16 = 89$$

$$(2,-2) \quad \textit{and} \quad (15,89)$$



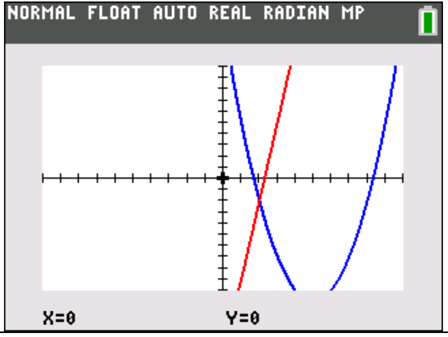
X	Y1	Y2
-1	25	-23
0	14	-16
1	5	-9
2	-2	-2
3	-7	5
4	-10	12
5	-11	19
6	-10	26
7	-7	33
8	-2	40
9	5	47

X=2



X	Y1	Y2
5	-11	19
6	-10	26
7	-7	33
8	-2	40
9	5	47
10	14	54
11	25	61
12	38	68
13	53	75
14	70	82
15	89	89

X=15



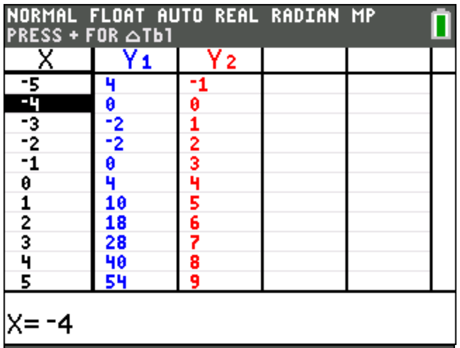
4.

$$y = x^2 + 5x + 4$$
$$y = x + 4$$

$$x^2 + 5x + 4 = x + 4$$
$$x^2 + 4x = x + 4$$
$$x(x+4)$$
$$x = \{0, -4\}$$

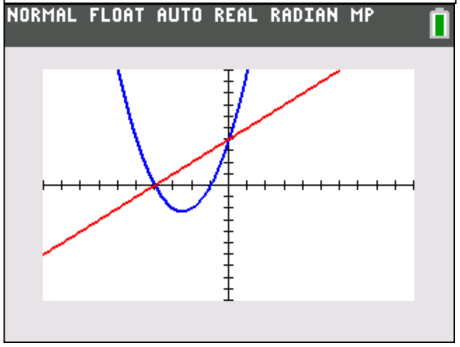
$$y = x + 4$$
$$y = (0) + 4 = 4$$
$$y = (-4) + 4 = 0$$

$$(0,4) \quad \textit{and} \quad (-4,0)$$



X	Y1	Y2
-5	4	-1
-4	0	0
-3	-2	1
-2	-2	2
-1	0	3
0	4	4
1	10	5
2	18	6
3	28	7
4	40	8
5	54	9

X= -4



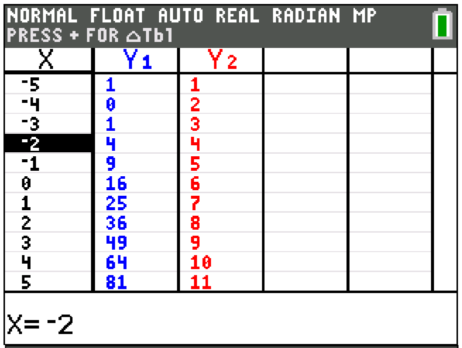
5.

$$y = x^2 + 8x + 16$$
$$y = x + 6$$

$$x^2 + 8x + 16 = x + 6$$
$$x^2 + 7x + 10 = 0$$
$$(x+5)(x+2) = 0$$
$$x = \{-5, -2\}$$

$$y = x + 6$$
$$y = (-5) + 6 = 1$$
$$y = (-2) + 6 = 4$$

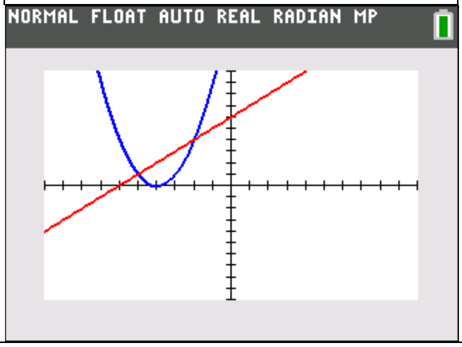
$$\boxed{(-5,1) \quad and \quad (-2,4)}$$



NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y₁	Y₂
-5	1	1
-4	0	2
-3	1	3
-2	4	4
-1	9	5
0	16	6
1	25	7
2	36	8
3	49	9
4	64	10
5	81	11

X= -2

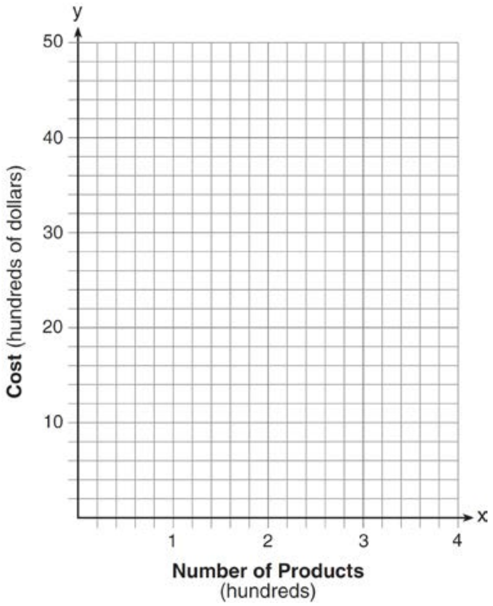


A.REI.C.7, A.REI.D.11: SOLVE QUADRATIC-LINEAR SYSTEMS

287) The graphs of $y = x^2 - 3$ and $y = 3x - 4$ intersect at approximately

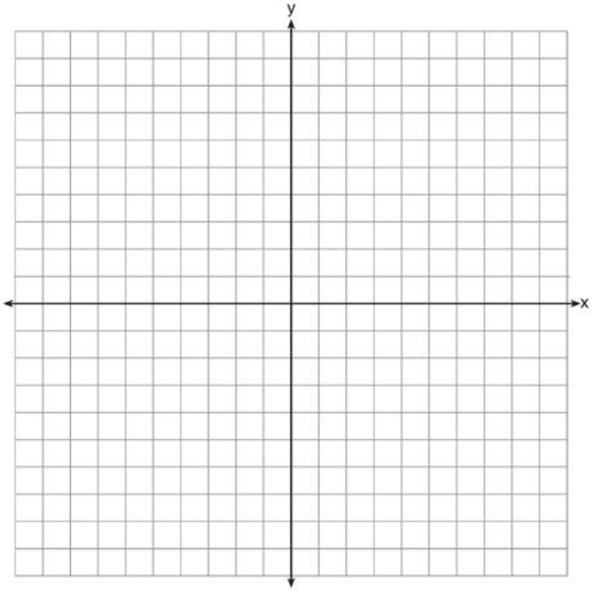
1) $(0.38, -2.85)$, only	3) $(0.38, -2.85)$ and $(2.62, 3.85)$
2) $(2.62, 3.85)$, only	4) $(0.38, -2.85)$ and $(3.85, 2.62)$

288) A company is considering building a manufacturing plant. They determine the weekly production cost at site *A* to be $A(x) = 3x^2$ while the production cost at site *B* is $B(x) = 8x + 3$, where *x* represents the number of products, *in hundreds*, and $A(x)$ and $B(x)$ are the production costs, *in hundreds of dollars*. Graph the production cost functions on the set of axes below and label them site *A* and site *B*.



State the positive value(s) of *x* for which the production costs at the two sites are equal. Explain how you determined your answer. If the company plans on manufacturing 200 products per week, which site should they use? Justify your answer.

289) Let $f(x) = -2x^2$ and $g(x) = 2x - 4$. On the set of axes below, draw the graphs of $y = f(x)$ and $y = g(x)$.



Using this graph, determine and state *all* values of x for which $f(x) = g(x)$.

290) John and Sarah are each saving money for a car. The total amount of money John will save is given by the function $f(x) = 60 + 5x$. The total amount of money Sarah will save is given by the function $g(x) = x^2 + 46$. After how many weeks, x, will they have the same amount of money saved? Explain how you arrived at your answer.

291) If $f(x) = x^2 - 2x - 8$ and $g(x) = \frac{1}{4}x - 1$, for which value of x is $f(x) = g(x)$?

1) -1.75 and -1.438
2) -1.75 and 4
3) -1.438 and 0
4) 4 and 0

292) If $f(x) = x^2$ and $g(x) = x$, determine the value(s) of x that satisfy the equation $f(x) = g(x)$.

293) Given: $g(x) = 2x^2 + 3x + 10$

$k(x) = 2x + 16$

Solve the equation $g(x) = 2k(x)$ algebraically for x, to the *nearest tenth*. Explain why you chose the method you used to solve this quadratic equation.

SOLUTIONS

287) ANS: 3

Strategy #1. Solve $y = x^2 - 3$ and $y = 3x - 4$ as a system of equations.

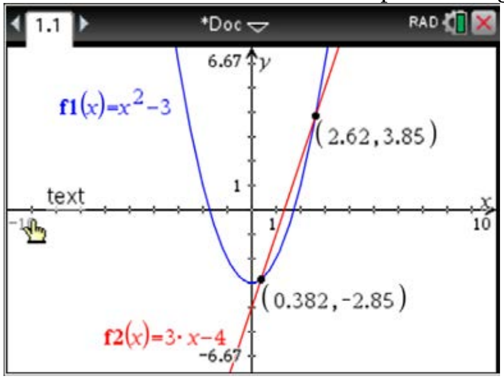
$y = x^2 - 3$ and $y = 3x - 4$

$$x^2 - 3 = 3x - 4$$
$$x^2 - 3x = -1$$
$$\left(x - \frac{3}{2}\right)^2 = -1 + \left(\frac{-3}{2}\right)^2$$
$$\left(x - \frac{3}{2}\right)^2 = -1 + \left(\frac{9}{4}\right)$$
$$\left(x - \frac{3}{2}\right)^2 = \frac{5}{4}$$

$$x - \frac{3}{2} = \pm\frac{\sqrt{5}}{2}$$
$$x = \frac{3 \pm \sqrt{5}}{2}$$

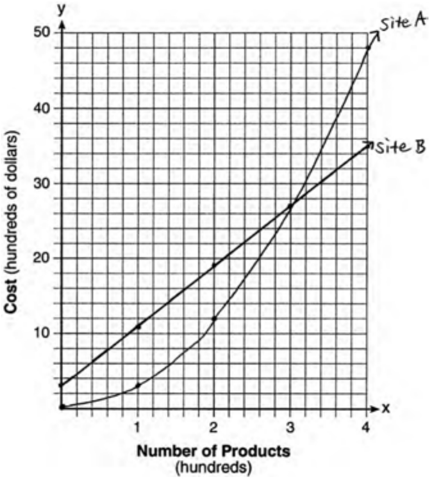
The values of x that satisfy the system are:
$x = \dfrac{3 + \sqrt{5}}{2} \approx 2.62$ and $x = \dfrac{3 - \sqrt{5}}{2} \approx .38$

Strategy #2. Use a graphing calculator to determine the inttercepts of the graphs of the two equations.



PTS: 2 NAT: A.REI.C.7 TOP: Quadratic-Linear Systems
KEY: algebraically

288) ANS:

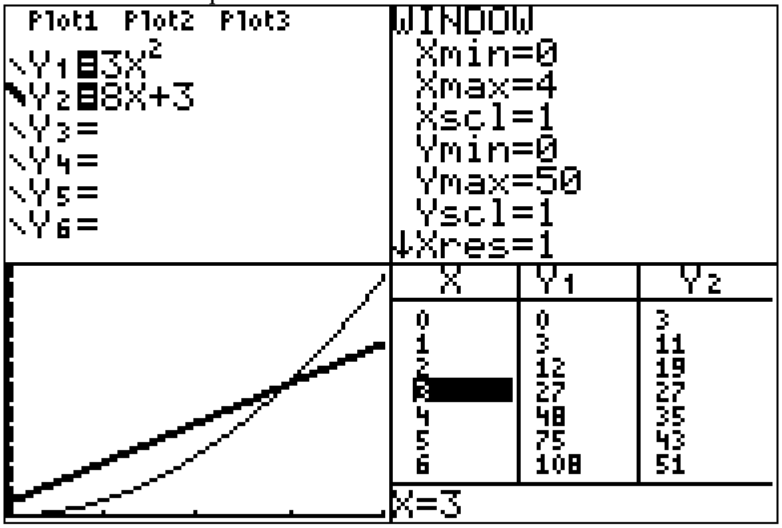


a)

b) The graphs of the production costs are equal when $x = 3$.

c) The company should use Site A, because the costs of Site A are lower when $x = 2$.

Strategy: Input both functions into a graphing calculator and use the table and graph views to construct the graph on paper and to answer the question.



Plot1 Plot2 Plot3

$Y_1 = 3X^2$

$Y_2 = 8X+3$

$Y_3 =$

$Y_4 =$

$Y_5 =$

$Y_6 =$

WINDOW

Xmin=0

Xmax=4

Xscl=1

Ymin=0

Ymax=50

Yscl=1

↓Xres=1

X	Y1	Y2
0	0	3
1	3	11
2	12	19
3	27	27
4	48	35
5	75	43
6	108	51

X=3

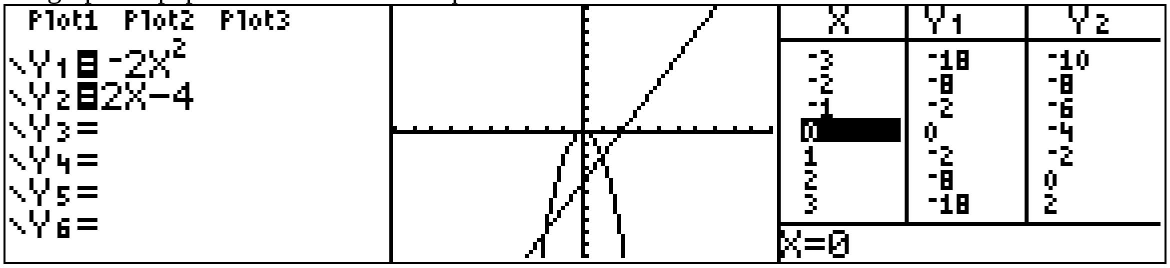
PTS: 6 NAT: A.REI.D.11 TOP: Quadratic-Linear Systems

289) ANS:

a) 

b) $f(x) = g(x)$ when $x = -2$ and $x = 1$.

Strategy: Input both functions into a graphing calculator and use the table and graph views to construct the graph on paper and to answer the question.



Plot1 Plot2 Plot3

$Y_1 = -2X^2$

$Y_2 = 2X-4$

$Y_3 =$

$Y_4 =$

$Y_5 =$

$Y_6 =$

X	Y1	Y2
-3	-18	-10
-2	-8	-8
-1	-2	-6
0	0	-4
1	-2	-2
2	-8	0
3	-18	2

X=0

PTS: 4 NAT: A.REI.D.11 TOP: Quadratic-Linear Systems

290) ANS:

John and Sarah will have the same amount of money saved at 7 weeks. I set the expressions representing their savings equal to each other and solved for the positive value of x by factoring.

Strategy: Set the expressions representing their savings equal to one another and solve for x.

$$f(x) = 60 + 5x \ \ and \ \ g(x) = x^2 + 46$$

$$Let \ \ f(x) = g(x)$$

$$x^2 + 46 = 60 + 5x$$

$$x^2 - 5x - 14 = 0$$

$$(x-7)(x+2) = 0$$

$$x = 7$$

DIMS? Does It Make Sense? Yes. After 7 weeks, John and Sarah will each have $95.00.

John’s Savings	Sarah’s Savings
$f(x) = 60 + 5x$	$g(x) = x^2 + 46$
$f(7) = 60 + 5(7)$	$g(7) = (7)^2 + 46$
$f(7) = 60 + 35$	$g(7) = 49 + 46$
$f(7) = 95$	$g(7) = 95$

PTS: 2 NAT: A.REI.D.11 TOP: Quadratic-Linear Systems

291) ANS: 2

Strategy: Set both expressions equal to one another and solve for *x*.

$$f(x) = x^2 - 2x - 8 \ \ and \ \ g(x) = \frac{1}{4}x - 1$$

$$Let\ f(x) = g(x)$$

$$x^2 - 2x - 8 = \frac{1}{4}x - 1$$

$$4x^2 - 8x - 32 = x - 4$$

$$4x^2 - 9x - 28 = 0$$

$$(4x + 7)(x - 4) = 0$$

$$x = -\frac{7}{4} \ \ and \ \ x = 4$$

$$f(-1.75) = g(-1.75)$$

$$and$$

$$f(4) = g(4)$$

PTS: 2 NAT: A.REI.D.11 TOP: Quadratic-Linear Systems

292) ANS:

$x = \{0, 1\}$

Given: $f(x) = x^2$ and $g(x) = x$, find $f(x) = g(x)$ as follows:

$$f(x) = g(x)$$

$$x^2 = x$$

$$x^2 - x = 0$$

$$x(x - 1) = 0$$

Therefore: $x = 0$ and $(x - 1) = 0$

$x = 1$

PTS: 2 NAT: A.REI.D.11 TOP: Quadratic-Linear Systems

KEY: AI

293) ANS:

$x \approx \{-3.1, 3.6\}$

$$g(x) = 2k(x)$$

$$2x^2 + 3x + 10 = 2(2x + 16)$$

$$2x^2 + 3x + 10 = 4x + 32$$

$$2x^2 - x - 22 = 0$$

The quadratic formula can be used to solve this quadratic in standard form, where $a = 2$, $b = -1$, and $c = -22$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-22)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{177}}{4}$$

$$x = \frac{1 + \sqrt{177}}{4} = 3.576033 \approx 3.6$$

$$x = \frac{1 - \sqrt{177}}{4} = -3.076033 \approx -3.1$$

The quadratic formula was chosen because it works with any quadratic equation.

PTS: 4 NAT: A.REI.D.11 TOP: Quadratic-Linear Systems
KEY: AI

SYSTEMS
Other Systems

Common Core Standard	**Next Generation Standard**
A-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations *y=f(x)* and *y=g(x)* intersect are the solutions of the equation *f(x)=g(x)*; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, ~~or find successive approximations~~. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential~~, and logarithmic functions~~. ~~PARCC: Tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. Finding the solutions approximately is limited to cases where f(x) and g(x) are polynomial functions.~~	**AI-A.REI.11** Given the equations *y = f(x)* and *y = g(x)*: i) **recognize** that each x-coordinate of the intersection(s) is the solution to the equation *f(x) = g(x)*; ii)find the solutions approximately using technology to graph the functions or make tables of values; and iii) **interpret the solution in context**. (Shared standard with Algebra II) **Notes: Algebra I tasks are limited to cases where *f(x)* and *g(x)* are linear, polynomial, absolute value, and exponential functions of the form** $f(x) = a(b)^x$ **where *a* > 0 and *b* > 0 (*b* ≠ 1).** **Students should be taught to find the solutions approximately by using technology to graph the functions *and* by making tables of values. When solving any problem, students can choose either strategy.**

LEARNING OBJECTIVES

Students will be able to:

1) use technology to create tables and graphs to find solutions of systems of equations involving linear and non-linear functions.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

absolute value equation
exponential equation
families of functions

linear equation
piecewise equation
quadratic equations

slope intercept form
solution of a system

BIG IDEAS

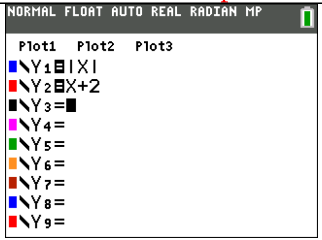
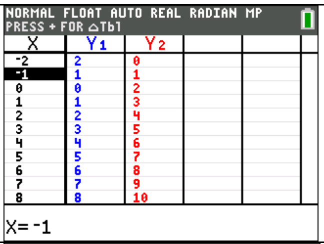
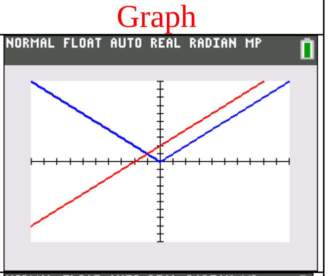
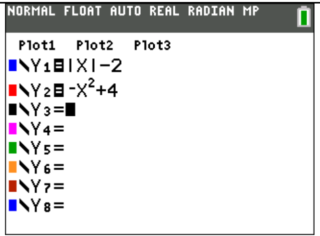
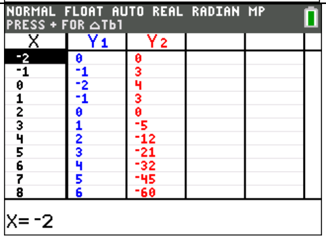
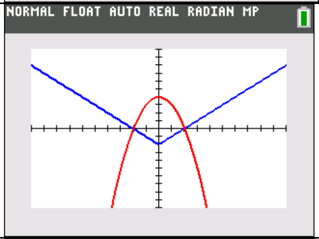
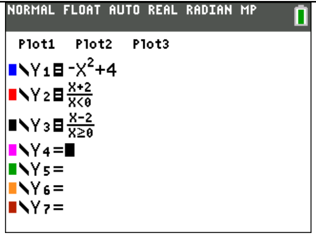
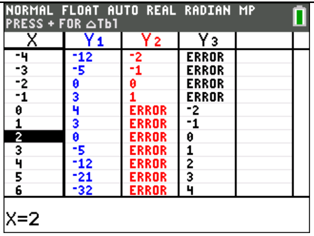
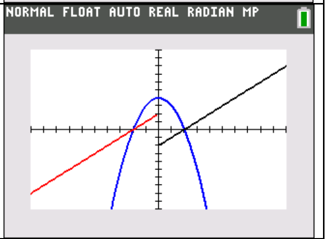
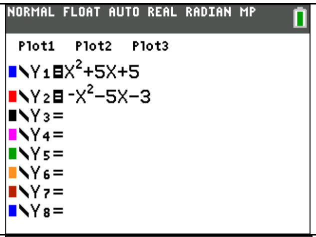
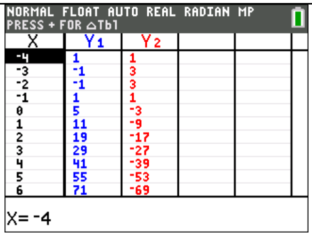
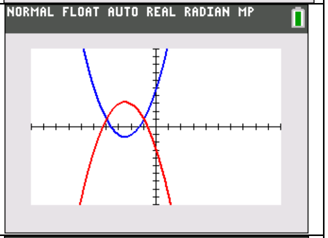
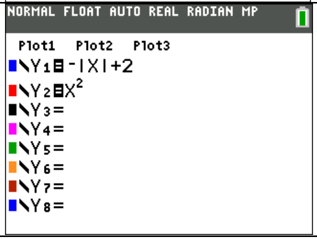
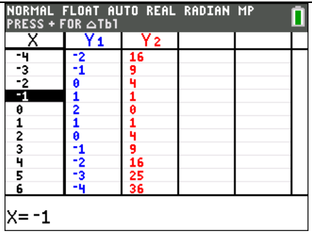
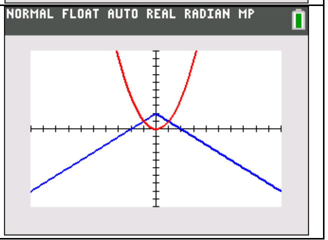
A **solution of a system** of equations makes each equation in the system true. This rule can be applied to systems involving different families of functions. Typically, graphing is the easiest way to solve systems of equations involving different families of functions. Algebraic solutions may also be used.

DEVELOPING ESSENTIAL SKILLS

Use technology to create a table of value and graph for each system, then state the solution(s) for each system.

1.	2.	3.	4.	5.
absolute linear	absolute quadratic	quadratic piecewise	quadratic quadratic	absolute quadratic
$y=\|x\|$ $y=x+2$	$y=\|x\|-2$ $y=-x^2+4$	$y=-x^2+4$ $y=\begin{cases} x+2 & x<0 \\ x-2 & x\geq 0 \end{cases}$	$y=x^2+5x+5$ $y=-x^2-5x-3$	$y=-\|x\|+2$ $y=x^2$

Answers

Solutions	Calculator Input	Table	Graph
1. $y=\lvert x\rvert$ $y=x+2$ (-1,1)	NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 Y1=|X| Y2=X+2 Y3= Y4= Y5= Y6= Y7= Y8= Y9=	NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR △Tbl X Y1 Y2 -2 2 0 -1 1 1 0 0 2 1 1 3 2 2 4 3 3 5 4 4 6 5 5 7 6 6 8 7 7 9 8 8 10 X=-1	
2. $y=\lvert x\rvert-2$ $y=-x^2+4$ (-2,0) and (2,0)	NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 Y1=|X|-2 Y2=-X²+4 Y3= Y4= Y5= Y6= Y7= Y8=	NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR △Tbl X Y1 Y2 -2 0 0 -1 -1 3 0 -2 4 1 -1 3 2 0 0 3 1 -5 4 2 -12 5 3 -21 6 4 -32 7 5 -45 8 6 -60 X=-2	
3. $y=-x^2+4$ $y=\begin{cases}x+2 & x<0\\ x-2 & x\geq 0\end{cases}$ (-2,0) and (2,0)	NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 Y1=-X²+4 Y2=$\frac{X+2}{X<0}$ Y3=$\frac{X-2}{X\geq 0}$ Y4= Y5= Y6= Y7=	NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR △Tbl X Y1 Y2 Y3 -4 -12 -2 ERROR -3 -5 -1 ERROR -2 0 0 ERROR -1 3 1 ERROR 0 4 ERROR -2 1 3 ERROR -1 2 0 ERROR 0 3 -5 ERROR 1 4 -12 ERROR 2 5 -21 ERROR 3 6 -32 ERROR 4 X=2	
4. $y=x^2+5x+5$ $y=-x^2-5x-3$ (-4,1) and (-1,1)	NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 Y1=X²+5X+5 Y2=-X²-5X-3 Y3= Y4= Y5= Y6= Y7= Y8=	NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR △Tbl X Y1 Y2 -4 1 1 -3 -1 3 -2 -1 3 -1 1 1 0 5 -3 1 11 -9 2 19 -17 3 29 -27 4 41 -39 5 55 -53 6 71 -69 X=-4	
5. $y=-\lvert x\rvert+2$ $y=x^2$ (-1,1) and (1,1)	NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 Y1=-|X|+2 Y2=X² Y3= Y4= Y5= Y6= Y7= Y8=	NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR △Tbl X Y1 Y2 -4 -2 16 -3 -1 9 -2 0 4 -1 1 1 0 2 0 1 1 1 2 0 4 3 -1 9 4 -2 16 5 -3 25 6 -4 36 X=-1	

A.REI.D.11: Other Systems

294) Two functions, $y = |x - 3|$ and $3x + 3y = 27$, are graphed on the same set of axes. Which statement is true about the solution to the system of equations?

1) $(3,0)$ is the solution to the system because it satisfies the equation $y = |x - 3|$.
2) $(9,0)$ is the solution to the system because it satisfies the equation $3x + 3y = 27$.
3) $(6,3)$ is the solution to the system because it satisfies both equations.
4) $(3,0)$, $(9,0)$, and $(6,3)$ are the solutions to the system of equations because they all satisfy at least one of the equations.

295) On the set of axes below, graph

$$g(x) = \frac{1}{2}x + 1$$

and

$$f(x) = \begin{cases} 2x + 1, & x \le -1 \\ 2 - x^2, & x > -1 \end{cases}$$

How many values of *x* satisfy the equation $f(x) = g(x)$? Explain your answer, using evidence from your graphs.

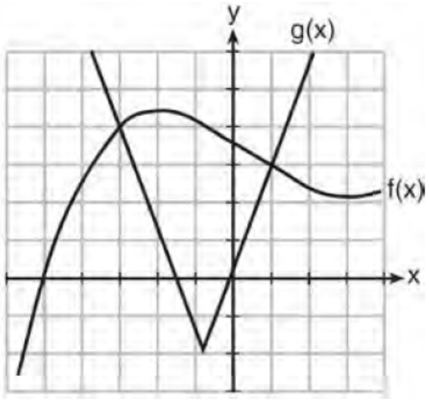
296) Given the functions $h(x) = \frac{1}{2}x + 3$ and $j(x) = |x|$, which value of *x* makes $h(x) = j(x)$?

1) -2
2) 2
3) 3
4) -6

297) The graphs of the functions $f(x) = |x - 3| + 1$ and $g(x) = 2x + 1$ are drawn. Which statement about these functions is true?

1) The solution to $f(x) = g(x)$ is 3.
2) The solution to $f(x) = g(x)$ is 1.
3) The graphs intersect when $y = 1$.
4) The graphs intersect when $x = 3$.

298) The graph below shows two functions, *f(x)* and *g(x)*. State the values of *x* for which $f(x) = g(x)$.



299) Which value of *x* results in equal outputs for $j(x) = 3x - 2$ and $b(x) = |x + 2|$?

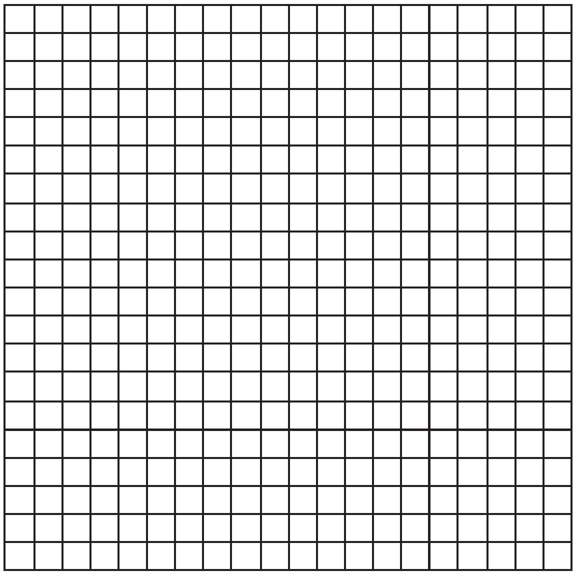
1) -2

3) $\frac{2}{3}$

2) 2

4) 4

300) Graph $f(x) = |x|$ and $g(x) = -x^2 + 6$ on the grid below. Does $f(-2) = g(-2)$? Use your graph to explain why or why not.



SOLUTIONS

294) ANS: 3

Strategy: Input both functions in a graphing calculator, then use the table and graph views of the function to select the correct answer.

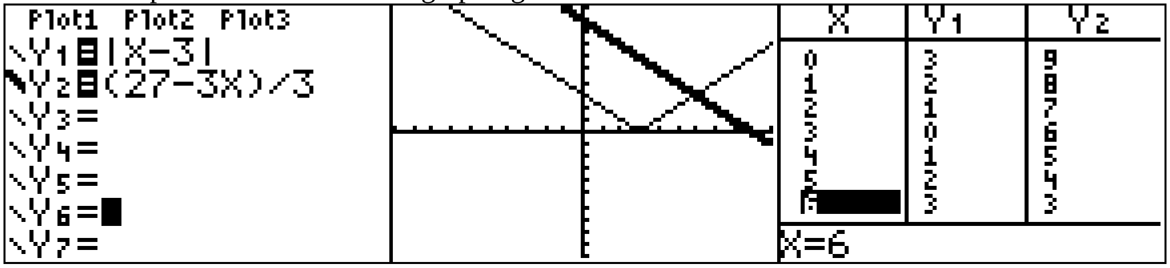
STEP 1. Transpose the second function for input into a graphing calculator.

$$3x + 3y = 27$$

$$3y = 27 - 3x$$

$$y = \frac{27 - 3x}{3}$$

STEP 2. Input both functions in a graphing calculator.



```
Plot1 Plot2 Plot3
\Y1=|X-3|
\Y2=(27-3X)/3
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
X    Y1   Y2
0    3    9
1    2    8
2    1    7
3    0    6
4    1    5
5    2    4
6    3    3
X=6
```

When $x = 6$, the value of y in both equations is 3. (6, 3) is the solution to this system.

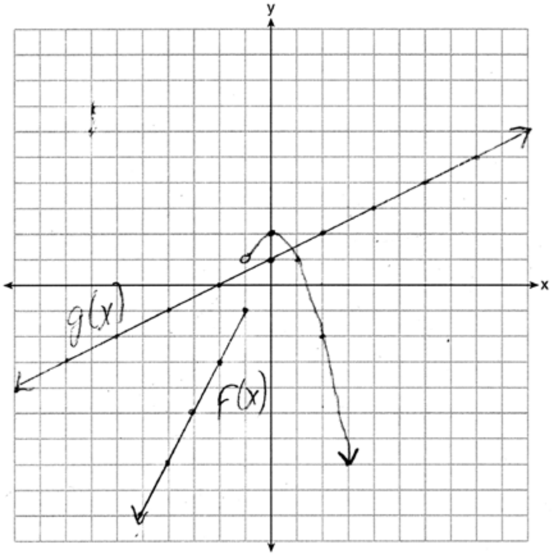
PTS: 2 NAT: A.REI.D.11 TOP: Nonlinear Systems

295) ANS:

Step 1. Plot $g(x) = \frac{1}{2}x + 1$

Step 2. Plot $f(x) = 2x + 1$ over the interval $x \leq -1$

Step 3. Plot $f(x) = 2 - x^2$ over the interval $x > -1$

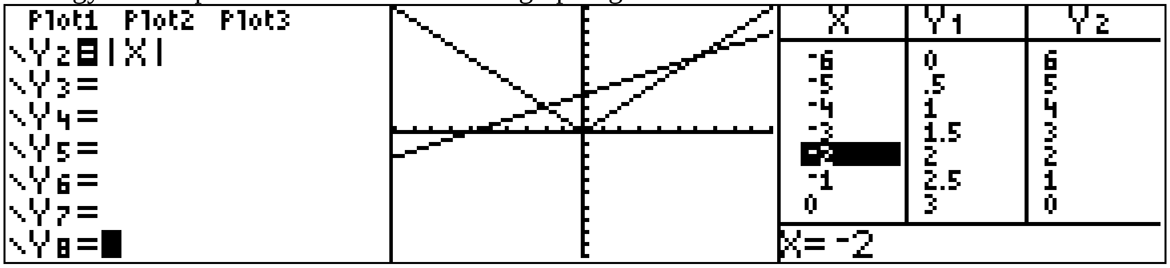


Only 1 value of x satisfies the equation $f(x) = g(x)$, because the graphs only intersect once.

PTS: 4 NAT: F.IF.C.7 TOP: Other Systems

296) ANS: 1

Strategy #1: Input both function rules in a graphing calculator.



```
 Plot1  Plot2  Plot3
\Y2=|X|
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
\Y8=
```

X	Y1	Y2
-6	0	6
-5	.5	5
-4	1	4
-3	1.5	3
-2	2	2
-1	2.5	1
0	3	0

X= -2

Strategy #2: Set the right expressions of both functions equal to one another. Then solve for the positive and negative values of $|x|$.

$$\frac{1}{2}x + 3 = |x|$$

$\frac{1}{2}x + 3 = x$ $x + 6 = 2x$ $6 = x$	$-\left(\frac{1}{2}x = 3\right) = x$ $-\frac{1}{2}x - 3 = x$ $-x - 6 = 2x$ $-6 = 3x$ $-2 = x$

Check:

$h(x) = \frac{1}{2}x + 3$ $h(-2) = \frac{1}{2}(-2) + 3$ $h(-2) = -1 + 3$ $h(-2) = 2$	$j(x) = \lvert x \rvert$ $j(-2) = \lvert -2 \rvert$ $j(x) = 2$

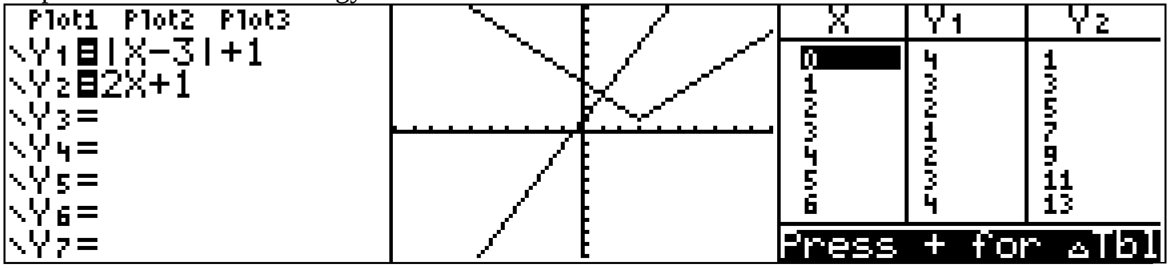
PTS: 2 NAT: A.REI.D.11 TOP: Other Systems

297) ANS: 2

Step 1. Understand that only of the answer choices is true.

Step 2. Strategy. Input both functions in a graphing calculator and explore the truth of each answer choice.

Step 3. Execution of Strategy.



```
 Plot1 Plot2 Plot3
\Y1■|X-3|+1
\Y2■2X+1
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

X	Y1	Y2
0	4	1
1	3	3
2	2	5
3	1	7
4	2	9
5	3	11
6	4	13

Press + for ΔTbl

The graph and table show that the solution for this system of equations is (1,3). This means that $f(1) = 3$ and $g(1) = 3$. Accordingly, when x is 1, $f(x) = g(x)$. The correct answer is choice b).

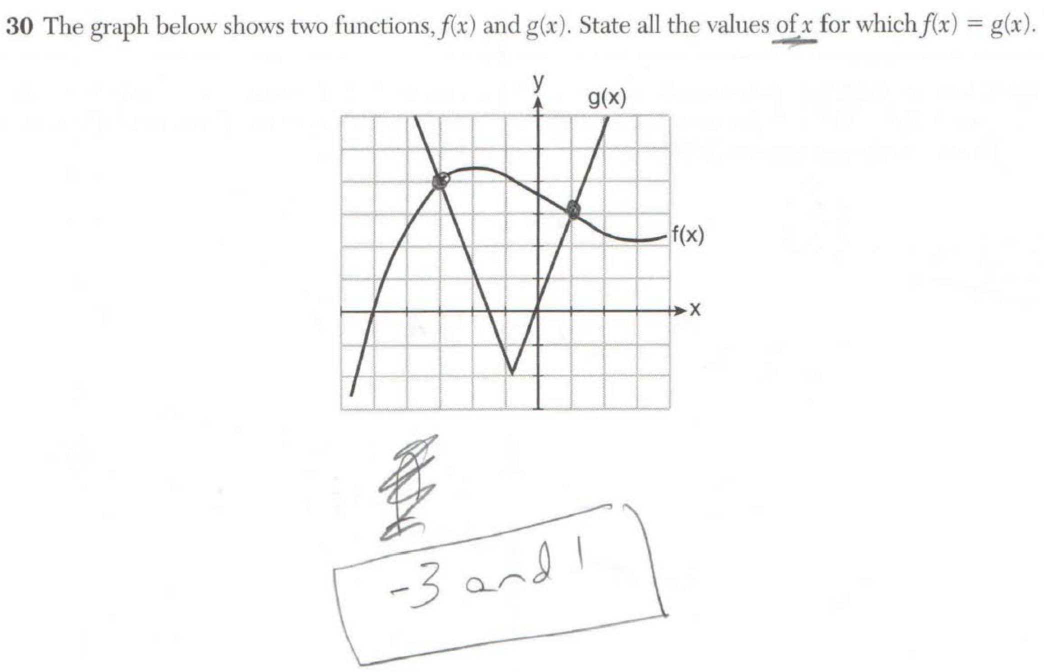
Step 4. Does is make sense? Yes. All of the other answer choices can be eliminated as wrong. The problem can be checked algebraically as follows:

Given: $f(x) = \lvert x - 3 \rvert + 1$ and $g(x) = 2x + 1$, find $f(x) = g(x)$

$\lvert x - 3 \rvert + 1 = 2x + 1$ $\lvert x - 3 \rvert = 2x$ $x - 3 = 2x$ $-3 = x$ This is an extraneous solution. $\lvert -3 - 3 \rvert + 1 = 2(-3) + 1$ $\lvert -6 \rvert + 1 = -6 + 1$ $6 + 1 = -6 + 1$ $7 \neq -5$	$\lvert x - 3 \rvert + 1 = 2x + 1$ $\lvert x - 3 \rvert = 2x$ $-x + 3 = 2x$ $3 = 3x$ $1 = x$ This solution checks. $\lvert 1 - 3 \rvert + 1 = 2(1) + 1$ $\lvert -2 \rvert + 1 = 2 + 1$ $2 + 1 = 2 + 1$ $3 = 3$

PTS: 2 NAT: A.REI.D.11 TOP: Other Systems

298) ANS:



30 The graph below shows two functions, $f(x)$ and $g(x)$. State all the values of x for which $f(x) = g(x)$.

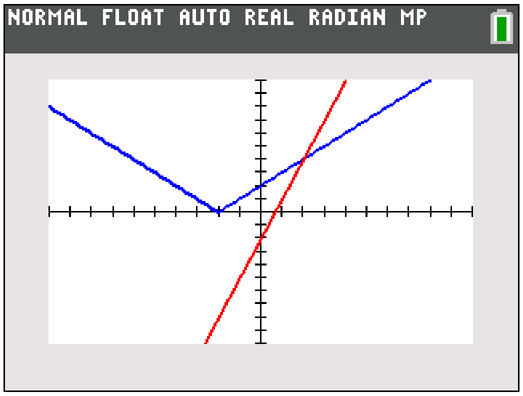
-3 and 1

PTS: 2 NAT: A.REI.D.11

299) ANS: 2

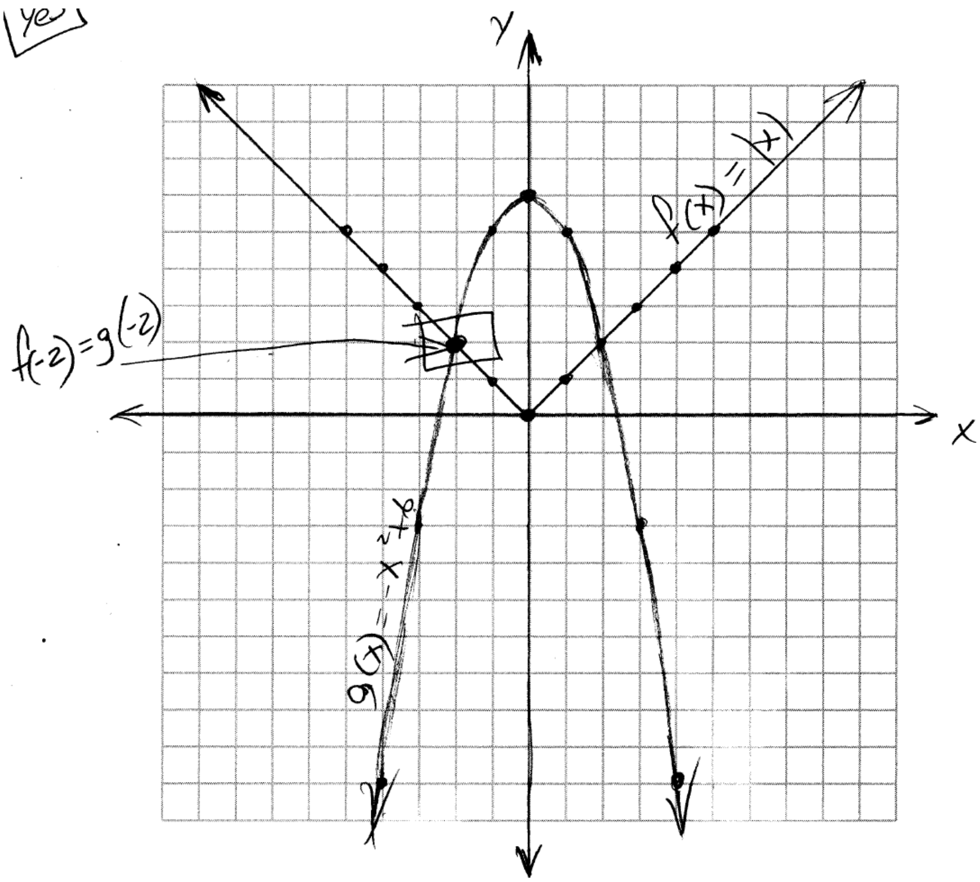
When $x = 2$, $f(x) = b(x)$.

$$f(x) = b(x)$$
$$3x - 2 = |x + 2|$$
$$3x - 2 = x + 2$$
$$2x = 4$$
$$x = 2$$
$$j(2) = 3(2) - 2$$
$$j(2) = 4$$
$$b(2) = |2 + 2|$$
$$b(2) = 4$$



PTS: 2 NAT: A.REI.D.11 TOP: Other Systems
KEY: AI

300) ANS:



Yes, because the graph of $f(x)$ intersects the graph of $g(x)$ at $x = -2$.

PTS: 4 NAT: A.REI.D.11 TOP: Other Systems
KEY: AI

J – Powers, Lesson 1, Modeling Exponential Functions (r. 2018)

POWERS

Modeling Exponential Functions

Common Core Standards	**Next Generation Standards**
A-SSE.B.3c Use the properties of exponents to transform expressions ~~for exponential functions.~~ ~~*For example, the expression 1.15_t can be rewritten as $(1.15_{1/12})_{12t} = 1.012_{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*~~ PARCC: Tasks are limited to exponential expressions with integer exponents. Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation.	**AI-A.SSE.3c** Use the properties of exponents to rewrite exponential expressions. (Shared standard with Algebra II) **e.g.,** • $3_{2x} = (3_2)_x = 9_x$ • $3_{2x+3} = 3_{2x} \cdot 3_3 = 9_x \cdot 27$ Note: Exponential expressions will include those with integer exponents, **as well as those whose exponents are linear expressions**. **Any linear term in those expressions will have an integer coefficient. Rational exponents are an expectation for Algebra II.**
A-CED.1 Create equations and inequalities in one variable ~~and use them to solve problems~~. *Include equations arising from linear and quadratic functions, ~~and simple rational~~ and exponential functions.* PARCC: Tasks are limited to linear, quadratic, or exponential equations with integer exponents.	**AI-A.CED.1** Create equations and inequalities in one variable to **represent a real-world context**. (Shared standard with Algebra II) Notes: • **This is strictly the development of the model (equation/inequality).** • **Limit equations to linear, quadratic, and exponentials of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).** • **Work with geometric sequences may involve an exponential equation/formula of the form $a_n = ar_{n-1}$, where a is the first term and r is the common ratio.** • **Inequalities are limited to linear inequalities.** • Algebra I tasks do not involve compound inequalities.
F-BF.A.1 Write a function that describes a relationship between two quantities.	**AI-F.BF.1** Write a function that describes a relationship between two quantities. (Shared standard with Algebra II)
F-LE.A.2 Construct linear and exponential functions, ~~including arithmetic and geometric sequences,~~ given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). PARCC: Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).	**AI-F.LE.2** Construct a linear or exponential function **symbolically** given: i) a graph; ii) a description of the relationship; iii) two input-output pairs (include reading these from a table). (Shared standard with Algebra II) Note: Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).
F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context. PARCC: Tasks have a real-world context. Exponential functions are limited to those with domains in the integers.	**AI-F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context. (Shared standard with Algebra II) Note: Tasks have a real-world context. **Exponential functions are limited to those with domains in the integers and are of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).**

LEARNING OBJECTIVES

Students will be able to:

1) Transform expressions and equations between equivalent exponential and radical forms.
2) Create and solve exponential functions based on real-world contexts.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

$A = P(1 \pm r)^t$
base
cycle
exponential decay
exponential growth
exponential regression
initial amount
power
rate of decay
rate of growth
rational exponents
root
scientific notation

BIG IDEAS

Rules for Rational Exponents:

Rule: For any nonzero number a, $a^0 = 1$, and $a^{-n} = \dfrac{1}{a^n}$

Rule: For any nonzero number a and any rational numbers m and n, $a^m \cdot a^n = a^{m+n}$

Rule: For any nonzero number a and any rational numbers m and n, $(a^m)^n = a^{mn}$

Rule: For any nonzero numbers a and b and any rational number n $(ab)^n = a^n b^n$

Rule: For any nonzero number a and any rational numbers m and n, $\dfrac{a^m}{a^n} = a^{m-n}$

A number is in **scientific notation** if it is written in the form $a \times 10^n$, where n is an integer and $1 \le |a| < 10$

Exponential Growth and Decay

$$A = P\left(1 \pm r\right)^t$$

A common formula for exponential growth or decay is

$$A = P\left(1 \pm r\right)^t$$

Where:

A is the *amount after* growth or decay.

P is the original *amount before* growth or decay.

$\left(1 \pm r\right)$ is 100% of the original plus-or-minus the *rate* of growth or decay.

- + is used to model growth.
- - is used to model decay.

t is the number of growth or decay cycles, usually measured in units of time.

Sample Problem

PROBLEM

The equation $A = 1500\left(1.03\right)^t$ can be used to find the amount of money in a bank account if the initial amount deposited was $1,500 and the money grows with interest compounded annually at the rate of 3%. Rewrite this equation to reflect a monthly rate of growth.

SOLUTION

The problem wants us to write a new equation in which the rate of growth is expressed in months instead of years.

STEP 1. The rate of growth will be smaller if interest compounds monthly rather than annually. Therefore, the base of the exponent (in parentheses) must be reduced.

Using the rule: For any nonzero number *a* and any rational numbers *m* and *n*, $(a^m)^n = a^{mn}$, rewrite the exponential base (in parentheses) and its power as follows:

$$\left(1.03\right)^t = \left(1.03^{\frac{1}{12}}\right)^{12t}.$$

NOTE: Both expressions reflect the amount after one year of growth.

- In the *left* expression, *t represents time in <u>years</u>*.
- In the *right* expression, *t represents time in <u>months</u>*.

STEP 2. Simplify the base of the exponent (the term in parentheses) as follows:

$$1.03^{\frac{1}{12}} = 1.00246627$$, which rounds to 1.0025

NOTE: *This reveals the approximate equivalent monthly interest rate.*

STEP 3. Rewrite the entire equation with the new exponent.

$$A = 1500(1.0025)^{12t}$$

NOTE: 12t represents one year, or 12 months. To find growth in months, eliminate the multiplication by 12.

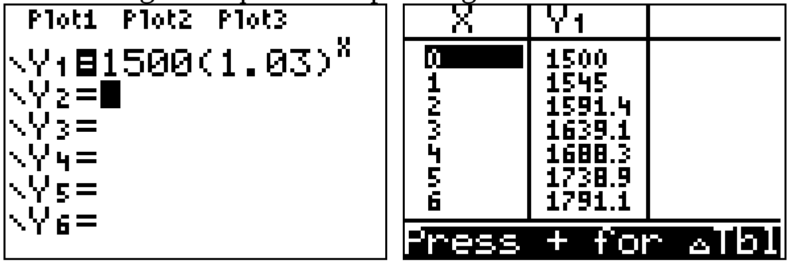
STEP 4. Write the new equation.

$$A = 1500(1.0025)^{t}$$

CHECK

If the new equation reflects the same mathematical relationship as the original equation, both equations should produce similar outputs for similar amounts of time.

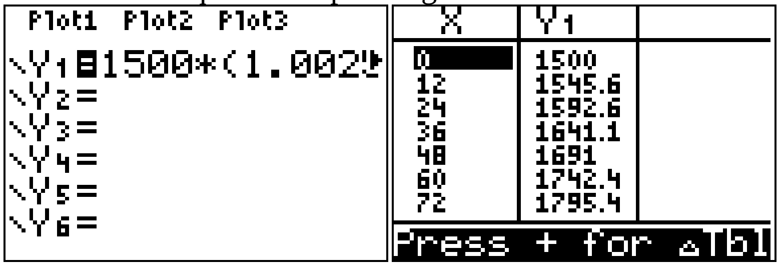
Original Equation Expressing Growth in Years



```
 Plot1  Plot2  Plot3
\Y1=1500(1.03)^X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

```
  X      Y1
  0      1500
  1      1545
  2      1591.4
  3      1639.1
  4      1688.3
  5      1738.9
  6      1791.1
Press + for ΔTbl
```

New Equation Expressing Growth in Months



```
 Plot1  Plot2  Plot3
\Y1=1500*(1.002▸
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

```
  X      Y1
  0      1500
  12     1545.6
  24     1592.6
  36     1641.1
  48     1691
  60     1742.4
  72     1795.4
Press + for ΔTbl
```

DEVELOPING ESSENTIAL SKILLS

Solve the following problems using exponential growth or exponential decay formulas.

	Problems	Solutions
1	Daniel's Print Shop purchased a new printer for $35,000. Each year it depreciates (loses value) at a rate of 5%. What will its approximate value be at the end of the fourth year?	$A = P(1 \pm r)^t$ $A = 35000(1-0.05)^4$ $A \approx \$28,507.72$
2	Kathy plans to purchase a car that depreciates (loses value) at a rate of 14% per year. The initial cost of the car is $21,000. Write an equation that represents the value, *v*, of the car after 3 years.	$A = P(1 \pm r)^t$ $v = 21000(1+0.14)^3$
3	A bank is advertising that new customers can open a savings account with a $3\frac{3}{4}\%$ interest rate compounded annually. Robert invests $5,000 in an account at this rate. If he makes no additional deposits or withdrawals on his account, find the amount of money he will have, to the *nearest cent*, after three years.	$A = P(1 \pm r)^t$ $A = 5000(1+0.0375)^3$ $A \approx \$5,583.86$
4	Cassandra bought an antique dresser for $500. If the value of her dresser increases 6% annually, what will be the value of Cassandra's dresser at the end of 3 years to the *nearest dollar*?	$A = P(1 \pm r)^t$ $A = 500(1+0.06)^3$ $A \approx \$596$
5	ooster Club raised $30,000 for a sports fund. No more money will be placed into the fund. Each year the fund will decrease by 5%. Determine the amount of money, to the *nearest cent*, that will be left in the sports fund after 4 years.	$A = P(1 \pm r)^t$ $A = 30000(1-.05)^4$ $A \approx \$24,435.19$

POWERS

A.SSE.B.3c, A.CED.A.1, F.BF.A.1, F.LE.A.2, F.LE.B.5: Modeling Exponential Functions

301) Miriam and Jessica are growing bacteria in a laboratory. Miriam uses the growth function $f(t) = n^{2t}$ while Jessica uses the function $g(t) = n^{4t}$, where *n* represents the initial number of bacteria and *t* is the time, in hours. If Miriam starts with 16 bacteria, how many bacteria should Jessica start with to achieve the same growth over time?

1) 32 3) 8
2) 16 4) 4

302) Jacob and Jessica are studying the spread of dandelions. Jacob discovers that the growth over *t* weeks can be defined by the function $f(t) = (8) \cdot 2^t$. Jessica finds that the growth function over *t* weeks is $g(t) = 2^{t+3}$.

Calculate the number of dandelions that Jacob and Jessica will each have after 5 weeks.

Based on the growth from both functions, explain the relationship between $f(t)$ and $g(t)$.

303) The growth of a certain organism can be modeled by $C(t) = 10(1.029)^{24t}$, where $C(t)$ is the total number of cells after *t* hours. Which function is approximately equivalent to $C(t)$?

1) $C(t) = 240(.083)^{24t}$ 3) $C(t) = 10(1.986)^t$
2) $C(t) = 10(.083)^t$ 4) $C(t) = 240(1.986)^{\frac{t}{24}}$

304) A computer application generates a sequence of musical notes using the function $f(n) = 6(16)^n$, where *n* is the number of the note in the sequence and $f(n)$ is the note frequency in hertz. Which function will generate the same note sequence as $f(n)$?

1) $g(n) = 12(2)^{4n}$ 3) $p(n) = 12(4)^{2n}$
2) $h(n) = 6(2)^{4n}$ 4) $k(n) = 6(8)^{2n}$

305) Mario's $15,000 car depreciates in value at a rate of 19% per year. The value, *V*, after *t* years can be modeled by the function $V = 15,000(0.81)^t$. Which function is equivalent to the original function?

1) $V = 15,000(0.9)^{9t}$ 3) $V = 15,000(0.9)^{\frac{t}{9}}$
2) $V = 15,000(0.9)^{2t}$ 4) $V = 15,000(0.9)^{\frac{t}{2}}$

306) Nora inherited a savings account that was started by her grandmother 25 years ago. This scenario is modeled by the function $A(t) = 5000(1.013)^{t+25}$, where $A(t)$ represents the value of the account, in dollars, *t* years after the inheritance. Which function below is equivalent to $A(t)$?

1) $A(t) = 5000[(1.013^t)]^{25}$ 3) $A(t) = (5000)^t(1.013)^{25}$
2) $A(t) = 5000[(1.013)^t + (1.013)^{25}]$ 4) $A(t) = 5000(1.013)^t(1.013)^{25}$

307) The Ebola virus has an infection rate of 11% per day as compared to the SARS virus, which has a rate of 4% per day. If there were one case of Ebola and 30 cases of SARS initially reported to authorities and cases are reported each day, which statement is true?

1) At day 10 and day 53 there are more Ebola cases.
2) At day 10 and day 53 there are more SARS cases.
3) At day 10 there are more SARS cases, but at day 53 there are more Ebola cases.
4) At day 10 there are more Ebola cases, but at day 53 there are more SARS cases.

308) Dylan invested \$600 in a savings account at a 1.6% annual interest rate. He made no deposits or withdrawals on the account for 2 years. The interest was compounded annually. Find, to the *nearest cent*, the balance in the account after 2 years.

309) Rhonda deposited \$3000 in an account in the Merrick National Bank, earning 4.2% interest, compounded annually. She made no deposits or withdrawals. Write an equation that can be used to find B, her account balance after t years.

310) Krystal was given \$3000 when she turned 2 years old. Her parents invested it at a 2% interest rate compounded annually. No deposits or withdrawals were made. Which expression can be used to determine how much money Krystal had in the account when she turned 18?

1) $3000(1+0.02)^{16}$
2) $3000(1-0.02)^{16}$
3) $3000(1+0.02)^{18}$
4) $3000(1-0.02)^{18}$

311) The country of Benin in West Africa has a population of 9.05 million people. The population is growing at a rate of 3.1% each year. Which function can be used to find the population 7 years from now?

1) $f(t)=(9.05\times10^{6})(1-0.31)^{7}$
2) $f(t)=(9.05\times10^{6})(1+0.31)^{7}$
3) $f(t)=(9.05\times10^{6})(1+0.031)^{7}$
4) $f(t)=(9.05\times10^{6})(1-0.031)^{7}$

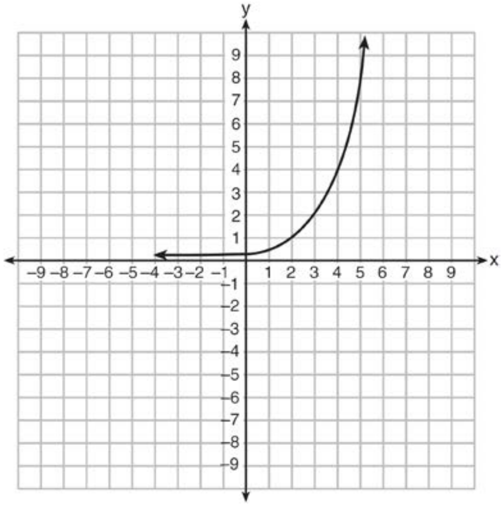
312) A student invests \$500 for 3 years in a savings account that earns 4% interest per year. No further deposits or withdrawals are made during this time. Which statement does not yield the correct balance in the account at the end of 3 years?

1) $500(1.04)^{3}$
2) $500(1-.04)^{3}$
3) $500(1+.04)(1+.04)(1+.04)$
4) $500+500(.04)+520(.04)+540.8(.04)$

313) Anne invested \$1000 in an account with a 1.3% annual interest rate. She made no deposits or withdrawals on the account for 2 years. If interest was compounded annually, which equation represents the balance in the account after the 2 years?

1) $A=1000(1-0.013)^{2}$
2) $A=1000(1+0.013)^{2}$
3) $A=1000(1-1.3)^{2}$
4) $A=1000(1+1.3)^{2}$

314) Write an exponential equation for the graph shown below.



Explain how you determined the equation.

315) The table below shows the temperature *T(m)*, of a cup of hot chocolate that is allowed to chill over several minutes, *m*.

Time, m (minutes)	0	2	4	6	8
Temperature, T(m) (°F)	150	108	78	56	41

Which expression best fits the data for *T(m)*?

1) $150(0.85)^{m}$
2) $150(1.15)^{m}$
3) $150(0.85)^{m-1}$
4) $150(0.85)^{m-1}$

316) Jill invests $400 in a savings bond. The value of the bond, $V(x)$, in hundreds of dollars after *x* years is illustrated in the table below.

x	**V(x)**
0	4
1	5.4
2	7.29
3	9.84

Which equation and statement illustrate the approximate value of the bond in hundreds of dollars over time in years?

1) $V(x) = 4(0.65)^{x}$ and it grows.
2) $V(x) = 4(0.65)^{x}$ and it decays.
3) $V(x) = 4(1.35)^{x}$ and it grows.
4) $V(x) = 4(1.35)^{x}$ and it decays.

317) The breakdown of a sample of a chemical compound is represented by the function $p(t) = 300(0.5)^{t}$, where $p(t)$ represents the number of milligrams of the substance and *t* represents the time, in years. In the function $p(t)$, explain what 0.5 and 300 represent.

318) Some banks charge a fee on savings accounts that are left inactive for an extended period of time. The equation $y = 5000(0.98)^x$ represents the value, *y*, of one account that was left inactive for a period of *x* years. What is the *y*-intercept of this equation and what does it represent?

1) 0.98, the percent of money in the account initially
2) 0.98, the percent of money in the account after *x* years
3) 5000, the amount of money in the account initially
4) 5000, the amount of money in the account after *x* years

319) The function $V(t) = 1350(1.017)^t$ represents the value $V(t)$, in dollars, of a comic book *t* years after its purchase. The yearly rate of appreciation of the comic book is

1) 17%
2) 1.7%
3) 1.017%
4) 0.017%

320) The number of carbon atoms in a fossil is given by the function $y = 5100(0.95)^x$, where *x* represents the number of years since being discovered. What is the percent of change each year? Explain how you arrived at your answer.

321) The equation $A = 1300(1.02)^7$ is being used to calculate the amount of money in a savings account. What does 1.02 represent in this equation?

1) 0.02% decay
2) 0.02% growth
3) 2% decay
4) 2% growth

322) Milton has his money invested in a stock portfolio. The value, v(x), of his portfolio can be modeled with the function $v(x) = 30,000(0.78)^x$, where *x* is the number of years since he made his investment. Which statement describes the rate of change of the value of his portfolio?

1) It decreases 78% per year.
2) It decreases 22% per year.
3) It increases 78% per year.
4) It increases 22% per year.

323) The 2014 winner of the Boston Marathon runs as many as 120 miles per week. During the last few weeks of his training for an event, his mileage can be modeled by $M(w) = 120(.90)^{w-1}$, where *w* represents the number of weeks since training began. Which statement is true about the model $M(w)$?

1) The number of miles he runs will increase by 90% each week.
2) The number of miles he runs will be 10% of the previous week.
3) $M(w)$ represents the total mileage run in a given week.
4) *w* represents the number of weeks left until his marathon.

324) The value, $v(t)$, of a car depreciates according to the function $v(t) = P(.85)^t$, where *P* is the purchase price of the car and *t* is the time, in years, since the car was purchased. State the percent that the value of the car *decreases* by each year. Justify your answer.

SOLUTIONS

301) ANS: 4

Understanding the Problem.

Miriam's exponential growth function is modeled by $f(t) = n^{2t}$. The problem tells us that *n* equals 16, so Miriam's exponential growth function can be rewritten as $f(t) = 16^{2t}$

Jessica's exponential growth function is modeled by $g(t) = n^{4t}$. The quantity *n* is unknown for Jessica's exponential growth function and the problem wants us to find the value of *n* that will make $f(t) = g(t)$.

Strategy: Substitute equivalent expresseions for $f(t)$ and $g(t)$, then solve for n.

$$f(t) = g(t)$$
$$16^{2t} = n^{4t}$$
$$16^{2t} = \left(n^2\right)^{2t}$$
$$16 = n^2$$
$$4 = n$$

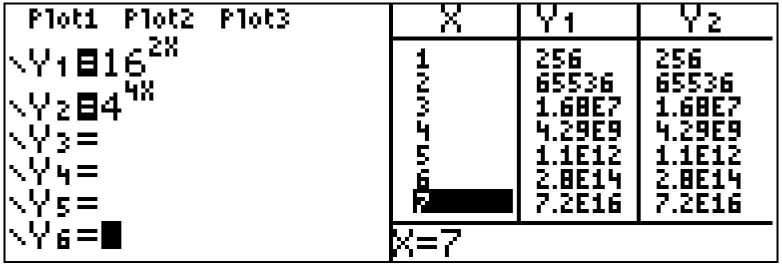
or

$$f(t) = g(t)$$
$$16^{2t} = n^{4t}$$
$$16^2 = n^4$$
$$256 = n^4$$
$$256^{\frac{1}{4}} = \left(n^4\right)^{\frac{1}{4}}$$
$$4 = n$$

or

$$f(t) = g(t)$$
$$16^{2t} = n^{4t}$$
$$16^2 = n^4$$
$$\sqrt{16^2} = \sqrt{n^4}$$
$$16 = n^2$$
$$4 = n$$

DIMS? Does It Make Sense? Yes. The outputs of $f(t) = 16^{2t}$ and $g(t) = 4^{4t}$ are identical.



```
Plot1  Plot2  Plot3
\Y1■16^(2X)
\Y2■4^(4X)
\Y3=
\Y4=
\Y5=
\Y6=
```

X	Y1	Y2
1	256	256
2	65536	65536
3	1.68E7	1.68E7
4	4.29E9	4.29E9
5	1.1E12	1.1E12
6	2.8E14	2.8E14
7	7.2E16	7.2E16

```
X=7
```

PTS: 2 NAT: A.SSE.B.3c TOP: Solving Exponential Equations

302) ANS:

Jacob and Jessica will both have 256 dandelions after 5 weeks.

$f(t) = 8 \cdot 2^t$	$g(t) = 2^{t+3}$
$f(5) = (8) \cdot 2^5$	$g(5) = 2^{5+3}$
$f(5) = 8 \cdot 32$	$g(5) = 2^8$
$f(5) = 256$	$g(5) = 256$

Both functions express the same mathematical relationships.

$$f(t) = g(t)$$
$$8 \cdot 2^t = 2^{t+3}$$
$$8 \cdot 2^t = 2^t \cdot 2^3$$
$$8 \cdot 2^t = 2^t \cdot 8$$

PTS: 2 NAT: A.SSE.B.3c TOP: Exponential Equations

303) ANS: 3

Step 1. Understand that this problem wants you to find the function in the answer choices that is equivalent to $C(t) = 10(1.029)^{24t}$.

Step 2. Strategy. Use properties of exponents to rewrite the expression.

Step 3. Execute the strategy.

$$C(t) = 10(1.029)^{24t}$$

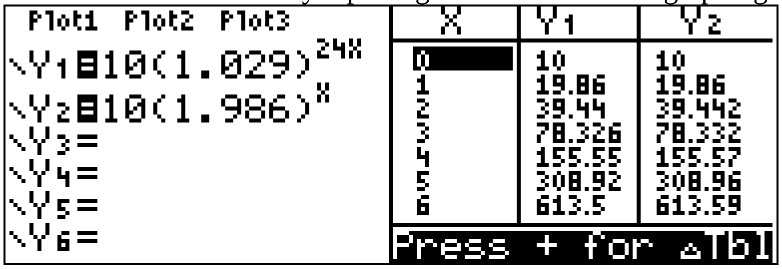
$$C(t) = 10(1.029^{24})^{t}$$

Use a calculator to find the value of 1.029^{24}

$$C(t) \approx 10(1.986)^{t}$$

Choice c is the correct answer.

Step 4. Does it make sense? Yes. Check by inputting both functions in a graphing calculator.



Plot1 Plot2 Plot3

$\backslash Y_1 = 10(1.029)^{24X}$

$\backslash Y_2 = 10(1.986)^{X}$

\Y3=

\Y4=

\Y5=

\Y6=

X	Y1	Y2
0	10	10
1	19.86	19.86
2	39.44	39.442
3	78.326	78.332
4	155.55	155.57
5	308.92	308.96
6	613.5	613.59

Press + for △Tbl

PTS: 2 NAT: A.SSE.B.3c TOP: Exponential Equations

304) ANS: 2

Strategy #1: Isolate the exponent n in each answer choice so that the structure of each function is identical, then eliminate any answer choice that is not equivalent to the original function..

$g(n) = 12(2)^{4n}$ $g(n) = 12(2^4)^n$ $g(n) = 12(16)^n$	$h(n) = 6(2)^{4n}$ $h(n) = 6(2^4)^n$ $h(n) = 6(16)^n$	$p(n) = 12(4)^{2n}$ $p(n) = 12(4^2)^n$ $p(n) = 12(16)^n$	$k(n) = 6(8)^{2n}$ $k(n) = 6(8^2)^n$ $k(n) = 6(64)^n$
Eliminate this choice.	Choose this, because $f(n) = h(n)$.	Eliminate this choice.	Eliminate this choice.

Strategy #2: Input the original function and all four answer choices in a graphing calculator. Choose the answer choice that produces the same function outputs (y-values) as the original function.

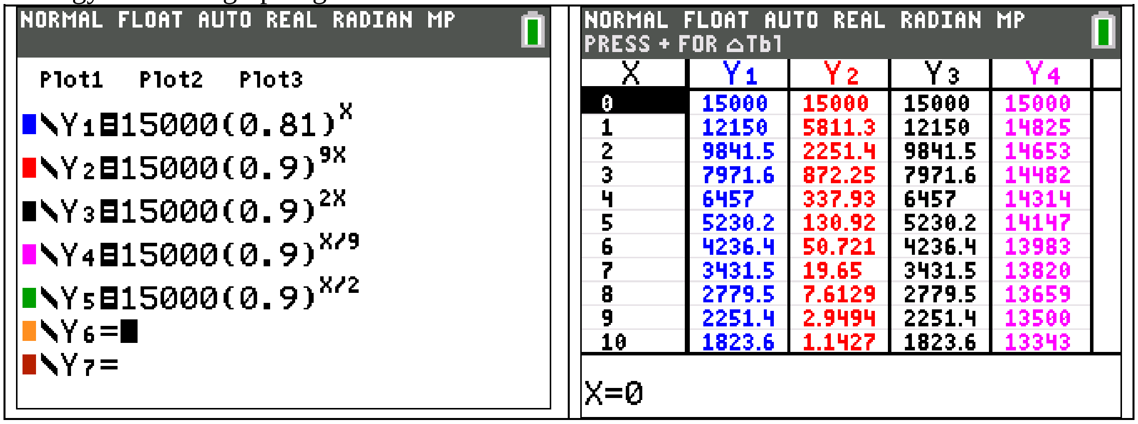
PTS: 2 NAT: A.SSE.B.3 TOP: Modeling Exponential Functions

305) ANS: 2

Strategy #1: Use properties of exponents.

$$V = 15,000(0.81)^{t} = 15,000((0.9)^{2})^{t} = 15,000(0.9)^{2t}$$

Strategy #2: Use graphing calculator.



NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

$\backslash Y_1 = 15000(0.81)^{X}$

$\backslash Y_2 = 15000(0.9)^{9X}$

$\backslash Y_3 = 15000(0.9)^{2X}$

$\backslash Y_4 = 15000(0.9)^{X/9}$

$\backslash Y_5 = 15000(0.9)^{X/2}$

\Y6=

\Y7=

NORMAL FLOAT AUTO REAL RADIAN MP

PRESS + FOR △Tbl

X	Y1	Y2	Y3	Y4
0	15000	15000	15000	15000
1	12150	5811.3	12150	14825
2	9841.5	2251.4	9841.5	14653
3	7971.6	872.25	7971.6	14482
4	6457	337.93	6457	14314
5	5230.2	130.92	5230.2	14147
6	4236.4	50.721	4236.4	13983
7	3431.5	19.65	3431.5	13820
8	2779.5	7.6129	2779.5	13659
9	2251.4	2.9494	2251.4	13500
10	1823.6	1.1427	1823.6	13343

X=0

$V = 15,000(0.9)^{2t}$ produces the same table of values as the original function $V = 15,000(0.81)^{t}$.

PTS: 2 NAT: A.SSE.B.3 TOP: Modeling Exponential Functions

306) ANS: 4

$$a^m a^n = a^{m+n}$$

$(1.013)^t(1.013)^{25} = (1.013)^{t+25}$

Therefore:

$5000(1.013)^{t+25} = 5000(1.013)^t(1.013)^{25}$

PTS: 2 NAT: A.SSE.B.3 TOP: Modeling Exponential Functions

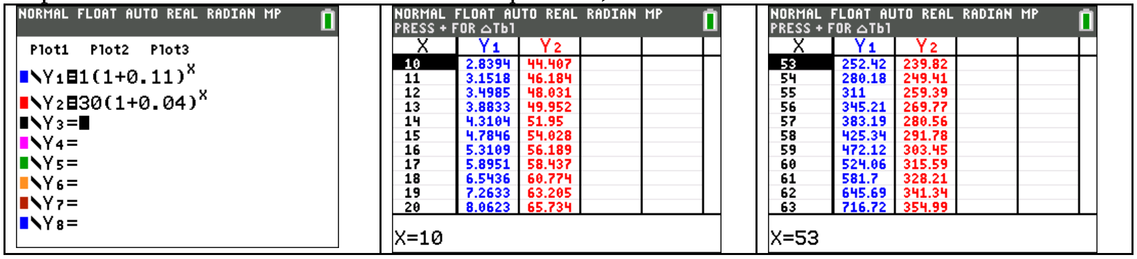
307) ANS: 3

Step 1. Use $A = P(1+r)^t$ to set up exponential growth equations to represent both viruses.

Ebola Virus: $E(t) = 1(1+0.11)^t$

SARS Virus: $S(t) = 30(1+0.04)^t$

Step 2. Find $t = 10$ and $t = 53$ for both equations, then choose the correct answer.



Y1=1(1+0.11)^X
Y2=30(1+0.04)^X
Y3=
Y4=
Y5=
Y6=
Y7=
Y8=

X	Y1	Y2
10	2.8394	44.407
11	3.1518	46.184
12	3.4985	48.031
13	3.8833	49.952
14	4.3104	51.95
15	4.7846	54.028
16	5.3109	56.189
17	5.8951	58.437
18	6.5436	60.774
19	7.2633	63.205
20	8.0623	65.734

X=10

X	Y1	Y2
53	252.42	239.82
54	280.18	249.41
55	311	259.39
56	345.21	269.77
57	383.19	280.56
58	425.34	291.78
59	472.12	303.45
60	524.06	315.59
61	581.7	328.21
62	645.69	341.34
63	716.72	354.99

X=53

At day 10, there are more SARS cases than Ebola cases.

$$E(10) = 1(1.11)^{10} \approx 3$$

$$S(10) = 30(1.04)^{10} \approx 44$$

At day 53, there are more Ebola cases than SARS cases.

$$E(53) = 1(1.11)^{53} \approx 252$$

$$S(53) = 30(1.04)^{53} \approx 239$$

PTS: 2 NAT: F.LE.A.2 TOP: Modeling Exponential Functions

308) ANS:

After 2 years, the balance in the account is $619.35.

Strategy: Write an exponential growth equation to model the problem. Then solve the equation for two years.

STEP 1: Exponential growth is modeled by the formula $A(t) = P(1+r)^t$, where:

A represents the amount after t cycles of growth,
P represents the starting amount, which is $600.
r represents the rate of growth, which is 1.6% or .016 as a decimal, and
t represents the number of cycles of growth, which are measured in years with annual compounding.

The equation is: $A(t) = 600(1+.016)^t$

STEP 2: Solve for two years growth.

$$A(t) = 600(1+.016)^t$$
$$A(2) = 600(1+.016)^2$$
$$A(2) = 600(1.016)^2$$
$$A(2) = 600(1.032256)$$
$$A(2) = 619.35$$

DIMS: Does It Make Sense? Yes. Each year, the interest on each \$100 is \$1.60, so the first year, there will be $6 \times \$1.60 = \$9.60$ interest. The second year interest will be another \$9.60 for the original \$600 plus 1.6% on the \$9.60. The total interest after two years will be $\$9.60 + \$9.60 + .016(\$9.60) \approx \$19.35$. Add this interest to the original \$600 and the amount in the account will be \$619.35.

PTS: 2 NAT: A.CED.A.1 TOP: Modeling Exponential Equations
NOT: NYSED classifies this problem as A.CED.A.1

309) ANS:
$B = 3000(1.042)^t$

Strategy: Use the formula for exponential growth to model the problem.

The formula for exponential **growth** is $y = a(1+r)^t$.
The formula for exponential **decay** is $y = a(1-r)^t$.
y = **final amount** after measuring growth/decay
a = **initial amount** before measuring growth/decay
r = growth/decay **rate** (usually a percent)
t = **number of time intervals** that have passed

The problem states that B should be used to represent the **final amount** after growth.
The problem states that \$3,000 is the **initial amount**.
The problem states that the **growth factor** is 4.2%, which is added to 1 and written as 1.042
The problem states that interest is compounded annually, so the number of time intervals is t years.
The final equation is written as $B = 3000(1.042)^t$

PTS: 2 NAT: F.BF.A.1 TOP: Modeling Exponential Equations

310) ANS: 1
Strategy 1: Use the formula for exponential growth to model the problem.

The formula for exponential **growth** is $y = a(1+r)^t$.
The formula for exponential **decay** is $y = a(1-r)^t$.
y = **final amount** after measuring growth/decay
a = **initial amount** before measuring growth/decay
r = growth/decay **rate** (usually a percent)
t = **number of time intervals** that have passed

The problem asks for the *right side expression for exponential growth*.
The problem states that \$3,000 is the **initial amount**.
The problem states that the **growth factor** is 2%, which is written as .02 and added to 1.
The problem states that interest is compounded annually from age 2 through age 18, so the number of time intervals is 16 years.

The final expression for the right side of the exponential growth equation is written as $3000(1+0.02)^{16}$.

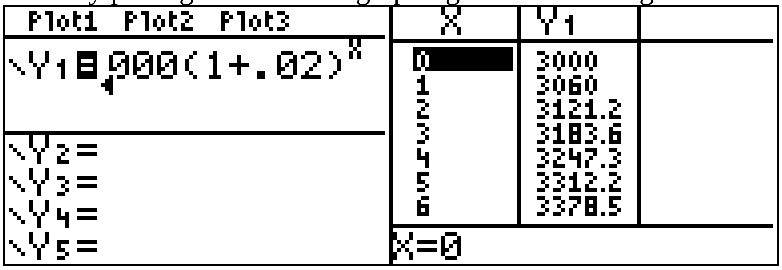
Strategy 2. Build a model and eliminate wrong answers.
Model the words using a table of values to see the pattern.

Krystal's Age	# Times Compounding	Amount
2	0	3000
3	1	3060
4	2	3121.2
5	3	3183.624
...	...	...
18	16	?

It is clear from the table that the number of times interest compounds is 2 less than Krystal's age. Eliminate answer choices *c* and *d*, because both show exponents of 18, which is Krystal's age, not the number of times the interest will compound.

The choices now are a and b. The table shows that the amounts are increasing, which is exponential growth, not exponential decay. Eliminate choice *b* because it shows exponential decay.

Check by putting choice a in a graphing calculator using x as the exponent.



```
Plot1  Plot2  Plot3
\Y1=3000(1+.02)^X

\Y2=
\Y3=
\Y4=
\Y5=
```

```
   X       Y1
   0       3000
   1       3060
   2       3121.2
   3       3183.6
   4       3247.3
   5       3312.2
   6       3378.5
X=0
```

Answer choice *a* creates the same table of values, and the amount of money on Krystal's 18th birthday will be $3000(1+0.02)^{16}$ dollars.

PTS: 2 NAT: F.BF.A.1 TOP: Modeling Exponential Equations

311) ANS: 3

Strategy: Use the formula for exponential growth: $A = P(1+r)^t$, where

A represents the amount after growth, which in this problem will be $f(t)$.

P represents the initial amount, which in this problem will be 9.05×10^6.

r represents the rate of growth expressed as a decimal, which in this problem will be 0.031 per year.

t represents the number of growth cycles, which in this problem will be 7

Use the exponential growth formula and substitution to write:

$$A = P(1+r)^t$$

$$f(t) = \left(9.05 \times 10^6\right)(1+0.031)^7$$

Answer choice *c* is correct.

PTS: 2 NAT: F.LE.A.2 TOP: Modeling Exponential Functions

312) ANS: 2

Step 1. Understand from the problem that only one of the answer choices will be different from the others, and one that is different will be the correct answer.
Step 2. Strategy: Use a graphing calculator to find the values of each expression.

Step 3. Execute the strategy.

a) $500(1.04)^3 = 562.432$

b) $500(1-.04)^3 = 442.368$

c) $500(1+.04)(1+.04)(1+.04) = 562.432$

d) $500 + 500(.04) + 520(.04) + 540.8(.04) = 562.432$

Answer choice b) is the correct answer, because produces a different value.

Step 4. Does it make sense? Yes. You can model an investment problem with compounding interest using the formula $A = P(1+r)^t$, where A is the amount, P is the initial amount invested, r is the interest rate expressed as a decimal, and t is the numer of compounding periods. Using this formula, the problem can be modelled as follows:

$$A = P(1+r)^t$$

$$A = 500(1+.04)^3$$

$$A = 500(1.04)^3$$

$$A = 562.432$$

PTS: 2 NAT: F.BF.A.1 TOP: Modeling Exponential Functions

313) ANS: 2

Use the formula $A = P(1+r)^t$, where A representes the amount in the accout, P represents the amount invested, r represents the rate, and t represents time.

Anne invested \$1000: $P = 1000$

1.3% annual interest rate: $r = .013$

2 years: $t = 2$

Write: $A = 1000(1+.013)^2$

Then, eliminate wrong answers.

a $A = 1000(1-0.013)^2$ The minus sign is wrong.

b $A = 1000(1+0.013)^2$ This is correct.

c $A = 1000(1-1.3)^2$ The minus sign is wrong and the annual interest rate is wrong.

d $A = 1000(1+1.3)^2$ The annual interest rate is wrong.

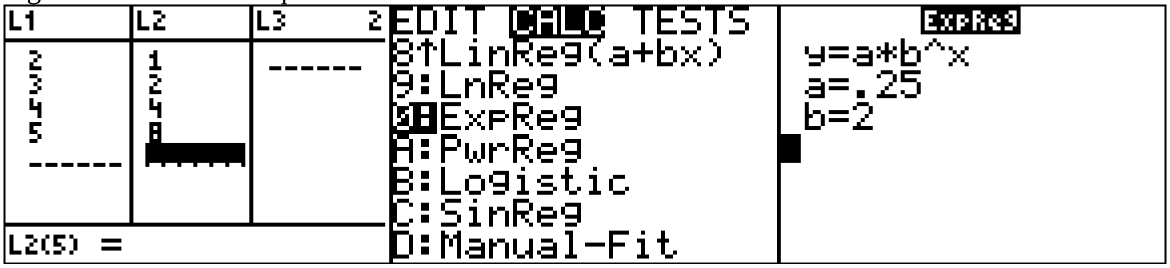
PTS: 2 NAT: F.BF.A.1 TOP: Modeling Exponential Functions

KEY: AI

314) ANS:

$y = 0.25(2)^x$.

Strategy: Input the four integral values from the graph into a graphing calculator and use exponental regression to find the equation.



L1	L2	L3
2	1	------
3	2	
4	4	
5	8	

L2(5) =

```
ExpReg
y=a*b^x
a=.25
b=2
```

Alternative Strategy: Use the standard form of an exponential equation, which is $y = ab^x$. Substitute the integral pairs of (2,1) and (3,2) from the graph into the standard form of an exponential equation and obtain the following: $1 = ab^2$ and $2 = ab^3$.

Therefore, $2ab^2 = ab^3$

$$2 = \frac{ab^3}{ab^2}$$

$$2 = b$$

Accordingly, the equation for the graph can now be written as $y = a \cdot 2^x$.
Substitute the integral pair (4,4) from the graph into the new equation and solve for *a*, as follows:

$$y = a \cdot 2^x$$

$$4 = a \cdot 2^4$$

$$4 = a \cdot 16$$

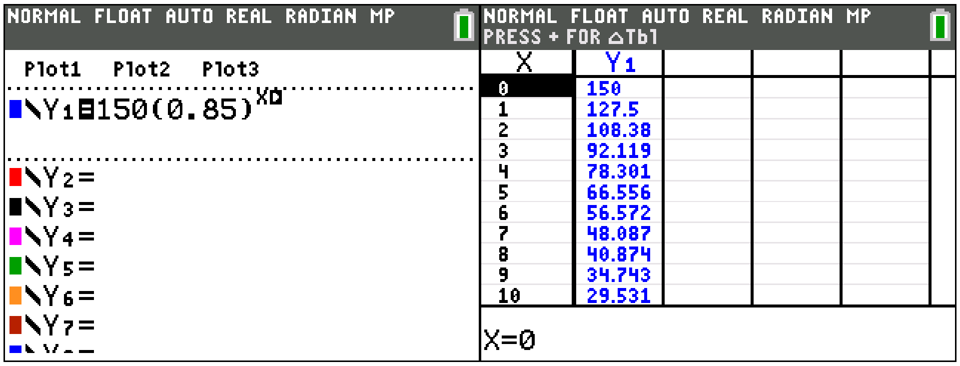
$$\frac{4}{16} = a$$

$$\frac{1}{4} = a$$

The graph of the equation can now be written as $y = \frac{1}{4}(2)^x$

PTS: 2 NAT: F.LE.A.2 TOP: Modeling Exponential Equations

315) ANS: 1
Strategy #1: Check each answer using a graphing calculator.



NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

$Y_1 = 150(0.85)^X$

$Y_2 =$
$Y_3 =$
$Y_4 =$
$Y_5 =$
$Y_6 =$
$Y_7 =$

NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1
0	150
1	127.5
2	108.38
3	92.119
4	78.301
5	66.556
6	56.572
7	48.087
8	40.874
9	34.743
10	29.531

X=0

Strategy #2: Use exponential regression to model the data in the table.

$$y = 149.58(0.8499)^x$$

PTS: 2 NAT: F.BF.A.1

316) ANS: 3
All of the answer choices involve exponential equations and are in the form of

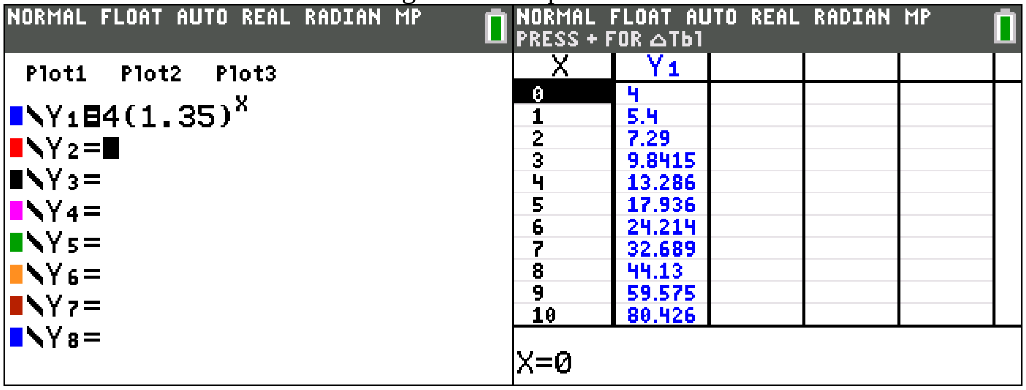
$$A = P(1 \pm r)^t$$

where A represents the current value, *P* represents the starting amount, *r* represents the rate of growth or decay, and *t* represents the number of times that growth occurs.

This answer choices involve two equations, $V(x) = 4(0.65)^x$ and $V(x) = 4(1.35)^x$ combined with the words growth and decay. The table shows that the value of V(x) is growing.

Therefore, any answer choices showing exponential decay must be eliminated and any answer choices where the value of $(1 \pm r) < 1$ must be eliminated. This leaves only $V(x) = 4(1.35)^x$ and it grows as the only correct answer.

This can be checked by inputting $V(x) = 4(1.35)^x$ into a graphing calculator and inspecting the resulting table to see it it matches the table given in the problem.



NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

\Y1=4(1.35)^X

\Y2=

\Y3=

\Y4=

\Y5=

\Y6=

\Y7=

\Y8=

NORMAL FLOAT AUTO REAL RADIAN MP

PRESS + FOR △Tbl

X	Y1
0	4
1	5.4
2	7.29
3	9.8415
4	13.286
5	17.936
6	24.214
7	32.689
8	44.13
9	59.575
10	80.426

X=0

PTS: 2 NAT: F.LE.A.2 TOP: Modeling Exponential Functions
KEY: AI

317) ANS:
0.5 represents the rate of decay and 300 represents the initial amount of the compound.

Strategy: Use information from the problem together with the standard formula for exponential decay, which is $A = P(1-r)^t$, where A represents the amount remaining, P represents the initial amount, r represents the rate of decay, and t represents the number of cycles of decay.

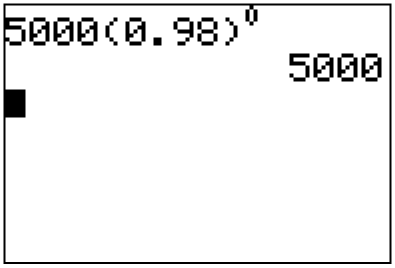
$$A = P(1-r)^t$$

$$p(t) = 300(0.5)^t$$

The structures of the equations show that $P = 300$ and $(1-r) = 0.5$.
Accordingly, 300 represents the initial amount of chemical substance in milligrams and 0.5 represents the rate of decay each year.

PTS: 2 NAT: F.LE.B.5 TOP: Modeling Exponential Equations

318) ANS: 3
Strategy 1: The y-intercept of a function occurs when the value of x is 0. The strategy is to evaluate the function $y = 5000(0.98)^x$ for $x = 0$

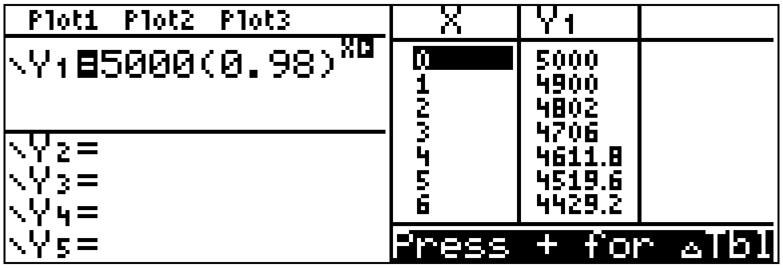


5000(0.98)^0

5000

This represents the amount of money in the account before exponential decay begins.

Strategy 2. Input the equation in a graphing calculator and view the table of values.



```
Plot1  Plot2  Plot3
\Y1=5000(0.98)^X
\Y2=
\Y3=
\Y4=
\Y5=
```

X	Y1	
0	5000	
1	4900	
2	4802	
3	4706	
4	4611.8	
5	4519.6	
6	4429.2	

Press + for ΔTbl

The table of values clearly shows the initial value of the account and its exponential decay.

PTS: 2 NAT: F.IF.C.8 TOP: Modeling Exponential Equations

319) ANS: 2

Strategy: Identify each of the parts of the function $V(t) = 1350(1.017)^t$, then answer the question.

$V(t)$ represents the current value of the comic book in dollars.
1350 represents the original value of the comic book when it was purchased.
(1.017) represents the growth factor, which consists of (1+r), where r is the rate of growth per year. The value of r is 0.017, which is found by subtracting 1 from (1.017).
t represents the number of years since its purchase.

The problem wants to know the value of r, which is 0.017. However, all of the answer choices are expressed as percents rather than decimals. A decimal may be converted to a percent as follows:

$$\frac{.017}{1} = \frac{x\%}{100\%}$$

$$.017 \times 100 = x\%$$

$$1.7\% = x\%$$

$$\frac{.017}{1} = \frac{1.7\%}{100\%}$$

The yearly appreciation rate of the comic book is 1.7% and the correct answer is b.

DIMS? Does It Make Sense? The appreciation rate seems to make sense, but it is difficult to understand why someone would originally pay $1,350 for a comic book.

PTS: 2 NAT: A.SSE.A.1 TOP: Modeling Exponential Equations

320) ANS:

The percent of change each year is 5%.

Strategy: Use information from the problem together with the standard formula for exponential decay, which is $A = P(1-r)^t$, where *A* represents the amount remaining, *P* represents the initial amount, *r* represents the rate of decay, and *t* represents the number of cycles of decay.

$$A = P(1-r)^t$$

$$y = 5100(0.95)^x$$

The structures of the equations show that $(1-r) = 0.95$.

Solving for r shows that $r = 0.05$, or 5%.

$$(1 - r) = 0.95$$
$$-r = 0.95 - 1$$
$$-r = -0.05$$
$$r = 0.05$$

PTS: 2 NAT: F.LE.B.5 TOP: Modeling Exponential Functions

321) ANS: 4

Strategy: Use the formula for exponential growth or decay, which is $A = P(1 \pm r)^t$, where
A represents the amount after *t* growth or decay cycles.
P represents the starting amount.
r represents the rate of growth expressed as a decimal, and
t represents the number of growth or decay cycles.

In the equation $A = 1300(1.02)^7$, the number 1.02 corresponds to $(1 \pm r)$, so write

$$1.02 = 1 \pm r$$
$$1.02 - 1 = r$$
$$.02 = r$$
$$2\% = r$$

1.02 means that the growth rate is 2%.

PTS: 2 NAT: A.SSE.A.1 TOP: Modeling Exponential Functions

322) ANS: 2

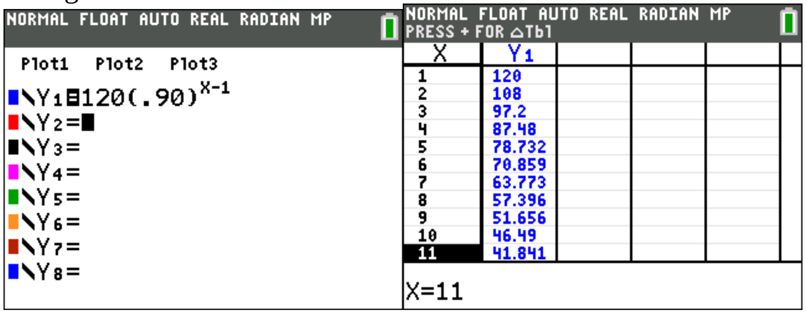
The function $v(x) = 30,000(0.78)^x$ is of the form $A = P(1 \pm r)^t$, which represents exponential growth or decay. The term in parenthesis (0.78) is equal to (1+r), so we can write and solve the following equation:

$$0.78 = 1 + 8$$
$$0.78 - 1 = r$$
$$-0.22 = r$$
$$-22\% = r$$

PTS: 2 NAT: F.LE.B.5

323) ANS: 3

Strategy: Input the function in a graphing calculator and study the graph and table views, then eliminate wrong answers.



$Y_1 = 120(.90)^{X-1}$
$Y_2 =$
$Y_3 =$
$Y_4 =$
$Y_5 =$
$Y_6 =$
$Y_7 =$
$Y_8 =$

X	Y1
1	120
2	108
3	97.2
4	87.48
5	78.732
6	70.859
7	63.773
8	57.396
9	51.656
10	46.49
11	41.841

X=11

~~a)~~ The number of miles he runs will ~~increase~~ by 90% each week.
~~b)~~ The number of miles he runs will be ~~10%~~ of the previous week.
c) $M(w)$ represents the total mileage run in a given week.
~~d)~~ *w* represents the number of weeks ~~left until his marathon~~.

PTS: 2 NAT: F.LE.B.5 TOP: Modeling Exponential Functions

324) ANS:
The percent that the value of the car decreases each year is 15%.

Strategy: Note that $v(t) = P(.85)^t$ is of the exponential growth/decay form $A = P(1 \pm r)^t$, and that the value (.85) in parentheses corresponds to the expression $(1 \pm r)$. Since the value of the car decreases, this is exponential decay. The relationship between the corresponding expressions can be written as $(.85) = (1 - r)$.
Solve for *r* as follows:

$$\begin{aligned}(.85) &= (1 - r)\\ .85 &= 1 - r\\ .85 - 1 &= -r\\ -.15 &= -r\\ .15 &= r\end{aligned}$$

PTS: 2 NAT: F.LE.B.5 TOP: Modeling Exponential Functions

POLYNOMIALS
Identifying Solutions

Common Core Standard	**Next Generation Standard**
A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, ~~often forming a curve (which could be a line)~~.	**AI-A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. **Note: Graphing linear equations is a fluency recommendation for Algebra I. Students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity; as well as modeling linear phenomena.**

LEARNING OBJECTIVES

Students will be able to:

1)

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

balanced equation
graph
ordered pair
solutions
table of values
true statement

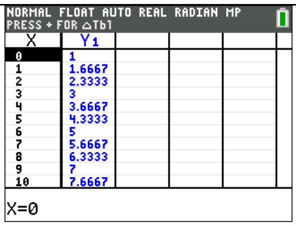
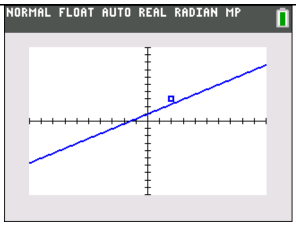
BIG IDEAS

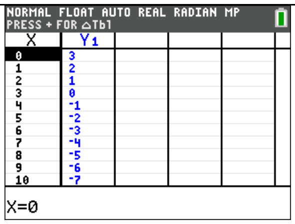
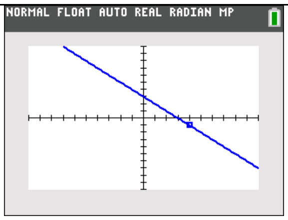
A solution to an equation is a value or values that satisfy the equation.

- In equations, solutions are those values that make the left expression equal to the right expression. When both the left and right expressions are equal in value, the equation is said to be balanced and the equation becomes a true statement.
- In tables of values, solutions appear in the form of ordered pairs that, when substituted into the equation, will make the equation balance.

- In a graph, solutions appear as points on the line. The graph of an equation represents the set of all points that satisfy the equation (make the equation balance). Each and every point on the graph of an equation represents a ordered pair that can be substituted into the equation to make the equation true. Thus, if a point is on the graph of the equation, the point is a solution to the equation.

MODELING ESSENTIAL SKILLS

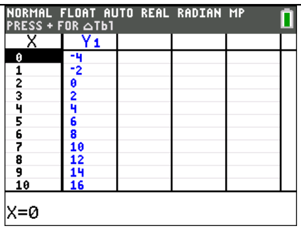
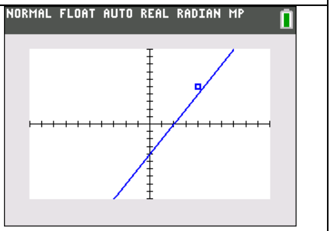
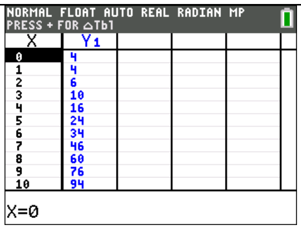
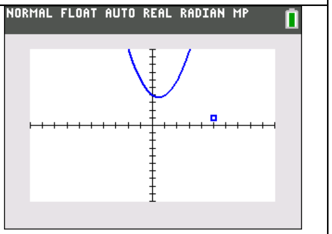
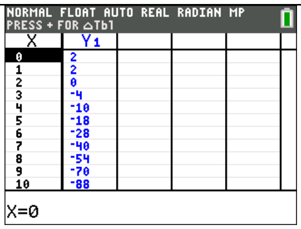
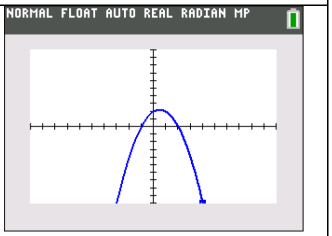
<table>
<tr><th>?Solution?
and Equation</th><th>Balanced
Equation</th><th>Table
of Values</th><th>Graph</th></tr>
<tr>
<td>Is (2,3) a solution of
$y=\frac{2}{3}x+1$</td>
<td>$y=\frac{2}{3}x+1$
$(3)=\frac{2}{3}(2)+1$
$3=\frac{4}{3}+1$
$9=4+3$
$9\neq 7$
No, the equation does not balance.</td>
<td>
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl
<table>
<tr><th>X</th><th>Y₁</th></tr>
<tr><td>0</td><td>1</td></tr>
<tr><td>1</td><td>1.6667</td></tr>
<tr><td>2</td><td>2.3333</td></tr>
<tr><td>3</td><td>3</td></tr>
<tr><td>4</td><td>3.6667</td></tr>
<tr><td>5</td><td>4.3333</td></tr>
<tr><td>6</td><td>5</td></tr>
<tr><td>7</td><td>5.6667</td></tr>
<tr><td>8</td><td>6.3333</td></tr>
<tr><td>9</td><td>7</td></tr>
<tr><td>10</td><td>7.6667</td></tr>
</table>
X=0
No, the table of values shows: when $x=2$, the values of y is 2.3333.</td>
<td>

No, the point (2,3) is not on the graph of the line</td>
</tr>
<tr>
<td>Is (4,-1) a solution of
$y=-x+3$</td>
<td>$y=-x+3$
$(-1)=-(4)+3$
$-1=-4+3$
$-1=-1$
Yes. The equation balances.</td>
<td>
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl
<table>
<tr><th>X</th><th>Y₁</th></tr>
<tr><td>0</td><td>3</td></tr>
<tr><td>1</td><td>2</td></tr>
<tr><td>2</td><td>1</td></tr>
<tr><td>3</td><td>0</td></tr>
<tr><td>4</td><td>-1</td></tr>
<tr><td>5</td><td>-2</td></tr>
<tr><td>6</td><td>-3</td></tr>
<tr><td>7</td><td>-4</td></tr>
<tr><td>8</td><td>-5</td></tr>
<tr><td>9</td><td>-6</td></tr>
<tr><td>10</td><td>-7</td></tr>
</table>
X=0
Yes. The table of values shows that when x = 4, the value of y is -1.</td>
<td>

Yes. The point (4,-1) is on the graph of the line</td>
</tr>
</table>

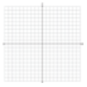
DEVELOPING ESSENTIAL SKILLS

Determine if the given ordered pair is a solution to the given equation using three different methods for identifying solutions.

?Solution? and Equation	Balanced Equation	Table of Values	Graph
Is (4,5) a solution of $y=2x-4$?	$y=2x-4$ $(5)=2(4)-4$ $5=8-4$ $5\neq 4$ No. The equation does not balance.	NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR ΔTbl X Y1 0 -4 1 -2 2 0 3 2 4 4 5 6 6 8 7 10 8 12 9 14 10 16 X=0 No. The table of values shows that when x = 4, the value of y is also 4.	 No, the point (4,5) is not on the graph of the line
Is (5,1) a solution of $y=x^2-x+4$?	$y=x^2-x+4$ $(1)=(5)^2-(5)+4$ $1=25-5+4$ $1\neq 24$ No. The equation does not balance.	NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR ΔTbl X Y1 0 4 1 4 2 6 3 10 4 16 5 24 6 34 7 46 8 60 9 76 10 94 X=0 No. The table of values shows that when x=5, the value of y is 24.	 No, the point (5,1) is not on the line.
Is (4,-10) a solution of $y=-x^2+x+2$?	$y=-x^2+x+2$ $(-10)=-(4)^2+(4)+2$ $-10=-16+4+2$ $-10=-10$ Yes. The equation balances.	NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR ΔTbl X Y1 0 2 1 2 2 0 3 -4 4 -10 5 -18 6 -28 7 -40 8 -54 9 -70 10 -88 X=0 Yes. The table of values shows that when x=4, the value of y is -10.	 Yes. The point (4, -10) is on the line.

A.REI.10: Identifying Solutions

325) On the set of axes below, draw the graph of the equation $y = -\frac{3}{4}x + 3$.



Is the point $(3,2)$ a solution to the equation? Explain your answer based on the graph drawn.

326) Which point is *not* on the graph represented by $y = x^2 + 3x - 6$?

1)	$(-6,12)$	3)	$(2,4)$
2)	$(-4,-2)$	4)	$(3,-6)$

327) The solution of an equation with two variables, *x* and *y*, is

1)	the set of all x values that make *y = 0.*	3)	the set of all ordered pairs (x,y), that makes the equation true.
2)	the set of all y values that make x = *0.*	4)	the set of all ordered pairs (x,y), where the graph of the equation crosses the *y*-axis

328) Which ordered pair would *not* be a solution to $y = x^3 - x$?

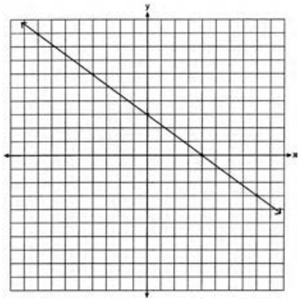
1) $(-4,-60)$
2) $(-3,-24)$
3) $(-2,-6)$
4) $(-1,-2)$

329) Which ordered pair below is *not* a solution to $f(x) = x^2 - 3x + 4$?

1) $(0,4)$
2) $(1.5,7.5)$
3) $(5,14)$
4) $(-1,6)$

SOLUTIONS

325) ANS:



No, because $(3,2)$ is not on the graph.

Strategy #1. Use the y-intercept and the slope to plot the graph of the line, then determine if the point $(3,2)$ is on the graph.

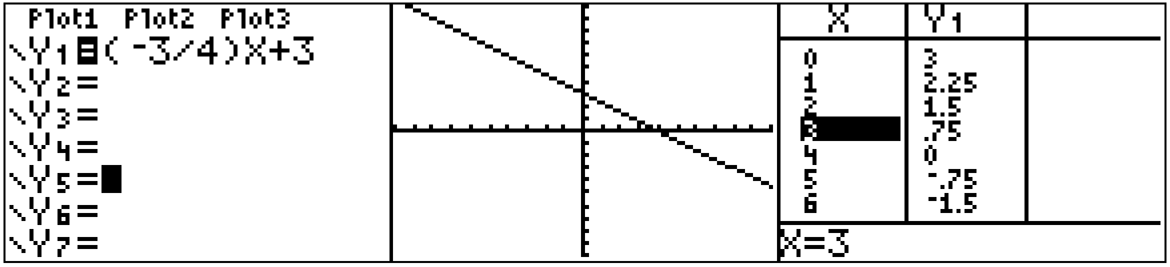
STEP 1. Plot the y-intercept.
Plot $(0,3)$. The given equation is in the slope intercept form of a line, $y = mx + b$, where b is the y-intercpet. The value of b is 3, so the graph of the equation crosses the y axis at $(0,3)$.

STEP 2. Use the slope of the line to find and plot a second point on the line. The given equation is in the slope intercept form of a line, $y = mx + b$, where m is the slopet. The value of m is $\frac{-3}{4}$, so the graph of the equation has a negative slope that goes down three units and across four units. Starting at the y-intercept, $(0,3)$, if you go down 3 and over 4, the graph of the line will pass through the point $(4,0)$.

STEP 3. Use a straightedge to draw a line that passes through the points $(0,3)$ and $(4,0)$.

STEP 4. Inspect the graph to determine if the point $(3,2)$ is on the line. It is not.

Strategy #2. Input the equation of the line into a graphing calculator, then use the table of values to plot the graph of the line and to determine if the point $(3,2)$ is on the line.



```
Plot1 Plot2 Plot3
\Y1=(-3/4)X+3
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

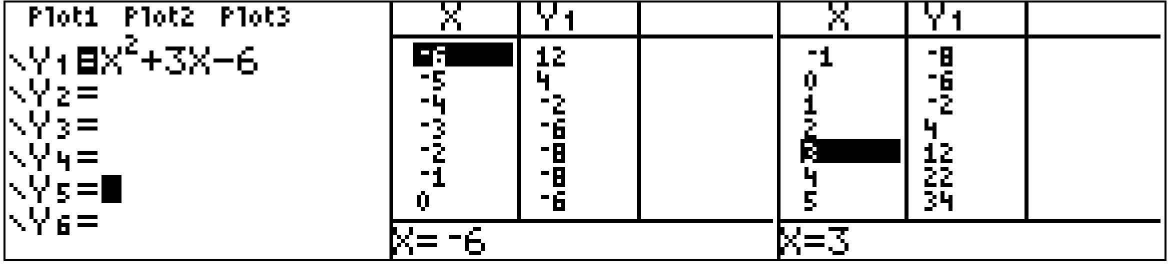
```
X     Y1
0     3
1     2.25
2     1.5
3     .75
4     0
5     -.75
6     -1.5
X=3
```

Be sure to explain your answer in terms of the graph and not in terms of the table of values or the function rule.

PTS: 2 NAT: A.REI.10 TOP: Graphing Linear Functions

326) ANS: 4
Straegy: Input the equation in a graphing calculator, then use the table of values to eliminate wrong answers.

STEP 1. Input the equation and look at the table view of the function.



```
Plot1 Plot2 Plot3
\Y1=X²+3X-6
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

```
X     Y1
-6    12
-5    4
-4    -2
-3    -6
-2    -8
-1    -8
0     -6
X=-6
```

```
X     Y1
-1    -8
0     -6
1     -2
2     4
3     12
4     22
5     34
X=3
```

STEP 2. Eliminate answers that are *on* the graph.
The point (-6, 12) is on the graph, so eliminate answer choice a.
The point (-4, -2) is on the graph, so eliminate answer choice b.
The point (2, 4) is on the graph, so eliminate answer choice c.
The point (3, -6) is *not* on the graph, so answer choice d is the correct answer.

PTS: 2 NAT: A.REI.D.10 TOP: Graphing Quadratic Functions

327) ANS: 3

1. Understanding: The problem is asking for the definition of solution as it relates to to equations with two variables.

2. Strategy: Examine each answer choice and eliminate wrong answers.

3. Execution of Strategy:

a) the set of all x-values that make $y = 0$ is used to find the x-intercepts of an equation.

b) the set of all y-values that make $x = 0$ is used to find the y-intercepts of an equation.

c) the set of all ordered pairs, (x,y) that makes the equation true is the best answer choice.

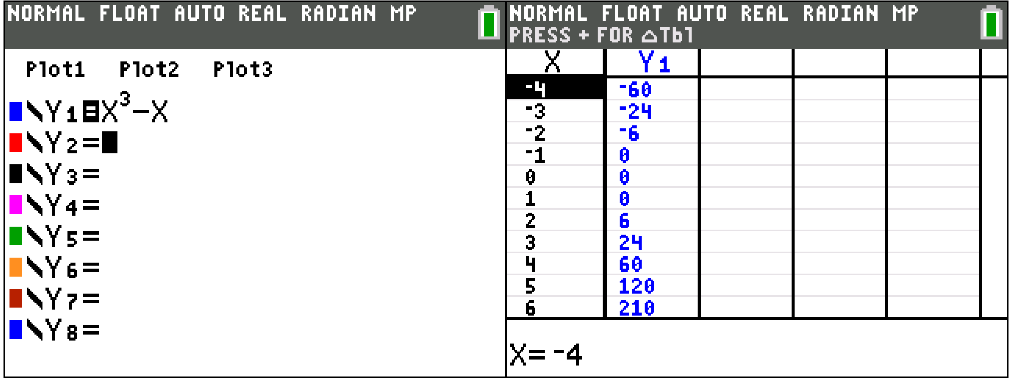
d) the set of all ordered pirs, (x,y) where the graph of the equation crosses the y-axis is too limiting.

4. Does it Make Sense? Yes. An equation is true if the left expression equals the right expression. If the equation has two variables, then the solution to the equation must have values for each variable.

PTS: 2 NAT: A.REI.D.11

328) ANS: 4

Strategy #1: Input the equation into a graphing calculator and inspect the table of values to see which answer choices are solutions to the equation.



NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

$Y_1 = X^3 - X$

$Y_2 =$

$Y_3 =$

$Y_4 =$

$Y_5 =$

$Y_6 =$

$Y_7 =$

$Y_8 =$

NORMAL FLOAT AUTO REAL RADIAN MP

PRESS + FOR △Tbl

X	Y_1
-4	-60
-3	-24
-2	-6
-1	0
0	0
1	0
2	6
3	24
4	60
5	120
6	210

X= -4

Note that $(-4,-60)$, $(-3,-24)$, and $(-2,-6)$ appear in the table and are, therefore, solutions to the equation $y = x^3 - x$. The ordered pair $(-1,-2)$ does not appear in the table and is, therefore, not a solution to the equation $y = x^3 - x$.

Strategy #2

Substitute each ordered pair into the equation $y = x^3 - x$ and see if the equation balances.

$(-4,-60)$ Equation balances. $y = x^3 - x$ $-60 = -4^3 - (-4)$ $-60 = -64 + 4$ $-60 = -60$	$(-2,-6)$ Equation balances. $y = x^3 - x$ $-6 = -2^3 - (-2)$ $-6 = -8 + 2$ $-6 = -6$
$(-3,-24)$ Equation balances. $y = x^3 - x$ $-24 = -3^3 - (-3)$ $-24 = -27 + 3$ $-24 = -24$	$(-1,-2)$ Equation does not balance. $y = x^3 - x$ $-2 \neq -1^3 - (-2)$ $-2 \neq -1 + 2$ $-2 \neq 1$

PTS: 2 NAT: A.REI.D.10 TOP: Identifying Solutions

329) ANS: 4

Strategy: Use the table of values view in a graphing calculator to find the ordered pair that does <u>not</u> satisfy the function.

STEP 1. Input $f(x) = x^2 - 3x + 4$ into a graphing calculator

STEP 2. Set table view to increase by half integers.

STEP 3. Inspect table and eliminate any answer choice that appears in the table.

```
 Plot1  Plot2  Plot3
\Y1=X²-3X+4
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

```
TABLE SETUP
 TblStart=0
 ΔTbl=.5
Indpnt: Auto  Ask
Depend:       Ask
```

```
  X     Y1
 -1     8
 -.5    5.75
 0      4
 .5     2.75
 1      2
 1.5    1.75
 2      2
X=-1
```

STEP 4. Select the ordered pair that does not appear in the table of values.

STEP 5. Check: The ordered pair (-1,6) does not appear in the table of values and will not satisfy the function rule $f(x) = x^2 - 3x + 4$.

$$f(x) = x^2 - 3x + 4$$
$$6 \neq (-1)^2 - 3(-1) + 4$$
$$6 \neq 1 + 3 + 4$$
$$6 \neq 8$$

PTS: 2 NAT: A.REI.D.10 TOP: Identifying Solutions

K – Polynomials, Lesson 2, Operations with Polynomials (r. 2018)

POLYNOMIALS

Operations with Polynomials

Common Core Standard	Next Generation Standard
A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	**AI-A.APR.1** Add, subtract, and multiply polynomials and recognize that the result of the operation is also a polynomial. This forms a system analogous to the integers. **Note: This standard is a fluency recommendation for Algebra I. Fluency in adding, subtracting and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions.**

LEARNING OBJECTIVES

Students will be able to:

1) add, subtact, and multiply polynomials.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

- **Polynomial:** A monomial or the sum of two or more monomials whose exponents are positive.
  Example: $5a^2 + ba - 3$
- **Monomial:** A polynomial with one term; it is a number, a variable, or the product of a number (the coefficient) and one or more variables
  Examples: $-\frac{1}{4}$, x^2, $4a^2b$, -1.2, $m^2n^3p^4$
- **Binomial:** An algebraic expression consisting of two terms
  Example $(5a + 6)$
- **Trinomial:** A polynomial with exactly three terms.
  Example $(a^2 + 2a - 3)$
- **Like Terms:** Like terms must have **exactly the same base and the same exponent**. Their coefficients may be different. Real numbers are like terms.
  Example: Given the expression

$1x^2 + 2y + 3x^2 + 4x + 5x^3 + 6y^2 + 7y + 8x^3 + 9y^2$,
the following are like terms:
$1x^2$ and $3x^2$
$2y$ and $7y$
$4x$ *has no other like terms in the expression*
$5x^3$ and $8x^3$
$6y^2$ and $9y^2$

Like terms in the same expression can be combined by adding their coefficients.

$1x^2$ and $3x^2 = 4x^2$
$2y$ and $7y = 9y$
$4x$ *has no other like terms in the expression = 4x*
$5x^3$ and $8x^3 = 13x^3$
$6y^2$ and $9y^2 = 15y^2$
$1x^2 + 2y + 3x^2 + 4x + 5x^3 + 6y^2 + 7y + 8x^3 + 9y^2 = 4x^2 + 9y + 4x + 13x^3 + 15y^2$

BIG IDEAS

Adding and Subtracting Polynomials

To add or subtract polynomials, arrange the polynomials one above the other with like terms in the same columns. Then, add or subtract the coefficients of the like terms in each column and write a new expression.

Addition Example	**Subtraction** Example
Add: $\left(3r^4 - 9r^3 - 8\right) + \left(4r^4 + 8r^3 - 8\right)$	Subtract: $\left(3r^4 - 9r^3 - 8\right) - \left(4r^4 + 8r^3 - 8\right)$
$3r^4 \qquad -9r^3 \qquad -8$	$3r^4 \qquad -9r^3 \qquad -8$
$\underline{4r^4 \qquad +8r^3 \qquad -8}$	$\underline{-\left(4r^4\right) \qquad -\left(+8r^3\right) \qquad -\left(-8\right)}$
$7r^4 \qquad -r^3 \qquad -16$	$-1r^4 \qquad -17r^3 \qquad +0$

Multiplying Polynomials

To multiply two polynomials, multiply each term in the first polynomial by each term in the second polynomial, then combine like terms.

Example:

Multiply: $\left(-8r^2 - 9r + 7\right)\left(-5r + 1\right)$

STEP 1: Multiply the first term in the first polynomial by each term in the second polynomial, as follows:

$$-8r^2\left(-5r+1\right)$$
$$-8r^2\left(-5r\right)+-8r^2\left(1\right)$$
$$\boxed{40r^3-8r^2}$$

STEP 2. Multiply the next term in the first polynomial by each term in the second polynomial, as follows:

$$\text{-9r}\left(\text{-5r+1}\right)$$

$$\text{-9r}\left(\text{-5r}\right)\text{+-9r}\left(1\right)$$

$$\boxed{\text{45r}^2\text{-9r}}$$

STEP 3. Multiply the next term in the first polynomial by each term in the second polynomial, as follows:

$$7\left(\text{-5r+1}\right)$$

$$7\left(\text{-5r}\right)+7\left(1\right)$$

$$\boxed{\text{-35r+7}}$$

STEP 4. Combine like terms from each step.

$$\text{40r}^3\ \text{-8r}^2+\text{45r}^2\ \text{-9r -35r +7}$$

$$\boxed{\text{40r}^3\quad \text{+37r}^2\qquad \text{-44r}\quad \text{+7}}$$

DEVELOPING ESSENTIAL SKILLS

1. When $3g^2-4g+2$ is subtracted from $7g^2+5g-1$, the difference is
   - a. $-4g^2-9g+3$
   - b. $4g^2+g+1$
   - c. $4g^2+9g-3$
   - d. $10g^2+g+1$
2. When $4x^2+7x-5$ is subtracted from $9x^2-2x+3$, the result is
   - a. $5x^2+5x-2$
   - b. $5x^2-9x+8$
   - c. $-5x^2+5x-2$
   - d. $-5x^2+9x-8$
3. The sum of $4x^3+6x^2+2x-3$ and $3x^3+3x^2-5x-5$ is
   - a. $7x^3+3x^2-3x-8$
   - b. $7x^3+3x^2+7x+2$
   - c. $7x^3+9x^2-3x-8$
   - d. $7x^6+9x^4-3x^2-8$
4. What is the result when $2x^2+3xy-6$ is subtracted from $x^2-7xy+2$?
   - a. $-x^2-10xy+8$
   - b. $x^2+10xy-8$
   - c. $-x^2-4xy-4$
   - d. $x^2-4xy-4$
5. When $5x+4y$ is subtracted from $5x-4y$, the difference is
   - a. 0
   - b. $10x$
   - c. $8y$
   - d. $-8y$
6. What is the sum of $-3x^2-7x+9$ and $-5x^2+6x-4$?
   - a. $-8x^2-x+5$
   - b. $-8x^4-x+5$
   - c. $-8x^2-13x+13$
   - d. $-8x^4-13x^2+13$
7. When $8x^2+3x+2$ is subtracted from $9x^2-3x-4$, the result is
   - a. x^2-2
   - b. $17x^2-2$
   - c. $-x^2+6x+6$
   - d. x^2-6x-6
8. The sum of $3x^2+5x-6$ and $-x^2+3x+9$ is
   - a. $2x^2+8x-15$
   - b. $2x^2+8x+3$
   - c. $2x^4+8x^2+3$
   - d. $4x^2+2x-15$
9. When $2x^2-3x+2$ is subtracted from $4x^2-5x+2$, the result is
   - a. $2x^2-2x$
   - c. $-2x^2-8x+4$

b. $-2x^2 + 2x$
d. $2x^2 - 8x + 4$

10. The sum of $8n^2 - 3n + 10$ and $-3n^2 - 6n - 7$ is
    a. $5n^2 - 9n + 3$
    b. $5n^2 - 3n - 17$
    c. $-11n^2 - 9n - 17$
    d. $-11n^2 - 3n + 3$
11. What is the result when $4x^2 - 17x + 36$ is subtracted from $2x^2 - 5x + 25$?
    a. $6x^2 - 22x + 61$
    b. $2x^2 - 12x + 11$
    c. $-2x^2 - 22x + 61$
    d. $-2x^2 + 12x - 11$
12. When $6x^2 - 4x + 3$ is subtracted from $3x^2 - 2x + 3$, the result is
    a. $3x^2 - 2x$
    b. $-3x^2 + 2x$
    c. $3x^2 - 6x + 6$
    d. $-3x^2 - 6x + 6$
13. What is the product of $(c + 8)$ and $(c - 5)$?
    a. $c^2 + 3c - 40$
    b. $c^2 - 3c - 40$
    c. $c^2 + 13c - 40$
    d. $c^2 - 40$
14. The expression $\left(a^2 + b^2\right)^2$ is equivalent to
    a. $a^4 + b^4$
    b. $a^4 + a^2b^2 + b^4$
    c. $a^4 + 2a^2b^2 + b^4$
    d. $a^4 + 4a^2b^2 + b^4$
15. The expression $(x - 6)^2$ is equivalent to
    a. $x^2 - 36$
    b. $x^2 + 36$
    c. $x^2 - 12x + 36$
    d. $x^2 + 12x + 36$
16. The length of a rectangle is represented by $x^2 + 3x + 2$, and the width is represented by $4x$. Express the perimeter of the rectangle as a trinomial. Express the area of the rectangle as a trinomial.
17. What is the product of $(3x + 2)$ and $(x - 7)$?
    a. $3x^2 - 14$
    b. $3x^2 - 5x - 14$
    c. $3x^2 - 19x - 14$
    d. $3x^2 - 23x - 14$
18. What is the product of $-3x^2y$ and $(5xy^2 + xy)$?
    a. $-15x^3y^3 - 3x^3y^2$
    b. $-15x^3y^3 - 3x^3y$
    c. $-15x^2y^2 - 3x^2y$
    d. $-15x^3y^3 + xy$

Answers

1. ANS: C
2. ANS: B
3. ANS: C
4. ANS: A
5. ANS: D
6. ANS: A
7. ANS: D
8. ANS: B
9. ANS: A
10. ANS: A
11. ANS: D
12. ANS: B
13. ANS: A
14. ANS: C
15. ANS: C
16. ANS:

$$P = 2(x^2 + 3x + 2) + 2(4x) = 2x^2 + 6x + 4 + 8x = 2x^2 + 14x + 4$$

$$A = 4x(x^2 + 3x + 2) = 4x^3 + 12x^2 + 8x$$

17. ANS: C
18. ANS: A

REGENTS EXAM QUESTIONS (through June 2018)

A.APR.A.1: Operations with Polynomials

330) If $A = 3x^2 + 5x - 6$ and $B = -2x^2 - 6x + 7$, then $A - B$ equals

1) $-5x^2 - 11x + 13$
2) $5x^2 + 11x - 13$
3) $-5x^2 - x + 1$
4) $5x^2 - x + 1$

331) Express the product of $2x^2 + 7x - 10$ and $x + 5$ in standard form.

332) Fred is given a rectangular piece of paper. If the length of Fred's piece of paper is represented by $2x - 6$ and the width is represented by $3x - 5$, then the paper has a total area represented by

1) $5x - 11$
2) $6x^2 - 28x + 30$
3) $10x - 22$
4) $6x^2 - 6x - 11$

333) Subtract $5x^2 + 2x - 11$ from $3x^2 + 8x - 7$. Express the result as a trinomial.

334) If the difference $(3x^2 - 2x + 5) - (x^2 + 3x - 2)$ is multiplied by $\frac{1}{2}x^2$, what is the result, written in standard form?

335) Which trinomial is equivalent to $3(x - 2)^2 - 2(x - 1)$?

1) $3x^2 - 2x - 10$
2) $3x^2 - 2x - 14$
3) $3x^2 - 14x + 10$
4) $3x^2 - 14x + 14$

336) When $(2x - 3)^2$ is subtracted from $5x^2$, the result is

1) $x^2 - 12x - 9$
2) $x^2 - 12x + 9$
3) $x^2 + 12x - 9$
4) $x^2 + 12x + 9$

337) The expression $3(x^2 - 1) - (x^2 - 7x + 10)$ is equivalent to

1) $2x^2 - 7x + 7$
2) $2x^2 + 7x - 13$
3) $2x^2 - 7x + 9$
4) $2x^2 + 7x - 11$

338) What is the product of $2x + 3$ and $4x^2 - 5x + 6$?

1) $8x^3 - 2x^2 + 3x + 18$
2) $8x^3 - 2x^2 - 3x + 18$
3) $8x^3 + 2x^2 - 3x + 18$
4) $8x^3 + 2x^2 + 3x + 18$

339) Which expression is equivalent to $2(3g - 4) - (8g + 3)$?

1) $-2g - 1$
2) $-2g - 5$
3) $-2g - 7$
4) $-2g - 11$

340) Express in simplest form: $(3x^2 + 4x - 8) - (-2x^2 + 4x + 2)$

341) Write the expression $5x + 4x^2(2x + 7) - 6x^2 - 9x$ as a polynomial in standard form.

342) Which polynomial is twice the sum of $4x^2 - x + 1$ and $-6x^2 + x - 4$?

1) $-2x^2 - 3$
2) $-4x^2 - 3$
3) $-4x^2 - 6$
4) $-2x^2 + x - 5$

SOLUTIONS

330) ANS: 2

Strategy: To subtract, change the signs of the subtrahend and add.

Given: $3x^2+5x-6$ $\underline{-\left(-2x^2-6x+7\right)}$	Change the signs and add: $\begin{array}{r} 3x^2+5x-\ 6 \\ \underline{+2x^2+\ 6x-\ 7} \\ 5x^2+11x-13 \end{array}$

PTS: 2 NAT: A.APR.A.1 TOP: Addition and Subtraction of Polynomials

KEY: subtraction

331) ANS:

$2x^3+17x^2+25x-50$

Strategy: Use the distribution property to multiply polynomials, then simplify.

STEP 1. Use the distributive property

$$(2x^2+7x-10)(x+5)$$

$$2x^3+10x^2+7x^2+35x-10x-50$$

$$2x^3+17x^2+25x-50$$

STEP 2. Simplify by combining like terms.

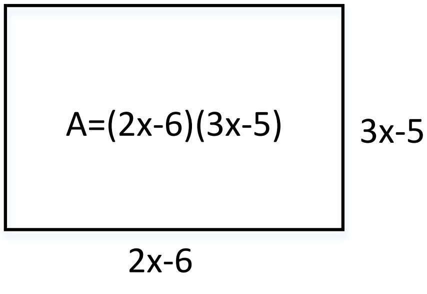
$$2x^3+10x^2+7x^2+35x-10x-50$$

$$2x^3+17x^2+25x-50$$

PTS: 2 NAT: A.APR.A.1 TOP: Multiplication of Polynomials

332) ANS: 2

Strategy: Draw a picture and use the area formula for a rectange: $A=lw$.



$$A=(2x-6)(3x-5)$$

$$A=6x^2-10x-18x+30$$

$$A=6x^2-28x+30$$

PTS: 2 NAT: A.APR.A.1 TOP: Multiplication of Polynomials

333) ANS:

Strategy: To subtract, change the signs of the subtrahend and add.

Given: $3x^2 + 8x - 7$ $\underline{-\left(5x^2 + 2x - 11\right)}$	Change the signs and add: $3x^2 + 8x - 7$ $\underline{-5x^2 - 2x + 11}$ $-2x^2 + 6x + 4$

PTS: 2 NAT: A.APR.A.1 TOP: Addition and Subtraction of Polynomials
KEY: subtraction

334) ANS:

$x^4 - \frac{5}{2}x^3 + \frac{7}{2}x^2$

Strategy. First, find the difference between $(3x^2 - 2x + 5) - (x^2 + 3x - 2)$, the use the distributive property to multiply the difference by $\frac{1}{2}x^2$. Simplify as necessary.

STEP 1. Find the difference between $(3x^2 - 2x + 5) - (x^2 + 3x - 2)$. To subtract polynomials, change the signs of the subtrahend and add.

Given: $(3x^2 - 2x + 5)$ $\underline{-(x^2 + 3x - 2)}$	Change the signs and add: $3x^2 - 2x + 5$ $\underline{-x^2 - 3x + 2}$ $2x^2 - 5x + 7$

STEP 2. Multiply $2x^2 - 5x + 7$ by $\frac{1}{2}x^2$.

$$\frac{1}{2}x^2\left(2x^2 - 5x + 7\right)$$

$$x^4 - \frac{5}{2}x^3 + \frac{7}{2}x^2$$

PTS: 2 NAT: A.APR.A.1 TOP: Operations with Polynomials
KEY: multiplication

335) ANS: 4

Strategy: Expand and simplify the expression $3(x-2)^2 - 2(x-1)$

STEP 1 Expand the expression.

$$3(x^2 - 4x + 4) - 2(x - 1)$$

$$3x^2 - 12x + 12 - 2x + 2$$

STEP 2: Simplify the expanded expression by combining like terms.

$$3x^2 - 12x + 12 - 2x + 2$$

$$3x^2 - 14x + 14$$

PTS: 2 NAT: A.APR.A.1 TOP: Operations with Polynomials
KEY: mixed

336) ANS: 3
Strategy: Expand the binomial, then subtract it from $5x^{2.}$

$$5x^2 - (2x-3)^2$$

$$5x^2 - (2x-3)(2x-3)$$

$$5x^2 - (4x^2 - 6x - 6x + 9)$$

$$5x^2 - (4x^2 - 12x + 9)$$

$$5x^2 - 4x^2 + 12x - 9$$

$$x^2 + 12x - 9$$

PTS: 2 NAT: A.APR.A.1 TOP: Operations with Polynomials
KEY: multiplication

337) ANS: 2

$$3(x^2 - 1) - (x^2 - 7x + 10)$$

$$3x^2 - 3 - x^2 + 7x - 10$$

$$2x^2 + 7x - 13$$

PTS: 2 NAT: A.APR.A.1 TOP: Operations with Polynomials
KEY: subtraction

338) ANS: 3
Strategy: Use the distributive property

$$(2x+3)\left(4x^2 - 5x + 6\right)$$

$$8x^3 - 10x^2 + 12x + 12x^2 - 15x + 18$$

$$8x^3\left(-10x^2 + 12x^2\right)(+12x - 15x) + 18$$

$$8x^3 + 2x^2 - 3x + 18$$

PTS: 2 NAT: A.APR.A.1

339) ANS: 4

Given	$2(3g-4)-(8g+3)$
Distributive Property	$6g-8-8g-3$
Combine Like Terms	-2g-11

PTS: 2 NAT: A.APR.A.1 TOP: Operations with Polynomials
KEY: subtraction

340) ANS:
$5x^2 - 10$

$$\begin{array}{r} 3x^2 + 4x - 8 \\ \underline{-(-2x^2 + 4x + 2)} \\ 5x^2 \qquad - 10 \end{array}$$

PTS: 2 NAT: A.APR.A.1 TOP: Operations with Polynomials
KEY: subtraction

341) ANS:

$$5x + 4x^2(2x + 7) - 6x^2 - 9x$$
$$5x + 8x^3 + 28x^2 - 6x^2 - 9x$$
$$8x^3 + 28x^2 - 6x^2 - 9x + 5x$$
$$8x^3 + 22x^2 - 4x \text{ Answer}$$

PTS: 2 NAT: A.APR.A.1 TOP: Operations with Polynomials
KEY: multiplication

342) ANS: 3

STEP 1. Solve for the sum of $4x^2 - x + 1$ and $-6x^2 + x - 4$.

$$\begin{array}{r} 4x^2 - x + 1 \\ \underline{-6x^2 + x - 4} \\ -2x^2 \qquad - 3 \end{array}$$

STEP 2. Solve for twice the sum of $-2x^2 - 3$.

$$2(-2x^2 - 3) = -4x^2 - 6$$

PTS: 2 NAT: A.APR.A.1 TOP: Operations with Polynomials
KEY: addition

K – Polynomials, Lesson 3, Factoring Polynomials (r. 2018)

POLYNOMIALS

Factoring Polynomials

Common Core Standard	Next Generation Standard
A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. PARCC: Tasks limited to numerical and polynomial expressions in one variable. Recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to -evaluate form (53+47)(53-47). See an opportunity to rewrite $a^2 + 9a + 14$ as (a+7)(a+2). NYSED: Does not include factoring by grouping and factoring the sum and difference of cubes.	**AI-A.SSE.2 Recognize** and use the structure of an expression to identify ways to rewrite it. (Shared standard with Algebra II) **e.g.,** $\mathbf{x^3 - x^2 - x = x(x^2 - x - 1)}$ $\mathbf{53^2 - 47^2 = (53 + 47)(53 - 47)}$ $\mathbf{16x^2 - 36 = (4x)^2 - (6)^2 = (4x + 6)(4x - 6) = 4(2x + 3)(2x - 3)}$ **or** $\mathbf{16x^2 - 36 = 4(4x^2 - 9) = 4(2x + 3)(2x - 3)}$ $\mathbf{-2x^2 + 8x + 10 = -2(x^2 - 4x - 5) = -2(x - 5)(x + 1)}$ $\mathbf{x^4 + 6x^2 - 7 = (x^2 + 7)(x^2 - 1) = (x^2 + 7)(x + 1)(x - 1)}$ Note: Algebra I expressions are limited to numerical and polynomial expressions in one variable. **Use factoring techniques such as factoring out a greatest common factor, factoring the difference of two perfect squares, factoring trinomials of the form ax2+bx+c with a lead coefficient of 1, or a combination of methods to factor completely.** Factoring will not involve factoring by grouping and factoring the sum and difference of cubes.

LEARNING OBJECTIVES

Students will be able to:

1) factor monomials
2) factor binomials, and
3) factor trinomials

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

binomial
factor completely
greatest common factor
monomial
perfect square
term
trinomial

BIG IDEAS

Factoring polynomials is one of four general methods taught in the Regents mathematics curriculum for finding the roots of a quadratic equation.   The other three methods are the quadratic formula, completing the square and graphing.

- The roots of a quadratic equation can found using the **factoring** method when the discriminant's value is equal to either zero or a perfect square.

Factoring Monomials:

$204x^2 = 2(102x^2) = 2 \bullet 2(51x^2) = 2 \bullet 2 \bullet 3(17x^2) = 2^2 \bullet 3 \bullet 17 \bullet x^2$

Factoring Binomials: *NOTE: This is the inverse of the distributive property.*

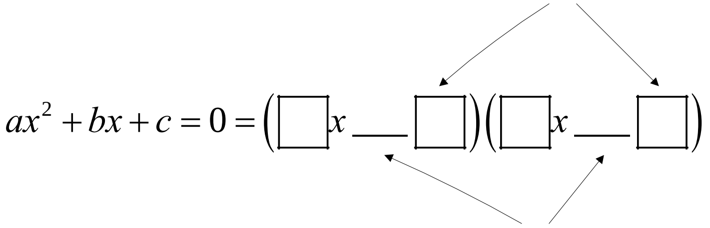
$3(x+2) = 3x+6$

$2x^2+6x = 2x(x+3)$

Factoring Trinomials

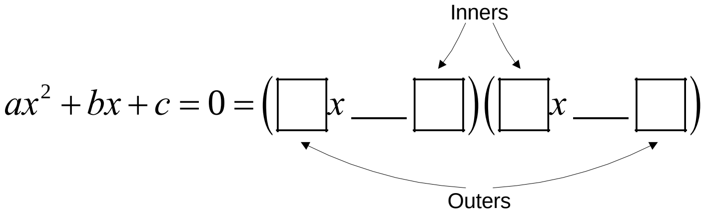
Standard Approach

Given a trinomial in the form $ax^2+bx+c=0$ whose discriminant equals zero or a perfect square, it may be factored as follows:



STEP 1. The product of these two numbers must equal c.

STEP 2. The signs of these two numbers are determined by the signs of b and c.

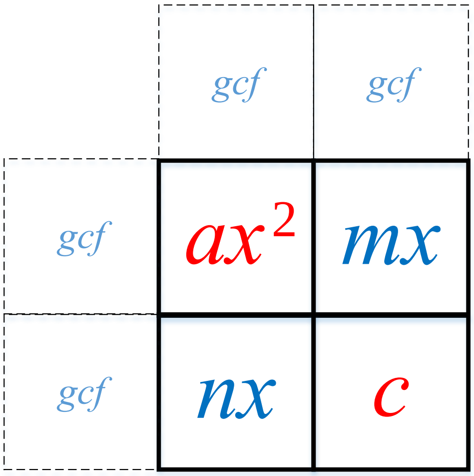


STEP 3. The product of the outer numbers plus the product of the inner numbers must sum to b.

Modeling:

$$x^2-5x+6=(x-2)(x-3)$$
$$2x^2-8x+6=(2x-2)(x-3)$$
$$4x^2-10x+6=(2x-2)(2x-3)$$

Box Method



The Box Method for Factoring a Trinomial

$$ax^2+bx+c=0$$

$$bx=mx+nx$$

INSTRUCTIONS	EXAMPLE
STEP 1 Start with a factorable quadratic in standard form: $ax^2+bx+c=0$ and a 2-row by 2-column table.	Solve by factoring: $6x^2-x-12=0$
STEP 2 Copy the quadratic term into the upper left box and the constant term into the lower right box.	$\begin{array}{\|c\|c\|}\hline 6x^2 & \\ \hline & -12 \\ \hline\end{array}$
STEP 3 Multiply the quadratic term by the constant term and write the product to the right of the table.	$\begin{array}{\|c\|c\|}\hline 6x^2 & \\ \hline & -12 \\ \hline\end{array}$ $6x^2\times-12=\boxed{-72x^2}$
STEP 4 Factor the product from STEP 3 until you obtain two factors that *sum* to the linear term (*bx*).	$1x\times-72x$ $-1x\times72x$ $2x\times-36x$ $-2x\times36x$ $3x\times-24x$ $-3x\times24x$ $4x\times-18x$ $-4x\times18x$ $6x\times-12x$ $-6x\times12x$ $8x\times-9x$ These two factors sum to bx $-8x\times9x$

<table>
<tr><td>STEP 5 Write one of the two factors found in STEP 4 in the upper right box and the other in the lower left box. Order does not matter.</td><td><table><tr><td>$6x^2$</td><td>-9x</td></tr><tr><td>8x</td><td>-12</td></tr></table></td></tr>
<tr><td>STEP 6 Find the greatest common factor of each row and each column and record these factors to the left of each row and above each column. Give each factor the same plus or minus value as the nearest term in a box.
NOTE: If all four of the greatest common factors share a common factor, reduce each factor by the common factor and add the common factor as a third factor. Eg. $(3x-9)(3x-15) \Rightarrow 3(x-3)(x-5)$</td><td><table><tr><td></td><td>2x</td><td>-3</td></tr><tr><td>3x</td><td>$6x^2$</td><td>-9x</td></tr><tr><td>4</td><td>8x</td><td>-12</td></tr></table></td></tr>
<tr><td>STEP 7 Write the expressions above and beside the box as binomial factors of the original trinomial.</td><td>$(2x-3)(3x+4)=0$</td></tr>
<tr><td>STEP 8 Check to see that the factored quadratic is the same as the original quadratic.</td><td>$(2x-3)(3x+4)=0$
$6x^2+8x-9x-12=0$
$6x^2-9x-12=0$ check</td></tr>
<tr><td>STEP 9 Convert the factors to zeros.</td><td>$(2x-3)=0$
$2x=3$
$x=\boxed{\frac{3}{2}}$
$(3x+4)=0$
$3x=-4$
$x=\boxed{-\frac{4}{3}}$</td></tr>
</table>

DEVELOPING ESSENTIAL SKILLS

1.Factored completely, the expression $2x^2+10x-12$ is equivalent to

a. $2(x-6)(x+1)$ c. $2(x+2)(x+3)$

b. $2(x+6)(x-1)$ d. $2(x-2)(x-3)$

2. Factored completely, the expression $3x^2-3x-18$ is equivalent to

a. $3(x^2-x-6)$ c. $(3x-9)(x+2)$

b. $3(x-3)(x+2)$ d. $(3x+6)(x-3)$

3. What are the factors of the expression x^2+x-20?

a. $(x+5)$ and $(x+4)$ c. $(x-5)$ and $(x+4)$

b. $(x+5)$ and $(x-4)$ d. $(x-5)$ and $(x-4)$

4. Factored completely, the expression $3x^3 - 33x^2 + 90x$ is equivalent to
   a. $3x(x^2 - 33x + 90)$
   b. $3x(x^2 - 11x + 30)$
   c. $3x(x + 5)(x + 6)$
   d. $3x(x - 5)(x - 6)$
5. Factor completely: $5x^3 - 20x^2 - 60x$
6. The greatest common factor of $3m^2n + 12mn^2$ is?
   a. $3n$
   b. $3m$
   c. $3mn$
   d. $3mn^2$
7. When factored completely, the expression $3x^2 - 9x + 6$ is equivalent to
   a. $(3x - 3)(x - 2)$
   b. $(3x + 3)(x - 2)$
   c. $3(x + 1)(x - 2)$
   d. $3(x - 1)(x - 2)$
8. Which is a factor of $x^2 + 5x - 24$?
   a. $(x + 4)$
   b. $(x - 4)$
   c. $(x + 3)$
   d. $(x - 3)$
9. Which expression is a factor of $x^2 + 2x - 15$?
   a. $(x - 3)$
   b. $(x + 3)$
   c. $(x + 15)$
   d. $(x - 5)$
10. Which expression is a factor of $n^2 + 3n - 54$?
    a. $n + 6$
    b. $n^2 + 9$
    c. $n - 9$
    d. $n + 9$
11. What are the factors of $x^2 - 10x - 24$?
    a. $(x - 4)(x + 6)$
    b. $(x - 4)(x - 6)$
    c. $(x - 12)(x + 2)$
    d. $(x + 12)(x - 2)$
12. If one factor of $56x^4y^3 - 42x^2y^6$ is $14x^2y^3$, what is the other factor?
    a. $4x^2 - 3y^3$
    b. $4x^2 - 3y^2$
    c. $4x^2y - 3xy^3$
    d. $4x^2y - 3xy^2$
13. If $3x$ is one factor of $3x^2 - 9x$, what is the other factor?
    a. $3x$
    b. $x^2 - 6x$
    c. $x - 3$
    d. $x + 3$
14. Factor completely: $3x^2 + 15x - 42$
15. Factored completely, the expression $2y^2 + 12y - 54$ is equivalent to
    a. $2(y + 9)(y - 3)$
    b. $2(y - 3)(y - 9)$
    c. $(y + 6)(2y - 9)$
    d. $(2y + 6)(y - 9)$
16. What are the factors of $x^2 - 5x + 6$?
    a. $(x + 2)$ and $(x + 3)$
    b. $(x - 2)$ and $(x - 3)$
    c. $(x + 6)$ and $(x - 1)$
    d. $(x - 6)$ and $(x + 1)$
17. The greatest common factor of $4a^2b$ and $6ab^3$ is
    a. $2ab$
    b. $2ab^2$
    c. $12ab$
    d. $24a^3b^4$

Answers

1. ANS: B
2. ANS: B

3. ANS: B
4. ANS: D

   $3x^3 - 33x^2 + 90x = 3x(x^2 - 11x + 30) = 3x(x - 5)(x - 6)$
5. ANS:

   $5x^3 - 20x^2 - 60x$

   $5x(x^2 - 4x - 12)$

   $5x(x + 2)(x - 6)$
6. ANS: C
7. ANS: D
8. ANS: D
9. ANS: A
10. ANS: D
11. ANS: C
12. ANS: A
13. ANS: C
14. ANS:

    $3(x + 7)(x - 2)$. $3x^2 + 15x - 42 = 3(x^2 + 5x - 14) = 3(x + 7)(x - 2)$
15. ANS: A
16. ANS: B
17. ANS: A

A.SSE.A.2: Factoring Polynomials

343) Which expression is equivalent to $x^4 - 12x^2 + 36$?

1) $(x^2 - 6)(x^2 - 6)$
2) $(x^2 + 6)(x^2 + 6)$
3) $(6 - x^2)(6 + x^2)$
4) $(x^2 + 6)(x^2 - 6)$

344) Four expressions are shown below.

I $2(2x^2 - 2x - 60)$
II $4(x^2 - x - 30)$
III $4(x + 6)(x - 5)$
IV $4x(x - 1) - 120$

The expression $4x^2 - 4x - 120$ is equivalent to

1) I and II, only
2) II and IV, only
3) I, II, and IV
4) II, III, and IV

345) When factored completely, $x^3 - 13x^2 - 30x$ is

1) $x(x + 3)(x - 10)$
2) $x(x - 3)(x - 10)$
3) $x(x + 2)(x - 15)$
4) $x(x - 2)(x + 15)$

346) Factor the expression $x^4 + 6x^2 - 7$ completely.

347) The trinomial $x^2 - 14x + 49$ can be expressed as

1) $(x - 7)^2$
2) $(x + 7)^2$
3) $(x - 7)(x + 7)$
4) $(x - 7)(x + 2)$

SOLUTIONS

343) ANS: 1

Strategy 1. Factor $x^4 - 12x^2 + 36$

$$x^4 - 12x^2 + 36$$

$$\left(x^2 - \_\_\_\right)\left(x^2 - \_\_\_\right)$$

The factors of 36 are:

1 and 36,

2 and 18,

3 and 12,

4 and 9,

6 and 6 (use these because they sum to 12)

$$\left(x^2 - 6\right)\left(x^2 - 6\right)$$

Strategy 2. Work backwards using the distributive property to check each answer choice.

a $(x^2-6)(x^2-6)$ $x^4-6x^2-6x^2+36$ x^4-12x^2+36 (correct)	c $(6-x^2)(6+x^2)$ $36+6x^2-6x^2-x^4$ $36-x^4$ (wrong)
b $(x^2+6)(x^2+6)$ $x^4+6x^2+6x^2+36$ x^4+12x^2+36 (wrong)	d $(x^2+6)(x^2-6)$ $x^4-6x^2+6x^2-36$ x^4-36 (wrong)

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

344) ANS: 3

Strategy: Use the distributive property to expand each expression, then match the expanded expressions to the answer choices.

I $2(2x^2-2x-60)$ $4x^2-4x-120$ *yes*	III $4(x+6)(x-5)$ $(4x+24)(x-5)$ $4x^2-20x+24x-120$ $4x^2+4x-120$ *no*
II $4(x^2-x-30)$ $4x^2-4x-120$ *yes*	IV $4x(x-1)-120$ $4x^2-4x-120$ *yes*

Answer choice *c* is correct.

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

345) ANS: 3

$$x^3-13x^2-30x$$
$$x(x^2-13x-30)$$
$$x(x+2)(x-15)$$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

346) ANS:

$\left(x^2+7\right)(x+1)(x-1)$

Strategy: Factor the trinomial, then factor the perfect square.

STEP 1. Factor the trinomial x^4+6x^2-7.

$x^4 + 6x^2 - 7$

$\left(x^2 + \_\_\_\right)\left(x^2 - \_\_\_\right)$

The factors of 7 are 1 and 7.

$\left(x^2 + 7\right)\left(x^2 - 1\right)$

STEP 2. Factor the perfect square.

$\left(x^2 + 7\right)\left(x^2 - 1\right)$

$\left(x^2 + 7\right)(x + 1)(x - 1)$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

347) ANS: 1

Strategy. Multiply binomials and eliminate wrong answers.

Choice 1: $(x - 7)^2$ Correct

$(x - 7)(x - 7)$

$x^2 - 7x - 7x + 49$

$x^2 - 14x + 49$

Choice 2: $(x + 7)^2$ Wrong: middle term has wrong sign.

$(x + 7)(x + 7)$

$x^2 + 7x + 7x + 49$

$x^2 + 14x + 49$

Choice 3: $(x - 7)(x + 7)$ Wrong: no middle term and second term has wrong sign.

$x^2 + 7x - 7x - 49$

$x^2 - 49$

Choice 4: $(x - 7)(x + 2)$ Wrong: middle term and third term have wrong coefficients.

$x^2 + 2x - 7x - 14$

$x^2 - 5x - 14$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: quadratic

K – Polynomials, Lesson 4, Factoring the Difference of Perfect Squares (r. 2018)

POLYNOMIALS
Factoring the Difference of Perfect Squares

Common Core Standard	**Next Generation Standard**
A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. PARCC: Tasks limited to numerical and polynomial expressions in one variable. Recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to -evaluate form (53+47)(53-47). See an opportunity to rewrite $a^2 + 9a + 14$ as (a+7)(a+2). NYSED: Does not include factoring by grouping and factoring the sum and difference of cubes.	**AI-A.SSE.2 Recognize** and use the structure of an expression to identify ways to rewrite it. (Shared standard with Algebra II) **e.g.,** $\mathbf{x^3 - x^2 - x = x(x^2 - x - 1)}$ $\mathbf{53^2 - 47^2 = (53 + 47)\ (53 - 47)}$ $\mathbf{16x^2 - 36 = (4x)^2 - (6)^2 = (4x + 6)\ (4x - 6) = 4(2x + 3)\ (2x - 3)}$ **or** $\mathbf{16x^2 - 36 = 4(4x^2 - 9) = 4(2x + 3)\ (2x - 3)}$ $\mathbf{-2x^2 + 8x + 10 = -2(x^2 - 4x - 5) = -2(x - 5)\ (x + 1)}$ $\mathbf{x^4 + 6x^2 - 7 = (x^2 + 7)(x^2 - 1) = (x^2 + 7)(x + 1)(x - 1)}$ Note: Algebra I expressions are limited to numerical and polynomial expressions in one variable. **Use factoring techniques such as factoring out a greatest common factor, factoring the difference of two perfect squares, factoring trinomials of the form ax_2+bx+c with a lead coefficient of 1, or a combination of methods to factor completely.** Factoring will not involve factoring by grouping and factoring the sum and difference of cubes.

LEARNING OBJECTIVES

Students will be able to:

1) factor the difference of perfect squares.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

Completely factor
Perfect square binomial

Square of a number
Square root of a number

BIG IDEA

General Rule

$$\left(a^2-b^2\right)=\left(a+b\right)\left(a-b\right)$$

Examples

$$x^2-4=\left(x+2\right)\left(x-2\right)$$

$$x^4-9=\left(x^2+3\right)\left(x^2-3\right)$$

DEVELOPING ESSENTIAL SKILLS

1. The expression x^2-16 is equivalent to
   a. $(x+2)(x-8)$
   b. $(x-2)(x+8)$
   c. $(x+4)(x-4)$
   d. $(x+8)(x-8)$
2. Factored, the expression $16x^2-25y^2$ is equivalent to
   a. $(4x-5y)(4x+5y)$
   b. $(4x-5y)(4x-5y)$
   c. $(8x-5y)(8x+5y)$
   d. $(8x-5y)(8x-5y)$
3. The expression $9x^2-100$ is equivalent to
   a. $(9x-10)(x+10)$
   b. $(3x-10)(3x+10)$
   c. $(3x-100)(3x-1)$
   d. $(9x-100)(x+1)$
4. Factor completely: $4x^3-36x$
5. Which expression is equivalent to $9x^2-16$?
   a. $(3x+4)(3x-4)$
   b. $(3x-4)(3x-4)$
   c. $(3x+8)(3x-8)$
   d. $(3x-8)(3x-8)$
6. If Ann correctly factors an expression that is the difference of two perfect squares, her factors could be
   a. $(2x+y)(x-2y)$
   b. $(2x+3y)(2x-3y)$
   c. $(x-4)(x-4)$
   d. $(2y-5)(y-5)$
7. Which expression is equivalent to $121-x^2$?
   a. $(x-11)(x-11)$
   b. $(x+11)(x-11)$
   c. $(11-x)(11+x)$
   d. $(11-x)(11-x)$
8. When a^3-4a is factored completely, the result is
   a. $(a-2)(a+2)$
   b. $a(a-2)(a+2)$
   c. $a^2(a-4)$
   d. $a(a-2)^2$
9. The expression x^2-36y^2 is equivalent to
   a. $(x-6y)(x-6y)$
   b. $(x-18y)(x-18y)$
   c. $(x+6y)(x-6y)$
   d. $(x+18y)(x-18y)$
10. Which expression represents $36x^2-100y^6$ factored completely?
    a. $2(9x+25y^3)(9x-25y^3)$
    b. $4(3x+5y^3)(3x-5y^3)$
    c. $(6x+10y^3)(6x-10y^3)$
    d. $(18x+50y^3)(18x-50y^3)$
11. Which expression is equivalent to $64-x^2$?
    a. $(8-x)(8-x)$
    b. $(8-x)(8+x)$
    c. $(x-8)(x-8)$
    d. $(x-8)(x+8)$
12. The expression $9a^2-64b^2$ is equivalent to
    a. $(9a-8b)(a+8b)$
    b. $(9a-8b)(a-8b)$
    c. $(3a-8b)(3a+8b)$
    d. $(3a-8b)(3a-8b)$
13. The expression $100n^2-1$ is equivalent to

a. $(10n+1)(10n-1)$
b. $(10n-1)(10n-1)$
c. $(50n+1)(50n-1)$
d. $(50n-1)(50n-1)$

14. When $9x^2-100$ is factored, it is equivalent to $(3x-b)(3x+b)$. What is a value for b?
a. 50
b. 10
c. 3
d. 100

15. Which expression is equivalent to $81-16x^2$?
a. $(9-8x)(9+8x)$
b. $(9-8x)(9+2x)$
c. $(9-4x)(9+4x)$
d. $(9-4x)(9-4x)$

16. One of the factors of $4x^2-9$ is
a. $(x+3)$
b. $(2x+3)$
c. $(4x-3)$
d. $(x-3)$

17. Factor completely: $3x^2-27$
a. $3(x-3)^2$
b. $3(x^2-27)$
c. $3(x+3)(x-3)$
d. $(3x+3)(x-9)$

18. Written in simplest factored form, the binomial $2x^2-50$ can be expressed as
a. $2(x-5)(x-5)$
b. $2(x-5)(x+5)$
c. $(x-5)(x+5)$
d. $2x(x-50)$

19. Expressed in factored form, the binomial $4a^2-9b^2$ is equivalent to
a. $(2a-3b)(2a-3b)$
b. $(2a+3b)(2a-3b)$
c. $(4a-3b)(a+3b)$
d. $(2a-9b)(2a+b)$

20. What is a common factor of x^2-9 and x^2-5x+6?
a. $x+3$
b. $x-3$
c. $x-2$
d. x^2

Answers

1. ANS: C
2. ANS: A
3. ANS: B
4. ANS:

$4x(x+3)(x-3)$. $4x^3-36x=4x(x^2-9)=4x(x+3)(x-3)$

5.ANS: A
6. ANS: B
7. ANS: C
8. ANS: B
9. ANS: C
10. ANS: B
11. ANS: B
12. ANS: C
13. ANS: A

14. ANS: B
15. ANS: C
16. ANS: B
17. ANS: C

18. ANS: B
19. ANS: B
20. ANS: B

A.SSE.A.2: Difference of Perfect Squares

348) When factored completely, the expression $p^4 - 81$ is equivalent to

1) $(p^2 + 9)(p^2 - 9)$
2) $(p^2 - 9)(p^2 - 9)$
3) $(p^2 + 9)(p + 3)(p - 3)$
4) $(p + 3)(p - 3)(p + 3)(p - 3)$

349) If the area of a rectangle is expressed as $x^4 - 9y^2$, then the product of the length and the width of the rectangle could be expressed as

1) $(x - 3y)(x + 3y)$
2) $(x^2 - 3y)(x^2 + 3y)$
3) $(x^2 - 3y)(x^2 - 3y)$
4) $(x^4 + y)(x - 9y)$

350) The expression $x^4 - 16$ is equivalent to

1) $(x^2 + 8)(x^2 - 8)$
2) $(x^2 - 8)(x^2 - 8)$
3) $(x^2 + 4)(x^2 - 4)$
4) $(x^2 - 4)(x^2 - 4)$

351) Which expression is equivalent to $36x^2 - 100$?

1) $4(3x - 5)(3x - 5)$
2) $4(3x + 5)(3x - 5)$
3) $2(9x - 25)(9x - 25)$
4) $2(9x + 5)(9x - 25)$

352) Which expression is equivalent to $16x^2 - 36$?

1) $4(2x - 3)(2x - 3)$
2) $4(2x + 3)(2x - 3)$
3) $(4x - 6)(4x - 6)$
4) $(4x + 6)(4x + 6)$

353) Which expression is equivalent to $16x^4 - 64$?

1) $(4x^2 - 8)^2$
2) $(8x^2 - 32)^2$
3) $(4x^2 + 8)(4x^2 - 8)$
4) $(8x^2 + 32)(8x^2 - 32)$

354) The expression $49x^2 - 36$ is equivalent to

1) $(7x - 6)^2$
2) $(24.5x - 18)^2$
3) $(7x - 6)(7x + 6)$
4) $(24.5x - 18)(24.5x + 18)$

355) Which expression is equivalent to $y^4 - 100$?

1) $(y^2 - 10)^2$
2) $(y^2 - 50)^2$
3) $(y^2 + 10)(y^2 - 10)$
4) $(y^2 + 50)(y^2 - 50)$

SOLUTIONS

348) ANS: 3
Strategy: Use difference of perfect squares.

STEP 1. Factor $p^4 - 81$
$p^4 - 81$
$\left(p^2 + 9\right)\left(p^2 - 9\right)$
STEP 2. Factor $p^2 - 9$
$\left(p^2 + 9\right)\left(p^2 - 9\right)$

$\left(p^2 + 9\right)\left(p + 3\right)\left(p - 3\right)$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

349) ANS: 2
Strategy: Use the distributive property to work backwards from the answer choices.

a. $(x - 3y)(x + 3y)$ $x^2 + 3xy - 3xy - 9y^2$ $x^2 - 9y^2$ (wrong)	c. $(x^2 - 3y)(x^2 - 3y)$ $x^4 - 3x^2y - 3x^2y + 9y^2$ $x^4 - 6x^2y + 9y^2$ (wrong)
b. $(x^2 - 3y)(x^2 + 3y)$ $x^4 + 3x^2y - 3x^2y - 9y^2$ $x^4 - 9y^2$ (correct)	d. $(x^4 + y)(x - 9y)$ $x^5 - 9x^4y + xy - 9y^2$ (wrong)

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

350) ANS: 3
Step 1. Understand the problem as a "difference of perfect squares", because the terms x^4 and 16 are both perfect squares and the operation is subtraction.
Step 2. Strategy: Use the pattern $a^2 - b^2 = (a + b)(a - b)$ to separate $x^4 - 16$ into two binomials.
Step3. Execution of Strategy

The square root of x^4 is x^2.
The square of 16 is 4.
$x^4 - 16 = (x^2 + 4)(x^2 - 4)$

Step 4. Does it make sense? Yes. You can show that $(x^2 + 4)(x^2 - 4) = x^4 - 16$ using the distributive property, as follows:

$$(x^2+4)(x^2-4)=x^4+16$$

$$x^4-4x^2+4x^2-16=x^4+16$$

$$x^4+16=x^4+16$$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring the Difference of Perfect Squares

351) ANS: 2

Strategy 1.

Recognize that the expression $36x^2-100$ is a difference of perfect squares. Therefore,

$$36x^2-100 \quad .$$

$$(6x+10)(6x-10)$$

Since this is not an answer choice, continue factoring, as follows:

$$(6x+10)(6x-10)$$

$$\left(2(3x+5)\right)\left(2(3x-5)\right)$$

$$4(3x+5)(3x-5)$$

Strategy 2.

Examine the answer choices, which begin with factors 4 and 2. Extract these factors first, as follows:

Start by extracting a 4	Start by extracting a 2
$36x^2-100$	$36x^2-100$
$4\left(9x^2-25\right)$	$2\left(18x^2-50\right)$
$4(3x+5)(3x-5)$	$(2)(2)\left(9x^2-25\right)$
	$(2)(2)(3x+5)(3x-5)$
	$4(3x+5)(3x-5)$

PTS: 2 NAT: A.SSE.A.2

352) ANS: 2

Strategy 1: Factor

$16x^2-36$

$4\left(4x^2-9\right)$

$4(2x+3)(2x-3)$

Strategy 2: Recognize that $16x^2-36$ appears to be a difference of perfect squares.

Recall that $a^2-b^2=(a+b)(a-b)$.

Eliminate any answers that do not take the form of $(a+b)(a-b)$, which leaves only one choice:

$4(2x+3)(2x-3)$

Check:

$$4(2x+3)(2x-3)$$

$$4\left[(2x+3)(2x-3)\right]$$

$$4\left[4x^2+6x-6x-9\right]$$

$$4\left[4x^2-9\right]$$

$$16x^2-36$$

$$\therefore\ 4(2x+3)(2x-3)=16x^2-36$$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring the Difference of Perfect Squares
KEY: quadratic

353) ANS: 3

Note that the expression $16x^4-64$ is the difference of perfect squares.

$$a^2-b^2=(a+b)(a-b)$$

$$16x^4-64=(4x^2+8)(4x^2-8)$$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring the Difference of Perfect Squares
KEY: higher power

354) ANS: 3

Note that $49x^2$ and 36 are both perfect squares. Therefore, $49x^2-36$ is the difference of perfect squares.

$$a^2-b^2=(a+b)(a-b)$$

$$49x^2-36=(7x+6)(7x-6)$$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring the Difference of Perfect Squares
KEY: quadratic

355) ANS: 3

y^4-100 is a difference of perfect squares. All polynomials in the form of a^2-b^2 can be factored into $(a+b)\left(a-b\right)$.

$$y^4-100$$

$$\left(y^2+10\right)\left(y^2-10\right)$$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring the Difference of Perfect Squares
KEY: higher power AI

K – Polynomials, Lesson 5, Zeros of Polynomials (r. 2018)

POLYNOMIALS

Zeros of Polynomials

Common Core Standard	**Next Generation Standard**
A-APR.3 Identify zeros of polynomials when suitable factorizations are available, ~~and use the zeros to construct a rough graph of the function defined by the polynomial~~. PARCC: Tasks are limited to quadratic and cubic polynomials in which linear and quadratic factors are available. *For example, find the zeros of* $(x-2)\ (x^2-9)$.	**AI-A.APR.3** Identify zeros of polynomial functions when suitable factorizations are available. (Shared standard with Algebra II) Note: Algebra I tasks will focus on identifying the zeros of quadratic and cubic polynomial functions. For tasks that involve finding the zeros of cubic polynomial functions, the linear and quadratic factors of the cubic polynomial function will be given (e.g., find the zeros of $P(x)=(x-2)\ (x^2-9)$).

LEARNING OBJECTIVES

Students will be able to:

1) Identify the zeros of a polynomial expression given its factors.
2) Identify the factors of a polynomial expression given its zeros.
3) Identify the zeros and factors of a polynomial expression given the graph of the expression.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

Multiplication Property of Zero: The **multiplication property of zero** says that if the product of two numbers or expressions is zero, then one or both of the numbers or expressions must equal zero. More simply, if $x \cdot y = 0$, then either $x = 0$ or $y = 0$, or, x and y both equal zero.

Factor: A **factor** is:

1) a whole number that is a **divisor** of another number, or
2) an algebraic expression that is a **divisor** of another algebraic expression.

Examples:

- 1, 2, 3, 4, 6, and 12 all divide the number 12, so 1, 2, 3, 4, 6, and 12 are all factors of 12.
- $(x-3)$ and $(x+2)$ will divide the trinomial expression x^2-x-6 ,

so $(x-3)$ and $(x+2)$ are both factors of the x^2-x-6 .

Zeros: A **zero** of an equation is a **solution** or **root** of the equation. The words **zero**, **solution**, and **root** all mean the same thing. The zeros of a polynomial expression are found by finding the value of x when the value of y is 0. This done by making and solving an equation with the value of the polynomial expression equal to zero.

Example:

- The **zeros** of the trinomial expression $x^2+2x-24$ can be found by writing and then factoring the equation:

$$x^2+2x-24=0$$

$$(x+6)(x-4)=0$$

After factoring the equation, use the **multiplication property of zero** to find the zeros, as follows:

$$(x+6)(x-4)=0$$

$$\therefore\ x+6=0\ \mathit{and/or}\ x-4=0$$

$$\mathit{If}\ x+6=0,\ \mathit{then}\ x=-6$$

$$\mathit{If}\ x-4=0,\ \mathit{then}\ x=+4$$

The zeros of the expression $x^2+2x-24=0$ are -6 and +4.

Check: You can check this by substituting both -6 or +4 into the expression, as follows:

Check for -6

$$x^2+2x-24$$

$$(-6)^2+2(-6)-24$$

$$36-12-24$$

$$0$$

Check for +4

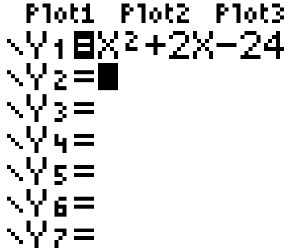
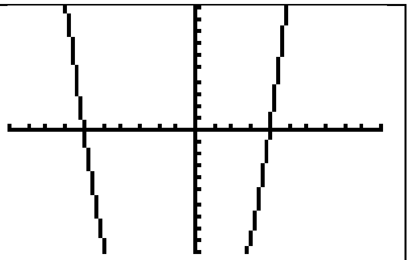
$$x^2+2x-24$$

$$(4)^2+2(4)-24$$

$$16+8-24$$

$$0$$

x-axis intercepts: The zeros of an expression can also be understood as the **x-axis intercepts** of the graph of the equation when $f(x)=0$. This is because the coordinates of the x-axis intercepts, by definition, have y-values equal to zero, and is the same as writing an equation where the expression is equal to zero.

The roots of $x^2+2x-24=0$ are $x=-6$ and $x=4$. These are the x coordinate values of the x-axis intercepts.	 Plot1 Plot2 Plot3 \Y1=X²+2X-24 \Y2= \Y3= \Y4= \Y5= \Y6= \Y7=	

BIG IDEA #1

Starting with Factors and Finding Zeros

Remember that the **factors** of an expression are *related to* the **zeros** of the expression by the **multiplication property of zero**. Thus, if you know the **factors**, it is easy to find the **zeros**.

Example: The factors of an expression are $(2x+2)$, $(x+3)$ and $\left(x-1\right)$.

The zeros are found as follows using the multiplication property of zero:

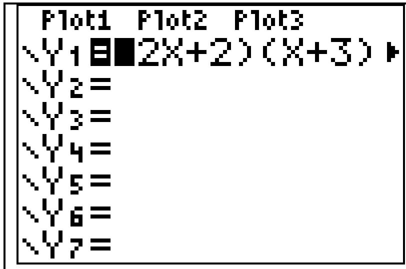
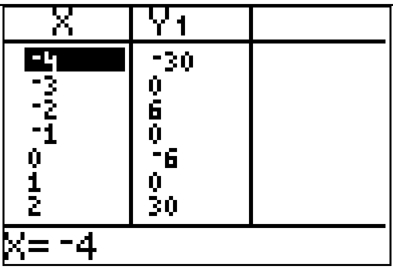
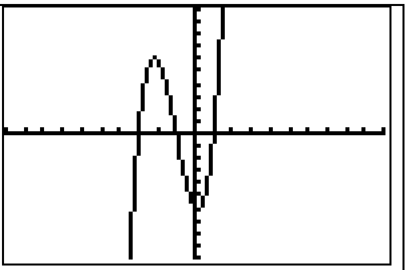
$$(2x+2)(x+3)(x-1)=0$$

$$\therefore\ 2x+2=0\ \mathit{and}\ x=-1$$

$$\mathit{and/or}\ x+3=0\ \mathit{and}\ x=-3$$

$$\mathit{and/or}\ x-1=0\ \mathit{and}\ x=1$$

The zeros are -3, -1, and +1.

 Plot1 Plot2 Plot3 \Y1=(2X+2)(X+3)▸ \Y2= \Y3= \Y4= \Y5= \Y6= \Y7= Be sure to input all factors.	 X Y1 -4 -30 -3 0 -2 6 -1 0 0 -6 1 0 2 30 X=-4	

BIG IDEA #2

Starting with Zeros and Finding Factors

If you know the **zeros** of an expression, you can work backwards using the **multiplication property of zero** to find the **factors** of the expression. For example, if you inspect the graph of an equation and find that it has **x-intercepts** at $x=3$ and $x=-2$, you can write:

$$x=3$$

$$\therefore\ (x-3)=0$$

$$\mathit{and}$$

$$x=-2$$

$$\therefore\ (x+2)=0$$

The equation of the graph has **factors** of $(x-3)$ and $(x+2)$, so you can write the equation:

$$(x-3)(x+2)=0$$

which simplifies to

$$x^2+2x-3x-6=f(x)$$

$$x^2-x-6=f(x)$$

With practice, you can probably move back and forth between the **zeros** of an expression and the **factors** of an expression with ease.

DEVELOPING ESSENTIAL SKILLS

Identify the factors, zeros, and x-axis intercepts of the following polynomials:

Polynomial	Factors	Zeros	x-axis Intercepts
$x^2 - x - 6 = 0$			
$x^2 + 7x + 6 = 0$			
$x^2 - 5x - 6 = 0$			
$x^2 - 2x - 15 = 0$			
$x^2 - 3x - 10 = 0$			
$x^2 - 2x - 8 = 0$			
$6x^2 + 5x - 6 = 0$			
$6x^2 - 15x - 36 = 0$			

ANSWERS

Identify the factors, zeros, and x-axis intercepts of the following polynomials:

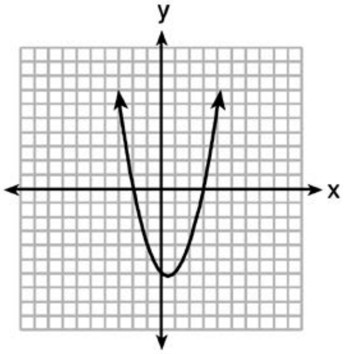
Polynomial	Factors	Zeros	x-axis Intercepts
$x^2 - x - 6 = 0$	$(x + 2)(x - 3)$	$x = \{-2, 3\}$	$x = \{-2, 3\}$
$x^2 + 7x + 6 = 0$	$(x + 1)(x + 6)$	$x = \{-6, 1\}$	$x = \{-6, 1\}$
$x^2 - 5x - 6 = 0$	$(x - 6)(x + 1)$	$x = \{-1, 6\}$	$x = \{-1, 6\}$
$x^2 - 2x - 15 = 0$	$(x - 5)(x + 3)$	$x = \{3, 5\}$	$x = \{3, 5\}$
$x^2 - 3x - 10 = 0$	$(x - 5)(x + 2)$	$x = \{2, 5\}$	$x = \{2, 5\}$
$x^2 - 2x - 8 = 0$	$(x - 4)(x + 2)$	$x = \{-2, 4\}$	$x = \{-2, 4\}$
$6x^2 + 5x - 6 = 0$	$(2x + 3)(3x - 2)$	$x = \left\{-\frac{3}{2}, \frac{2}{3}\right\}$	$x = \left\{-\frac{3}{2}, \frac{2}{3}\right\}$
$6x^2 - 15x - 36 = 0$	$-3(x - 4)(2x + 3)$	$x = \left\{-\frac{3}{2}, 4\right\}$	$x = \left\{-\frac{3}{2}, 4\right\}$

REGENTS EXAM QUESTIONS (through June 2018)

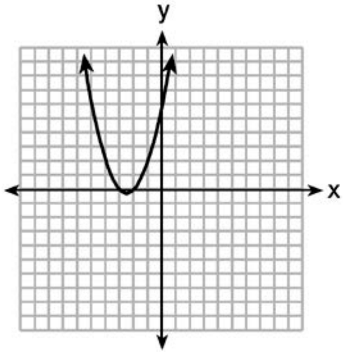
A.APR.B.3: Zeros of Polynomials

356) The graphs below represent functions defined by polynomials. For which function are the zeros of the polynomials 2 and -3?

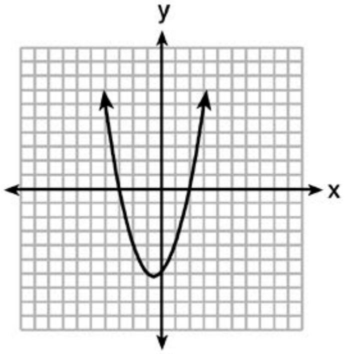
1)



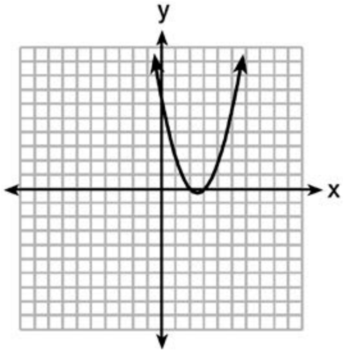
2)



3)



4)

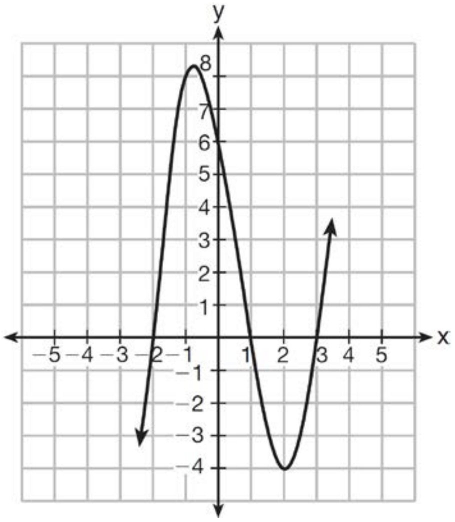


357) Which equation(s) represent the graph below?

I $y = (x + 2)(x^2 - 4x - 12)$

II $y = (x - 3)(x^2 + x - 2)$

III $y = (x - 1)(x^2 - 5x - 6)$



1) I, only
2) II, only
3) I and II
4) II and III

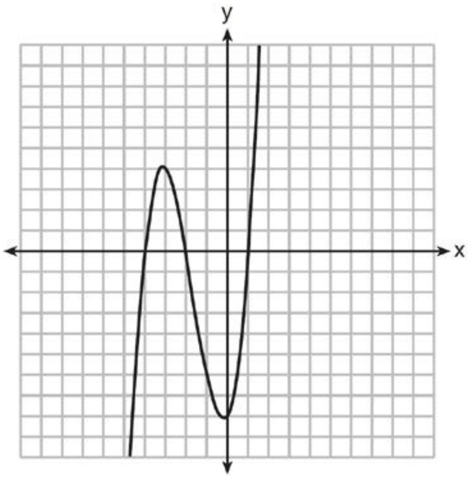
358) For which function defined by a polynomial are the zeros of the polynomial -4 and -6?

1) $y = x^2 - 10x - 24$
2) $y = x^2 + 10x + 24$
3) $y = x^2 + 10x - 24$
4) $y = x^2 - 10x + 24$

359) The zeros of the function $f(x) = (x + 2)^2 - 25$ are

1) -2 and 5
2) -3 and 7
3) -5 and 2
4) -7 and 3

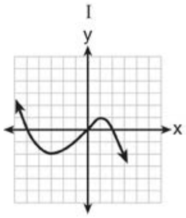
360) The graph of $f(x)$ is shown below.

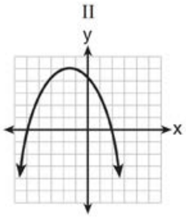


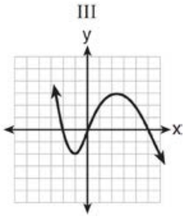
Which function could represent the graph of $f(x)$?

1) $f(x) = (x + 2)(x^2 + 3x - 4)$
2) $f(x) = (x - 2)(x^2 + 3x - 4)$
3) $f(x) = (x + 2)(x^2 + 3x + 4)$
4) $f(x) = (x - 2)(x^2 + 3x + 4)$

361) A polynomial function contains the factors x, $x - 2$, and $x + 5$. Which graph(s) below could represent the graph of this function?







1) I, only
2) II, only
3) I and III
4) I, II, and III

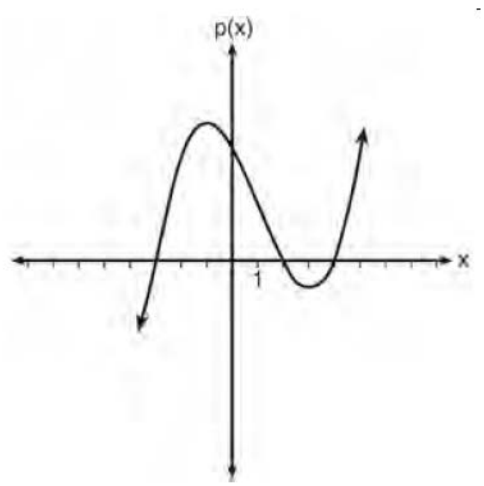
362) What are the zeros of the function $f(x) = x^2 - 13x - 30$?

1) -10 and 3
2) 10 and -3
3) -15 and 2
4) 15 and -2

363) The zeros of the function $f(x) = x^2 - 5x - 6$ are

1) -1 and 6
2) 1 and -6
3) 2 and -3
4) -2 and 3

364) Based on the graph below, which expression is a possible factorization of $p(x)$?

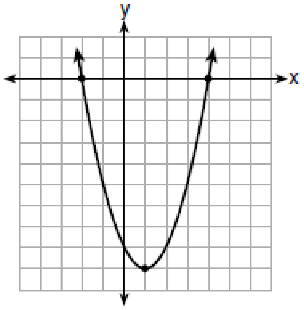


1) $(x+3)(x-2)(x-4)$
2) $(x-3)(x+2)(x+4)$
3) $(x+3)(x-5)(x-2)(x-4)$
4) $(x-3)(x+5)(x+2)(x+4)$

365) Which function has zeros of -4 and 2?

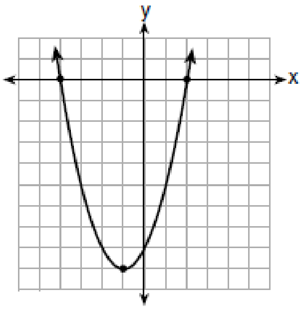
1) $f(x) = x^2 + 7x - 8$

2)



3) $g(x) = x^2 - 7x - 8$

4)

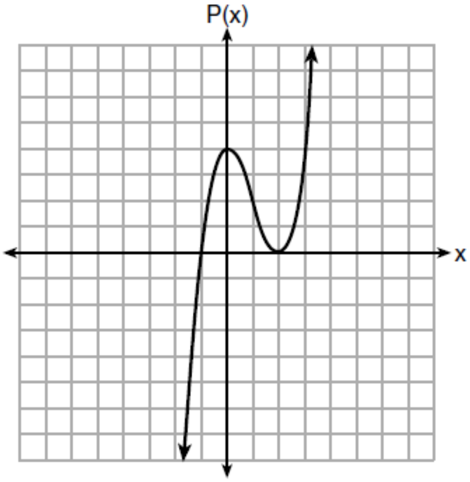


366) Which polynomial function has zeros at -3, 0, and 4?
1) $f(x) = (x+3)(x^2+4)$
2) $f(x) = (x^2-3)(x-4)$
3) $f(x) = x(x+3)(x-4)$
4) $f(x) = x(x-3)(x+4)$

367) The zeros of the function $f(x) = 2x^3 + 12x - 10x^2$ are
1) $\{2, 3\}$
2) $\{-1, 6\}$
3) $\{0, 2, 3\}$
4) $\{0, -1, 6\}$

368) Determine all the zeros of $m(x) = x^2 - 4x + 3$, algebraically.

369) Wenona sketched the polynomial $P(x)$ as shown on the axes below.



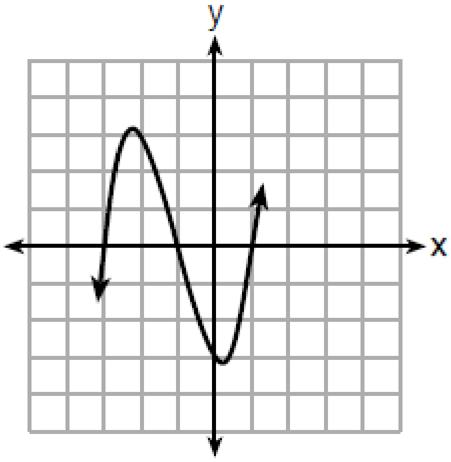
Which equation could represent $P(x)$?

1) $P(x) = (x+1)(x-2)^2$
2) $P(x) = (x-1)(x+2)^2$
3) $P(x) = (x+1)(x-2)$
4) $P(x) = (x-1)(x+2)$

370) The zeros of the function $p(x) = x^2 - 2x - 24$ are

1) -8 and 3
2) -6 and 4
3) -4 and 6
4) -3 and 8

371) A cubic function is graphed on the set of axes below.



Which function could represent this graph?

1) $f(x) = (x-3)(x-1)(x+1)$
2) $g(x) = (x+3)(x+1)(x-1)$
3) $h(x) = (x-3)(x-1)(x+3)$
4) $k(x) = (x+3)(x+1)(x-3)$

SOLUTIONS

356) ANS: 3

Strategy: Look for the coordinates of the x-intercepts (where the graph crosses the x-axis). The zeros are the x-values of those coordinates.

Answer c is the correct choice. The coordinates of the x-intercepts of the graph are (2, 0) and (-3, 0).
The zeros of the polynomial are 2 and -3.

PTS: 2 NAT: A.APR.B.3 TOP: Zeros of Polynomials
KEY: bimodalgraph

357) ANS: 2

Strategy: Factor the trinomials in each equation, then convert the factors into zeros and select the equations that have zeros at -2, 1, and 3.

STEP 1.

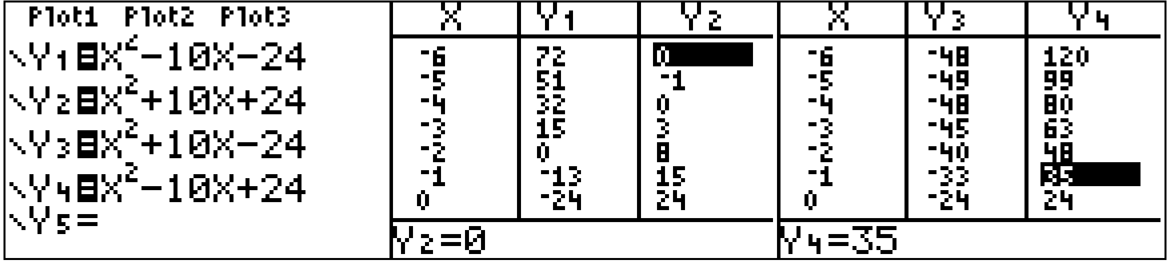
I	II	III
$y = (x+2)(x^2-4x-12)$	$y = (x-3)(x^2+x-2)$	$y = (x-1)(x^2-5x-6)$
$y = (x+2)(x-6)(x+2)$	$y = (x-3)(x+2)(x-1)$	$y = (x-1)(x-6)(x+1)$
Zeros at -2, 6, and -2	Zeros at 3, -2, and 1	Zeros at 1, 6, and -1
(Wrong Choice)	(Correct Choice)	(Wrong Choice)

The correct answer choice is *b*.

PTS: 2 NAT: A.APR.B.3 TOP: Zeros of Polynomials

358) ANS: 2

Strategy. Input each function in a graphing calculator and look at the table views to find the values of x when y equals zero.



Plot1 Plot2 Plot3
\Y1=X²−10X−24
\Y2=X²+10X+24
\Y3=X²+10X−24
\Y4=X²−10X+24
\Y5=

X	Y1	Y2
-6	72	0
-5	51	-1
-4	32	0
-3	15	3
-2	0	8
-1	-13	15
0	-24	24

Y2=0

X	Y3	Y4
-6	-48	120
-5	-49	99
-4	-48	80
-3	-45	63
-2	-40	48
-1	-33	35
0	-24	24

Y4=35

Answer choice b, enterred as Y_2, has zeros at $x = -4$ and $x = -6$.

PTS: 2 NAT: A.APR.B.3 TOP: Zeros of Polynomials

359) ANS: 4

Strategy: Use root operations to solve $f(x) = (x+2)^2 - 25$ for $f(x) = 0$.

$$f(x) = (x+2)^2 - 25$$

$$0 = (x+2)^2 - 25$$

$$25 = (x+2)^2$$

$$\sqrt{25} = \sqrt{(x+2)^2}$$

$$\pm 5 = x+2$$

$$-2 \pm 5 = x$$

$$-7 \text{ and } 3 = x$$

PTS: 2 NAT: F.IF.C.8 TOP: Zeros of Polynomials

360) ANS: 1

Strategy:

STEP 1. Identify the zeros and convert them into factors.

The graph has zeros at -4, -2, and 1. Convert these zeros of the function into the following factors: (x+4)(x+2)(x-1). The functiuon rule is f(x)= (x+4)(x+2)(x-1)

STEP 2. Eliminate wrong answers. Choices b and d can be eliminated because (x-2) is not a factor.

b.	d.
$f(x) = (x-2)(x^2+3x-4)$	$f(x) = (x-2)(x^2+3x+4)$
$(x-2)$ is not a factor.	$(x-2)$ is not a factor.
(Wrong Choice)	(Wrong Choice)

STEP 3. Choose between remaining choices by factoring the trinomials.

a.	c.
$f(x) = (x+2)(x^2+3x-4)$	$f(x) = (x+2)(x^2+3x+4)$
$f(x) = (x+2)(x+4)(x-1)$	(x^2+3x+4) cannot be factored into $(x+4)(x-1)$
Contains all three factors.	(Wrong Choice)
(Correct Choice)	

PTS: 2 NAT: A.APR.B.3 TOP: Zeros of Polynomials

361) ANS: 1

Stategy 1. Convert the factors to zeros, then find the graph(s) with the corresponding zeros.

STEP 1. Convert the factors to zeros.

A factor of $x-0$ equates to a zero of the polynomial at x=0.

A factor of $x-2$ equates to a zero of the polynomial at x=2.

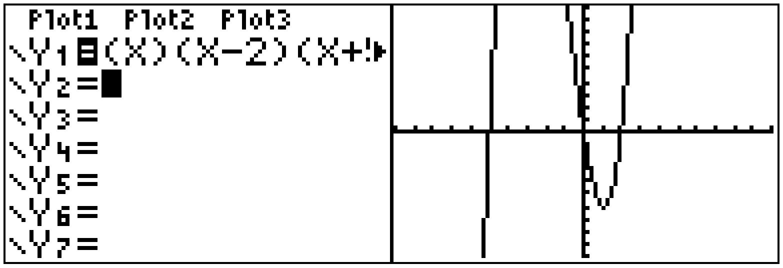
A factor of $x+5$ equates to a zero of the polynomial at x=-5.

STEP 2. Find the zeros of the graphs.

Graph I has zeros at -5, 0, and 2.
Graph II has zeros at -5 and 2.
Graph III has zeros at -2, 0, and 5.

Answer choice *a* is correct.

Strategy 2: Input the factors into a graphing calculator and view the graph of the function $y = (x)(x-2)(x+5)$.



```
Plot1 Plot2 Plot3
\Y1=(X)(X-2)(X+5▸
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Note: This graph has the same zeros as graph I, but the end behaviors of the graph are reversed. This graph is a reflection in the x-axis of graph I and the reversal is caused by a change in the sign of the leading coefficient in the expansion of $y = (x)(x-2)(x+5)$. It makes no difference in answering this problem. The zeros are the same and the correct answer choice is answer choice *a*.

PTS: 2 NAT: A.APR.B.3 TOP: Zeros of Polynomials

362) ANS: 4

Strategy: Find the factors of $f(x) = x^2 - 13x - 30$, then convert the factors to zeros.

STEP 1. Find the factors of $f(x) = x^2 - 13x - 30$.

$$f(x) = x^2 - 13x - 30$$

$$f(x) = \left(x - \_\_\_\right)\left(x + \_\_\_\right)$$

The factors of 30 are

1 and 30

2 and 15 (*use these*)

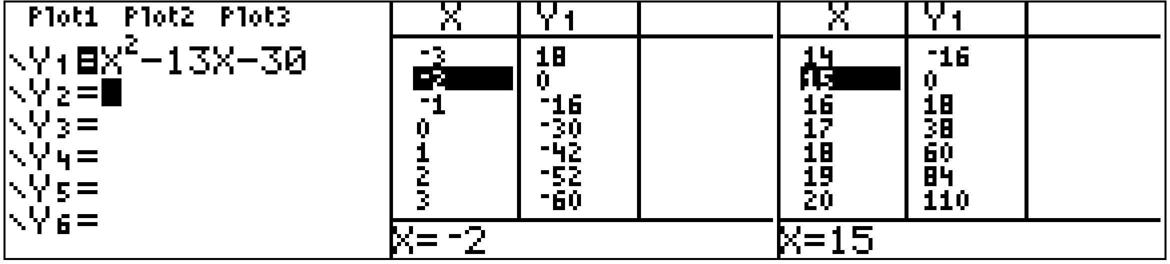
$$f(x) = (x - 15)(x + 2)$$

STEP 2. Convert the factors to zeros.

If the factors are $(x - 15)$ and $(x + 2)$,

then the zeros are at $x = 15$ and $x = -2$.

DIMS? Does It Make Sense? Yes. Check by inputting $f(x) = x^2 - 13x - 30$ into a graphing calculator and verify that there are zeros when $x = 15$ and $x = -2$.



Plot1 Plot2 Plot3

$\backslash Y_1 = X^2 - 13X - 30$

$\backslash Y_2 =$

$\backslash Y_3 =$

$\backslash Y_4 =$

$\backslash Y_5 =$

$\backslash Y_6 =$

X	Y1
-3	18
-2	0
-1	-16
0	-30
1	-42
2	-52
3	-60

X=-2

X	Y1
14	-16
15	0
16	18
17	38
18	60
19	84
20	110

X=15

PTS: 2 NAT: A.SSE.B.3 TOP: Zeros of Polynomials

363) ANS: 1

Step 1. Understand that the zeros of a function are the x values when $f(x) = 0$.

Step 2. Strategy: Solve for x when $f(x) = 0$.

Step 3. Execute the strategy

$$f(x) = x^2 - 5x - 6$$

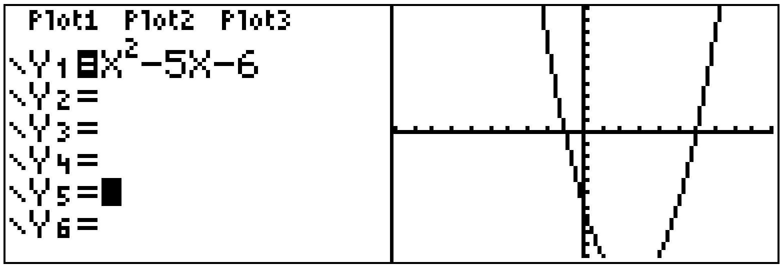
$$0 = x^2 - 5x - 6$$

$$0 = (x + 1)(x - 6)$$

$$x = -1$$

$$x = 6$$

Step 4. Does it make sense? Yes. Check by inputting the function in a graphing calculator and inspecting the graph and table of values.



Plot1 Plot2 Plot3

$\backslash Y_1 = X^2 - 5X - 6$

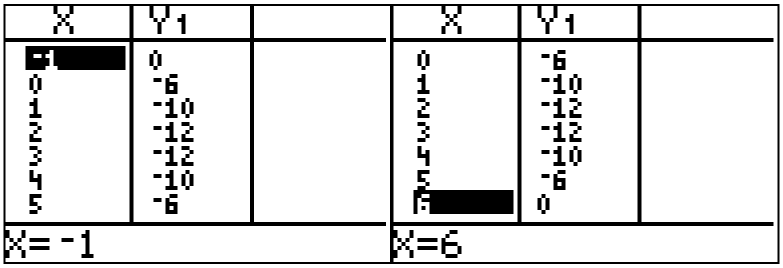
$\backslash Y_2 =$

$\backslash Y_3 =$

$\backslash Y_4 =$

$\backslash Y_5 =$

$\backslash Y_6 =$



X	Y1	
-1	0	
0	-6	
1	-10	
2	-12	
3	-12	
4	-10	
5	-6	

X=-1

X	Y1	
0	-6	
1	-10	
2	-12	
3	-12	
4	-10	
5	-6	
6	0	

X=6

PTS: 2 NAT: A.APR.B.3 TOP: Zeros of Polynomials

364) ANS: 1

Strategy: Convert the zeros of the function to factors.

Zeros occur at	Factors are:
(-3, 0)	(x+3)
(2, 0)	(x-2)
(4, 0)	(x-4)

PTS: 2 NAT: A.APR.B.3

365) ANS: 4

The zeros of a function are the x values when y = 0.

Strategy: Eliminate wrong answers.

~~a)~~ Solve for $0 = x^2 + 7x - 8$ Eliminate this choice.

$0 = (x+8)(x-1)$

$x = -8 \text{ and } x = 1$

~~b)~~ Solve for $0 = x^2 - 7x - 8$ Eliminate this choice.

$0 = (x-8)(x+1)$

$x = 8 \text{ and } x = -1$

c) The graph shows x-axis intercepts at $(-2, 0)$ and at $(4, 0)$, so the zeros are -2 and 4. Eliminate this choice.

d) The graph shows x-axis intercepts at $(-4, 0)$ and at $(2, 0)$, so the zeros are -4 and 2. This is the correct choice.

PTS: 2 NAT: A.APR.B.3 TOP: Zeros of Polynomials

366) ANS: 3

The zeros of a function are the x-values when $y = 0$.

Strategy: Convert the zeros to factors, then combine the factors to write the function.

Zeros	Factors
$x = -3$	$(x+3)$
$x = 0$	(x)
$x = 4$	$(x-4)$

$f(x) = (x+3)(x)(x-4)$

Check by inputting the function in a graphing calculator and inspecting the zeros

PTS: 2 NAT: A.APR.B.3 TOP: Zeros of Polynomials

367) ANS: 3

Strategy #1. Find the factors and use the multiplication property of zero to find the zeros.

$$2x^3 + 12x - 10x^2 = 0$$

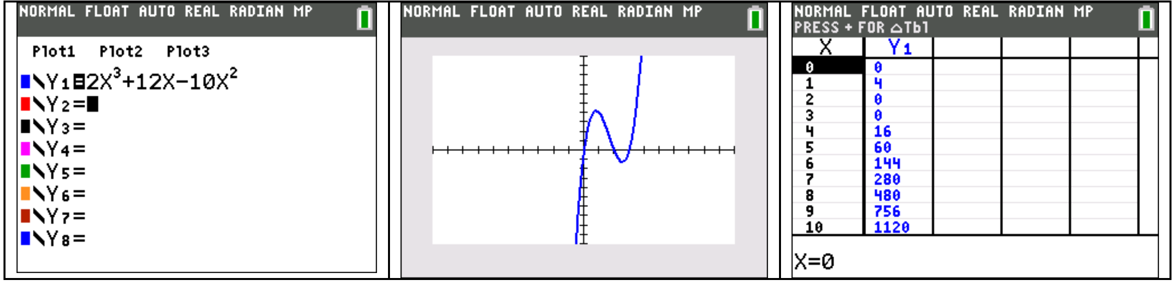
$$2x^3 - 10x^2 + 12x = 0$$

$$2x(x^2 - 5x + 6) = 0$$

$$2x(x-3)(x-2) = 0$$

If the factors are 2x, x-3, and x-2, the zeros are 0, 2, and 3.

Strategy #2: Use a graphing calculator.



```
Y1=2X^3+12X-10X^2
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
Y8=
```

X	Y1
0	0
1	4
2	0
3	0
4	16
5	60
6	144
7	280
8	480
9	756
10	1120

X=0

PTS: 2 NAT: A.APR.B.3 TOP: Zeros of Polynomials

368) ANS:

Strategy 1: Use factoring.

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = \{1, 3\}$$

Strategy 2: Use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 12}}{2}$$

$$x = \frac{4 \pm 2}{2}$$

$$x = 2 \pm 1$$

$$x = \{1, 3\}$$

Strategy 3. Complete the square

$$x^2 - 4x + 3 = 0$$

$$x^2 - 4x = -3$$

$$(x-2)^2 = -3 + (-2)^2$$

$$(x-2)^2 = -3 + 4$$

$$(x-2)^2 = 1$$

$$\sqrt{(x-2)^2} = \sqrt{1}$$

$$x - 2 = \pm 1$$

$$x = 2 \pm 1$$

$$x = \{1, 3\}$$

PTS: 2 NAT: A.APR.B.3 TOP: Zeros of Polynomials

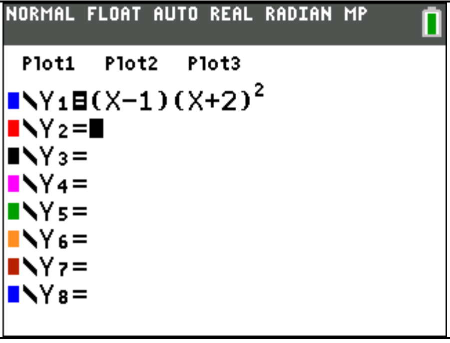
369) ANS: 1

Note that the zeros (x-intercepts) occur at -1 and +2. This means that the factors of the equation are (x+1) and (x-2). Eliminate $P(x) = (x-1)(x+2)^2$ and $P(x) = (x-1)(x+2)$ because they have the wrong factors.

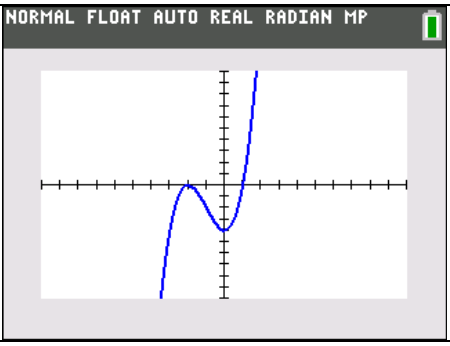
The choice is between $P(x) = (x+1)(x-2)^2$ and $P(x) = (x+1)(x-2)$. $P(x) = (x+1)(x-2)^2$ is a third degree equation and $P(x) = (x+1)(x-2)$ is a second degree (quadratic) equation.

The graph is definitely not a parabola, so it cannot be the graph of a quadratic function. Eliminate $P(x) = (x+1)(x-2)$. The correct answer is $P(x) = (x+1)(x-2)^2$.

Check in a graphing calculator.



```
NORMAL FLOAT AUTO REAL RADIAN MP
 Plot1  Plot2  Plot3
\Y1=(X-1)(X+2)²
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
\Y8=
```



PTS: 2 NAT: A.APR.B.3 TOP: Zeros of Polynomials
KEY: AI

370) ANS: 3

Strategy: Let $p(x) = 0$ and solve the quadratic.

Notes	Left Expression	Sign	Right Expression
Given	$p(x)$	=	$x^2 - 2x - 24$
Let $p(x) = 0$	0	=	$x^2 - 2x - 24$
Factor	0	=	$(x-6)(x+4)$

By the zero property of multiplication: If $0 = (x-6)$, then $x = 6$.
By the zero property of multiplication: If $0 = (x+4)$, then $x = -4$.

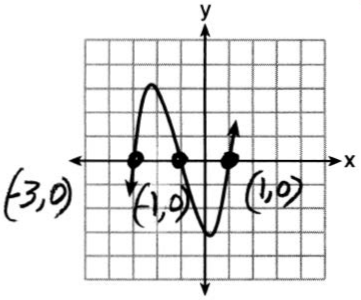
NOTE: The zero property of multiplication says that if the product of two numbers is zero, then one or both of those numbers must be zero.

PTS: 2 NAT: A.APR.B.3 TOP: Zeros of Polynomials

371) ANS: 2

Strategy: Find the zeros of the cubic function, then convert the zeros to factors.

STEP 1

	Zeros	Conversions to Factors	Factors
	$x = -3$	$x + 3 = 0$	$(x + 3)$
	$x = -1$	$x + 1 = 0$	$(x + 1)$
	$x = 1$	$x - 1 = 0$	$(x - 3)$

STEP 2: Combine all factors into one expression.

$$(x + 3)(x + 1)(x - 1)$$

The correct answer choice is $g(x) = (x + 3)(x + 1)(x - 1)$

PTS: 2 NAT: A.APR.B.3 TOP: Zeros of Polynomials

K – Polynomials, Lesson 6, Graphing Polynomial Functions (r. 2018)

POLYNOMIALS

Graphing Polynomial Functions

Common Core Standard	**Next Generation Standard**
F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, ~~$f(kx)$~~, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing ~~even and odd~~ functions from their graphs and algebraic expressions for them. ~~PARCC: Identifying the effect on the graph of replacing $f(x)$ by $f(x) +k$, $kf(x)$, and $f(x+k)$ for specific values of k (both positive and negative) is limited to linear and quadratic functions. Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. Tasks do not involve recognizing even and odd functions.~~	**AI-F.BF.3a** Using $f(x) + k$, $k\,f(x)$, and $f(x + k)$: i) identify the effect on the graph when replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, and $f(x + k)$ for specific values of k (both positive and negative); ii) find the value of k given the graphs; **iii) write a new function using the value of *k*;** and iv) use technology to experiment with cases and explore the effects on the graph. (Shared standard with Algebra II) **Note: Tasks are limited to linear, quadratic, square root, and absolute value functions; and exponential functions of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).**

LEARNING OBJECTIVES

Students will be able to:

1) Use a constant k in the equation of the parabola to move the graph of parabolas up, down, left, and/or right.
2) Use a constant k in the equation of the parabola to make the parabola open upward or downward.
3) Use a constant k in the equation of the parabola to make the parabola narrower or wider.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

constant
narrower
scalar
translation
vertex
wider

BIG IDEAS

The graph of a function is changed when either $f(x)$ or x is multiplied by a scalar, or when a constant is added to or subtracted from either $f(x)$ or x. A graphing calculator can be used to explore the translations of graph views of functions.

Up and Down

The addition or subtraction of a constant **outside the parentheses** moves the graph up or down by the value of the constant.

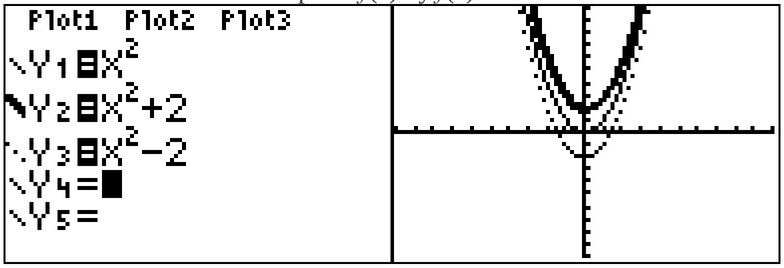
$f(x) \Leftrightarrow f(x) \pm k$ moves the graph up or down k units $\updownarrow$.

+k moves the graph up.

-k moves the graph down.

Examples:

Replace *f*(*x*) by *f*(*x*) + *k*



$Y_1 = X^2$

$Y_2 = X^2+2$

$Y_3 = X^2-2$

$Y_4 =$

$Y_5 =$

Left and Right

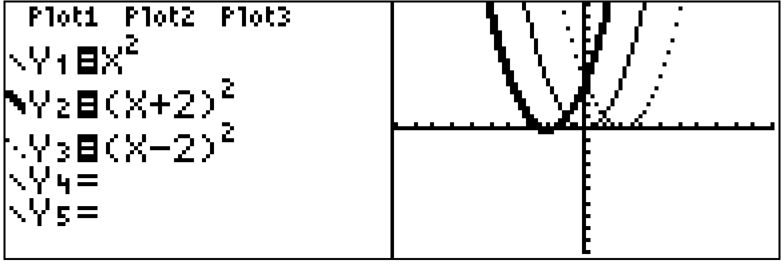
The addition or subtraction of a constant **inside the parentheses** moves the graph left or right by the value of the constant.

$f(x) \Leftrightarrow f(x \pm k)$ moves the graph left or right k units $\updownarrow$.

+k moves the graph leftg k units.

-k moves the graph right k units.

Replace *f*(*x*) by *f*(*x* + *k*)



$Y_1 = X^2$

$Y_2 = (X+2)^2$

$Y_3 = (X-2)^2$

$Y_4 =$

$Y_5 =$

Width and Direction of a Parabola

Changing the value of *a* in a quadratic affects the width and direction of a parabola. The bigger the absolute value of *a*, the narrower the parabola.

$f\left(x\right) \Leftrightarrow f\left(kx\right)$ changes the direction and width of a parabola.

+k opens the parabola upward.

-k opens the parabola downward.

If k is a fraction less than 1, the parabola will get wider.

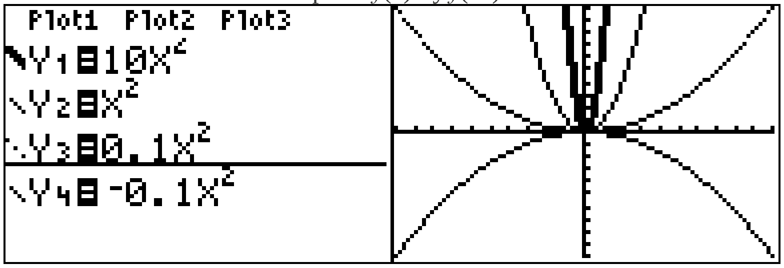
As k approaches zero, the parabola approaches a straight horizontal line.

If k is a number greater than 1, the parabola will get narrower.

As k approaches infinity, the parabola approaches a straight vertical line.

Examples:

Replace *f*(*x*) by *f*(*kx*)



Plot1 Plot2 Plot3

$Y_1 = 10X^2$

$Y_2 = X^2$

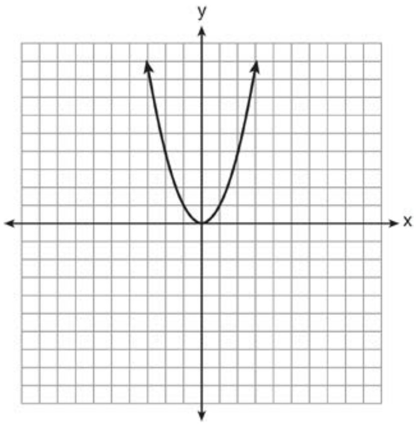
$Y_3 = 0.1X^2$

$Y_4 = -0.1X^2$

DEVELOPING ESSENTIAL SKILLS

1. Consider the graph of the equation $y = ax^2 + bx + c$, when $a \neq 0$. If *a* is multiplied by 3, what is true of the graph of the resulting parabola?
   a. The vertex is 3 units above the vertex of the original parabola.
   b. The new parabola is 3 units to the right of the original parabola.
   c. The new parabola is wider than the original parabola.
   d. The new parabola is narrower than the original parabola.
2. Melissa graphed the equation $y = x^2$ and Dave graphed the equation $y = -3x^2$ on the same coordinate grid. What is the relationship between the graphs that Melissa and Dave drew?
   a. Dave's graph is wider and opens in the opposite direction from Melissa's graph.
   b. Dave's graph is narrower and opens in the opposite direction from Melissa's graph.
   c. Dave's graph is wider and is three units below Melissa's graph.
   d. Dave's graph is narrower and is three units to the left of Melissa's graph.
3. The graph of a parabola is represented by the equation $y = ax^2$ where *a* is a positive integer. If *a* is multiplied by 2, the new parabola will become
   a. narrower and open downward
   b. narrower and open upward
   c. wider and open downward
   d. wider and open upward
4. How is the graph of $y = x^2 + 4x + 3$ affected when the coefficient of x^2 is changed to a smaller positive number?
   a. The graph becomes wider, and the *y*-intercept changes.
   b. The graph becomes wider, and the *y*-intercept stays the same.
   c. The graph becomes narrower, and the *y*-intercept changes.
   d. The graph becomes narrower, and the *y*-intercept stays the same.
5. Which is the equation of a parabola that has the same vertex as the parabola represented by $y = x^2$, but is wider?
   a. $y = x^2 + 2$
   b. $y = x^2 - 2$
   c. $y = 2x^2$
   d. $y = \frac{1}{2}x^2$

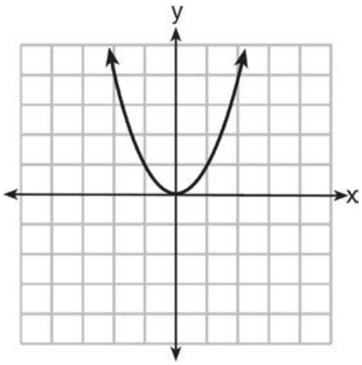
6.The graph of the equation $y = x^2$ is shown below.



Which statement best describes the change in this graph when the coefficient of x^2 is multiplied by 4?

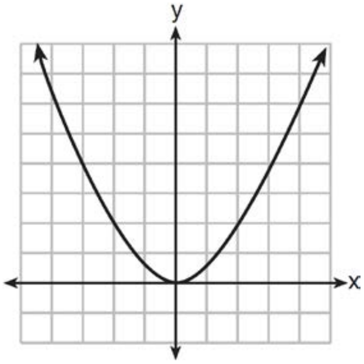
a. The parabola becomes wider.
b. The parabola becomes narrower.
c. The parabola will shift up four units.
d. The parabola will shift right four units.

7. The graph of $y = x^2$ is shown below.

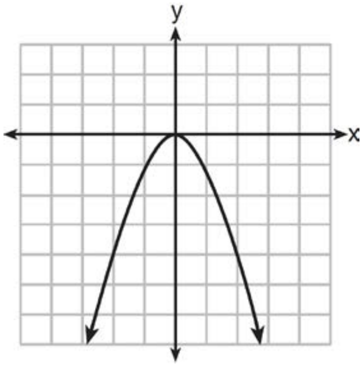


Which graph represents $y = 2x^2$?

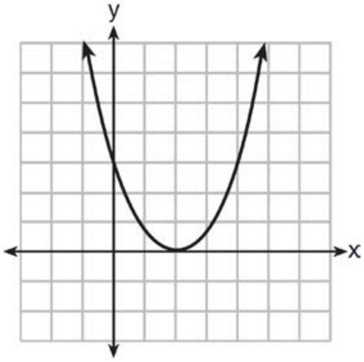
a.



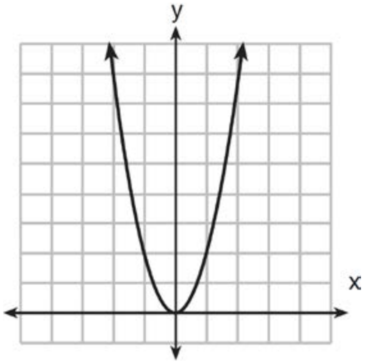
c.



b.



d.

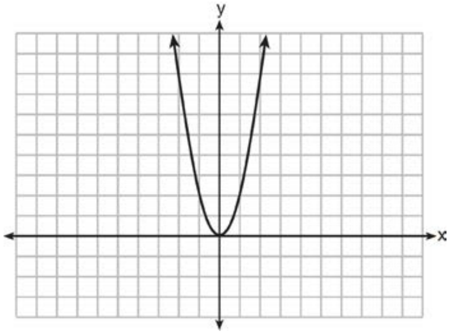


ANSWERS

1. ANS: D
2. ANS: B
3. ANS: B
4. ANS: B
5. ANS: D
6. ANS: B
7. ANS: D

F.BF.B.3: Graphing Polynomial Functions

372) The graph of the equation $y = ax^2$ is shown below.



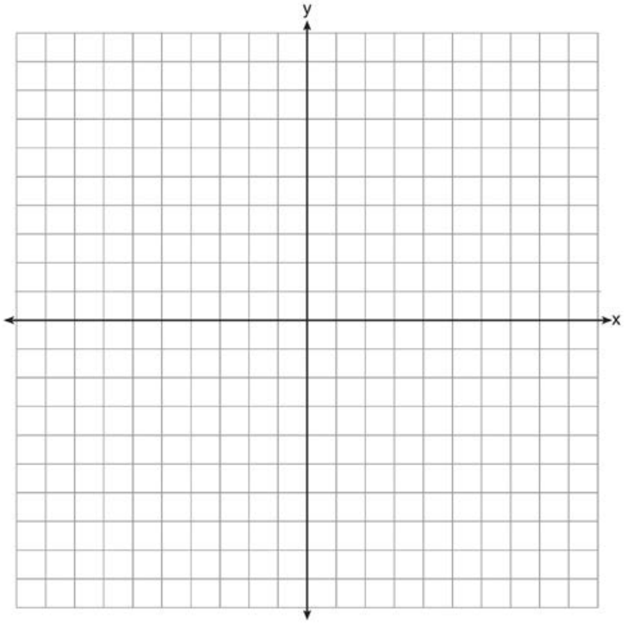
If a is multiplied by $-\frac{1}{2}$, the graph of the new equation is

1) wider and opens downward
2) wider and opens upward
3) narrower and opens downward
4) narrower and opens upward

373) How does the graph of $f(x) = 3(x-2)^2 + 1$ compare to the graph of $g(x) = x^2$?

1) The graph of $f(x)$ is wider than the graph of $g(x)$, and its vertex is moved to the left 2 units and up 1 unit.
2) The graph of $f(x)$ is narrower than the graph of $g(x)$, and its vertex is moved to the right 2 units and up 1 unit.
3) The graph of $f(x)$ is narrower than the graph of $g(x)$, and its vertex is moved to the left 2 units and up 1 unit.
4) The graph of $f(x)$ is wider than the graph of $g(x)$, and its vertex is moved to the right 2 units and up 1 unit.

374) The vertex of the parabola represented by $f(x) = x^2 - 4x + 3$ has coordinates $(2,-1)$. Find the coordinates of the vertex of the parabola defined by $g(x) = f(x-2)$. Explain how you arrived at your answer. [The use of the set of axes below is optional.]



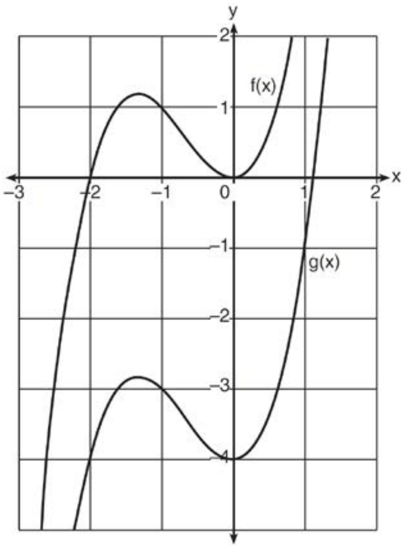
375) Given the graph of the line represented by the equation $f(x) = -2x + b$, if *b* is increased by 4 units, the graph of the new line would be shifted 4 units

1) right
2) up
3) left
4) down

376) When the function $f(x) = x^2$ is multiplied by the value *a*, where $a > 1$, the graph of the new function, $g(x) = ax^2$

1) opens upward and is wider
2) opens upward and is narrower
3) opens downward and is wider
4) opens downward and is narrower

377) In the diagram below, $f(x) = x^3 + 2x^2$ is graphed. Also graphed is $g(x)$, the result of a translation of $f(x)$.



Determine an equation of $g(x)$. Explain your reasoning.

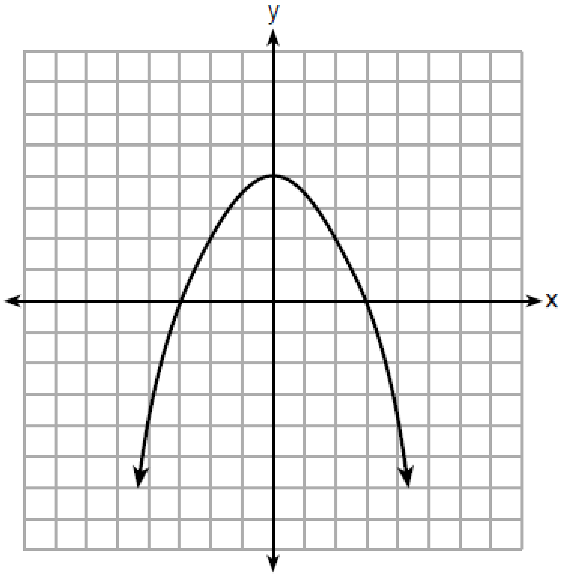
378) In the functions $f(x) = kx^2$ and $g(x) = |kx|$, *k* is a positive integer. If *k* is replaced by $\frac{1}{2}$, which statement about these new functions is true?

1) The graphs of both $f(x)$ and $g(x)$ become wider.
2) The graph of $f(x)$ becomes narrower and the graph of $g(x)$ shifts left.
3) The graphs of both $f(x)$ and $g(x)$ shift vertically.
4) The graph of $f(x)$ shifts left and the graph of $g(x)$ becomes wider.

379) If the original function $f(x) = 2x^2 - 1$ is shifted to the left 3 units to make the function $g(x)$, which expression would represent $g(x)$?

1) $2(x-3)^2 - 1$
2) $2(x+3)^2 - 1$
3) $2x^2 + 2$
4) $2x^2 - 4$

380) The graph of the function $p(x)$ is represented below. On the same set of axes, sketch the function $p(x+2)$.

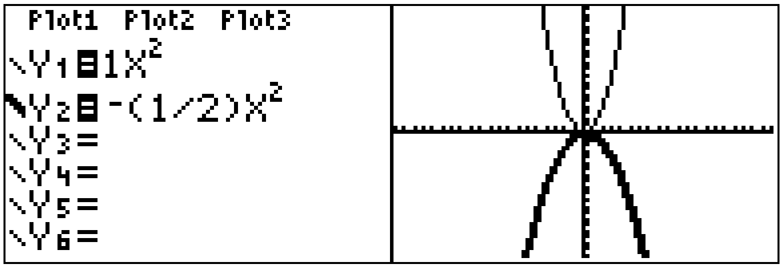


SOLUTIONS

372) ANS: 1

Strategy: Use the following general rules for quadratics, then check with a graphiong calculator.
As the value of a approaches 0, the parabola gets wider.
A positive value of a opens upward.
A negative value of a opens downward.

Check with graphing calculator:
Assume $a = 1$, then $y_1 = 1x^2$
If a is multiplied by $-\frac{1}{2}$, then $y_2 = -\frac{1}{2}x^2$.
Input both equations in a graphing calculator, as follows:



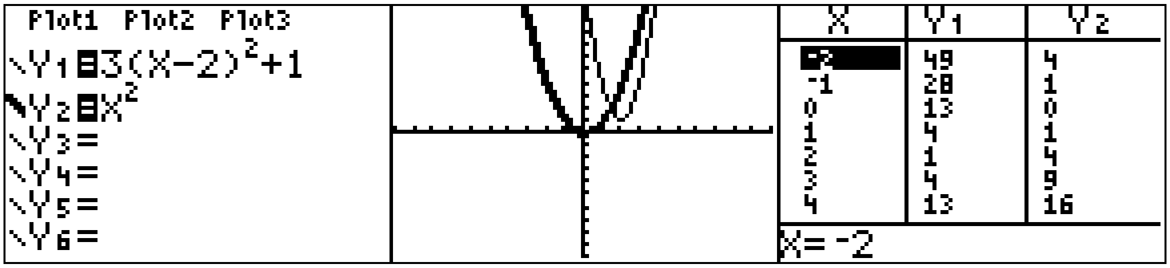
```
 Plot1 Plot2 Plot3
\Y1=1X^2
\Y2=-(1/2)X^2
\Y3=
\Y4=
\Y5=
\Y6=
```

PTS: 2 NAT: F.BF.B.3 TOP: Transformations with Functions and Relations

373) ANS: 2

Strategy: Input both functions in a graphing calculator and compare them.

Let the graph of Y_1 be the graph of $f(x) = 3(x-2)^2 + 1$
Let the graph of Y_2 be the graph of $g(x) = x^2$
Input both functions in a graphing calculator.
g(x) is the thick line and *f(x)* is the thin line.



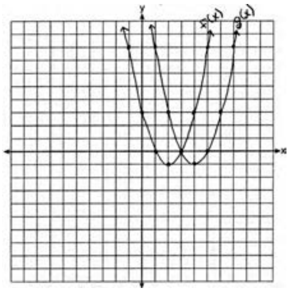
```
Plot1 Plot2 Plot3
\Y1=3(X-2)²+1
\Y2=X²
\Y3=
\Y4=
\Y5=
\Y6=
```

X	Y1	Y2
-2	49	4
-1	28	1
0	13	0
1	4	1
2	1	4
3	4	9
4	13	16

X= -2

PTS: 2 NAT: F.BF.B.3 TOP: Transformations with Functions and Relations

374) ANS:



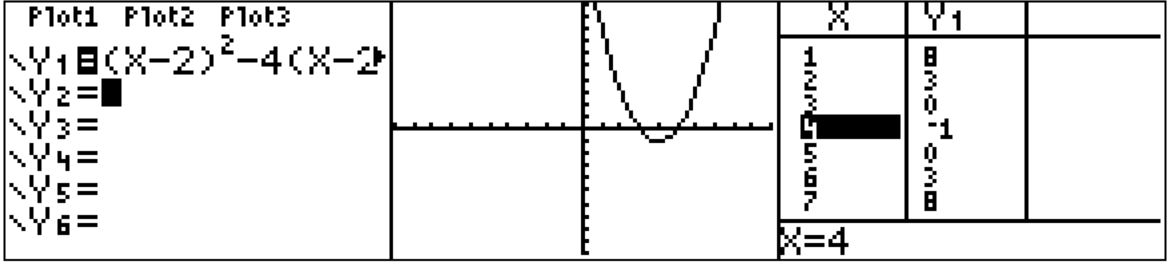
$(4,-1)$. $f(x-2)$ is a horizontal shift two units to the right

Strategy 1: Compose a new function, find the axis of symmetry, solve for g(x) at axis of symmetry, as follows:

$f(x)=x^2-4x+3$ and $g(x)=f(x-2)$ Therefore: $g(x)=(x-2)^2-4(x-2)+3$ $g(x)=x^2-4x+4-4x+8+3$ $g(x)=x^2-8x+15$	$axis\ of\ symmetry=\frac{-b}{2a}=\frac{-(-8)}{2(1)}=\frac{8}{2}=4$ $g(x)=x^2-8x+15$ $g(4)=(4)^2-8(4)+15$ $g(4)=16-32+15$ $g(4)=-1$

The coordinates of the vertex of *g(x)* are (4,-1)

Strategy #2. Input the new function in a graphing calculator and identify the vertex.



```
Plot1 Plot2 Plot3
\Y1=(X-2)²-4(X-2▸
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

X	Y1	
1	8	
2	3	
3	0	
4	-1	
5	0	
6	3	
7	8	

X=4

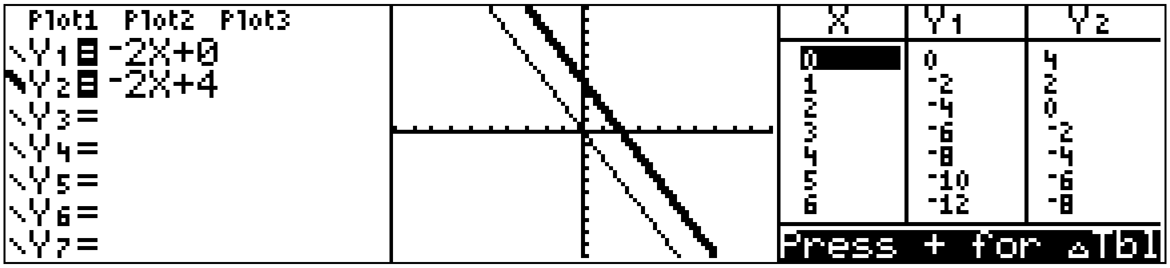
PTS: 2 NAT: F.BF.B.3 TOP: Transformations with Functions and Relations

375) ANS: 2

Strategy: Use the characteristics of the slope intercept form of a line, which is $y=mx+b$, where y is the dependent variable, m is the slope, x is the dependent variable, and b is the y-inctercept.

If b (the y-intercept) is increased by four, the slope remains the same and the new line is shifted up 4 units.

Check using a graphing calculator.



Plot1 Plot2 Plot3

Y1=-2X+0

Y2=-2X+4

Y3=

Y4=

Y5=

Y6=

Y7=

X	Y1	Y2
0	0	4
1	-2	2
2	-4	0
3	-6	-2
4	-8	-4
5	-10	-6
6	-12	-8

Press + for ΔTbl

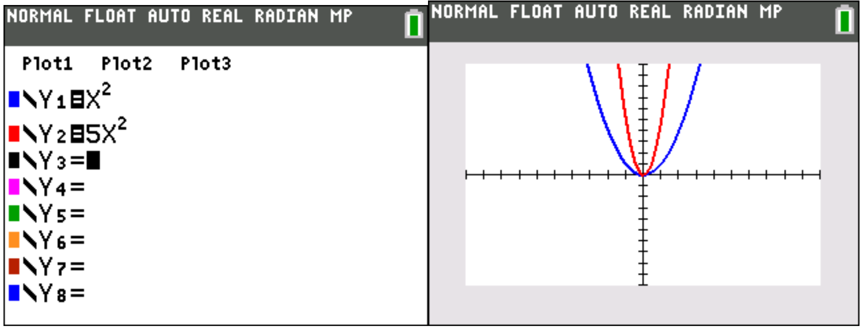
PTS: 2 NAT: F.BF.B.3 TOP: Transformations with Functions and Relations

376) ANS: 2

Strategy: Eliminate wrong answers.

Step 1. Since $a > 1$, a must be positive and the graph of $g(x) = ax^2$ must open upward. Eliminate any choice that opens downward.

Step 2. Determine if the graph gets wider or narrower by selecting a number larger than 1 for a, then input both functions in a graphing calculator and compare their graphs. The graph gets narrower, so eliminate any answer that indicates wider.



NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

Y1=X^2

Y2=$5X^2$

Y3=

Y4=

Y5=

Y6=

Y7=

Y8=

NORMAL FLOAT AUTO REAL RADIAN MP

~~a)~~ opens upward and is ~~wider~~

b) opens upward and is narrower

~~c)~~ ~~opens downward~~ and is ~~wider~~

~~d)~~ ~~opens downward~~ and is narrower

PTS: 2 NAT: F.BF.B.3 STA: A.G.5 TOP: Graphing Polynomial Functions

377) ANS:

$$g(x) = x^3 + 2x^2 - 4$$

f(x) has a y-intercept of 0.

g(x) has a y-intercept of -4.

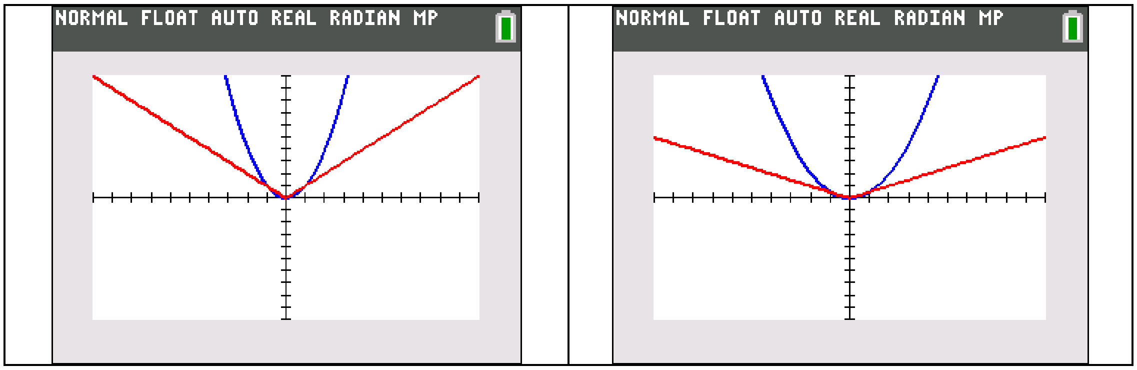
Every point on f(x) is a translation down 4 units to create g(x).

PTS: 2 NAT: F.BF.B.3 TOP: Graphing Polynomial Functions

378) ANS: 1

Since k is a positive integer, the lowest possible value for k is 1. If k is replaced by $\frac{1}{2}$, the graphs of both $f(x)$ and $g(x)$ will become wider.

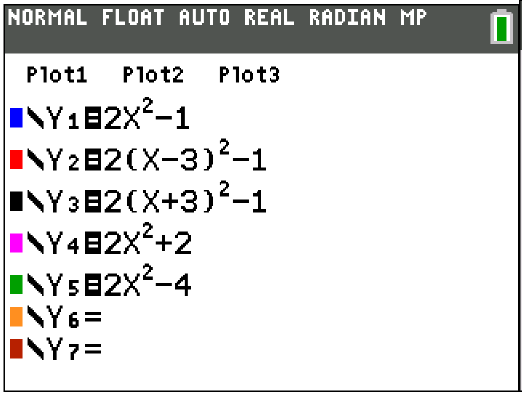
Let k = 1	Let $k = 1$ be replaced by $k = \frac{1}{2}$



PTS: 2 NAT: F.BF.B.3 TOP: Graphing Polynomial Functions

379) ANS: 2

Strategy: Input the orginal function and the four answer choices in a graphing calculator. Then, select the function rule for the graph that is shifted 3 units to the left.



Plot1 Plot2 Plot3

$Y_1=2X^2-1$

$Y_2=2(X-3)^2-1$

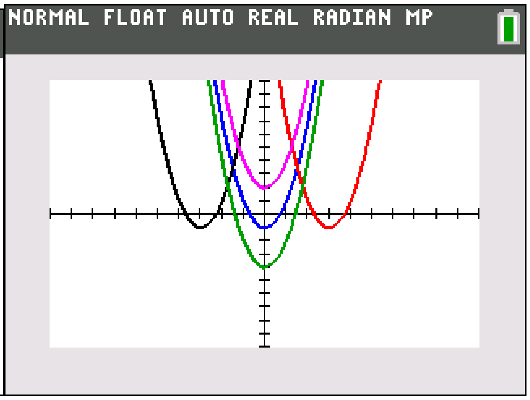
$Y_3=2(X+3)^2-1$

$Y_4=2X^2+2$

$Y_5=2X^2-4$

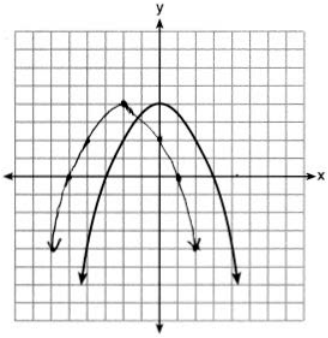
$Y_6=$

$Y_7=$



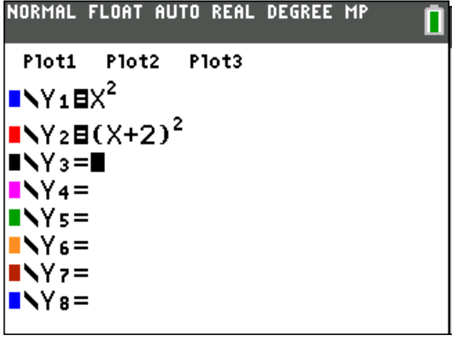
PTS: 2 NAT: F.BF.B.3 TOP: Graphing Polynomial Functions

380) ANS:



Strategy: Solve a simpler problem - pick a simple quadratic function, such as $y = x^2$ and see what happens to the graph when the function is changed to $y = (x + 2)^2$.

STEP 1. Input both in functions in a graphing calculaor.



Plot1 Plot2 Plot3

$Y_1=X^2$

$Y_2=(X+2)^2$

$Y_3=$

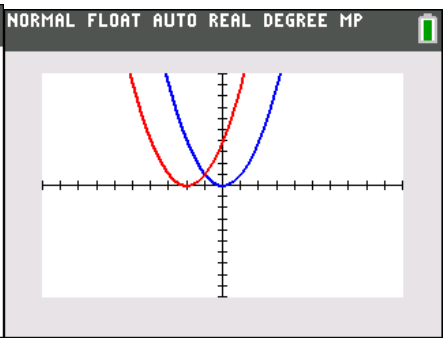
$Y_4=$

$Y_5=$

$Y_6=$

$Y_7=$

$Y_8=$



STEP 2. Observe that the graph moves two units to the left.
STEP 3. Move every point of the original function two units to the left.

PTS: 2 NAT: F.BF.B.3 TOP: Graphing Polynomial Functions

L – Radicals, Lesson 1, Operations with Radicals (r. 2018)

RADICALS

Operations with Radicals

Common Core Standard	**Next Generation Standard**
N-RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	**AI-N.RN.3** Use properties and operations to understand the different forms of rational and irrational numbers. a.) Perform all four arithmetic operations and apply properties to generate equivalent forms of rational numbers and square roots. Note: Tasks include rationalizing numerical denominators of the form $\frac{a}{\sqrt{b}}$ where a is an integer and b is a natural number. b.) Categorize the sum or product of rational or irrational numbers. • The sum and product of two rational numbers is rational. • The sum of a rational number and an irrational number is irrational. • The product of a nonzero rational number and an irrational number is irrational. • The sum and product of two irrational numbers could be either rational or irrational.

LEARNING OBJECTIVES

Students will be able to:

1) Perform addition, subtraction, multiplication and division with radical numbers (prior skill).
2) Identify if the sum or product of two numbers is rational or irrational and explain why.

Overview of Lesson

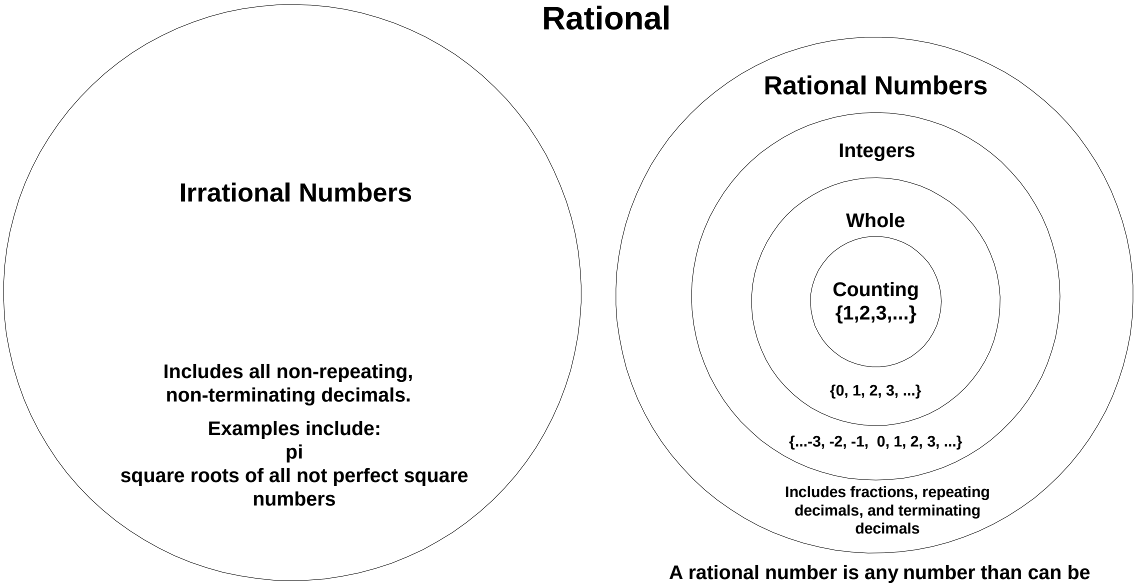
Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

Decimal form
Equivalent
Irrational
Perfect square
Prime number
Radical form
Rational
Simplest radical form

BIG IDEAS

The Set of Real Numbers
includes two major classifications of numbers
Irrational and
Rational



An irrational number is any number that cannot be expressed as the ratio of two integers.

A rational number is any number than can be expressed as the ratio of two integers.

Is a Number Irrational or Rational?

Irrational Numbers	Rational Numbers
If a decimal does not repeat or terminate, it is an irrational number. Numbers with names, such as π and e are irrational. They are given names because it is impossible to state their infinitely long values. The square roots of all numbers (that are not perfect squares) are irrational. If a term reduced to simplest form contains an irrational number, the term is irrational. .	If a number is an integer, it is rational, since it can be expressed as a ratio with the integer as the numerator and 1 as the denominator. If a decimal is a repeating decimal, it is a rational number. If a decimal terminates, it is a rational number.

Operations with Irrational and Rational Numbers

Addition and Subtraction:

When two rational numbers are added or subtracted, the result is rational.
When two irrational numbers are added or subtracted, the result is irrational.
When an irrational number and a rational number are added or subtracted, the sum is irrational.

Multiplication and Division:

When two rational numbers are multiplied or divided, the product is rational.
When an irrational number and a non-zero rational number are multiplied or divided, the product is irrational.
When two irrational numbers are multiplied or divided, the product is sometimes rational and sometimes irrational.

Example of Rational Product $\sqrt{7}\times\sqrt{28}$ $\sqrt{7}\times\left(\sqrt{4}\times\sqrt{7}\right)$ $\left(\sqrt{7}\sqrt{7}\right)\sqrt{4}$ $7\times 2 = 14$ $\frac{14}{1}$	Example of Irrational Product $\sqrt{7}\times\sqrt{3}$ $\sqrt{21}$ 4.582575695...... NOTE: Be careful using a calculator to decide if a number is irrational. The calculator stops when it runs out of room to display the numbers, and the whole number may continue beyond the calculator display.
Rational Quotient $\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{\frac{20}{5}} = \sqrt{4} = 2 = \frac{2}{1}$	Irrational Quotient $\frac{\sqrt{10}}{\sqrt{5}} = \sqrt{\frac{10}{5}} = \sqrt{2}$

DEVELOPING ESSENTIAL SKILLS

Question	Answer	Is Answer Rational or Irrational?
1. Express the product of $3\sqrt{20}(2\sqrt{5}-7)$ in simplest radical form.		
2. The expression $6\sqrt{50}+6\sqrt{2}$ written in simplest radical form is:		
3. The expression $\sqrt{72}-3\sqrt{2}$ written in simplest radical form is		
4. Express $\frac{16\sqrt{21}}{2\sqrt{7}}-5\sqrt{12}$ in simplest radical form.		
5. Express $\frac{3\sqrt{75}+\sqrt{27}}{3}$ in simplest radical form.		
6. Express $\sqrt{25}-2\sqrt{3}+\sqrt{27}+2\sqrt{9}$ in simplest radical form.		
7. Express $\frac{\sqrt{84}}{2\sqrt{3}}$ in simplest radical form.		
8. Perform the indicated operations and express the answer in simplest radical form. $3\sqrt{7}\left(\sqrt{14}+4\sqrt{56}\right)$		
9. The expression $\sqrt{90}\cdot\sqrt{40}-\sqrt{8}\cdot\sqrt{18}$ simplifies to		
10. The expression $\frac{6\sqrt{20}}{3\sqrt{5}}$ is equivalent to		

ANSWERS

Question	Answer	Is Answer Rational or Irrational?
1. Express the product of $3\sqrt{20}(2\sqrt{5}-7)$ in simplest radical form.	$60-42\sqrt{5}$	Irrational
2. The expression $6\sqrt{50}+6\sqrt{2}$ written in simplest radical form is:	$36\sqrt{2}$	Irrational
3. The expression $\sqrt{72}-3\sqrt{2}$ written in simplest radical form is	$3\sqrt{2}$	Irrational
4. Express $\dfrac{16\sqrt{21}}{2\sqrt{7}}-5\sqrt{12}$ in simplest radical form.	$-2\sqrt{3}$	Irrational
5. Express $\dfrac{3\sqrt{75}+\sqrt{27}}{3}$ in simplest radical form.	$6\sqrt{3}$	Irrational
6. Express $\sqrt{25}-2\sqrt{3}+\sqrt{27}+2\sqrt{9}$ in simplest radical form.	$11+\sqrt{3}$	Irrational
7. Express $\dfrac{\sqrt{84}}{2\sqrt{3}}$ in simplest radical form.	$\sqrt{7}$	Irrational
8. Perform the indicated operations and express the answer in simplest radical form. $3\sqrt{7}\left(\sqrt{14}+4\sqrt{56}\right)$	$189\sqrt{2}$	Irrational
9. The expression $\sqrt{90}\cdot\sqrt{40}-\sqrt{8}\cdot\sqrt{18}$ simplifies to	48	Rational
10. The expression $\dfrac{6\sqrt{20}}{3\sqrt{5}}$ is equivalent to	4	Rational

REGENTS EXAM QUESTIONS (through June 2018)

N.RN.B.3: Operations with Radicals

381) Given: $L = \sqrt{2}$

$M = 3\sqrt{3}$

$N = \sqrt{16}$

$P = \sqrt{9}$

Which expression results in a rational number?

1) $L + M$
2) $M + N$
3) $N + P$
4) $P + L$

382) Which statement is *not* always true?

1) The product of two irrational numbers is irrational.
2) The product of two rational numbers is rational.
3) The sum of two rational numbers is rational.
4) The sum of a rational number and an irrational number is irrational.

383) Ms. Fox asked her class "Is the sum of 4.2 and $\sqrt{2}$ rational or irrational?" Patrick answered that the sum would be irrational. State whether Patrick is correct or incorrect. Justify your reasoning.

384) Which statement is *not* always true?

1) The sum of two rational numbers is rational.
2) The product of two irrational numbers is rational.
3) The sum of a rational number and an irrational number is irrational.
4) The product of a nonzero rational number and an irrational number is irrational.

385) For which value of P and W is $P + W$ a rational number?

1) $P = \frac{1}{\sqrt{3}}$ and $W = \frac{1}{\sqrt{6}}$
2) $P = \frac{1}{\sqrt{4}}$ and $W = \frac{1}{\sqrt{9}}$
3) $P = \frac{1}{\sqrt{6}}$ and $W = \frac{1}{\sqrt{10}}$
4) $P = \frac{1}{\sqrt{25}}$ and $W = \frac{1}{\sqrt{2}}$

386) Given the following expressions:

I. $-\frac{5}{8} + \frac{3}{5}$ III. $(\sqrt{5}) \cdot (\sqrt{5})$

II. $\frac{1}{2} + \sqrt{2}$ IV. $3 \cdot (\sqrt{49})$

Which expression(s) result in an irrational number?

1) II, only
2) III, only
3) I, III, IV
4) II, III, IV

387) Determine if the product of $3\sqrt{2}$ and $8\sqrt{18}$ is rational or irrational. Explain your answer.

388) Is the sum of $3\sqrt{2}$ and $4\sqrt{2}$ rational or irrational? Explain your answer.

389) Jakob is working on his math homework. He decides that the sum of the expression $\frac{1}{3} + \frac{6\sqrt{5}}{7}$ must be rational because it is a fraction. Is Jakob correct? Explain your reasoning.

390) State whether $7-\sqrt{2}$ is rational or irrational. Explain your answer.

391) A teacher wrote the following set of numbers on the board:

$$a=\sqrt{20} \qquad b=2.5 \qquad c=\sqrt{225}$$

Explain why $a+b$ is irrational, but $b+c$ is rational.

392) The product of $\sqrt{576}$ and $\sqrt{684}$ is

1) irrational because both factors are irrational
2) rational because both factors are rational
3) irrational because one factor is irrational
4) rational because one factor is rational

393) Is the product of $\sqrt{16}$ and $\frac{4}{7}$ rational or irrational? Explain your reasoning.

SOLUTIONS

381) ANS: 3

$\sqrt{16}+\sqrt{9}=\frac{7}{1}$ may be expressed as the ratio of two integers.

Strategy: Recall that under the operation of addition, the addition of two irrational numbers and the addition of an irrational number and a rational number will always result in a sum that is irrational. To get a rational number as a sum, you must add two rational numbers.

STEP 1 Determine whether numbers L, M, N, and P are ratiional, then reject any answer choice that does not contain two rational numbers.

$L=\sqrt{2}$ is irrational

$M=3\sqrt{3}$ is irrational

$N=\sqrt{16}=4$ and is rational

$P=\sqrt{9}=3$ and is rational

STEP 2 Reject any answer choice that does not include $N+P$. Choose answer choice c.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

382) ANS: 1

Strategy: Find a counterexample to prove one of the answer choices is *not* always true.

Answer choice a is not always true because: $\sqrt{3}$ and $\sqrt{12}$ are both irrational numbers, but $\sqrt{3}\times\sqrt{12}=\sqrt{3\times 12}=\sqrt{36}=6$, and 6 is a rational number, so the product of two irrational numbers is not *always* irrational.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

383) ANS:

Patrick is correct. The sum of a rational and irrational is irrational.

Strategy: Determine whether 4.2 and $\sqrt{2}$ are rational or irrational numbers, then apply the rules of operations on rational and irrational numbers.

4.2 is rational because it can be expressed as $\frac{42}{10}$, which is the ratio of two integers.

$\sqrt{2}$ is irrational because it cannot be expressed as the ratio of two integers.

The rules of addition and subtraction of rational and irrational numbers are:
When two rational numbers are added or subtracted, the result is rational.
When two irrational numbers are added or subtracted, the result is irrational.
When an irrational number and a rational number are added or subtracted, the sum is irrational.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

384) ANS: 2

Strategy: Find a counterexample to prove one of the answer choices is *not* always true. This will usually involve the *product* or *quotient* of two irrational numbers since the outcomes of addition and subtraction of irrational numbers are more predictable.

Answer choice b is not always true because: $\sqrt{2}$ and $\sqrt{3}$ are both irrational numbers, but $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$, and $\sqrt{6}$ is an rational number, so the product of two irrational numbers is not *always* rational.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

385) ANS: 2

$$\frac{1}{\sqrt{4}} + \frac{1}{\sqrt{9}} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

Strategy: Recall that under the operation of addition, the addition of two irrational numbers and the addition of an irrational number and a rational number will always result in a sum that is irrational. To get a rational number as a sum, you must add two rational numbers. Reject any answer choice that does not contain two rational numbers.

Reject answer choice a because $\frac{1}{\sqrt{3}}$ is irrational.

Choose answer choice b because both $P = \frac{1}{\sqrt{4}}$ and $W = \frac{1}{\sqrt{9}}$ can be expressed as rational numbers, as shown above.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

386) ANS: 1

Strategy: Eliminate wrong answers.

Expression I results in a rational number because the set of rational numbers is closed under addition.

$$-\frac{5}{8} + \frac{3}{5} = \frac{-25}{40} + \frac{24}{40} = \frac{-1}{40}$$

Expression II is is correct because the additon of a rational number and an irrational number always results in an irrational number.

$$\frac{1}{2} + \sqrt{2} = 0.5 + 1.414203562... = 1.914203562...$$

Expression III results in a rational number because $\left(\sqrt{5}\right) \cdot \left(\sqrt{5}\right) = \sqrt{5 \cdot 5} = \sqrt{25} = 5 = \frac{5}{1}$, which is the ratio of two integers.

Expression IV results in a rational number because $3 \cdot \left(\sqrt{49}\right) = 3 \cdot 7 = 21 = \frac{21}{1}$, which the ratio of two integers.

Expression II is the only expression that results in an irrational number, so Choice (a) is the correct answer.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

387) ANS:

$$3\sqrt{2}\cdot 8\sqrt{18}$$

$$3\times 8\times \sqrt{2}\times \sqrt{18}$$

$$24\sqrt{36}$$

$$144$$

The product is 144, which is rational, because it can be written as $\frac{144}{1}$, a ratio of two integers.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

388) ANS:

Irrational

$3\sqrt{2}+4\sqrt{2}=7\sqrt{2}$

$7\sqrt{2}$ is irrational because it is the product of a rational number and an irrational number.

7 is rational because it can be expressed as the ratio of two integers ($\frac{7}{1}$)

$\sqrt{2}$ is irrational because the square roots of all prime numbers are irrational.

PTS: 2 NAT: N.RN.B

389) ANS:

Jakob is incorrect. The sum of a rational number and an irrational number is irrational.

$$\frac{1}{3}+\frac{6\sqrt{5}}{7}=\frac{7+18\sqrt{5}}{21}$$

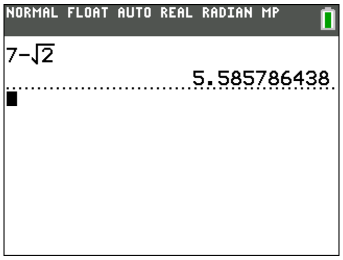
Note the square root of 5 in the sum. The square root of any prime is irrational.

PTS: 2 NAT: N.RN.B.3 TOP: Classifying Numbers

390) ANS:

Irrational

A rational number and an irrational number under addition or subtraction will always be irrational.



```
7-√2
                5.585786438
```

Note that the answer does not appear to repeat or end.

is irrational because it can not be written as the ratio of two integers.

PTS: 2 NAT: N.RN.B.3 TOP: Operations with Radicals

KEY: classify

391) ANS:

The sum of *a* and *b* is irrational because the sum of an irrational number and a rational number is always irrational.

The sum of *b* and *c* is rational because the sum of a rational number and another rational number is always rational.

$\sqrt{20}$ is an irrational number that can be simplified to $2\sqrt{5}$, but cannot be expressed as the ratio of two integers or as a never-ending, never-repeating decimal.

2.5 is a rational number because it can be expressed as the ratio of two integers, such as $\frac{25}{10}$.

$\sqrt{225}$ is a rational number that can be simplified to 15 and expressed as the ratio of two integers, such as $\frac{15}{1}$.

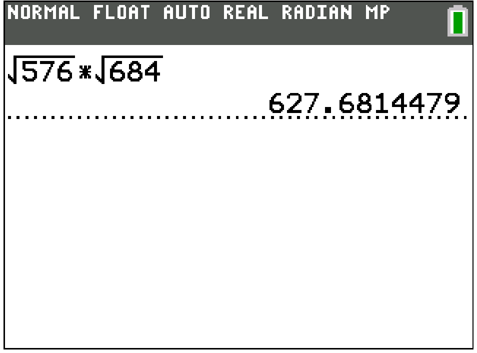
PTS: 2 NAT: N.RN.B.3 TOP: Operations with Radicals
KEY: classify

392) ANS: 3

$\sqrt{576} = 24$, which can be expressed as the ratio $\frac{24}{1}$, which means that $\sqrt{576}$ is a rational number.

$\sqrt{684}$ cannot be expressed as a rational number. It can be simplified to $6\sqrt{19}$, but it cannot be expressed as the ratio of two integers. Therefore, $\sqrt{684}$ is an irrational number.

The product of a rational number and an irrational number is always irrational.



```
√576*√684
                    627.6814479
```

Note that the product of $\sqrt{576}$ and $\sqrt{684}$ appears to be a never ending, non-repeating decimal, which indicates that the product is an irrational number.

PTS: 2 NAT: N.RN.B.3 TOP: Operations with Radicals
KEY: classify

393) ANS:

Answer: The product of $\sqrt{16}$ and $\frac{4}{7}$ is rational.

Explanation: A rational number is a number that can be expressed as the ratio of two intergers, in the form of $\frac{a}{b}$, where both *a* and *b* are integers. An irrational number is a number that cannot be expressed as the ratio of two integers.

$\sqrt{16}$ is a rational number because $\sqrt{16}$ can be expressed as $\frac{4}{1}$, which is a ratio of two integers.

$\frac{4}{7}$ is a rational number because it is already expressed as a ratio of two integers.

$\frac{4}{1} \times \frac{4}{7} = \frac{16}{7}$, and $\frac{16}{7}$ is a ratio of two integers.

The product of any two rational numbers will always be a rational number.

PTS: 2 NAT: N.RN.B.3 TOP: Operations with Radicals
KEY: classify

RADICALS
Graphing Root Functions

Common Core Standard	**Next Generation Standard**
F-IF.7b Graph square root, ~~cube root~~, and piecewise-defined functions, including step functions and absolute value functions.	**AI-F.IF.7b** Graph square root, and piecewise-defined functions, including step functions and absolute value functions and **show key features.** **Note: Algebra I key features include the following: intercepts, zeros; intervals where the function is increasing, decreasing, positive, or negative; maxima, minima; and symmetries.**

LEARNING OBJECTIVES

Students will be able to:

1) Graph functions involving square roots.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

square root cube root nth root

BIG IDEAS

NOTE: All of the functions in this lesson require special consideration for the domain of the independent variable (the x-axis).

ROOT FUNCTIONS

Root functions are associated with equations involving square roots, cube roots, or nth roots. The easiest way to graph a root function is to use the three views of a function that are associated with a graphing calculator.

STEP 1. Input the root function in the y-editor of the calculator.

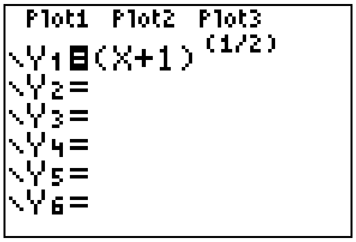
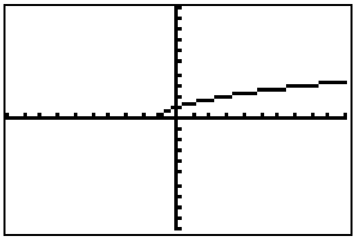
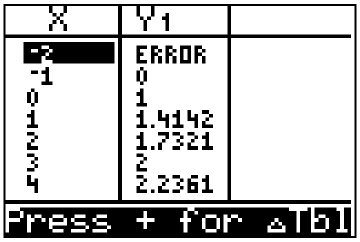
(Note: The use of rational exponents is recommended, i.e.

$\sqrt{x} = x^{\frac{1}{2}}$ $\sqrt[3]{x} = x^{\frac{1}{3}}$ $\sqrt[4]{x} = x^{\frac{1}{4}}$

STEP 2. Look at the graph of the function.

STEP 3. Use the table of values to transfer coordinate pairs to graph paper.

Example: Graph the root function $f(x) = \sqrt{x+1}$

STEP 1 Input the function rule in the y-editor of your graphing calculator	STEP 2. Look at the graph view of the function.	STEP 3. Select coordinate pairs from the table view to create your graph.
 Plot1 Plot2 Plot3 \Y1=(X+1)^(1/2) \Y2= \Y3= \Y4= \Y5= \Y6=		 X Y1 -2 ERROR -1 0 0 1 1 1.4142 2 1.7321 3 2 4 2.2361 Press + for ΔTbl

DEVELOPING ESSENTIAL SKILLS

Use technology to graph the following the following functions:

$$y = \sqrt{x}$$

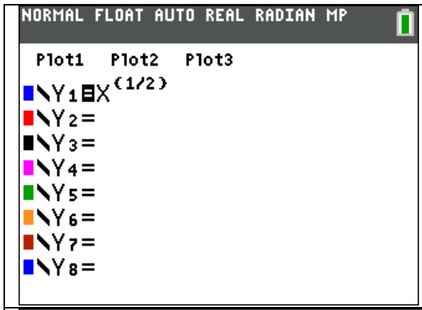
$$y = -\sqrt{x}$$

$$y = \sqrt{x+3}^{(1/2)}$$

$$y = x^{(1/2)} + 3$$

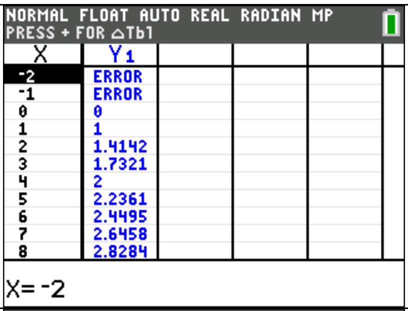
$$y = \sqrt[3]{x}$$

ANSWERS


NORMAL FLOAT AUTO REAL RADIAN MP

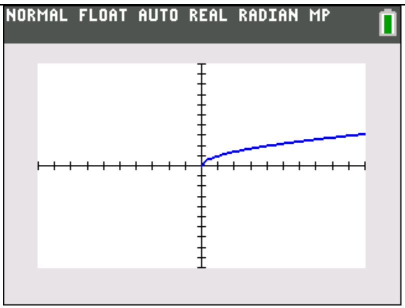
Plot1 Plot2 Plot3

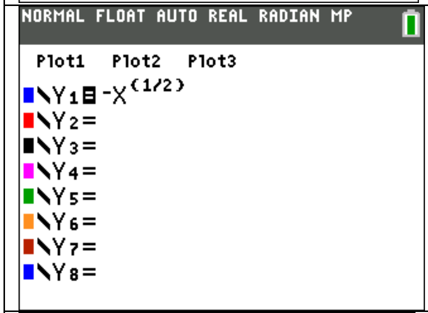
$Y_1 = X^{(1/2)}$
$Y_2 =$
$Y_3 =$
$Y_4 =$
$Y_5 =$
$Y_6 =$
$Y_7 =$
$Y_8 =$


NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1
-2	ERROR
-1	ERROR
0	0
1	1
2	1.4142
3	1.7321
4	2
5	2.2361
6	2.4495
7	2.6458
8	2.8284

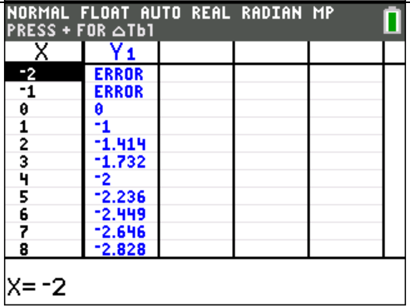
X= -2




NORMAL FLOAT AUTO REAL RADIAN MP

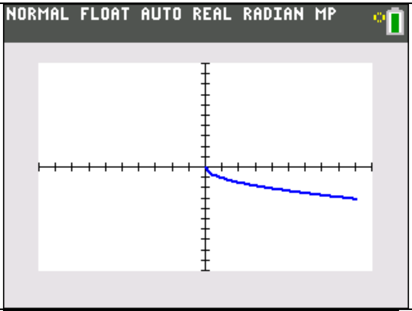
Plot1 Plot2 Plot3

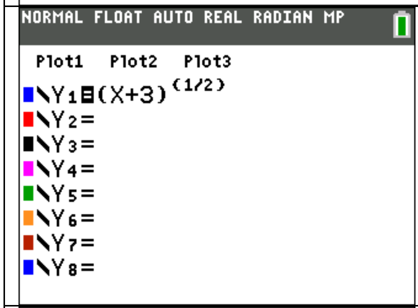
$Y_1 = -X^{(1/2)}$
$Y_2 =$
$Y_3 =$
$Y_4 =$
$Y_5 =$
$Y_6 =$
$Y_7 =$
$Y_8 =$


NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1
-2	ERROR
-1	ERROR
0	0
1	-1
2	-1.414
3	-1.732
4	-2
5	-2.236
6	-2.449
7	-2.646
8	-2.828

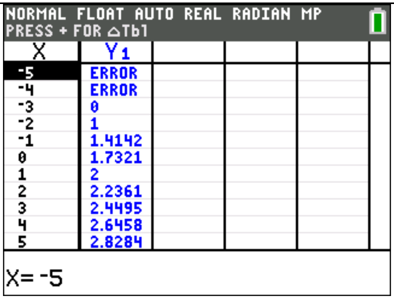
X= -2




NORMAL FLOAT AUTO REAL RADIAN MP

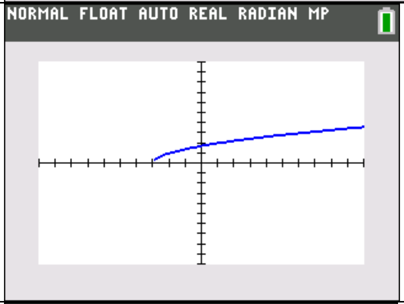
Plot1 Plot2 Plot3

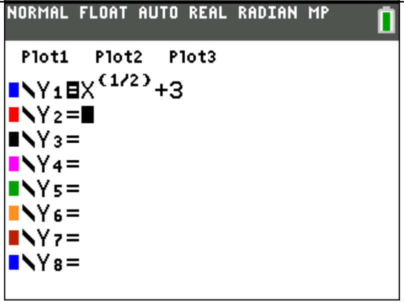
$Y_1 = (X+3)^{(1/2)}$
$Y_2 =$
$Y_3 =$
$Y_4 =$
$Y_5 =$
$Y_6 =$
$Y_7 =$
$Y_8 =$


NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1
-5	ERROR
-4	ERROR
-3	0
-2	1
-1	1.4142
0	1.7321
1	2
2	2.2361
3	2.4495
4	2.6458
5	2.8284

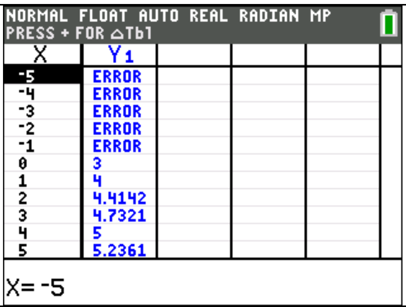
X= -5




NORMAL FLOAT AUTO REAL RADIAN MP

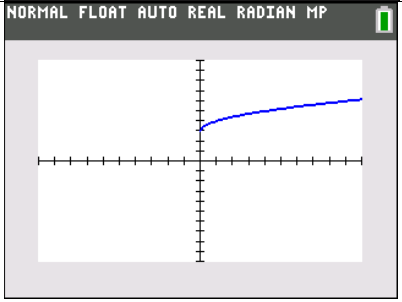
Plot1 Plot2 Plot3

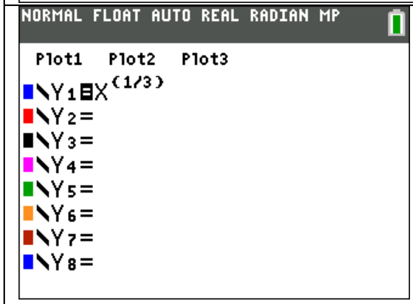
$Y_1 = X^{(1/2)}+3$
$Y_2 =$
$Y_3 =$
$Y_4 =$
$Y_5 =$
$Y_6 =$
$Y_7 =$
$Y_8 =$


NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1
-5	ERROR
-4	ERROR
-3	ERROR
-2	ERROR
-1	ERROR
0	3
1	4
2	4.4142
3	4.7321
4	5
5	5.2361

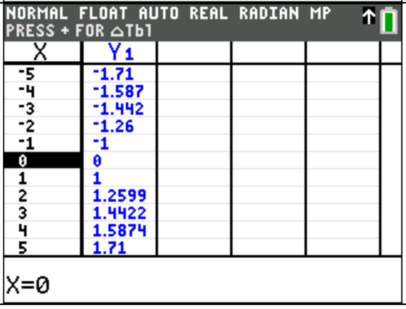
X= -5




NORMAL FLOAT AUTO REAL RADIAN MP

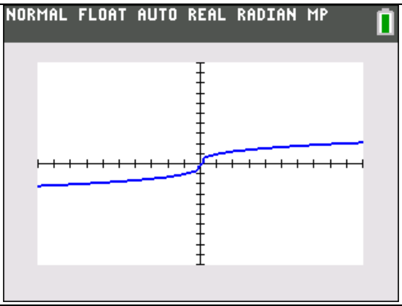
Plot1 Plot2 Plot3

$Y_1 = X^{(1/3)}$
$Y_2 =$
$Y_3 =$
$Y_4 =$
$Y_5 =$
$Y_6 =$
$Y_7 =$
$Y_8 =$


NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1
-5	-1.71
-4	-1.587
-3	-1.442
-2	-1.26
-1	-1
0	0
1	1
2	1.2599
3	1.4422
4	1.5874
5	1.71

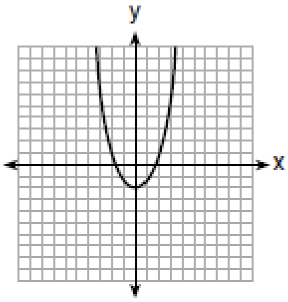
X=0



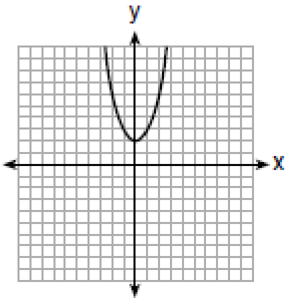
F.IF.C.7: Graphing Root Functions

394) Which graph represents $y = \sqrt{x - 2}$?

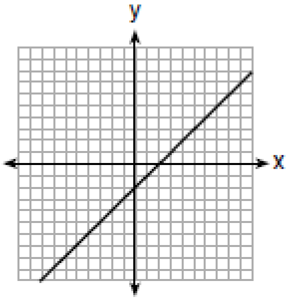
1)



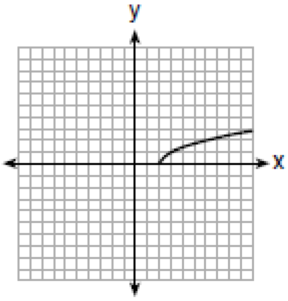
3)



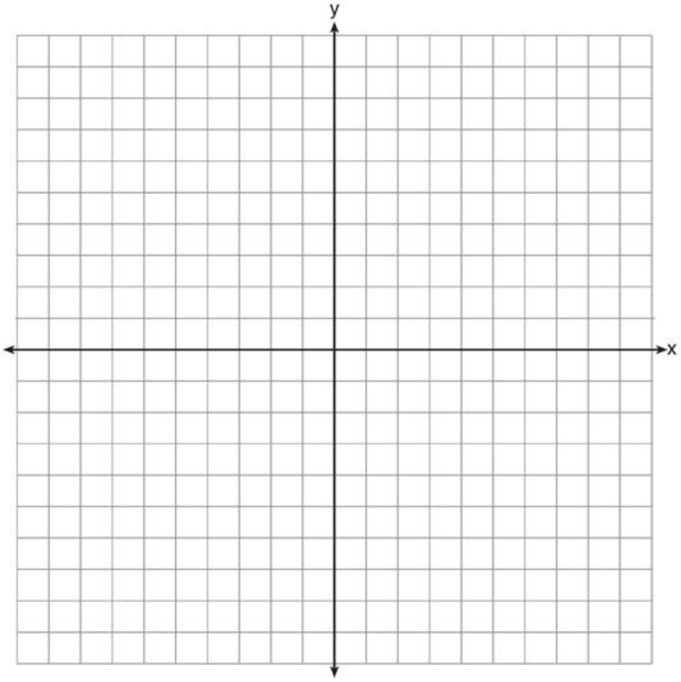
2)



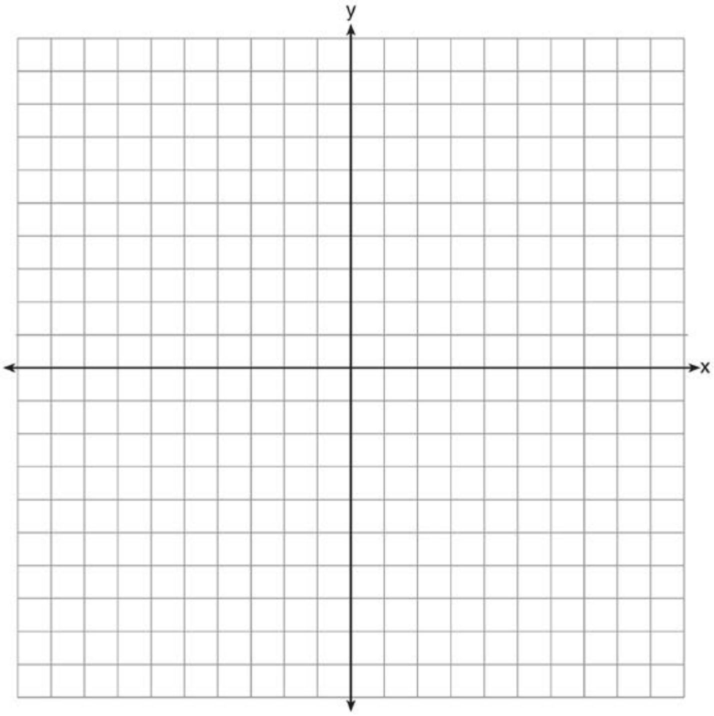
4)



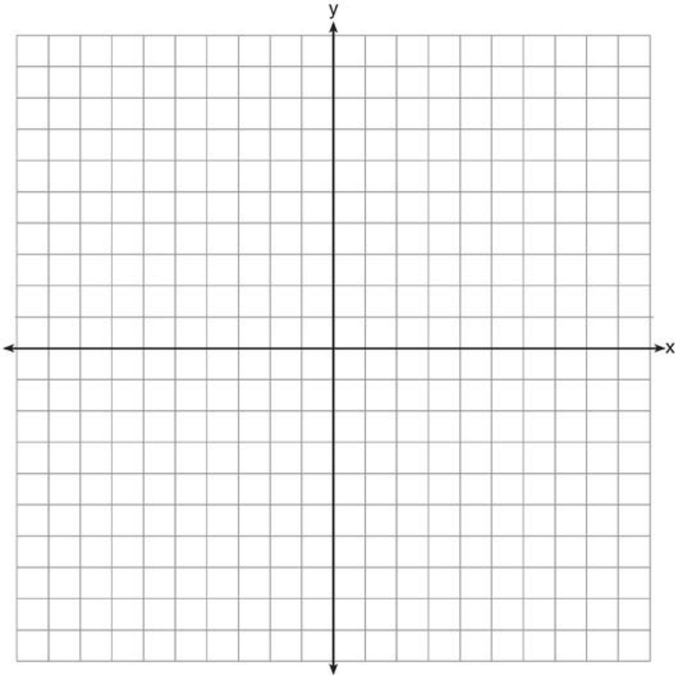
395) On the set of axes below, graph the function represented by $y = \sqrt[3]{x-2}$ for the domain $-6 \le x \le 10$.



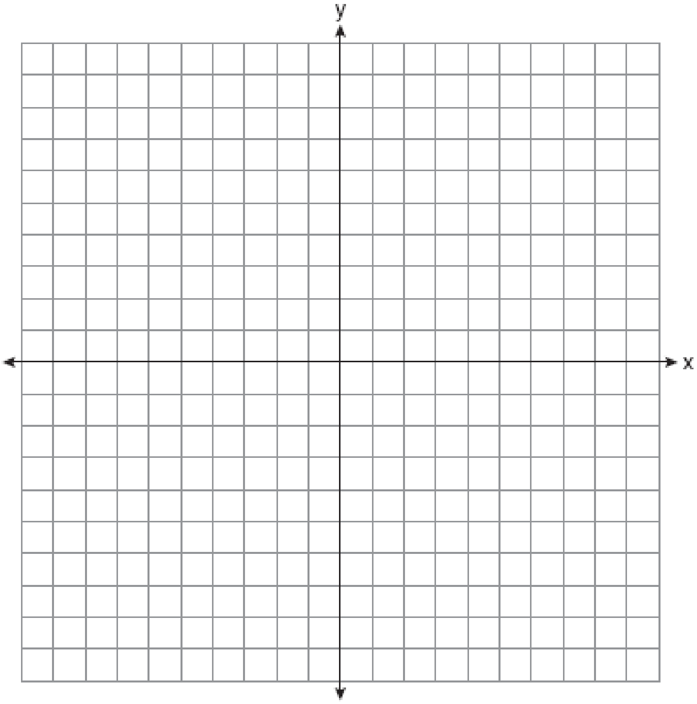
396) Draw the graph of $y = \sqrt{x} - 1$ on the set of axes below.



397) Graph the function $y = -\sqrt{x + 3}$ on the set of axes below.



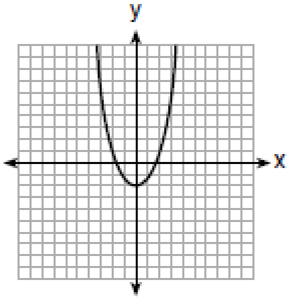
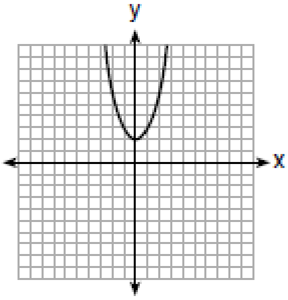
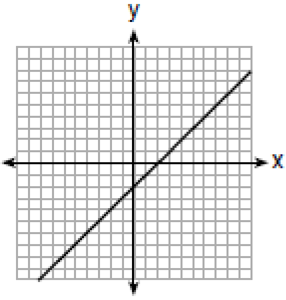
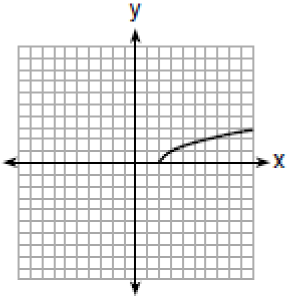
398) Graph $f(x) = \sqrt{x + 2}$ over the domain $-2 \leq x \leq 7$.



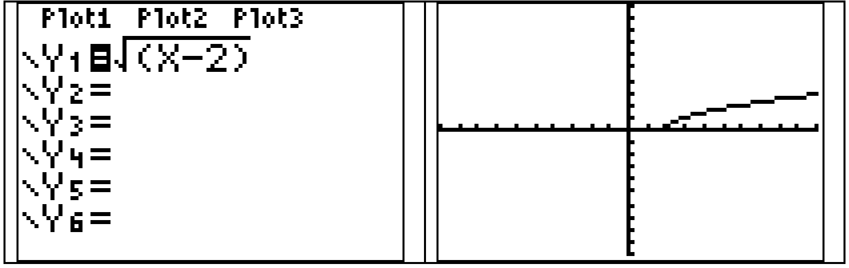
SOLUTIONS

394) ANS: 4

$y = \sqrt{x-2}$ is a root function, so its graph must look like a root function.

 a) This is a quadratic function.	 c) This is a quadratic function.
 b) This is a linear function.	 d) By the process of elimination, this is the only root function.

You can also solve this problem by inputting the equation $y = \sqrt{x-2}$ into a graphing calcualtor and looking at the graph, as fopllows:

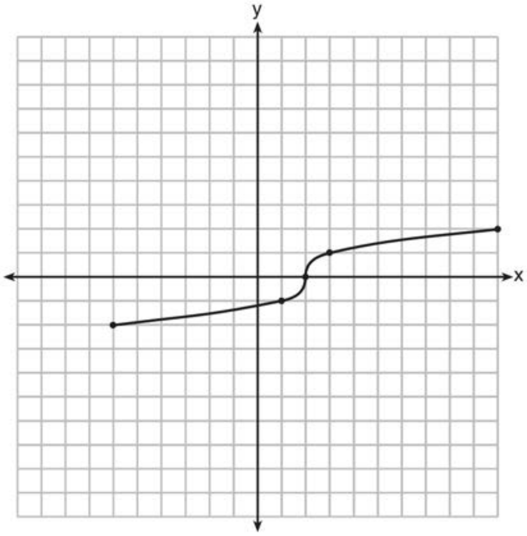


```
 Plot1  Plot2  Plot3
\Y1 =√(X-2)
\Y2 =
\Y3 =
\Y4 =
\Y5 =
\Y6 =
```

PTS: 2 NAT: F.IF.C.7 TOP: Graphing Root Functions

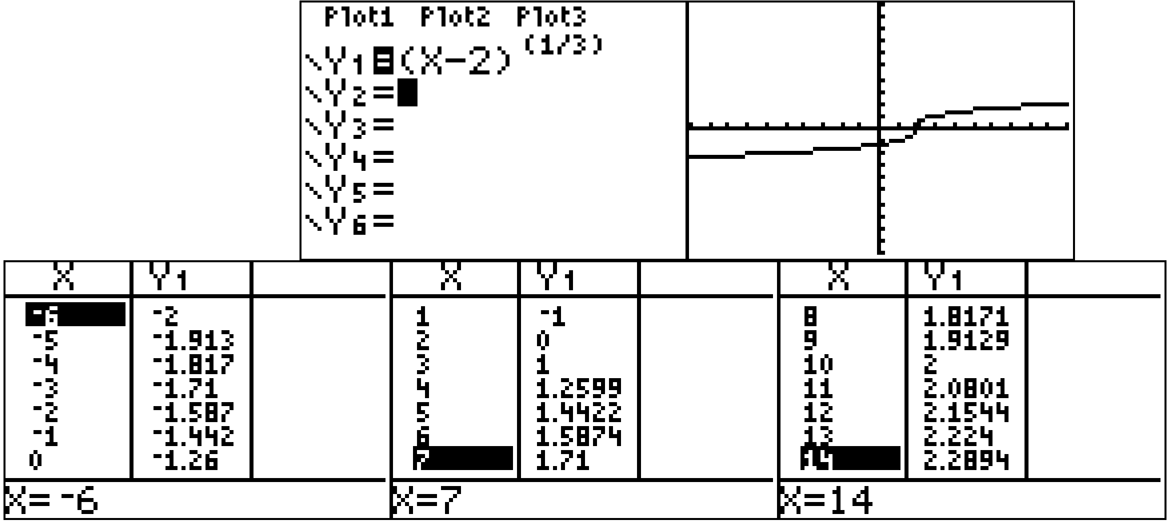
KEY: bimodalgraph

395) ANS:



Strategy: Input the function in a graphing calculator, then use the graph and table views to construct the graph on paper. Limit the domain of the graph to $-6 \le x \le 10$.

STEP 1: Use exponential notation to input the function into the graphing calculator, where $\sqrt[3]{x-2} = (x-2)^{(1/3)}$. Then use the table and graph views to reproduce the graph on paper.



```
 Plot1  Plot2  Plot3
\Y1=(X-2)^(1/3)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

X	Y1	
-6	-2	
-5	-1.913	
-4	-1.817	
-3	-1.71	
-2	-1.587	
-1	-1.442	
0	-1.26	

X= -6

X	Y1	
1	-1	
2	0	
3	1	
4	1.2599	
5	1.4422	
6	1.5874	
7	1.71	

X=7

X	Y1	
8	1.8171	
9	1.9129	
10	2	
11	2.0801	
12	2.1544	
13	2.224	
14	2.2894	

X=14

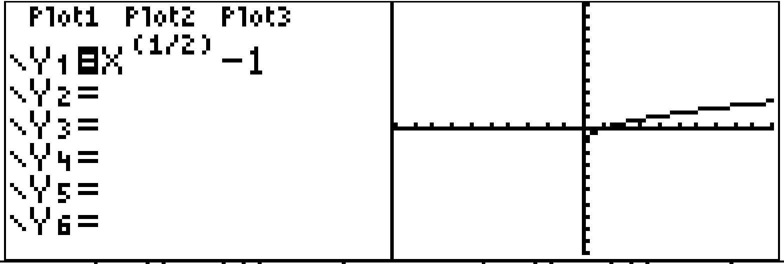
STEP 2: Limit the domain of the function to $-6 \le x \le 10$. Used closed dots to show the ends of the function at coordinates (-6, -2) and for (10, 2).

PTS: 2 NAT: F.IF.C.7 TOP: Graphing Root Functions

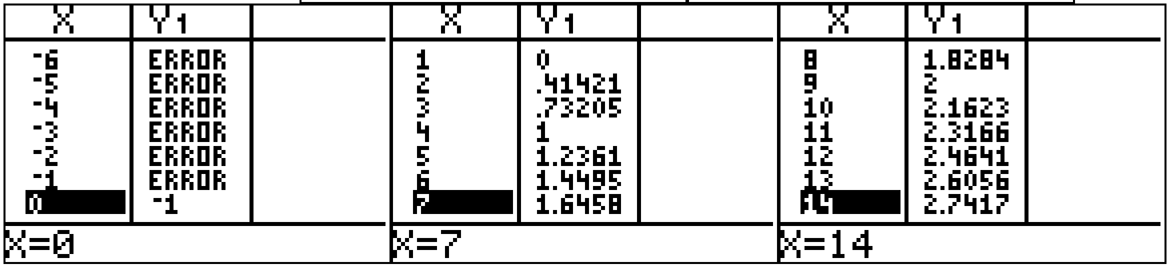
396) ANS:

Strategy: Input the function in a graphing calculator, then use the graph and table views to construct the graph on paper.

STEP 1: Use exponential notation to input the function into the graphing calculator, where $\sqrt{x}-1=x^{(1/2)}-1$. Then use the table and graph views to reproduce the graph on paper.



```
 Plot1  Plot2  Plot3
\Y1=X^(1/2)-1
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```



X	Y1
-6	ERROR
-5	ERROR
-4	ERROR
-3	ERROR
-2	ERROR
-1	ERROR
0	-1

X=0

X	Y1
1	0
2	.41421
3	.73205
4	1
5	1.2361
6	1.4495
7	1.6458

X=7

X	Y1
8	1.8284
9	2
10	2.1623
11	2.3166
12	2.4641
13	2.6056
14	2.7417

X=14

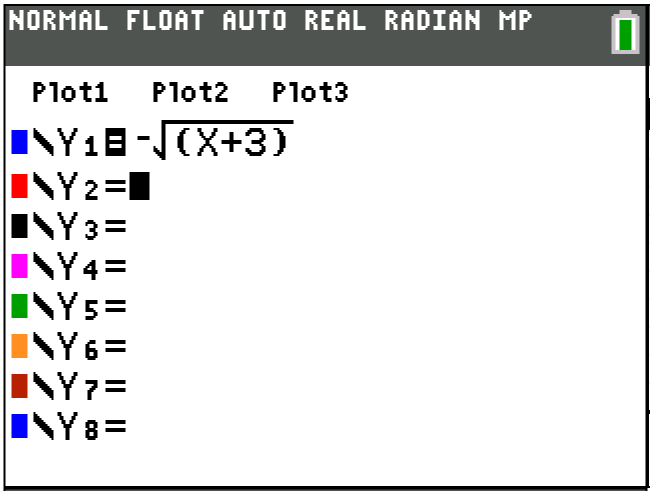
Note: Do nopt plot coordinates with errors. Focus on plotting coordinates with integer values and estimate the graph between the points with integer values when drawing the graph.

STEP 2: Limit the domain of the function to $-6 \le x \le 10$. Used closed dots to show the ends of the function at coordinates (-6, -2) and for (10, 2).

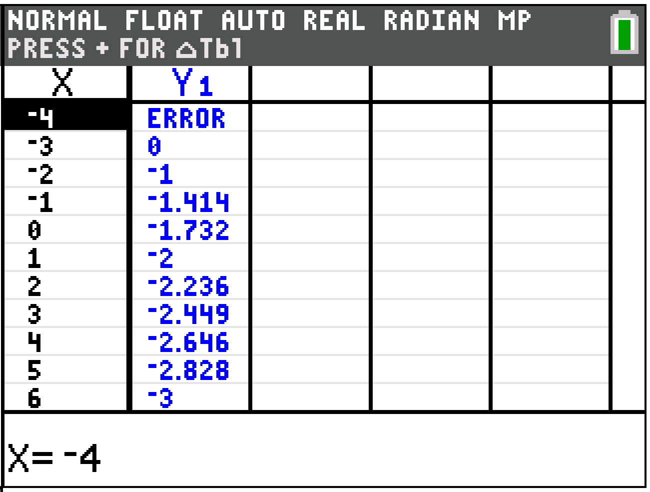
PTS: 2 NAT: F.IF.C.7 TOP: Graphing Root Functions

397) ANS:

Strategy: Input the equation in a graphing calculator. Plot the coordinates with integer values. Complete the graph.



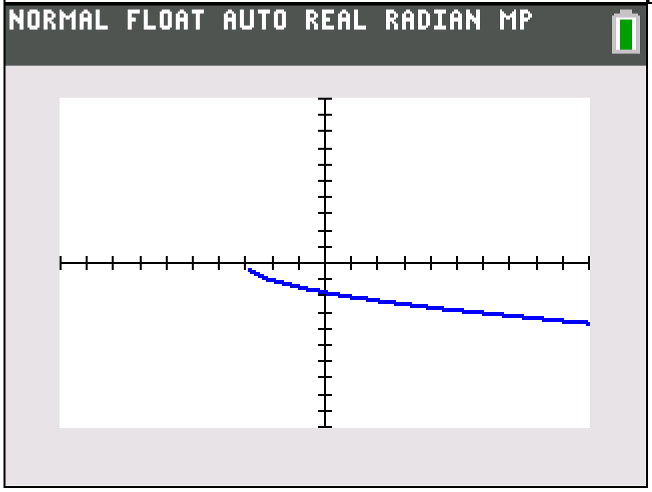
```
NORMAL FLOAT AUTO REAL RADIAN MP

 Plot1  Plot2  Plot3
\Y1=-√(X+3)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
\Y8=
```



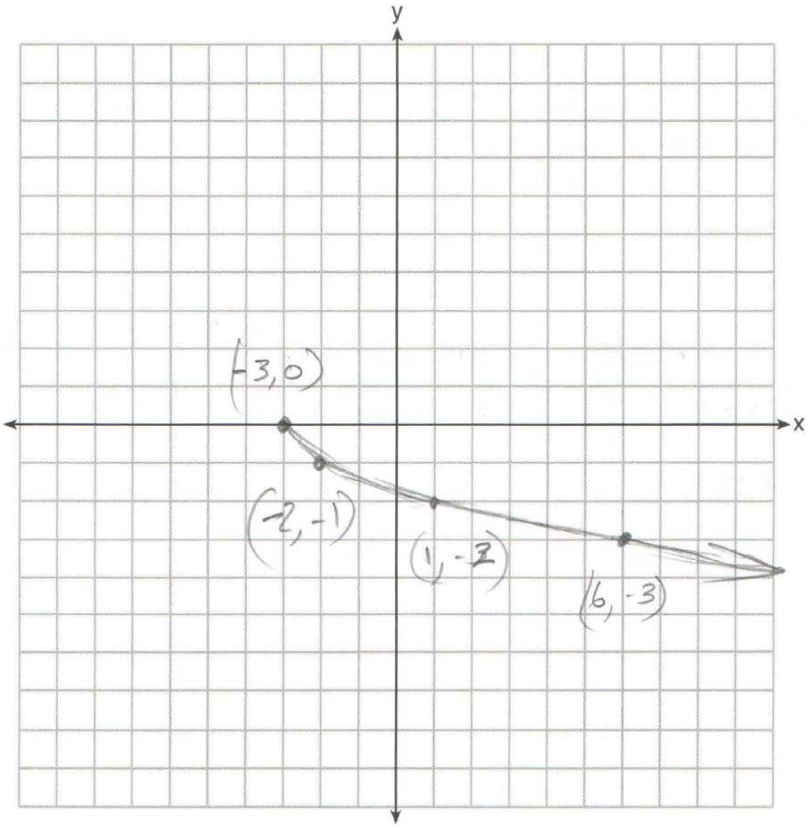
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1
-4	ERROR
-3	0
-2	-1
-1	-1.414
0	-1.732
1	-2
2	-2.236
3	-2.449
4	-2.646
5	-2.828
6	-3

X= -4

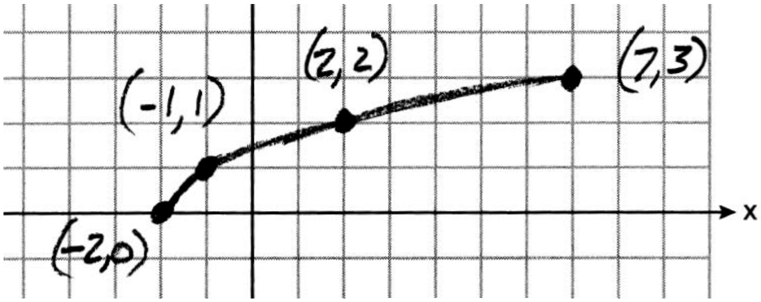


25 Graph the function $y = -\sqrt{x+3}$ on the set of axes below.

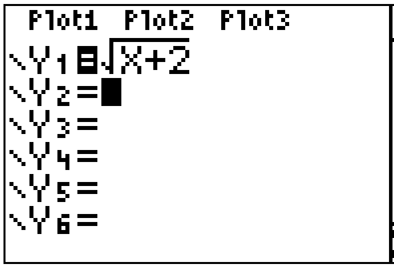


PTS: 2 NAT: F.IF.C.7

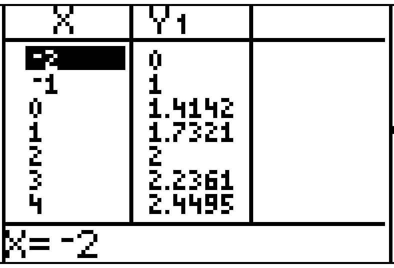
398) ANS:



Strategy: Input the function $f(x) = \sqrt{x+2}$ in a graphing calculator and use the table of values and graph views to plot the graph for integer values.

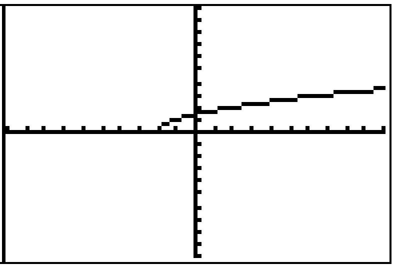


```
 Plot1  Plot2  Plot3
\Y1=√(X+2)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```



X	Y1
-2	0
-1	1
0	1.4142
1	1.7321
2	2
3	2.2361
4	2.4495

X= -2



PTS: 2 NAT: F.IF.C.7 TOP: Graphing Root Functions

FUNCTIONS

Defining Functions

Common Core Standard	**Next Generation Standard**
F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.	**AI-F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x).* **Note: Domain and range can be expressed using inequalities, set builder notation, verbal description, and interval notations for functions of subsets of real numbers to the real numbers.**

LEARNING OBJECTIVES

Students will be able to:

1) Define and identify functions.

Overview of Lesson

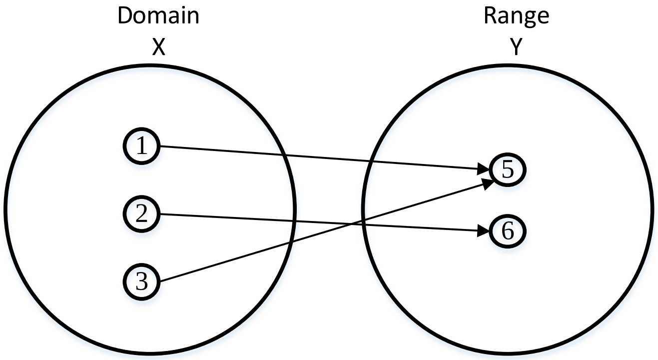
Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

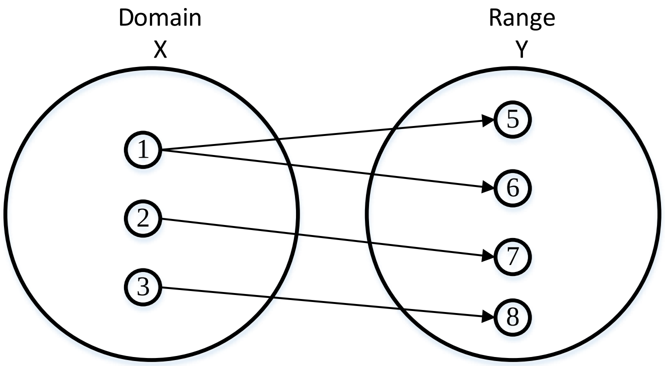
VOCABULARY

Function: A rule that assigns to each number x in the **function's** *domain* (x-axis) a unique number $f(x)$ in the function's *range (y-axis)*. A function takes the input value of an independent variable and pairs it with *one and only one* output value of a dependent variable.

BIG IDEAS

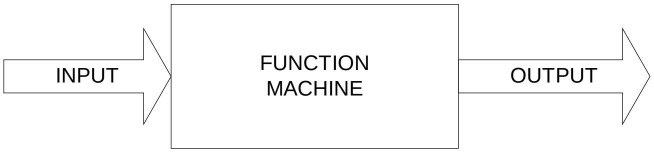
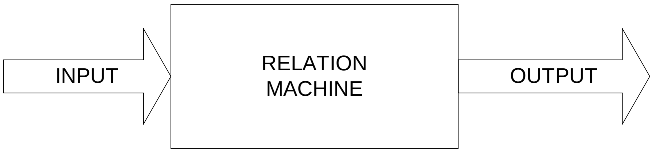
Function





Not a Function

Expressed as ordered Pairs:
Function: (1,5) (2,6) (3,5)
Not a Function: (1,5) (2,7) (3,8) (1,6)

	**Function**: A **function** is a relation that assigns exactly one value of the dependent variable to each value of the independent variable. A **function** is always a relation. Example: y=2x
	**Relation**: A relation may produce more than one output for a given input. A relation may or may not be a function. Example: $y^2 = x$ $y = \sqrt{x}$ This is not a function, because when x=16, there is more than one y-value. $\sqrt{16} = \pm 4$.

A function can be represented four ways. These are:

- a context (verbal description)
- a function rule (equation)
- a table of values
- a graph.

Function Rules show the relationship between dependent and independent variables in the form of an equation with two variables.

- The *independent* variable is the *input* of the function and is typically denoted by the x-variable.
- The *dependent* variable is the *output* of the function and is typically denoted by the y-variable.

All linear equations in the form $y = mx + b$ are functions except vertical lines.
2nd degree and higher equations may or may not be functions.

Tables of Values show the relationship between dependent and independent variables in the form of a table with columns and rows:

- The *independent variable is the input* of the function and is typically shown in the left column of a vertical table or the top row of a horizontal table.
- The *dependent variable is the output* of the function and is typically shown in the right column of a vertical table or the bottom row of a horizontal table.

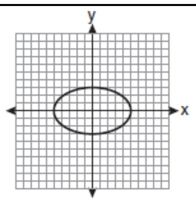
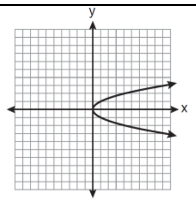
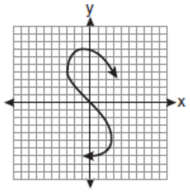
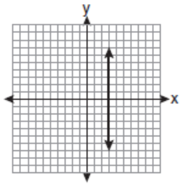
Function			Not A Function	
x	y		x	y
1	5		1	5
2	6		2	6
3	7		3	7
4	8		4	8
5	9		2	9

Graphs show the relationship between dependent and independent variables in the form of line or curve on a coordinate plane:

- The value of independent variable is input of the function and is typically shown on the x-axis (horizontal axis) of the coordinate plane.
- The value of the dependent variable is the output of the function and is typically shown on the y-axis (vertical axis) of the coordinate plane.

Vertical Line Test: If a vertical line passes through a graph of an equation more than once, the graph is *not* a graph of a function.

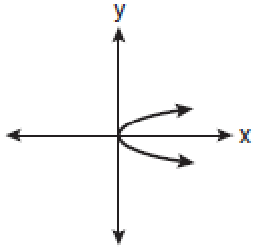
If you can draw a vertical line through any value of x in a relation, and the vertical line intersects the graph in two or more places, the relation is not a function.

 Circles and Ellipses …are **not functions**.	 Parabola-like graphs that open to the side …are **not functions**.	 S-Curves …are **not functions**	 Vertical lines …are **not functions**.

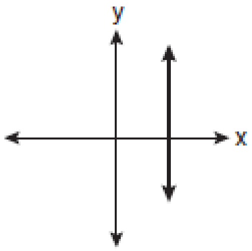
DEVELOPING ESSENTIAL SKILLS

1. Which graph represents a function?

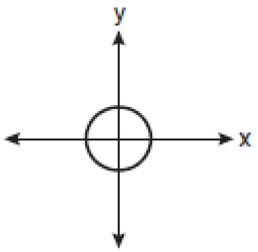
a.



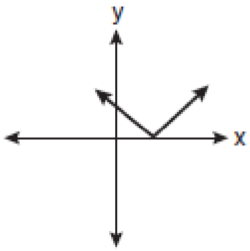
b.



c.



d.



2. Which relation is *not* a function?

a. $\{(1,5),(2,6),(3,6),(4,7)\}$

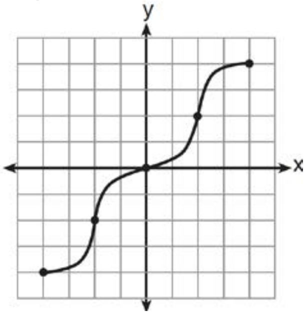
b. $\{(4,7),(2,1),(-3,6),(3,4)\}$

c. $\{(-1,6),(1,3),(2,5),(1,7)\}$

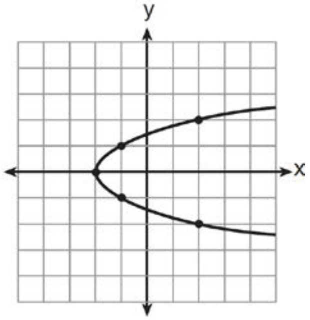
d. $\{(-1,2),(0,5),(5,0),(2,-1)\}$

3. Which graph represents a function?

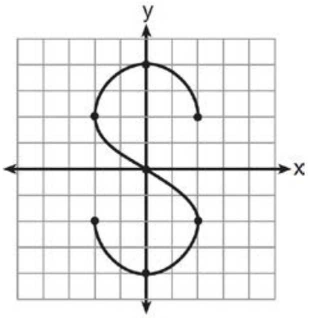
a.



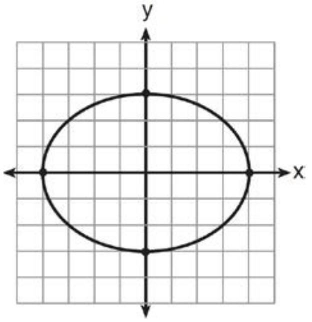
b.



c.



d.



4. Which relation is *not* a function?

a. $\{(2,4),(1,2),(0,0),(-1,2),(-2,4)\}$

b. $\{(2,4),(1,1),(0,0),(-1,1),(-2,4)\}$

c. $\{(2,2),(1,1),(0,0),(-1,1),(-2,2)\}$

d. $\{(2,2),(1,1),(0,0),(1,-1),(2,-2)\}$

5. Which relation is a function?

a. $\{(2,1),(3,1),(4,1),(5,1)\}$

b. $\{(1,2),(1,3),(1,4),(1,5)\}$

c. $\{(2,3),(3,2),(4,2),(2,4)\}$

d. $\{(1,6),(2,8),(3,9),(3,12)\}$

6. Which set is a function?

a. $\{(3,4),(3,5),(3,6),(3,7)\}$

b. $\{(1,2),(3,4),(4,3),(2,1)\}$

c. $\{(6,7),(7,8),(8,9),(6,5)\}$

d. $\{(0,2),(3,4),(0,8),(5,6)\}$

ANSWERS

1. ANS: D
2. ANS: C
3. ANS: A
4. ANS: D
5. ANS: A
6. ANS: B

REGENTS EXAM QUESTIONS (through June 2018)

F.IF.A.1: Defining Functions

399) Which table represents a function?

1)

x	2	4	2	4
f(x)	3	5	7	9

2)

x	0	−1	0	1
f(x)	0	1	−1	0

3)

x	3	5	7	9
f(x)	2	4	2	4

4)

x	0	1	−1	0
f(x)	0	−1	0	1

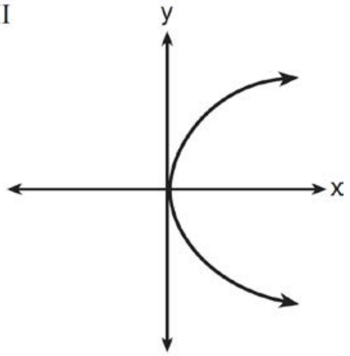
400) The function f has a domain of $\{1,3,5,7\}$ and a range of $\{2,4,6\}$. Could f be represented by $\{(1,2),(3,4),(5,6),(7,2)\}$? Justify your answer.

401) Which representations are functions?

I

x	y
2	6
3	–12
4	7
5	5
2	–6

III

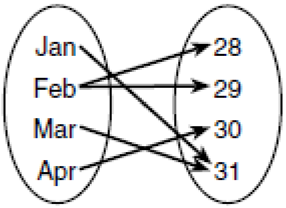


II $\{(1,1),(2,1),(3,2),(4,3),(5,5),(6,8),(7,13)\}$

IV $y=2x+1$

1) I and II
2) II and IV
3) III, only
4) IV, only

402) A mapping is shown in the diagram below.



This mapping is

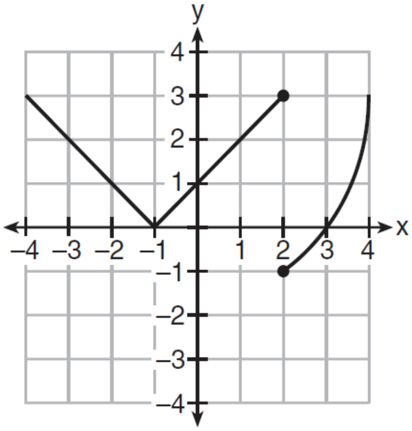
1) a function, because Feb has two outputs, 28 and 29
2) a function, because two inputs, Jan and Mar, result in the output 31
3) not a function, because Feb has two outputs, 28 and 29
4) not a function, because two inputs, Jan and Mar, result in the output 31

403) A function is shown in the table below.

x	f(x)
–4	2
–1	–4
0	–2
3	16

If included in the table, which ordered pair, $(-4,1)$ or $(1,-4)$, would result in a relation that is no longer a function? Explain your answer.

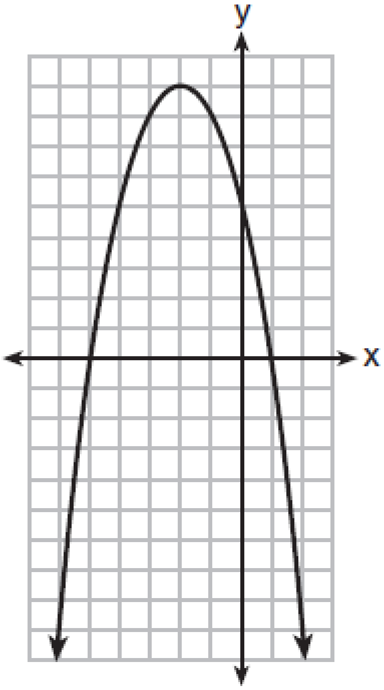
404) Marcel claims that the graph below represents a function.



State whether Marcel is correct. Justify your answer.

405) Nora says that the graph of a circle is a function because she can trace the whole graph without picking up her pencil. Mia says that a circle graph is *not* a function because multiple values of x map to the same y-value. Determine if either one is correct, and justify your answer completely.

406) A relation is graphed on the set of axes below.



Based on this graph, the relation is

1)	a function because it passes the horizontal line test	3)	not a function because it fails the horizontal line test
2)	a function because it passes the vertical line test	4)	not a function because it fails the vertical line test

407) A function is defined as $\{(0,1),(2,3),(5,8),(7,2)\}$. Isaac is asked to create one more ordered pair for the function. Which ordered pair can he add to the set to keep it a function?

1)	$(0,2)$	3)	$(7,0)$
2)	$(5,3)$	4)	$(1,3)$

SOLUTIONS

399) ANS: 3

Strategy: Eliminate wrong answers. A function is a relation that assigns exactly one value of the dependent variable to each value of the independent variable.

Answer choice a ***is not*** a function because there are two values of y when $x = 2$.
Answer choice b ***is not*** a function because there are two values of y when $x = 0$.
Answer choice c ***is*** a function because only one value of y is paired with each value of x.
Answer choice d ***is not*** a function because there are two values of y when $x = 0$.

.

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions

400) ANS:
Yes, because every element of the domain is assigned one unique element in the range.

Strategy: Determine if any value of *x* has more that one associated value of *y*. A function has one and only one value of *y* for every value of *x*.

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions

401) ANS: 2

Strategy: Determine if each of the for views are functions, then select from the answer choices. A function is a relation that assigns exactly one value of the dependent variable to each value of the independent variable.

I ***is not*** a function because when $x = 2$, y can equal both 6 and -6.
II is a function because there are no values of x that have more than one value of y.
III ***is not*** a function because it fails the vertical line test, which means there are values of x that have more than one value of y.
IV ***is*** a function because it is a straight line that is not vertical.

Answer choice b is the correct answer.

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions

402) ANS: 3

A function has one and only one output for each input. The diagram shows that February maps to two different output numbers, so the diagram cannot represent a function.

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions
KEY: ordered pairs

403) ANS:

$(-4, 1)$, because then every element of the domain is not assigned one unique element in the range.

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions

404) ANS:

Marcel is not correct, because the relation does not pass the vertical line test. If you draw the vertical line $x = 2$, there will be more than one value of y. A function can have one and only one value of y for every value of x.

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions
KEY: graphs

405) ANS:

Neither is correct.

Nora’s reason is wrong since a circle is not a function because it fails the vertical line test.

Although Mia correctly states that a circle is not a function, her reasoning is wrong. She confuses the variables in the definition of a function, which states that a function has one and only one value of y for each value of x. It is okay for a y to be associated with multiple values of x in a function. It is not okay for an x to be associated with multiple values of y.

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions
KEY: graphs

406) ANS: 2

A function has one and only one value of y for each value of x. A graph represents a function if there are no vertical lines that intersect the graph at more than one point.

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions
KEY: graphs

407) ANS: 4

Strategy. The the definition of a function to eliminate wrong answers. (i.e. for each value of x in a function, there can be one and only one value of y).

Choice 1: $(0, 2)$ Wrong, because 0 is already paired with $y = 1$.
Choice 2: $(5, 3)$ Wrong, because 5 is already paired with $y = 8$.
Choice 3: $(7, 0)$ Wrong, because 7 is already paired with $y = 2$.
Choice 4: $(1, 3)$ Correct, because 1 is not paired with any other value of y.

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions
KEY: ordered pairs

M – Functions, Lesson 2, Function Notation, Evaluating Functions (r. 2018)

FUNCTIONS
Function Notation, Evaluating Functions

Common Core Standard	Next Generation Standard
F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	**AI-F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

LEARNING OBJECTIVES

Students will be able to:

1) use function notation,
2) evaluate functions for specific input values, and
3) use function notation in context.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

function notation
dependent variable
independent variable
composition of functions

BIG IDEAS

Function Notation

In function notation, $f(x)$ is used instead of the letter y to denote the dependent variable. It is read as "f of x" or "the value f(x) is a function of x," which is the independent variable. Other letters may also be used.

There are four primary advantages to using function notation:

1) The use of function notation indicates that the relationship is a function.
2) The use of function notation explicitly defines which variable is the dependent variable and which variable is the independent variable.
3) The use of function notation simplifies evaluation of the dependent variable for specific values of the independent variable.

Example: If $f(x) = 2x,\ \mathit{then}$

$$f(2) = 2(2) = 4,\ \mathit{and}$$

$$f(3) = 2(3) = 6,\ \mathit{and}$$

$$f(4) = 2(4) = 8,\ \mathit{etc.}$$

4) The use of function notation allows greater flexibility and specificity in naming variables.

   Example #1: If total cost is a function of the number of pencils bought, a function rule might begin with *C(p)=*.

   Example #2: If miles driven at a constant speed is a function of hours driving, a function rule might begin with *M(h)=*.

When graphing using function notation, the label of the y-axis is changed to reflect the function notation being used.

Evaluating Functions

To evaluate a function for a specific input, simply replace the dependent variable with the desired input throughout the function.

Example: Given the function $f(x) = 3x^2 + 4$, find the value of $f(5)$ as follows:

$$f(x) = 3x^2 + 4$$

$$f(5) = 3(5)^2 + 4$$

$$f(5) = 3(25) + 4$$

$$f(5) = 75 + 4$$

$$f(5) = 79$$

Composition of Functions

Some functions are defined using other functions. Such functions are called compositions of functions. For example, if $f(x) = 2x$ and $g(x) = 3f(x)$, then the function $g(x)$ is defined in terms of the function $f(x)$. Since we know that $f(x) = 2x$, we can use substitution to write $g(x) = 3(2x)$.

DEVELOPING ESSENTIAL SKILLS

Evaluate the following functions for the given input values:

$f(x) = 2x + 3$	$f(x) = 3x - 1$
$f(1) =$	$f(1) =$
$f(2) =$	$f(2) =$
$f(3) =$	$f(3) =$
$f(4) =$	$f(4) =$
$f(5) =$	$f(5) =$

$f(x)=x^2+2x+3$ $f(1)=$ $f(2)=$ $f(3)=$ $f(4)=$ $f(5)=$	$f(x)=2x+3$ $g(x)=f(x)^2$ $g(1)=$ $g(2)=$ $g(3)=$ $g(4)=$ $g(5)=$

ANSWERS

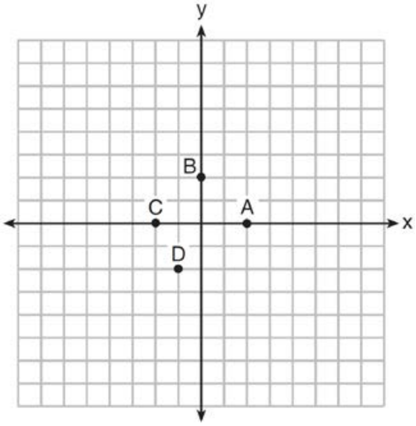
$f(x)=2x+3$ $f(1)=5$ $f(2)=7$ $f(3)=9$ $f(4)=11$ $f(5)=13$	$f(x)=3x-1$ $f(1)=2$ $f(2)=5$ $f(3)=8$ $f(4)=11$ $f(5)=14$
$f(x)=x^2+2x+3$ $f(1)=6$ $f(2)=11$ $f(3)=18$ $f(4)=27$ $f(5)=28$	$f(x)=2x+3$ $g(x)=f(x)^2$ $g(1)=25$ $g(2)=49$ $g(3)=81$ $g(4)=121$ $g(5)=169$

REGENTS EXAM QUESTIONS (through June 2018)

F.IF.A.2: Function Notation, Evaluating Functions

408) Given that $f(x)=2x+1$, find $g(x)$ if $g(x)=2[f(x)]^2-1$.

409) The graph of $y=f(x)$ is shown below.



Which point could be used to find $f(2)$?

1) *A*
2) *B*
3) *C*
4) *D*

410) The value in dollars, $v(x)$, of a certain car after *x* years is represented by the equation $v(x) = 25,000(0.86)^x$. To the *nearest dollar*, how much more is the car worth after 2 years than after 3 years?

1) 2589
2) 6510
3) 15,901
4) 18,490

411) If $f(n) = (n-1)^2 + 3n$, which statement is true?

1) $f(3) = -2$
2) $f(-2) = 3$
3) $f(-2) = -15$
4) $f(-15) = -2$

412) The equation to determine the weekly earnings of an employee at The Hamburger Shack is given by $w(x)$, where *x* is the number of hours worked.

$$w(x) = \begin{cases} 10x, & 0 \le x \le 40 \\ 15(x-40)+400, & x > 40 \end{cases}$$

Determine the difference in salary, *in dollars*, for an employee who works 52 hours versus one who works 38 hours. Determine the number of hours an employee must work in order to earn $445. Explain how you arrived at this answer.

413) If $f(x) = \frac{\sqrt{2x+3}}{6x-5}$, then $f\left(\frac{1}{2}\right) =$

1) 1
2) -2
3) -1
4) $-\frac{13}{3}$

414) Lynn, Jude, and Anne were given the function $f(x) = -2x^2 + 32$, and they were asked to find $f(3)$. Lynn's answer was 14, Jude's answer was 4, and Anne's answer was ±4. Who is correct?

1) Lynn, only
2) Jude, only
3) Anne, only
4) Both Lynn and Jude

415) If $f(x) = \frac{1}{2}x^2 - \left(\frac{1}{4}x + 3\right)$, what is the value of $f(8)$?

1) 11
2) 17
3) 27
4) 33

416) For a recently released movie, the function $y = 119.67(0.61)^x$ models the revenue earned, y, in millions of dollars each week, x, for several weeks after its release. Based on the equation, how much more money, in millions of dollars, was earned in revenue for week 3 than for week 5?

1) 37.27
2) 27.16
3) 17.06
4) 10.11

417) If $k(x) = 2x^2 - 3\sqrt{x}$, then $k(9)$ is

1) 315
2) 307
3) 159
4) 153

SOLUTIONS

408) ANS:
Step 1. Understand this as a composition of functions problem.
Step 2. Strategy: Substitute the expression for f(x) into the equation for g(x).
Step 3. Execution of Strategy.

$$f(x) = 2x + 1 \quad \text{and} \quad g(x) = 2[f(x)]^2 - 1$$

$$g(x) = 2(2x+1)^2 - 1 \quad (\textit{answer})$$

$$g(x) = 2(4x^2 + 4x + 1) - 1 \quad (\textit{alternate answer})$$

$$g(x) = 8x^2 + 8x + 2 - 1 \quad (\textit{alternate answer})$$

$$g(x) = 8x^2 + 8x + 1 \quad (\textit{alternate answer})$$

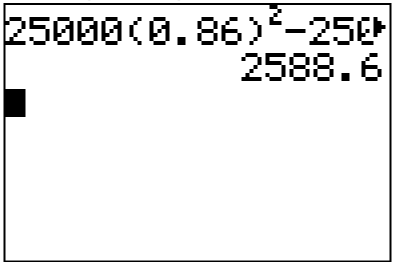
PTS: 2 NAT: F.IF.A.2 TOP: Functional Notation Evaluating Functions

409) ANS: 1
Strategy: Understand that the meaning of $f(2)$ is the value of y when $x = 2$, then eliminate wrong answers.

Choose answer choice A because represents $f(2)$ with coordinates $(2,0)$. $f(2) = 0$.
Answer choice b is wrong because if represents $f(0)$. $f(0) = 2$
Answer choice c is wrong because if represents $f(-2)$. $f(-2) = 0$
Answer choice d is wrong because if represents $f(-1)$. $f(-1) = -2$

PTS: 2 NAT: F.IF.A.2 TOP: Functional Notation Evaluating Functions

410) ANS: 1
Strategy #1
Input $25,000(0.86)^2 - 25,000(0.86)^3$ into a graphing calculator and press enter.

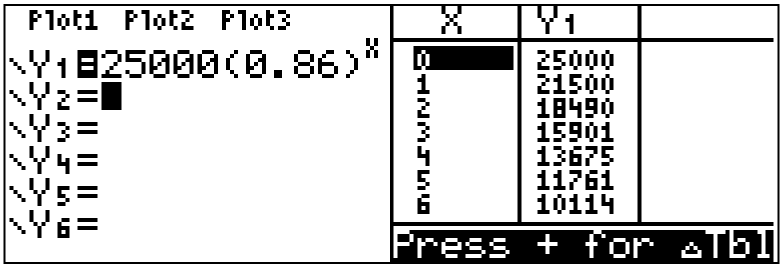


```
25000(0.86)²-250▸
           2588.6
█
```

$$25,000(0.86)^2 - 25,000(0.86)^3 = 18490 - 15901.40 = 2588.60$$

Strategy #2: Input the function rule in a graphing calculator and obtain the value of the car after 2 years and 3 years from the table of values. Then, compute the difference.

STEP 1: Input the function rule and obtain data from the table of values.



```
Plot1 Plot2 Plot3
\Y1=25000(0.86)^X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

```
X    Y1
0    25000
1    21500
2    18490
3    15901
4    13675
5    11761
6    10114
Press + for ΔTbl
```

STEP 2: Compare the value of the car after 2 years and after 3 years.
The car is worth $18,490 after 2 years.
The car is worth $15,901 after 3 years.
The difference is $18490 - 15901 = 2589$

$25,000(0.86)^2 - 25,000(0.86)^3 = 18490 - 15901.40 = 2588.60$

PTS: 2 NAT: F.IF.A.2 TOP: Functional Notation Evaluating Functions

411) ANS: 2

Strategy #1: Input $f(n) = (n-1)^2 + 3n$ into a graphing calculator and inspect the table of values.

x	*f(x)*
3	13
-2	3
-15	211

Strategy #2: Manually calculate the answer.

$$f(n) = (n-1)^2 + 3n$$
$$f(-2) = (-2-1)^2 + 3(-2)$$
$$f(-2) = (-3)^2 - 6$$
$$f(-2) = 9 - 6$$
$$f(-2) = 3$$

PTS: 2 NAT: F.IF.A.2 TOP: Functional Notation Evaluating Functions

412) ANS:

a) The difference in salary, *in dollars*, for an employee who works 52 hours versus one who works 38 hours, is $200.

b) An employee must work 43 hours in order to earn $445. See work below.

Strategy: Part a: Use the piecewise function to first determine the salaries of 1) an employee who works 52 hours, and 2) an employee who works 38 hours. Then, find the difference of the two salaries.

Working 38 Hours	Working 52 Hours

$x = 38$ $w(x) = \begin{cases} 10x, & 0 \le x \le 40 \\ 15(x-40)+400, & x > 40 \end{cases}$ $w(38) = \begin{cases} 10(38), & 0 \le x \le 40 \\ \text{not applicable}, & x > 40 \end{cases}$ $w(38) = \begin{cases} 10(38), & 0 \le x \le 40 \end{cases}$ $w(38) = 380$	$x = 52$ $w(x) = \begin{cases} 10x, & 0 \le x \le 40 \\ 15(x-40)+400, & x > 40 \end{cases}$ $w(52) = \begin{cases} \text{not applicable}, & 0 \le x \le 40 \\ 15(52-40)+400, & x > 40 \end{cases}$ $w(52) = \begin{cases} 15(52-40)+400, & x > 40 \end{cases}$ $w(52) = \begin{cases} 15(12)+400, & x > 40 \end{cases}$ $w(52) = \begin{cases} 180+400, & x > 40 \end{cases}$ $w(52) = 580$

The difference between the values of $w(38)$ and $w(52)$ is $200.

Strategy: Part b: The employee must work more than 40 hours, and compensation for hours worked in excess of 40 hours is found in the second formula and is equal to $15 per hour. The compensation worked in excess of 40 hours is $\$445 - \$400 = \$45$, so

$$\frac{45 \text{ dollars}}{15 \text{ dollars/hour}} = 3 \text{ hours}$$

The employee must work a total of 43 hours. The employee receives $400 for the first 40 hours and $45 for the 3 hours in excess of 40 hours.

PTS: 4 NAT: F.IF.A.2 TOP: Functional Notation Evaluating Functions

413) ANS: 3

Strategy: Substitute $\frac{1}{2}$ for x, and solve.

$$f(x) = \frac{\sqrt{2x+3}}{6x-5}$$

$$f\left(\frac{1}{2}\right) = \frac{\sqrt{2\left(\frac{1}{2}\right)+3}}{6\left(\frac{1}{2}\right)-5}$$

$$f\left(\frac{1}{2}\right) = \frac{\sqrt{4}}{-2}$$

$$f\left(\frac{1}{2}\right) = \frac{2}{-2}$$

$$f\left(\frac{1}{2}\right) = -1$$

PTS: 2 NAT: F.IF.A.2 TOP: Functional Notation Evaluating Functions

414) ANS: 1

$$f(x) = -2(x)^2 + 32$$
$$f(3) = -2(3)^2 + 32$$
$$f(3) = -2(9) + 32$$
$$f(3) = -18 + 32$$
$$f(3) = 14$$

PTS: 2 NAT: F.IF.A.2 TOP: Functional Notation

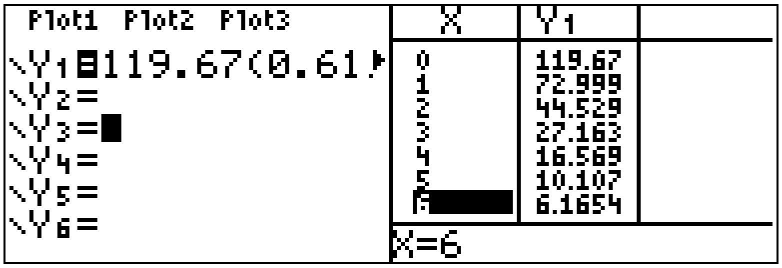
415) ANS: 3

$$f(x) = \frac{1}{2}x^2 - \left(\frac{1}{4}x + 3\right)$$
$$f(8) = \frac{1}{2}8^2 - \left(\frac{1}{4}(8) + 3\right)$$
$$f(8) = \frac{1}{2}(64) - (2 + 3)$$
$$f(8) = 32 - (5)$$
$$f(8) = 27$$

PTS: 2 NAT: F.IF.A.2 TOP: Functional Notation

416) ANS: 3

Strategy #1. Input the function rule in a graphing calculator, then use the table of values to identify the revenues earned in weeks 3 and 5, then compute the difference.



```
Plot1 Plot2 Plot3
\Y1=119.67(0.61▸
\Y2=
\Y3=█
\Y4=
\Y5=
\Y6=
```

```
X      Y1
0      119.67
1      72.999
2      44.529
3      27.163
4      16.569
5      10.107
6      6.1654
X=6
```

The table of values shows that the movie earned 27.163 million dollars in week 3.
The table of values shows that the movie earned 10.107 million dollars in week 5.
The difference is $(27.163 - 10.107) = 17.056$

Strategy #2. Use a graphing calculator to evaluate the expression $119.67(0.61)^5 - 119.67(0.61)^3$, which equals 17.056..

PTS: 2 NAT: F.IF.A.2 TOP: Functional Notation Evaluating Functions

417) ANS: 4

Strategy: Substitute and solve.

Notes	Left Expression	Sign	Right Expression
Given	$k(x)$	=	$2x^2 - 3\sqrt{x}$
Substitute 9 for x	$k(9)$	=	$2(9)^2 - 3\sqrt{9}$
Exponents and Radicals	$k(9)$	=	$2(81) - 3\sqrt{3}$
Simplify	$k(9)$	=	162-9

Simplify	$k(9)$	=	153

PTS: 2 NAT: F.IF.A.2 TOP: Functional Notation

FUNCTIONS
Domain and Range

CC Standard	**NG Standard**
F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*	**AI-F.IF.5 Determine** the domain of a function from its graph and, where applicable, **identify** the appropriate domain for a function in context.

LEARNING OBJECTIVES

Students will be able to:

1) Determine the domain of a function from its graph.
2) Identify appropriate sets of numbers for the domain and range of a function.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

continuous
counting numbers
discrete
domain
integers
natural numbers
range
rational numbers
real numbers
whole numbers

BIG IDEAS

The **domain of x** and the **range of y**.

The coordinate plane consists of two perpendicular number lines, which are commonly referred to as the x-axis and the y-axis. Each number line represents the set of real numbers. The x-axis represents the independent variable (inputs) and the y-axis represents the dependent variable (outputs).

The domain of a function is that part (or parts) of the x-axis number line required for the funtion's input values. This can be an interval of all real numbers, or limited to specific subsets of real numbers, such as positive or negative integers.

The range of a function is that part (or parts) of the y-axis number line required for the funtion's output values. This can be an interval of all real numbers, or limited to specific subsets of real numbers, such as positive or negative integers.

A function maps an element of the **domain** onto one and only one element of the **range**.

Choosing Appropriate Domains and Ranges

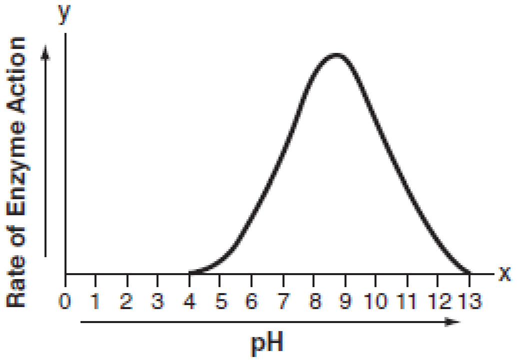
Many functions make sense only when a subset of all the Real Numbers are used as inputs. This subset of the Real Numbers that makes sense is known as the domain of the function.

Example: If a store makes $2.00 profit on each sandwich sold, total profits might be modeled by the function $P(s) = 2s$, where $P(s)$ represents total profits and s represents the number of sandwiches sold. The entire set of real numbers, including fractions and irrational numbers, make no sense for this function, because the store only sells whole sandwiches. In this example, the domain of the function $P(s) = 2s$ should be restricted to the set of whole numbers. Likewise, the range of a function can also be limited to a well-defined subset of the Real Numbers on the y-axis.

Domains and **ranges** can be either **continuous** or **discrete**.

DEVELOPING ESSENTIAL SKILLS

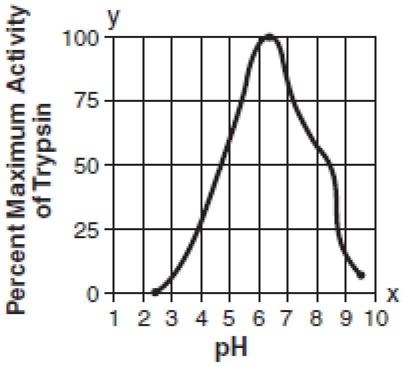
1. The effect of pH on the action of a certain enzyme is shown on the accompanying graph.



   What is the domain of this function?

   a. $4 \le x \le 13$ c. $x \ge 0$
   b. $4 \le y \le 13$ d. $y \ge 0$

2. Data collected during an experiment are shown in the accompanying graph.



   What is the range of this set of data?

   a. $2.5 \le y \le 9.5$ c. $0 \le y \le 100$
   b. $2.5 \le x \le 9.5$ d. $1 \le x \le 10$

3. What is the range of the relation $y = 2x^2 + 3x$ if the domain is the set $\{-2, -1, 0\}$?

   a. $\{2, 1, 0\}$ c. $\{-1, -5, 0\}$
   b. $\{2, -1, 0\}$ d. $\{10, 1, 0\}$

4. The domain for $\mathrm{f}(x) = 3x + 2$ is $-3 \le x \le 2$. The greatest value in the range of $\mathrm{f}(x)$ is

   a. -7 c. 8
   b. 2 d. 11

5. The domain of $\mathrm{f}(x) = x^2 + 2x + 1$ is $-3 \le x \le 3$. The largest value in the range of $\mathrm{f}(x)$ is

   a. 20 c. 3
   b. 16 d. 4

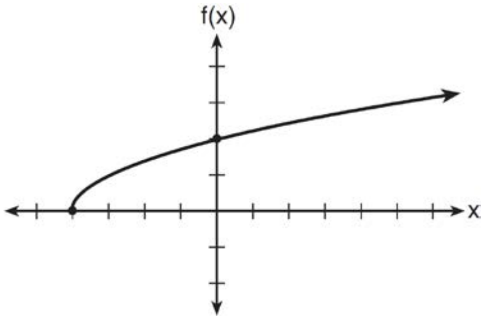
ANSWERS

1. ANS: A
2. ANS: C
3. ANS: B
4. ANS: C
5. ANS: B

REGENTS EXAM QUESTIONS (through June 2018)

F.IF.B.5: Domain and Range

418) The graph of the function $f(x) = \sqrt{x+4}$ is shown below.



The domain of the function is

1) $\{x\vert x > 0\}$	3) $\{x\vert x > -4\}$
2) $\{x\vert x \ge 0\}$	4) $\{x\vert x \ge -4\}$

419) If $f(x) = \frac{1}{3}x + 9$, which statement is always true?

1) $f(x) < 0$	3) If $x < 0$, then $f(x) < 0$.
2) $f(x) > 0$	4) If $x > 0$, then $f(x) > 0$.

420) Let f be a function such that $f(x) = 2x - 4$ is defined on the domain $2 \le x \le 6$. The range of this function is

1) $0 \le y \le 8$	3) $2 \le y \le 6$
2) $0 \le y < \infty$	4) $-\infty < y < \infty$

421) The range of the function defined as $y = 5^x$ is

1) $y < 0$	3) $y \le 0$
2) $y > 0$	4) $y \ge 0$

422) The range of the function $f(x) = x^2 + 2x - 8$ is all real numbers

1) less than or equal to -9	3) less than or equal to -1
2) greater than or equal to -9	4) greater than or equal to -1

423) What is the domain of the relation shown below?

$$\{(4,2),(1,1),(0,0),(1,-1),(4,-2)\}$$

1) $\{0,1,4\}$	3) $\{-2,-1,0,1,2,4\}$
2) $\{-2,-1,0,1,2\}$	4) $\{-2,-1,0,0,1,1,1,2,4,4\}$

424) If the domain of the function $f(x) = 2x^2 - 8$ is $\{-2, 3, 5\}$, then the range is

1) $\{-16, 4, 92\}$
2) $\{-16, 10, 42\}$
3) $\{0, 10, 42\}$
4) $\{0, 4, 92\}$

425) Officials in a town use a function, *C*, to analyze traffic patterns. $C(n)$ represents the rate of traffic through an intersection where *n* is the number of observed vehicles in a specified time interval. What would be the most appropriate domain for the function?

1) $\{\ldots -2, -1, 0, 1, 2, 3, \ldots\}$
2) $\{-2, -1, 0, 1, 2, 3\}$
3) $\{0, \frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}\}$
4) $\{0, 1, 2, 3, \ldots\}$

426) The function $h(t) = -16t^2 + 144$ represents the height, $h(t)$, in feet, of an object from the ground at *t* seconds after it is dropped. A realistic domain for this function is

1) $-3 \le t \le 3$
2) $0 \le t \le 3$
3) $0 \le h(t) \le 144$
4) all real numbers

427) Which domain would be the most appropriate set to use for a function that predicts the number of household online-devices in terms of the number of people in the household?

1) integers
2) whole numbers
3) irrational numbers
4) rational numbers

428) A store sells self-serve frozen yogurt sundaes. The function $C(w)$ represents the cost, in dollars, of a sundae weighing *w* ounces. An appropriate domain for the function would be

1) integers
2) rational numbers
3) nonnegative integers
4) nonnegative rational numbers

429) A construction company uses the function $f(p)$, where *p* is the number of people working on a project, to model the amount of money it spends to complete a project. A reasonable domain for this function would be

1) positive integers
2) positive real numbers
3) both positive and negative integers
4) both positive and negative real numbers

430) An online company lets you download songs for $0.99 each after your have paid a $5 membership fee. Which domain would be most appropriate to calculate the cost to download songs?

1) rational numbers greater than zero
2) whole numbers greater than or equal to one
3) integers less than or equal to zero
4) whole numbers less than or equal to one

431) The daily cost of production in a factory is calculated using $c(x) = 200 + 16x$, where *x* is the number of complete products manufactured. Which set of numbers best defines the domain of $c(x)$?

1) integers
2) positive real numbers
3) positive rational numbers
4) whole numbers

432) At an ice cream shop, the profit, $P(c)$, is modeled by the function $P(c) = 0.87c$, where *c* represents the number of ice cream cones sold. An appropriate domain for this function is

1) an integer ≤ 0
2) an integer ≥ 0
3) a rational number ≤ 0
4) a rational number ≥ 0

433) If $f(x) = x^2 + 2$, which interval describes the range of this function?

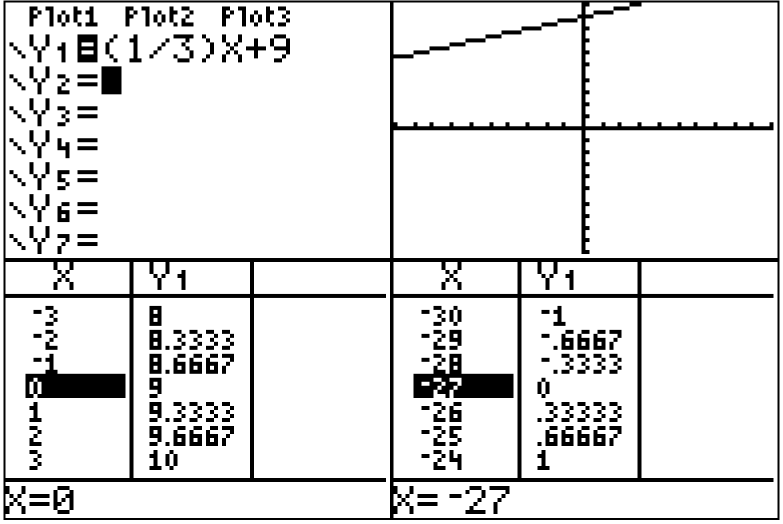
1) $(-\infty, \infty)$
2) $[0, \infty)$
3) $[2, \infty)$
4) $(-\infty, 2]$

SOLUTIONS

418) ANS: 4
Strategy: Use the number line of the x-axis, the fact that the graph begins with a solid dot, indicating that -4 is included in the domain, and the fact that the graph includes an arrow indicating that the graph continues to positive infinity, to select answer choice d.

PTS: 2 NAT: F.IF.A.1 TOP: Domain and Range

419) ANS: 4
Strategy: Inspect the function rule in a graphing calculator, then eliminate wrong answers.



```
Plot1 Plot2 Plot3
\Y1=(1/3)X+9
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
  X     Y1                X     Y1
 -3     8                -30    -1
 -2     8.3333           -29    -.6667
 -1     8.6667           -28    -.3333
 0      9                -27    0
 1      9.3333           -26    .33333
 2      9.6667           -25    .66667
 3      10               -24    1
X=0                     X=-27
```

Answer choice *a* can be eliminated because the table clearly shows $f(x)$ values greater than zero.
Answer choice *b* can be eliminated because the table clearly shows $f(x)$ values less than zero.
Answer choice *c* can be eliminated because if x is greater than -27, then $f(x) > 0$.
Choose answer choice *d* because the graph and table clearly show that all values of $f(x)$ are positive when values of x are positive.

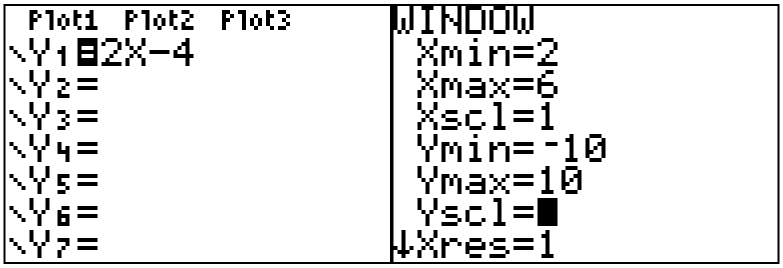
PTS: 2 NAT: F.IF.A.2 TOP: Domain and Range

420) ANS: 1

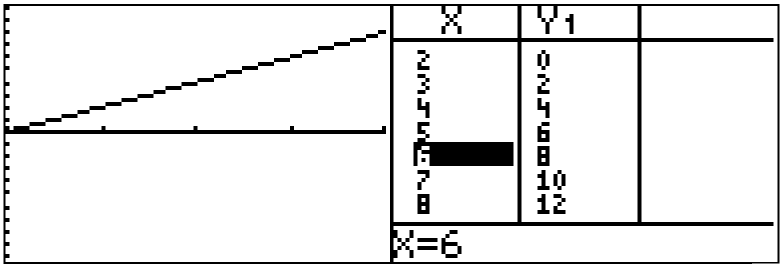
$f(2) = 0$

$f(6) = 8$

Strategy: Inspect the function rule in a graphing calculator over the domain $2 \le x \le 6$, eliminate wrong answers.



```
Plot1 Plot2 Plot3        WINDOW
\Y1=2X-4                  Xmin=2
\Y2=                      Xmax=6
\Y3=                      Xscl=1
\Y4=                      Ymin=-10
\Y5=                      Ymax=10
\Y6=                      Yscl=
\Y7=                     ↓Xres=1
```



X	Y1
2	0
3	2
4	4
5	6
6	8
7	10
8	12

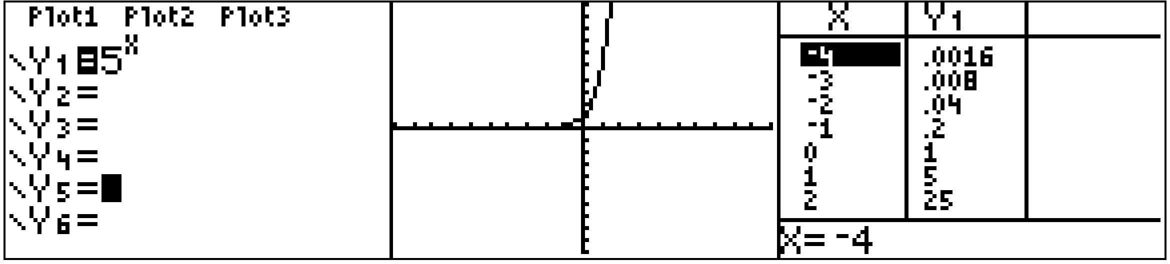
X=6

Choose answer choice a because the table of values and the graph clearly show that $f(2) = 0$ and $f(6) = 8$, and all values of y between $x = 2$ and $x = 6$ are between 0 and 8.
Eliminate answer choice *b* because infinity is clearly bigger than 8.
Eliminate answer choice *c* because these are the domain of x, not the range of y.
Eliminate answer choice *d* because negative infinity is clearly less than 0.

PTS: 2 NAT: F.IF.A.2 TOP: Domain and Range

421) ANS: 2

Strategy: Input the function in a graphing calculator and inspect the graph and table views..



Plot1 Plot2 Plot3
\Y1=5^X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=

X	Y1
-4	.0016
-3	.008
-2	.04
-1	.2
0	1
1	5
2	25

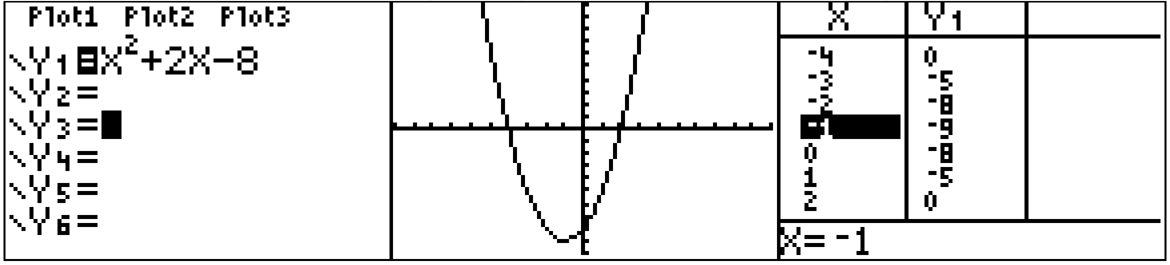
X= -4

The value of y approaches zero, but never reachers zero, as the value of x decreases.
The the range of $y = 5^x$ is $y > 0$.

PTS: 2 NAT: F.IF.A.2 TOP: Domain and Range
KEY: real domain, exponential

422) ANS: 2

Strategy: Input the function into a graphing calculator and inspect the range of y-values.



Plot1 Plot2 Plot3
\Y1=X^2+2X-8
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=

X	Y1
-4	0
-3	-5
-2	-8
-1	-9
0	-8
1	-5
2	0

X= -1

The graph and the table of values show that all values of f(x) are greater than or equal to -9. Choice b) is the correct answer.

PTS: 2 NAT: F.IF.A.2 TOP: Domain and Range
KEY: real domain, quadratic

423) ANS: 1

Domain refers to the x-axis while range refers to the y-axis. This question is asking what values on the x-axis are required by this relation.
Strategy: Underline all the x-values of the relation, then organize the unique values.

$$\{(\underline{4},2),(\underline{1},1),(\underline{0},0),(\underline{1},-1),(\underline{4},-2)\}$$

$$\{4,1,0,1,4\}$$

$$\{0,1,4\}$$

You could graph the entire relation if you have x-values of 0, 1, and 4.

PTS: 2 NAT: F.IF.A.2 TOP: Domain and Range
KEY: limited domain

424) ANS: 3

Substitute each value of the domain into the function and solve for the range for each value.

$f(-2) = 2(-2)^2 - 8$ $f(3) = 2(3)^2 - 8$ $f(5) = 2(5)^2 - 8$

$f(-2) = 0$ $f(3) = 10$ $f(5) = 42$

PTS: 2 NAT: F.IF.A.2 TOP: Domain and Range
KEY: limited domain

425) ANS: 4

Strategy: Examine each answer choice and eliminate wrong answers.

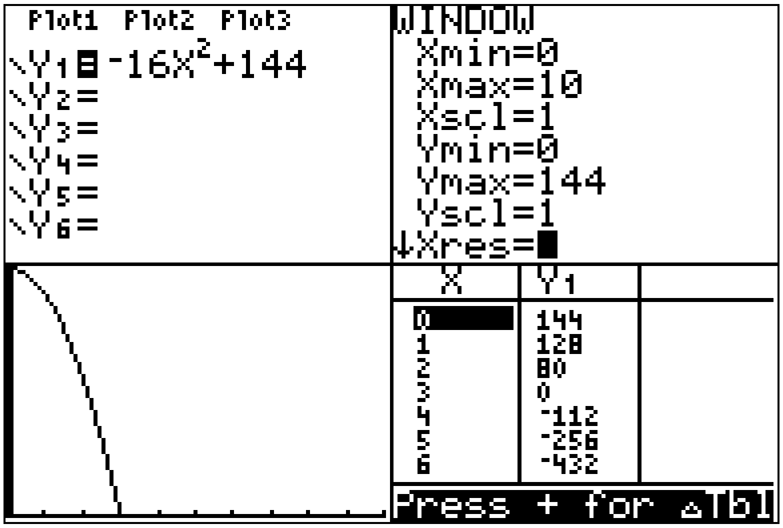
Eliminate answer choices *a* and *b* because *negative numbers* of cars observed do not make sense.
Eliminate answer choice *c* because *fractional numbers* of cars observed do not make sense.
Choose answer choice *d* because it is the only choice that makes sense. The number of cars observed must be either zero or some counting number.

PTS: 2 NAT: F.IF.B.5 TOP: Domain and Range

426) ANS: 2

Strategy: Input the function into a graphing calculator and examine it to determine a realistic range.

First, transform $h(t) = -16t^2 + 144$ to $Y_1 = -16x^2 + 144$ for input.



```
Plot1  Plot2  Plot3
\Y1 = -16X²+144
\Y2 =
\Y3 =
\Y4 =
\Y5 =
\Y6 =
```

```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=144
 Yscl=1
↓Xres=■
```

```
  X   | Y1
  0   | 144
  1   | 128
  2   | 80
  3   | 0
  4   | -112
  5   | -256
  6   | -432
Press + for ΔTbl
```

The graph and table of values show that it takes 3 seconds for the object to reach the ground. Therefore, a realistic domain for this function is $0 \le t \le 3$.

$t = 0$ represents the time when the object is dropped.

$t = 3$ represents the time when the object hits the ground.

Answer choice *b* is correct.

PTS: 2 NAT: F.IF.B.5 TOP: Domain and Range

427) ANS: 2

Strategy: Eliminate wrong answers.

Eliminate answer choice *a* because the set of integers contains negative numbers, which do not make sense when counting the number of appliances in a household.
Choose answer choice *b* because the set of whole numbers is defined as {0, 1, 2, 3, ...}. This does make sense when counting the number of appliances in a household.

Eliminate answer choice *c* because the set of irrational numbers includes numbers like π and $\sqrt{7}$, which do not make sense when counting the number of appliances in a household.

Eliminate answer choice *d* because the set of rational numbers includes fractions such as $\frac{3}{4}$ and $\frac{15}{23}$, which do not make sense when counting the number of appliances in a household.

PTS: 2 NAT: F.IF.B.5 TOP: Domain and Range

428) ANS: 4

Step 1. Understand that the problem is asking for a set of numbers that would be appropriate x-values to measure the weight (in ounces) of frozen yogurt sundaes.
Step 2. Strategy. Eliminate wrong answers.
Step 3. Execution of Strategy.
a) Integers would not be an appropriate domain because there is no need for negative whole numbers. It makes no sense to have a yogurt sundae that weighs -4 ounces.
b) Rational numbers would not be an appropriate domain because, once again, there is no need for negative numbers. It makes no sense to have a yogurt sundae that weighs $-\frac{7}{2}$ ounces.
c) Nonnegative Integers could work except for zero, which is a non-negative integer. It makes no sense to have a yogurt sundae that weighs zero ounces.
d) Nonnegative rational numbers are the best choice.
Step 4. Does it make sense? Yes. You could weigh yogurt sundaes by the ounce, half ounce, quarter ounce, or any other nonnegative fraction.

PTS: 2 NAT: F.IF.B.5 TOP: Domain and Range

429) ANS: 1

Strategy: Eliminate wrong answers. The number of people must be counting numbers, since it makes no sense to have a half a person or a quarter person.
The **positive integers** are 1, 2, 3, 4, ..., which makes sense.
Positive real numbers should be eliminated because positive real numbers include fractions, and fractions make no sense for the number of workers.
Both positive and negative integers should be eliminated because it makes no sense to have negative numbers of workers.
Both positive and negative real numbers should also be eliminated because it makes no sense to have negative numbers of workers.
The correct choice is **positive integers**.

PTS: 2 NAT: F.IF.B.5 TOP: Domain and Range

430) ANS: 2

Understand the Question: Cost is a function of the number of songs downloaded, so cost is the dependent variable and the number of songs is the independent variable. The domain of a function refers to the independent variable (x-axis), so the problem is asking which numbers are most appropriate for the number of songs downloaded.
Then, eliminate wrong answers:
Eliminate: Rational numbers greater than zero because there is no need for fractions.
Choose: Whole numbers greater than or equal to one because you only need positive whole numbers.
Eliminate: Integers less than or equal to zero because you would not download a negative number of songs.
Eliminate: Whole numbers less than or equal to one because you would not download a negative number of songs.

PTS: 2 NAT: F.IF.B.5

431) ANS: 4

Reason: If x represents the number of complete products manufactured, there is no need for fractions or negative numbers.
Strategy: Eliminate wrong answers:

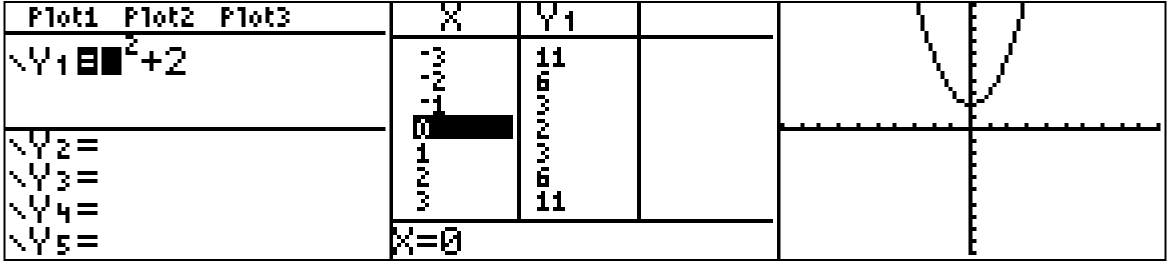
~~a) integers~~ There is no need for negative numbers.
~~b) positive real numbers~~ There is no need for fractions and/or irrational numbers.
~~c) positive rational numbers~~ There is no need for fractions.
d) whole numbers A complete product can be represented by a whole number.

PTS: 2 NAT: F.IF.B.5 TOP: Domain and Range

432) ANS: 2
Strategy: Eliminate wrong answers.
1. Eliminate an integer ≤ 0 because all of the integers less than or equal to zero are negative numbers and you cannot sell a negative number of ice cream cones.
2. Select an integer ≥ 0 because these are the whole numbers and you can only sell a whole ice cream cone.
3. Eliminate a rational number ≤ 0 because you cannot sell a negative number of ice cream cones or negatives fractions of ice cream cones.
4. Eliminate a rational number ≥ 0 because you cannot sell fractional parts of ice cream cones.

PTS: 2 NAT: F.IF.B.5 TOP: Domain and Range

433) ANS: 3
Strategy: Inspect the table and graph views of this function in a graphing calculator to find the <u>range</u> (not the domain).



```
Plot1 Plot2 Plot3
\Y1=■²+2
\Y2=
\Y3=
\Y4=
\Y5=
```

X	Y1
-3	11
-2	6
-1	3
0	2
1	3
2	6
3	11

X=0

The table of values and the graph both show the smalles value of f(x) is 2, which occurs when $x = 0$. The maximum value of f(x) is infiniy. Therefore, the range of the function is $[2,\infty)$.

NOTE: $(-\infty,\infty)$ is the domain of the function. Don't confuse domain and range.

PTS: 2 NAT: F.IF.A.2 TOP: Domain and Range
KEY: real domain, quadratic

M – Functions, Lesson 4, Operations with Functions (r. 2018)

FUNCTIONS

Operations with Functions

Common Core Standard	Next Generation Standard
A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	**AI-A.APR.1** Add, subtract, and multiply polynomials and recognize that the result of the operation is also a polynomial. This forms a system analogous to the integers. **Note: This standard is a fluency recommendation for Algebra I. Fluency in adding, subtracting and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions.**

LEARNING OBJECTIVES

Students will be able to:

1)

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

LEARNING OBJECTIVES

Students will be able to:

1) Use the output of one function as the input for another function.
2) Substitute expressions from one function into another.

BIG IDEAS

Polynomial expressions can be substituted into equations and functions.

Example:

Given that: $f(x) = g(x) - 2h(x)$ and $g(x) = 3x + 4$, then $f(x) = (3x+4) - 2(5x-6)$

$h(x) = 5x - 6$

Functions can be multiplied or divided if each and every term in both expressions is multiplied or divided by the same value.

$$2(y = 3x + 4)$$

Example: $$2(y) = 2(3x) + 2(4)$$

$$2y = 6x + 8$$

DEVELOPING ESSENTIAL SKILLS

1. If f and g are two functions defined by $f(x) = 3x + 5$ and $g(x) = x^2 + 1$, then $g(f(x))$ is
   - a. $x^2 + 3x + 6$
   - b. $9x^2 + 30x + 26$
   - c. $3x^2 + 8$
   - d. $9x^2 + 26$

2. If $f(x) = -2x + 7$ and $g(x) = x^2 - 2$, then $f(g(3))$ is equal to
   - a. -7
   - b. -3
   - c. -1
   - d. 7

3. The accompanying tables define functions f and g.

x	1	2	3	4	5
$f(x)$	3	4	5	6	7

x	3	4	5	6	7
$g(x)$	4	6	8	10	12

   What is $g(f(3))$?
   - a. 6
   - b. 2
   - c. 8
   - d. 4

4. If $f(x) = x^2 + 4$ and $g(x) = \sqrt{1-x}$, what is the value of $f(g(-3))$?
   - a. $2i\sqrt{3}$
   - b. 2
   - c. 8
   - d. 13

5. If $f(x) = x^2 + 4$ and $g(x) = 2x + 3$, find $f(g(-2))$.

ANSWERS

1. ANS: B

$$f(x) = 3x + 5$$

$$g(3x+5) = (3x+5)^2 + 1$$

$$= 9x^2 + 30x + 26$$

2. ANS: A

$$g(3) = 3^2 - 2$$

$$= 7$$

$$f(7) = -2(7) + 7$$

$$-7$$

3. ANS: C

f(3) = 5, g(5) = 8

4. ANS: C

$$g(-3) = \sqrt{1-x} = \sqrt{1-(-3)} = 2$$

$$f(2) = 2^2 + 4 = 8$$

5. ANS:

5. $g(-2) = 2(-2) + 3 = -1$. $f(-1) = (-1)^2 + 4 = 5$.

A.APR.A.1: Operations with Functions

434) A company produces x units of a product per month, where $C(x)$ represents the total cost and $R(x)$ represents the total revenue for the month. The functions are modeled by $C(x) = 300x + 250$ and $R(x) = -0.5x^2 + 800x - 100$. The profit is the difference between revenue and cost where $P(x) = R(x) - C(x)$. What is the total profit, $P(x)$, for the month?

1) $P(x) = -0.5x^2 + 500x - 150$
2) $P(x) = -0.5x^2 + 500x - 350$
3) $P(x) = -0.5x^2 - 500x + 350$
4) $P(x) = -0.5x^2 + 500x + 350$

SOLUTION

434) ANS: 2

Strategy: Substitute $R(x)$ and $C(x)$ into $P(x) = R(x) - C(x)$.

Given:

$$P(x) = R(x) - C(x)$$

$$R(x) = -0.5x^2 + 800x - 100$$

$$C(x) = 300x + 250$$

Therefore:

$$P(x) = \left(-0.5x^2 + 800x - 100\right) - (300x + 250)$$

$$P(x) = -0.5x^2 + 800x - 100 - 300x - 250$$

$$P(x) = -0.5x^2 + 500x - 350$$

PTS: 2 NAT: A.APR.A.1 TOP: Addition and Subtraction of Polynomials
KEY: subtraction

M – Functions, Lesson 5, Families of Functions (r. 2018)

FUNCTIONS
Families of Functions

Common Core Standards	Next Generation Standards
F-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.	**AI-F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.
F-LE.A.1a ~~Prove~~ that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.	**AI-F.LE.1a Justify** that a function is linear because it grows by equal differences over equal intervals, and that a function is exponential because it grows by equal factors over equal intervals.
F-LE.A.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	**AI-.F.LE.1b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another, **and therefore can be modeled linearly.** **e.g., A flower grows two inches per day.**
F-LE.A.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	**AI-F.LE.1c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another, **and therefore can be modeled exponentially.** **e.g., A flower doubles in size after each day.**
F-LE.A.2 Construct linear and exponential functions, ~~including arithmetic and geometric sequences~~, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). PARCC: Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).	**AI-F.LE.2** Construct a linear or exponential function **symbolically** given: i) a graph; ii) a description of the relationship; iii) two input-output pairs (include reading these from a table). (Shared standard with Algebra II) Note: Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).
F-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	**AI-F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

LEARNING OBJECTIVES

Students will be able to:

1) Describe characteristics of linear, exponential and quadratic functions.
2) Associate linear functions with constant rates of change.
3) Associate exponential and quadratic functions with variable rates of change.

Overview of Lesson

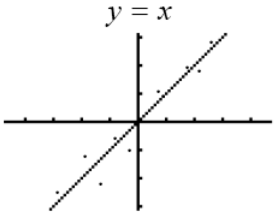
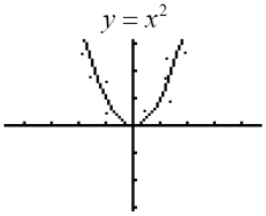
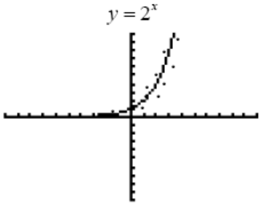
Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

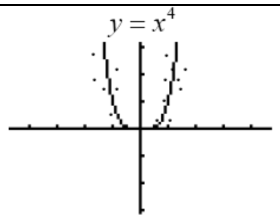
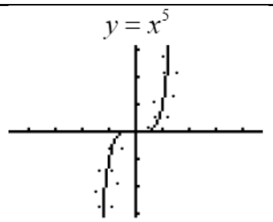
VOCABULARY

exponential
families of functions
linear
parabloa
quadratic
rate of change

BIG IDEAS

Families of Functions

The Linear Family	The Quadratic Family	The Exponential Family
 If the graph is a straight line, the function is in the family of **linear functions.** All **first degree functions** are linear functions, except those lines that are vertical. All linear functions can be expressed as $y = mx + b$, where *m* is a constant defined slope and *b* is the y-intercept. A **constant rate of change** indicates a linear function.	 If the graph is a parabola, the function is in the family of **quadratic functions**. All **quadratic functions** have an exponent of 2 or can be factored into a single factor with an exponent of 2. Examples: $x^2 + 6x + 9 = (x + 3)^2$ $x^{16} + 6x^8 + 9 = \left(x^8 + 3\right)^2$	 If the graph is a curve that approaches a horizontal limit on one end and gets steeper on the other end, the function is in the family of **exponential functions**. An **exponential function** is a function that contains a variable for an exponent. Example: $y = 2^x$ Exponential growth and decay can be modeled using the general formula $A = P(1 + r)^t$

 NOTE: All functions in the form of $y = ax^n$, where $a \neq 0$ and n is an ***even number >1***, take the form of parabolas. The larger the value of *n*, the wider the flat part at the bottom/top.	 NOTE: All functions in the form of $y = ax^n$, where $a \neq 0$ and n is an ***odd number >1***, take the form of hyperbolas. These are not quadratic functions.

Rates of Change Can be Used to Identify a Function's Family

Linear functions have **constant** rates of change.
Quadratic functions have **both** negative and positive **varying** rates of change.
Exponential functions have **either** negative or positive **varying** rates of change. (NOTE: A quantity increasing exponentially will eventually exceeds a quantity increasing linearly or quadratically.)

When the rate of change is not constant, it is called a **variable rate** of change.

,

Finding Rates of Change from Tables

The slope formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ is used to find the rate of change in table views of a function. When applying the slope formula to tables, it may be helpful to think of the formual as

$$m = \frac{\Delta y}{\Delta x}.$$

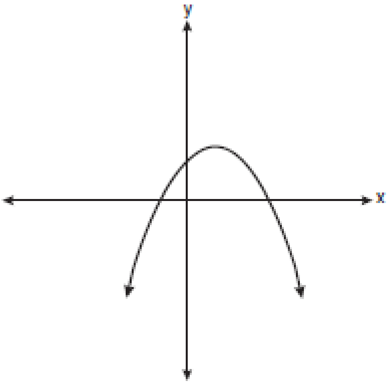
Simply add two extra columns titleD Δx and Δy to the table, then find the differences between any two y values in the table and their corresponding x values.
Example:

Δx	x	y	Δy
2-1=1	1	3	6-3=3
	2	6	
	4	12	
9-7=2	7	21	27-21=6
	9	27	

The above table is the table view of the function $y = 3x$. The ratio of $\frac{\Delta y}{\Delta x}$ always reduces to $\frac{3}{1}$, regardless of which coordinate pairs are selected. This means that the above table represents a linear function.

DEVELOPING ESSENTIAL SKILLS

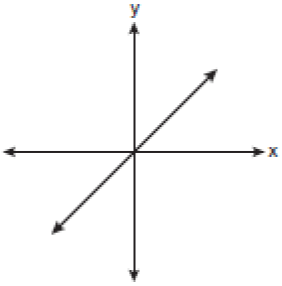
1. Which type of graph is shown in the diagram below?



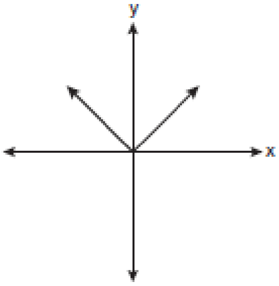
a. absolute value
b. exponential
c. linear
d. quadratic

2. Which graph represents a linear function?

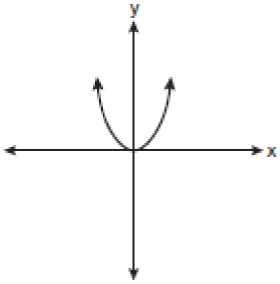
a.



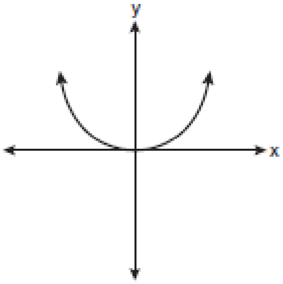
c.



b.

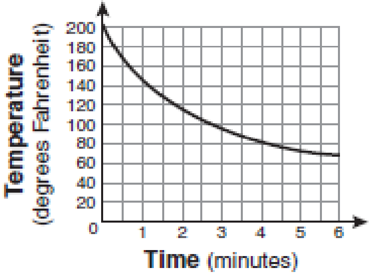


d.

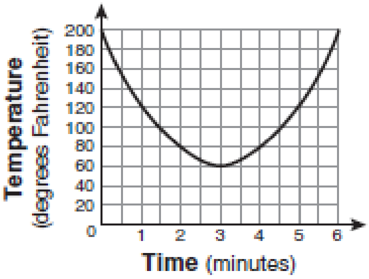


3. Antwaan leaves a cup of hot chocolate on the counter in his kitchen. Which graph is the best representation of the change in temperature of his hot chocolate over time?

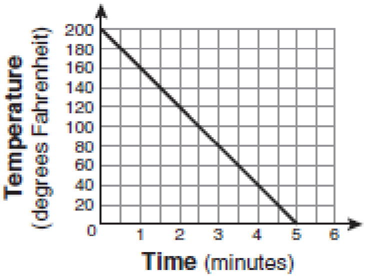
a.



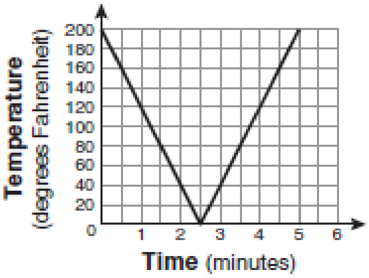
c.



b.

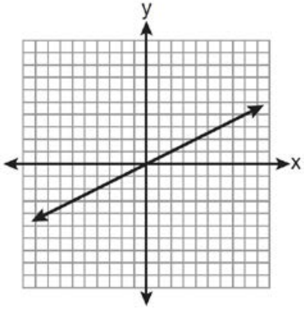


d.

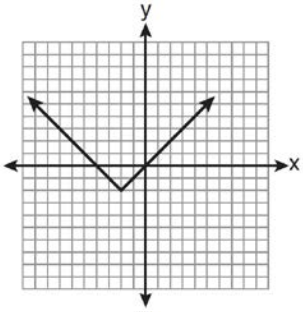


4. Which graph represents an exponential equation?

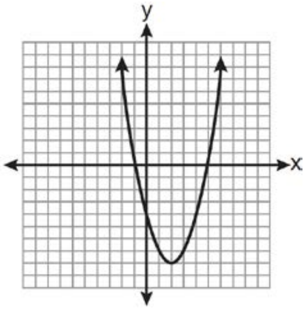
a.



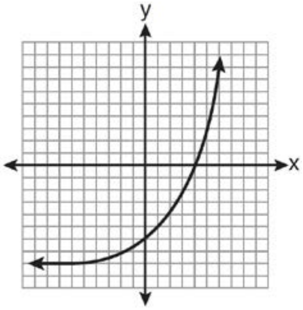
c.



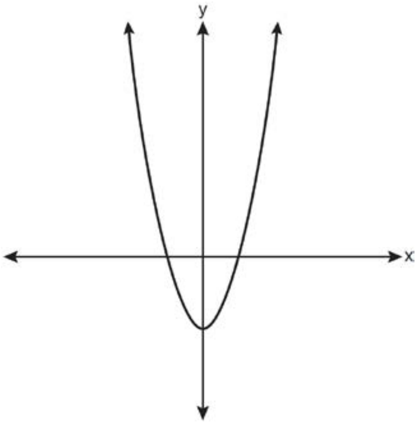
b.



d.



5. Which type of function is represented by the graph shown below?



a. absolute value
b. exponential
c. linear
d. quadratic

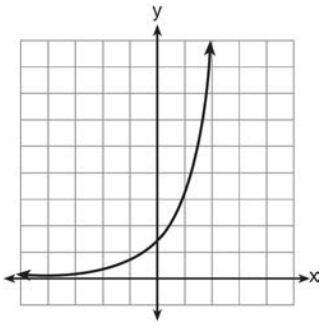
6. Which equation represents a quadratic function?

a. $y = x + 2$
c. $y = x^2$

b. $y = |x + 2|$

d. $y = 2^x$

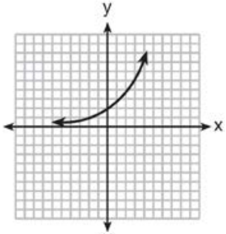
7. Which type of function is graphed below?



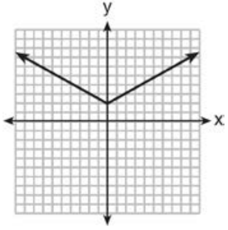
a. linear
b. quadratic
c. exponential
d. absolute value

8. Which graph represents an absolute value equation?

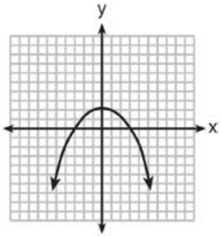
a.



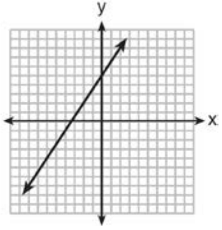
b.



c.



d.



ANSWERS

1. ANS: D
2. ANS: A
3. ANS: A
4. ANS: D
5. ANS: D
6. ANS: C
7. ANS: C
8. ANS: B

REGENTS EXAM QUESTIONS (through June 2018)

F.LE.A.1, F.LE.A.2, F.LE.A.3: Model Families of Functions

435) Which situation could be modeled by using a linear function?

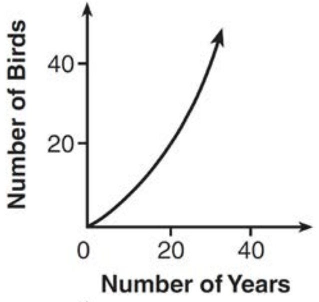
1) a bank account balance that grows at a rate of 5% per year, compounded annually
2) a population of bacteria that doubles every 4.5 hours
3) the cost of cell phone service that charges a base amount plus 20 cents per minute
4) the concentration of medicine in a person’s body that decays by a factor of one-third every hour

436) Sara was asked to solve this word problem: "The product of two consecutive integers is 156. What are the integers?" What type of equation should she create to solve this problem?

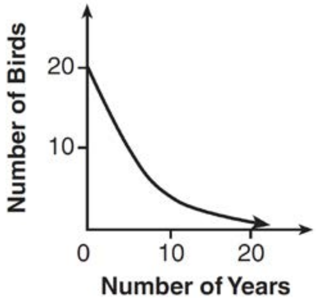
1) linear
2) quadratic
3) exponential
4) absolute value

437) A population that initially has 20 birds approximately doubles every 10 years. Which graph represents this population growth?

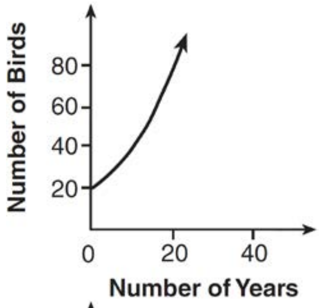
1)



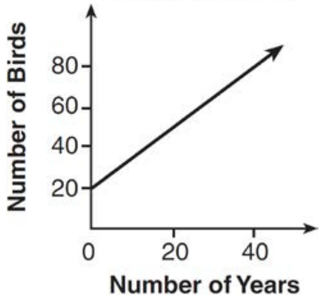
2)



3)



4)



438) Which table of values represents a linear relationship?

1)

x	f(x)
−1	−3
0	−2
1	1
2	6
3	13

3)

x	f(x)
−1	−3
0	−1
1	1
2	3
3	5

2)

x	f(x)
−1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

4)

x	f(x)
−1	−1
0	0
1	1
2	8
3	27

439) The table below shows the average yearly balance in a savings account where interest is compounded annually. No money is deposited or withdrawn after the initial amount is deposited.

Year	Balance, in Dollars
0	380.00
10	562.49
20	832.63
30	1232.49
40	1824.39
50	2700.54

Which type of function best models the given data?

1) linear function with a negative rate of change
2) linear function with a positive rate of change
3) exponential decay function
4) exponential growth function

440) Rachel and Marc were given the information shown below about the bacteria growing in a Petri dish in their biology class.

Number of Hours, x	1	2	3	4	5	6	7	8	9	10
Number of Bacteria, $B(x)$	220	280	350	440	550	690	860	1070	1340	1680

Rachel wants to model this information with a linear function. Marc wants to use an exponential function. Which model is the better choice? Explain why you chose this model.

441) The function, $t(x)$, is shown in the table below.

x	t(x)
–3	10
–1	7.5
1	5
3	2.5
5	0

Determine whether $t(x)$ is linear or exponential. Explain your answer.

442) The tables below show the values of four different functions for given values of *x*.

x	f(x)
1	12
2	19
3	26
4	33

x	g(x)
1	–1
2	1
3	5
4	13

x	h(x)
1	9
2	12
3	17
4	24

x	k(x)
1	–2
2	4
3	14
4	28

Which table represents a linear function?

1) $f(x)$
2) $g(x)$
3) $h(x)$
4) $k(x)$

443) Grisham is considering the three situations below.

I. For the first 28 days, a sunflower grows at a rate of 3.5 cm per day.
II. The value of a car depreciates at a rate of 15% per year after it is purchased.
III. The amount of bacteria in a culture triples every two days during an experiment.

Which of the statements describes a situation with an equal difference over an equal interval?

1) I, only
2) II, only
3) I and III
4) II and III

444) Consider the pattern of squares shown below:

Which type of model, linear or exponential, should be used to determine how many squares are in the *n*th pattern? Explain your answer.

445) Which scenario represents exponential growth?

1) A water tank is filled at a rate of 2 gallons/minute.
2) A vine grows 6 inches every week.
3) A species of fly doubles its population every month during the summer.
4) A car increases its distance from a garage as it travels at a constant speed of 25 miles per hour.

446) One characteristic of all linear functions is that they change by

1) equal factors over equal intervals
2) unequal factors over equal intervals
3) equal differences over equal intervals
4) unequal differences over equal intervals

447) The highest possible grade for a book report is 100. The teacher deducts 10 points for each day the report is late. Which kind of function describes this situation?

1) linear
2) quadratic
3) exponential growth
4) exponential decay

448) Ian is saving up to buy a new baseball glove. Every month he puts $10 into a jar. Which type of function best models the total amount of money in the jar after a given number of months?

1) linear
2) exponential
3) quadratic
4) square root

449) If a population of 100 cells triples every hour, which function represents $p(t)$, the population after *t* hours?

1) $p(t) = 3(100)^t$
2) $p(t) = 100(3)^t$
3) $p(t) = 3t + 100$
4) $p(t) = 100t + 3$

450) The table below represents the function *F*.

x	3	4	6	7	8
F(x)	9	17	65	129	257

The equation that represents this function is

1) $F(x) = 3^x$
2) $F(x) = 3x$
3) $F(x) = 2^x + 1$
4) $F(x) = 2x + 3$

451) A laboratory technician studied the population growth of a colony of bacteria. He recorded the number of bacteria every other day, as shown in the partial table below.

t (time, in days)	0	2	4
f(t) (bacteria)	25	15,625	9,765,625

Which function would accurately model the technician's data?

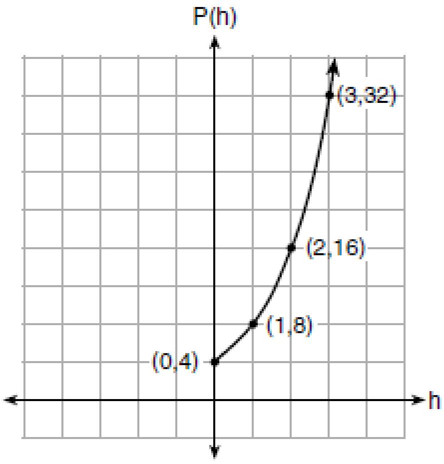
1) $f(t) = 25^t$
2) $f(t) = 25^{t+1}$
3) $f(t) = 25t$
4) $f(t) = 25(t + 1)$

452) Which function is shown in the table below?

x	f(x)
–2	$\frac{1}{9}$
–1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

1) $f(x) = 3x$
2) $f(x) = x + 3$
3) $f(x) = -x^3$
4) $f(x) = 3^x$

453) Vinny collects population data, $P(h)$, about a specific strain of bacteria over time in hours, *h*, as shown in the graph below.



Which equation represents the graph of $P(h)$?

1) $P(h) = 4(2)^h$
2) $P(h) = \frac{46}{5}h + \frac{6}{5}$
3) $P(h) = 3h^2 + 0.2h + 4.2$
4) $P(h) = \frac{2}{3}h^3 - h^2 + 3h + 4$

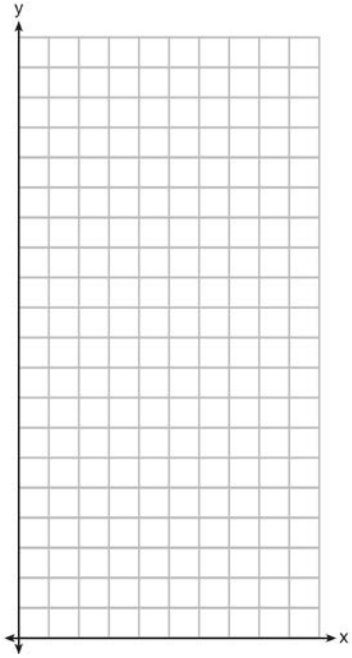
454) If $f(x) = 3^x$ and $g(x) = 2x + 5$, at which value of x is $f(x) < g(x)$?

1) -1
2) 2
3) -3
4) 4

455) Alicia has invented a new app for smart phones that two companies are interested in purchasing for a 2-year contract. Company *A* is offering her $10,000 for the first month and will increase the amount each month by $5000. Company *B* is offering $500 for the first month and will double their payment each month from the previous month. Monthly payments are made at the end of each month. For which monthly payment will company *B*'s payment first exceed company *A*'s payment?

1) 6
2) 7
3) 8
4) 9

456) Graph $f(x) = x^2$ and $g(x) = 2^x$ for $x \ge 0$ on the set of axes below.



State which function, $f(x)$ or $g(x)$, has a greater value when $x = 20$. Justify your reasoning.

457) What is the largest integer, x, for which the value of $f(x) = 5x^4 + 30x^2 + 9$ will be greater than the value of $g(x) = 3^x$?

1) 7
2) 8
3) 9
4) 10

458) As x increases beyond 25, which function will have the largest value?

1) $f(x) = 1.5^x$
2) $g(x) = 1.5x + 3$
3) $h(x) = 1.5x^2$
4) $k(x) = 1.5x^3 + 1.5x^2$

459) Michael has \$10 in his savings account. Option 1 will add \$100 to his account each week. Option 2 will double the amount in his account at the end of each week. Write a function in terms of *x* to model each option of saving. Michael wants to have at least \$700 in his account at the end of 7 weeks to buy a mountain bike. Determine which option(s) will enable him to reach his goal. Justify your answer.

460) Caleb claims that the ordered pairs shown in the table below are from a nonlinear function.

x	**f(x)**
0	2
1	4
2	8
3	16

State if Caleb is correct. Explain your reasoning.

461) Which situation is *not* a linear function?

1) A gym charges a membership fee of \$10.00 down and \$10.00 per month.
2) A cab company charges \$2.50 initially and \$3.00 per mile.
3) A restaurant employee earns \$12.50 per hour.
4) A \$12,000 car depreciates 15% per year.

SOLUTIONS

435) ANS: 3

Strategy: Eliminate wrong answers.

a) Eliminate answer choice *a* because it describes exponential growth of money in a bank account.
b) Eliminate answer choice *b* because is describes exponential growth of bacteria.
c) Choose answer choice *c* because it can be modeled using the slope intercept formula as follows:

$$y = mx + b$$

$$\text{cost of cell phone service} = \$0.20 \times \text{ number of minutes plus the base cost}$$

d) Eliminate answer choice *d* because it describes exponential decay of medicine in the body.

PTS: 2 NAT: F.LE.A.1 TOP: Families of Functions

436) ANS: 2

1. Understand the question as asking what type of equation is needed to solve a product of consecutive integers problem.
2. Step 2. Strategy. Write the equation, then decide it it is linear, quadratic, exponential, or absolute value.
3. Step 3. Execution of Strategy.

Let x represent the first consecutive integer.
Let (x+1) represent the second consecutive integer.
Write the equation $x(x+1) = 156$
This is a quadratic equation because it will have an exponent of 2.

$$x(x+1) = 156$$

$$x^2 + x = 156$$

$$x^2 + x - 156 = 0$$

Step 4. Does it make sense? Yes. All of the other answer choices can be eliminated as wrong.

PTS: 2 NAT: A.CED.A.1 TOP: Families of Functions

437) ANS: 3

Strategy: Build a second model of the problem using a table of values.

If a population starts with 20 birds and doubles every ten years, the following table of values can be created:

Number of Years	Population of Birds
0	20
10	40
20	80
30	160
40	320

Choice a can be eliminated because it shows 20 birds after 20 years.
Choice b can be eliminated because it shows 0 birds after 20 years.
Choice c looks good because it shows 80 birds after 20 years.
Choice d can be eliminated because it shows 40 birds after 20 years.

PTS: 2 NAT: F.LE.A.2 TOP: Families of Functions
KEY: bimodalgraph

438) ANS: 3

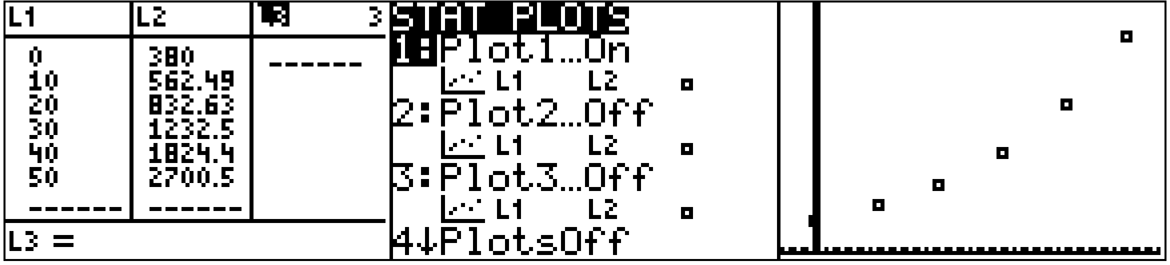
Strategy: Use $\frac{\Delta Y}{\Delta X}$ (the slope formula) to determine which table represents a constant rate of change. A linear function will have a constant rate of change.

Answer Choice	First set of coordinates	Second set of coordinates
a eliminate because *slope is not constant*	(1,1) and (2,6) $slope = \frac{6-1}{2-1} = 5$	(2,6) and (3, 13) $slope = \frac{13-6}{3-2} = 7$
b eliminate because *slope is not constant*	(1,2) and (2,4) $slope = \frac{4-2}{2-1} = 2$	(2,4) and (3, 8) $slope = \frac{8-4}{3-2} = 4$
c choose because ***slope is constant***	(1,1) and (2,3) $slope = \frac{3-1}{2-1} = 2$	(2,3) and (3, 5) $slope = \frac{5-3}{3-2} = 2$
d eliminate because *slope is not constant*	(1,1) and (2,8) $slope = \frac{8-1}{2-1} = 7$	(2,8) and (3, 27) $slope = \frac{27-8}{3-2} = 19$

PTS: 2 NAT: F.LE.A.1 TOP: Families of Functions

439) ANS: 4

Strategy: Input the table into the stats editor of a graphing calculator, then plot the points and examine the shape of the scatterplot.



L1	L2	L3
0	380	------
10	562.49	
20	832.63	
30	1232.5	
40	1824.4	
50	2700.5	
------	------	

L3 =

The data in this table creates a scatterplot that appears to model an exponential growth function.

DIMS? Does It Make Sense? Yes. Savings accounts are excellent exemplars of exponential growth.

PTS: 2 NAT: F.LE.A.1 TOP: Modeling Exponential Equations

440) ANS:

Exponential, because the function does not grow at a constant rate.

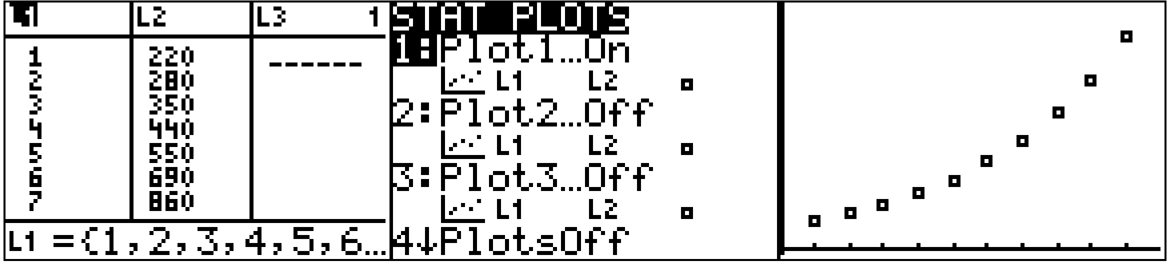
Strategy 1.

Compare the rates of change for different pairs of data using the slope formula.

Rate of change between (1, 220) and (5, 550): $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{550 - 220}{5 - 1} = \dfrac{330}{4} = 82.5$

Rate of change between (6, 690) and (10, 1680): $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1680 - 690}{10 - 6} = \dfrac{990}{4} = 247.5$

Strategy 2: Use stat plots in a graphing calculator to create a scatterplot view of the multivariate data.



```
L1    L2    L3    1
1     220   ------
2     280
3     350
4     440
5     550
6     690
7     860
L1 ={1,2,3,4,5,6…
```

```
STAT PLOTS
1:Plot1…On
  L1  L2
2:Plot2…Off
  L1  L2
3:Plot3…Off
  L1  L2
4↓PlotsOff
```

The graph view of the data clearly shows that the data is not linear.

PTS: 2 NAT: S.ID.B.6a TOP: Comparing Linear and Exponential Functions

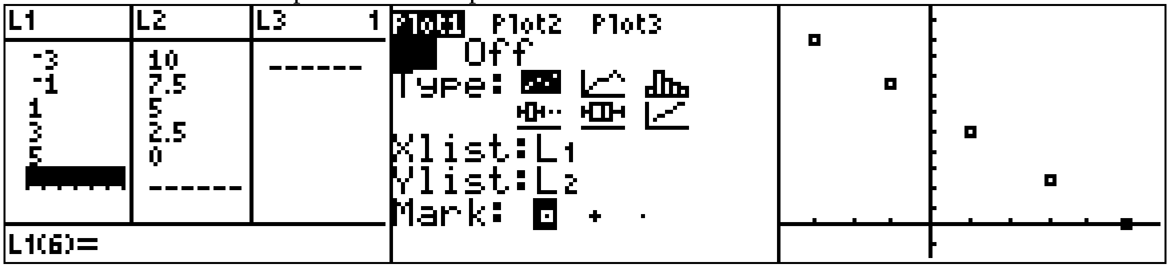
441) ANS:

Strategy #1. Calculate the change in x and the change in y for each ordered pair in the table. If the ratio of $\dfrac{\Delta y}{\Delta x}$ is constant, the function is linear.

Δx	x	t(x)	$\Delta t(x)$
	-3	10	
+2<	-1	7.5	>-2.5
+2<	1	5	>-2.5
+2<	3	2.5	>-2.5
+2<	5	0	>-2.5

This table shows a linear function, because the ratio of $\dfrac{\Delta y}{\Delta x}$ can always be expressed as $\dfrac{-2.5}{2}$.

Strategy #2. Input values from the table into the stats editor of a graphing calculator, turn stats plot on, then use zoom stat to inspect the scatterplot.



```
L1    L2    L3    1
-3    10    ------
-1    7.5
1     5
3     2.5
5     0
      ------
L1(6)=
```

```
Plot1 Plot2 Plot3
On Off
Type:
Xlist:L1
Ylist:L2
Mark: □ + ·
```

The scatterplot shows a linear relationship.

PTS: 2 NAT: F.LE.A.1 TOP: Families of Functions

442) ANS: 1

Step 1. Notice that in each of the tables, the values of the indepedendent variable (x) are 1, 2, 3, and 4, while the dependent variables are different. The question asks which table represents a linear function and, bu definition, a linear function must have a constant rate of change.
Step 2. Use the slope formula and data from each table to determine which table represents a constant rate of change.
Step 3. Execute the strategy.

f(x) $\text{rate of change} = \dfrac{f(x)_2 - f(x)_1}{x_2 - x_1}$. Every time x increases by 1, f(x) increases by 7. This is a constant rate of change, so f(x) is a linear function.

g(x) $\text{rate of change} = \dfrac{g(x)_2 - g(x)_1}{x_2 - x_1}$. Every time x increases by 1, g(x) increases by a different amount. This is not a constant rate of change, so g(x) is not a linear function.

h(x) $\text{rate of change} = \dfrac{h(x)_2 - h(x)_1}{x_2 - x_1}$. Every time x increases by 1, h(x) increases by a different amount. This is not a constant rate of change, so h(x) is not a linear function.

k(x) $\text{rate of change} = \dfrac{k(x)_2 - k(x)_1}{x_2 - x_1}$. Every time x increases by 1, k(x) increases by a different amount. This is not a constant rate of change, so k(x) is not a linear function.

Step 4. Does it make sense? Yes. Only one table shows a constant rate of change.

PTS: 2 NAT: F.LE.A.1 TOP: Families of Functions

443) ANS: 1

Interpreting the Question: Equal differences over equal intervals suggests a constant rate of change, which would be a linear relationship.
Strategy: Model each situation with a function rule, then select the linear functions.
I. For the first 28 days, a sunflower grows at a rate of 3.5 cm per day.
This can be modeled with the **linear** function $h = 3.5d$, where h represents the height of the sunflower and d represents the number of days. Since this function is linear, it represents a situation with an equal difference over an equal interval.
II. The value of a car depreciates at a rate of 15% per year after it is purchased.
This can be modeled with the **exponential decay** function $V = P(1 - .15)^t$, where V represents the value of the car, P represents its price when purchased, -.15 represents the annual depreciation rate, and t represents the number of years after purchase. This is an exponential decay function, so it does not represent a situation with an equal difference over an equal interval.
III. The amount of bacteria in a culture triples every two days during an experiment.
This can be modeled with the **exponential growth** function $A = B(3)^{\frac{d}{2}}$, where A represents the amount of bacteria, B represents starting amount of bacteria, 3 represents the growth rate, and $\dfrac{d}{2}$ represents the number of growth cyles. This is an exponential growth function, so it does not represent a situation with an equal difference over an equal interval.
The only choice that represents a situation with an equal difference over an equal interval is the first situation.

PTS: 2 NAT: F.LE.A.1 TOP: Families of Functions

444) ANS:

Exponential. The rate of change is not constant, so a linear model must be eliminated.

Strategy: Build a table of values, as follows:

n	1	2	3	4	5	6	7	n
f(n)	2	4	8	16	32	64	128	2^n

The pattern can be modeled using the exponential function $f(n) = 2^n$.

PTS: 2 NAT: F.LE.A.1

445) ANS: 3

Strategy: Eliminate wrong ansers.

a) A water tank is filled at a rate of 2 gallons/minute. A rate of 2 gallons a minute is a constant rate of change, so this cannot be an exponential function.

b) A vine grows 6 inches every week. A rate of 6 inches every week is a constant rate of change, so this cannot be an exponential function.

c) A species of fly doubles its population every month during the summer. A rate of change that doubles every month is not constant. The population of flies could be modeled with the following exponential equation.

$$\text{Population} = \text{starting amount}(2)^{\#\text{months}}$$

d) A car increases its distance from a garage as it travels at a constant speed of 25 miles per hour.

PTS: 2 NAT: F.LE.A.1 TOP: Families of Functions

446) ANS: 3

All linear functions must have constant rates of change, which means equal differences over equal intervals.

PTS: 2 NAT: F.LE.A.1 TOP: Families of Functions

447) ANS: 1

The rate of change is constant (-10 points per day), so it must be a linear function.

PTS: 2 NAT: F.LE.A.1 TOP: Families of Functions

448) ANS: 1

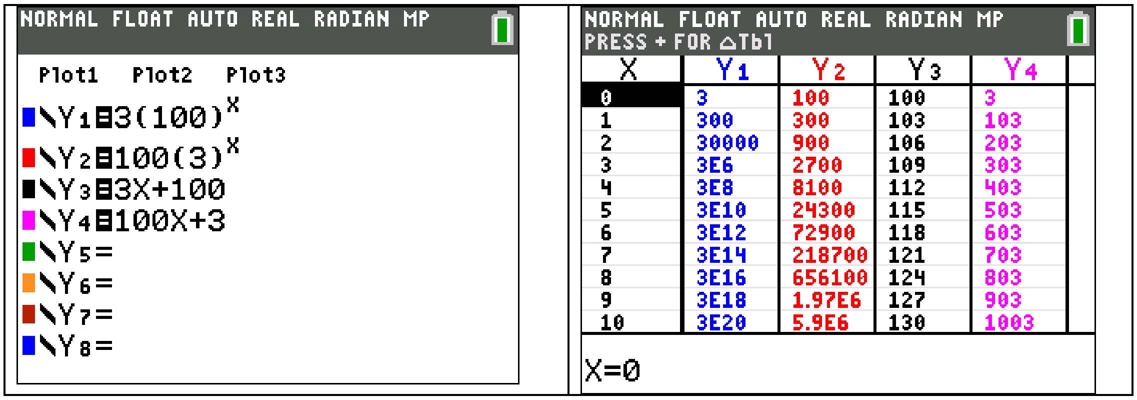
The sentence “Every month he puts $10 into a jar” indicates a constant rate of change. Linear functions represent constant rates of change.

PTS: 2 NAT: F.LE.A.1 TOP: Families of Functions

449) ANS: 2

Build a table that models the growth, then test the answer choices to see which one produces the table.

t	0	1	2	3
p(t)	100	300	900	2700



NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

$Y_1 = 3(100)^X$

$Y_2 = 100(3)^X$

$Y_3 = 3X+100$

$Y_4 = 100X+3$

$Y_5 =$

$Y_6 =$

$Y_7 =$

$Y_8 =$

NORMAL FLOAT AUTO REAL RADIAN MP

PRESS + FOR △Tbl

X	Y1	Y2	Y3	Y4
0	3	100	100	3
1	300	300	103	103
2	30000	900	106	203
3	3E6	2700	109	303
4	3E8	8100	112	403
5	3E10	24300	115	503
6	3E12	72900	118	603
7	3E14	218700	121	703
8	3E16	656100	124	803
9	3E18	1.97E6	127	903
10	3E20	5.9E6	130	1003

X=0

$p(t) = 100(3)^t$ is the correct answer because it reproduces the table view of the function.

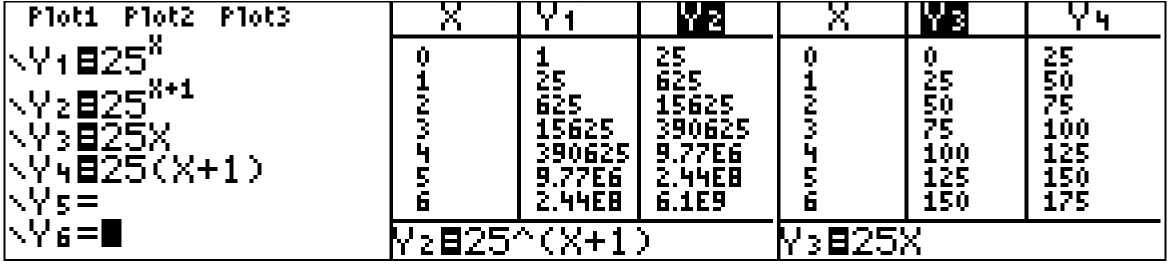
PTS: 2 NAT: F.LE.A.2 TOP: Families of Functions
KEY: AI

450) ANS: 3
Strategy: Test each function to see if it fits the table:

Choice	Equation	(3,9)	(6,65)	(8,257)
a	$F(x) = 3^x$	$F(3) = 3^3 = 27$ (eliminate)		
b	$F(x) = 3x$	$F(3) = 3(3) = 9$ (correct)	$F(6) = 3(6) = 18$ (eliminate)	
c	$F(x) = 2^x + 1$	$F(3) = 2^3 + 1 = 9$ (correct)	$F(6) = 2^6 + 1 = 65$ (correct)	$F(8) = 2^8 + 1 = 257$ (correct)
d	$F(x) = 2x + 3$	$F(3) = 2(3) + 3 = 9$ (correct)	$F(6) = 2(6) + 3 = 15$ (eliminate)	

PTS: 2 NAT: F.LE.A.2 TOP: Modeling Exponential Equations

451) ANS: 2
Strategy: Input all four functions into a graphing calculator and compare the table of values.



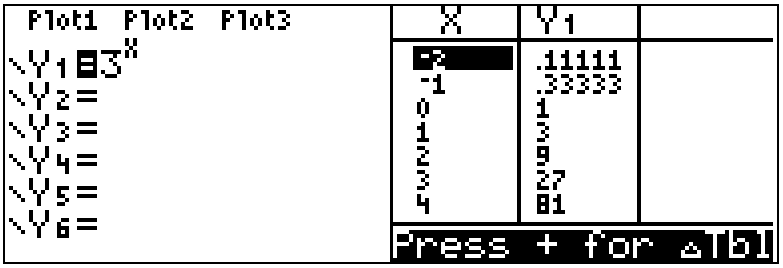
```
 Plot1  Plot2  Plot3
\Y1=25^X
\Y2=25^(X+1)
\Y3=25X
\Y4=25(X+1)
\Y5=
\Y6=
```

```
   X  |   Y1   |   Y2         X  |  Y3  |  Y4
   0  | 1      | 25           0  | 0    | 25
   1  | 25     | 625          1  | 25   | 50
   2  | 625    | 15625        2  | 50   | 75
   3  | 15625  | 390625       3  | 75   | 100
   4  | 390625 | 9.77E6       4  | 100  | 125
   5  | 9.77E6 | 2.44E8       5  | 125  | 150
   6  | 2.44E8 | 6.1E9        6  | 150  | 175
Y2=25^(X+1)                Y3=25X
```

Answer choice *b* produces a table of values that agrees with the table of values in the problem.

PTS: 2 NAT: F.LE.A.2 TOP: Modeling Linear and Exponential Equations

452) ANS: 4
Strategy: Put the functions in a graphing calculator and inspect the table view. The correct answer is $f(x) = 3^x$.



```
 Plot1  Plot2  Plot3
\Y1=3^X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

```
   X  |  Y1
  -2  | .11111
  -1  | .33333
   0  | 1
   1  | 3
   2  | 9
   3  | 27
   4  | 81
Press + for ΔTbl
```

PTS: 2 NAT: F.LE.A.2 TOP: Families of Functions

453) ANS: 1
Note that the graph represents an exponential function.

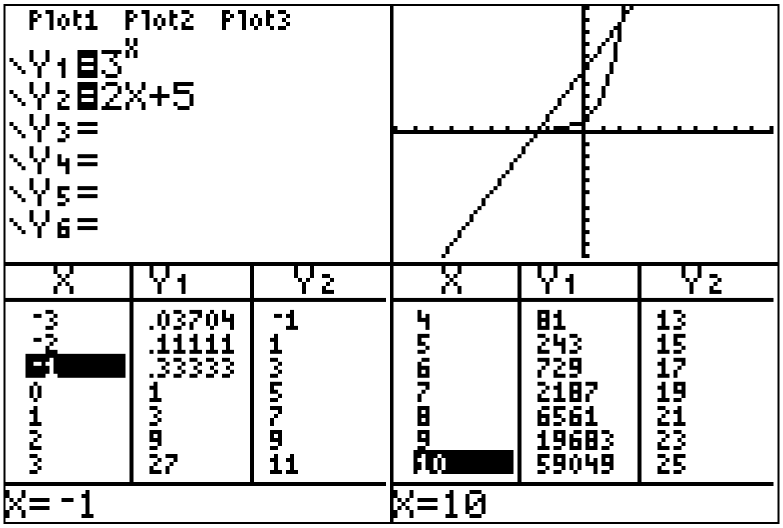
Choice	Family of Functions	Standard Form
a $P(h) = 4(2)^h$	Exponential	$y = ab^x$
b $P(h) = \frac{46}{5}h + \frac{6}{5}$	~~Linear~~	$y = mx + b$
c $P(h) = 3h^2 + 0.2h + 4.2$	~~Quadratic~~	$y = ax^2 + bx + c$

d $P(h) = \frac{2}{3}h^3 - h^2 + 3h + 4$	~~Cubic~~	$y = ax^3 + bx^2 + cx + d$

PTS: 2 NAT: F.LE.A.2 TOP: Families of Functions

454) ANS: 1

Strategy: Input both functions in a graphing calculator and compares the values of y for various values of x.



```
Plot1  Plot2  Plot3
\Y1=3^X
\Y2=2X+5
\Y3=
\Y4=
\Y5=
\Y6=

X     Y1      Y2        X     Y1      Y2
-3    .03704  -1        4     81      13
-2    .11111  1         5     243     15
-1    .33333  3         6     729     17
0     1       5         7     2187    19
1     3       7         8     6561    21
2     9       9         9     19683   23
3     27      11        10    59049   25
X=-1                    X=10
```

The table of values shows:

When $x = -1$, $f(x) < g(x)$

When $x = 2$, $f(x) = g(x)$

When $x = -3$, $f(x) > g(x)$

When $x = 4$, $f(x) > g(x)$

PTS: 2 NAT: F.LE.A.3 TOP: Families of Functions

455) ANS: 3

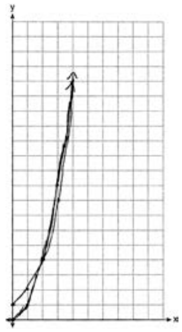
Strategy: Build a table of values for the integer values of the domain $6 \le x \le 9$ to compare both offers.

x	$A = 5000x + 10000$	$B = 500(2)^{x-1}$
6	40,000	16,000
7	45,000	32,000
8	50,000	64,000
9	55,000	128,000

Offer B is greater than offer A when $x = 8$.

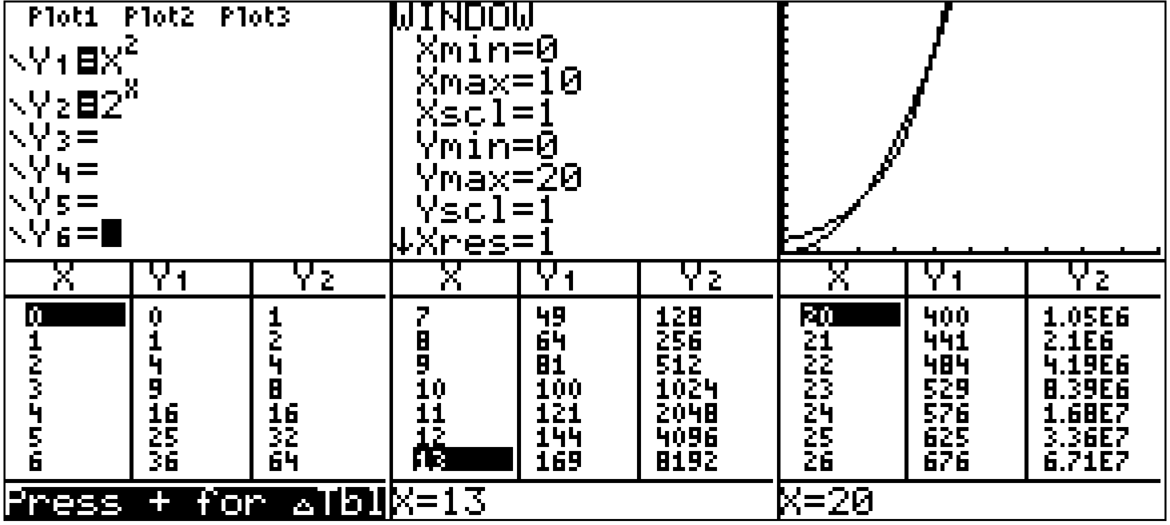
PTS: 2 NAT: F.LE.A.3 TOP: Comparing Linear and Exponential Functions

456) ANS:



$g(x)$ has a greater value: $2^{20} > 20^2$

Strategy: Input both functions in a graphing calculator, use the table of values to create the paper graph, and to compare the values of *y* for various values of *x*.



```
Plot1  Plot2  Plot3
\Y1=X²
\Y2=2^X
\Y3=
\Y4=
\Y5=
\Y6=
```

```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=20
 Yscl=1
↓Xres=1
```

```
X     Y1    Y2
0     0     1
1     1     2
2     4     4
3     9     8
4     16    16
5     25    32
6     36    64
Press + for ΔTbl
```

```
X     Y1    Y2
7     49    128
8     64    256
9     81    512
10    100   1024
11    121   2048
12    144   4096
13    169   8192
X=13
```

```
X     Y1    Y2
20    400   1.05E6
21    441   2.1E6
22    484   4.19E6
23    529   8.39E6
24    576   1.68E7
25    625   3.36E7
26    676   6.71E7
X=20
```

The table of values shows that when $x = 20$, $g(x) > f(x)$.

DIMS? Does It Make Sense? Yes. $2^{20} > 20^2$

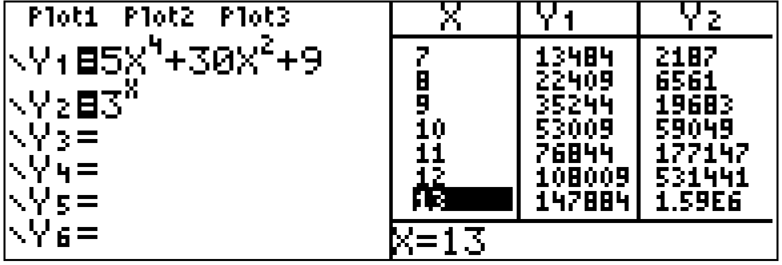
PTS: 4 NAT: F.LE.A.3 TOP: Comparing Quadratic and Exponential Functions

457) ANS: 3

Step 1. Understand that the problem asks you to select the largest value of x where the value of f(x) will be greater than the value of g(x).

Step 2. Strategy. Input both functions in a graphing calculator and explore the table of values.

Step 3. Execution of Strategy.



```
Plot1  Plot2  Plot3
\Y1=5X⁴+30X²+9
\Y2=3^X
\Y3=
\Y4=
\Y5=
\Y6=
```

```
X     Y1      Y2
7     13484   2187
8     22409   6561
9     35244   19683
10    53009   59049
11    76844   177147
12    108009  531441
13    147884  1.59E6
X=13
```

The table shows that f(x) is greater than g(x) when $x = 7$, $x = 8$, and $x = 9$, but not when $x = 10$. The largest integer for which f(x) is greater than g(x) is 9.

Step 4. Does it make sense? Yes. $f(x) = 5x^4 + 30x^2 + 9$ is a quadratic function and $g(x) = 3^x$ is an exponential function. Exponential growth evetually outpaces quadratic growth.

PTS: 2 NAT: F.LE.A.3 TOP: Families of Functions

458) ANS: 1

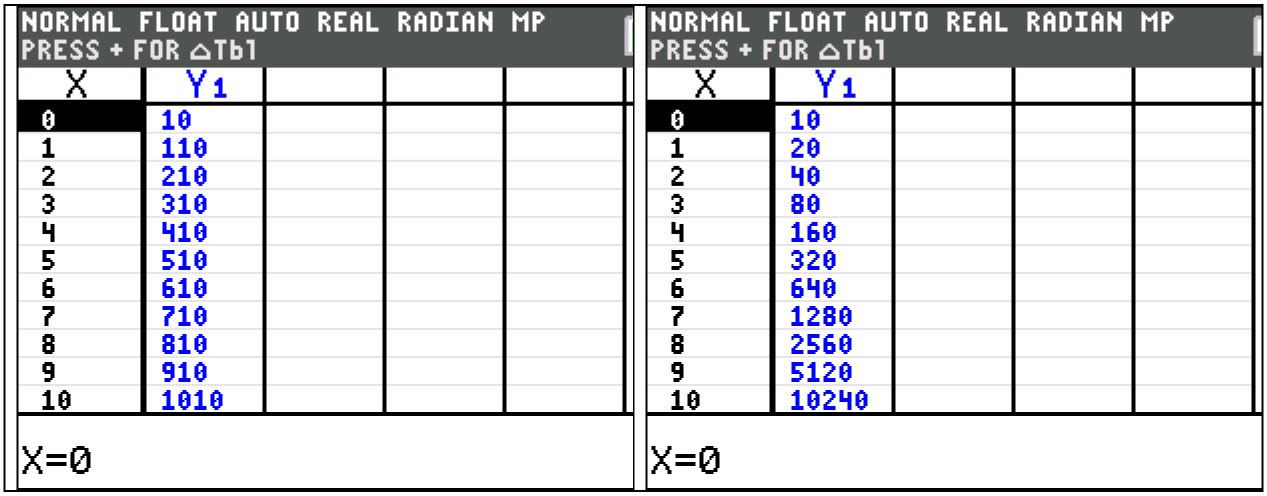
Strategy: Input all functions in a graphing calculator and inspect the table of values.

x	f(x)	g(x)	h(x)	k(x)
25	25,251	40.5	937.5	24,375

PTS: 2 NAT: F.LE.A.3

459) ANS:

Option 1 can be modeled by the function $A(x) = 10 + 100x$	Option 2 can be modeled by the function $B(x) = 10(2)^x$



```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl
X     Y1
0     10
1     110
2     210
3     310
4     410
5     510
6     610
7     710
8     810
9     910
10    1010
X=0
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl
X     Y1
0     10
1     20
2     40
3     80
4     160
5     320
6     640
7     1280
8     2560
9     5120
10    10240
X=0
```

Either option will allow Michael to have at least $700 in his account at the end of 7 weeks.

$$A(7) = 10 + 100(7) \quad \text{and} \quad B(7) = 10(2)^7$$

$$A(7) = 710 \qquad\qquad B(7) = 1280$$

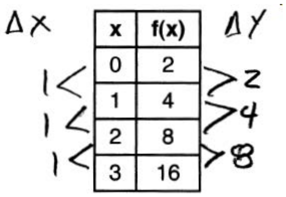
$f(x) = 10 + 100x$, $g(x) = 10(2)^x$; both, since $f(7) = 10 + 100(7) = 710$ and $g(7) = 10(2)^7 = 1280$

PTS: 4 NAT: F.LE.A.3 TOP: Families of Functions

460) ANS:

Yes. Caleb is correct because the function does not have a constant rate of change. A linear function must have a constant rate of change.

Strategy: Compare the rate of change $\left(\frac{\Delta y}{\Delta x}\right)$ between each row of the table of values.



x	f(x)
0	2
1	4
2	8
3	16

$$\frac{\Delta y}{\Delta x} \Big| \quad \frac{2}{1} \neq \frac{4}{1} \neq \frac{8}{1}$$

PTS: 2 NAT: F.LE.A.1 TOP: Families of Functions

461) ANS: 4

Strategy: A linear function will have a constant rate of change, so find the function that does not have a constant rate of change.

Choice 1: $10.00 per month is a constant rate, so eliminate this choice.

Choice 2: $3.00 per mile is a constant rate, so eliminate this choice

Choice 3: $12.50 per hour is a constant rate, so eliminate this choice

Choice 4: depreciates 15% per year is an exponential rate of change. The amount of change gets smaller each year as the car depreciates in value. This is the correct answer because it is not a constant rate of change and all linear functions must have constant rates of change.

PTS: 2 NAT: F.LE.A.1 TOP: Families of Functions

M – Functions, Lesson 6, Transformations with Functions (r. 2018)

FUNCTIONS
Transformations with Functions

Common Core Standard	**Next Generation Standard**
F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, ~~$f(kx)$~~, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing ~~even and odd~~ functions from their graphs and algebraic expressions for them. ~~PARCC: Identifying the effect on the graph of replacing $f(x)$ by $f(x) +k$, $kf(x)$, and $f(x+k)$ for specific values of k (both positive and negative) is limited to linear and quadratic functions. Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. Tasks do not involve recognizing even and odd functions.~~	**AI-F.BF.3a** Using $f(x) + k$, $k\,f(x)$, and $f(x + k)$: i) identify the effect on the graph when replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, and $f(x + k)$ for specific values of k (both positive and negative); ii) find the value of k given the graphs; **iii) write a new function using the value of *k*;** and iv) use technology to experiment with cases and explore the effects on the graph. (Shared standard with Algebra II) **Note: Tasks are limited to linear, quadratic, square root, and absolute value functions; and exponential functions of the form** $f(x) = a(b)^x$ **where *a* > 0 and *b* > 0 (*b* ≠ 1).**

NOTE: This lesson is related to Polynomials, Lesson 6, Graphing Polynomial Functions

LEARNING OBJECTIVES

Students will be able to:

1)

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡸Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

down
function
left
right
transform
up

BIG IDEAS

Transforming Any Function

The graph of any function is changed when either $f(x)$ or x is multiplied by a scalar, or when a constant is added to or subtracted from either $f(x)$ or x. A graphing calculator can be used to explore the translations of graph views of functions.

Up and Down

The addition or subtraction of a constant **outside the parentheses** moves the graph up or down by the value of the constant.

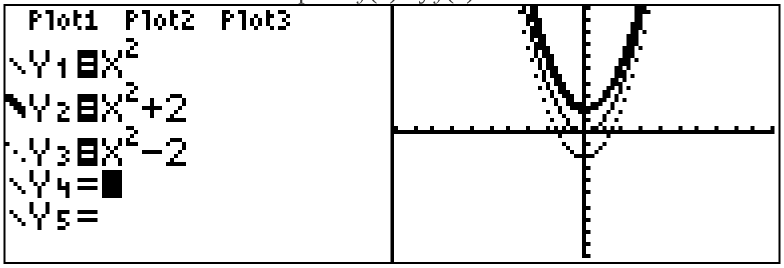
$f(x) \Leftrightarrow f(x) \pm k$ moves the graph up or down k units $\updownarrow$.

+k moves the graph up.

-k moves the graph down.

Examples:

Replace $f(x)$ by $f(x) + k$



```
 Plot1  Plot2  Plot3
\Y1■X²
\Y2■X²+2
\Y3■X²-2
\Y4=■
\Y5=
```

Left and Right

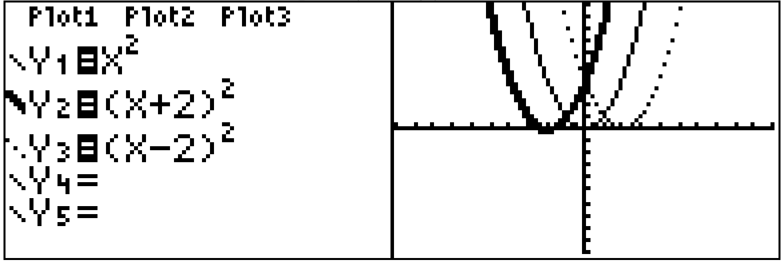
The addition or subtraction of a constant **inside the parentheses** moves the graph left or right by the value of the constant.

$f(x) \Leftrightarrow f(x \pm k)$ moves the graph left or right k units $\updownarrow$.

+k moves the graph leftg k units.

-k moves the graph right k units.

Replace $f(x)$ by $f(x + k)$



```
 Plot1  Plot2  Plot3
\Y1■X²
\Y2■(X+2)²
\Y3■(X-2)²
\Y4=
\Y5=
```

Width and Direction of a Parabola

Changing the value of *a* in a quadratic affects the width and direction of a parabola. The bigger the absolute value of *a*, the narrower the parabola.

$f(x) \Leftrightarrow f(kx)$ changes the direction and width of a parabola.

+k opens the parabola upward.

-k opens the parabola downward.

If k is a fraction less than 1, the parabola will get wider.

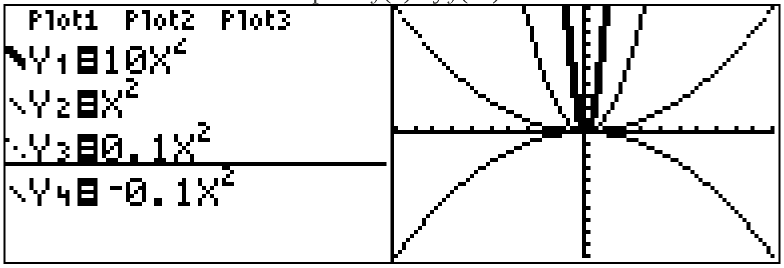
As k approaches zero, the parabola approaches a straight horizontal line.

If k is a number greater than 1, the parabola will get narrower.

As k approaches infinity, the parabola approaches a straight vertical line.

Examples:

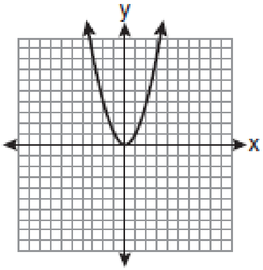
Replace *f*(*x*) by *f*(*kx*)



```
 Plot1  Plot2  Plot3
\Y1=10X²
\Y2=X²
\Y3=0.1X²
\Y4= -0.1X²
```

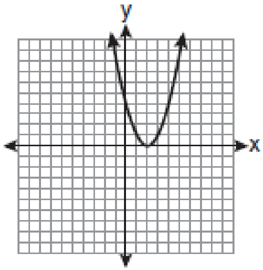
DEVELOPING ESSENTIAL SKILLS

1. The graph below shows the function $f(x)$.

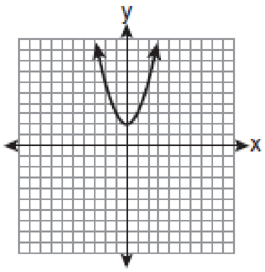


Which graph represents the function $f(x+2)$?

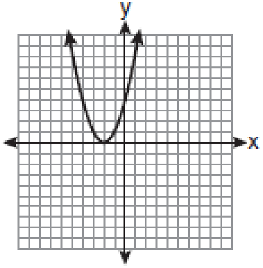
a.



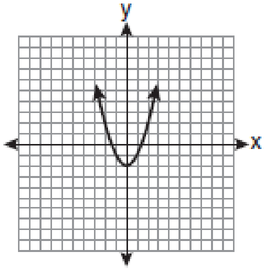
c.



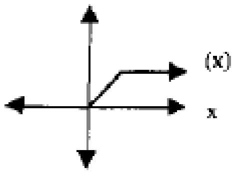
b.



d.

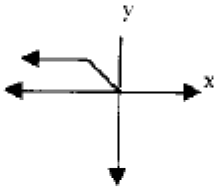


2.The graph below represents $f(x)$.

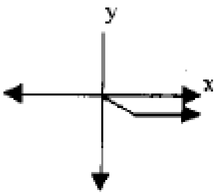


Which of the following is the graph of $-f(x)$?

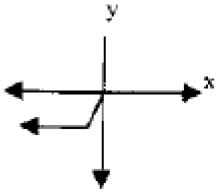
a.



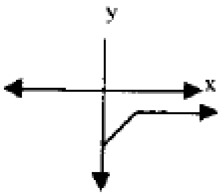
c.



b.



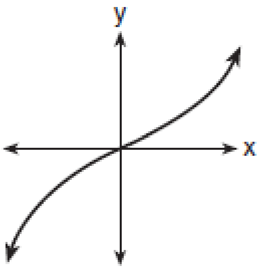
d.



3. The minimum point on the graph of the equation $y = f(x)$ is $(-1,-3)$. What is the minimum point on the graph of the equation $y = f(x) + 5$?

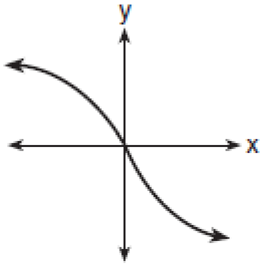
a.	$(-1,2)$	c.	$(4,-3)$
b.	$(-1,-8)$	d.	$(-6,-3)$

4. The graph below represents $f(x)$.

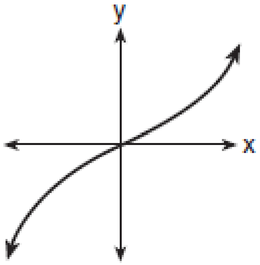


Which graph best represents $f(-x)$?

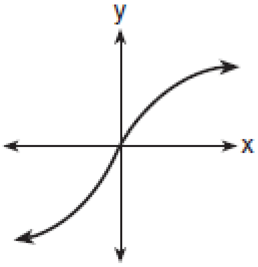
a.



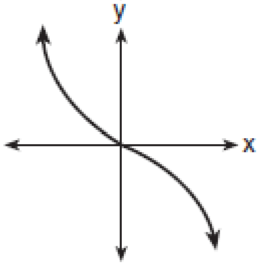
c.



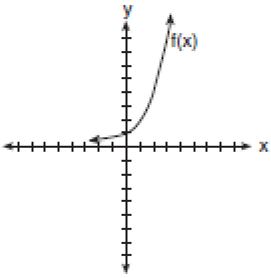
b.



d.

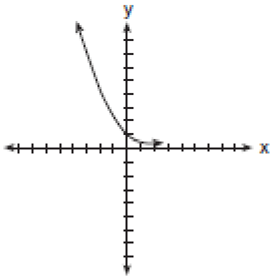


5. The graph of $\mathrm{f}(x)$ is shown in the accompanying diagram

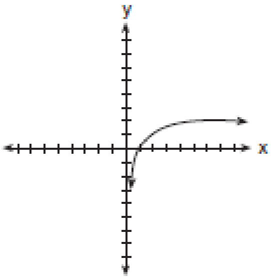


Which graph represents $\mathrm{f}(x)_{r_{x-\text{axis}} \circ r_{y-\text{axis}}}$?

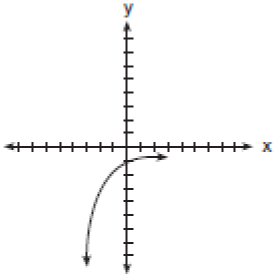
a.



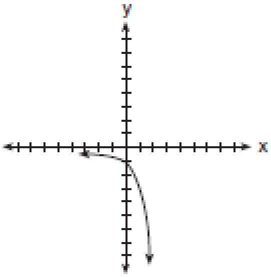
c.



b.



d.



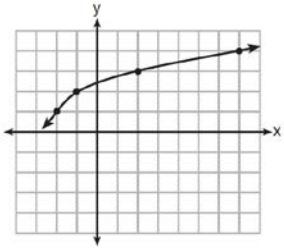
ANSWERS

1. ANS: B
2. ANS: C
3. ANS: A
4. ANS: D
5. ANS: B

REGENTS EXAM QUESTIONS (through June 2018)

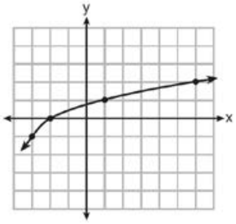
F.BF.B.3: Transformations with Functions

462) The graph of $y = f(x)$ is shown below.

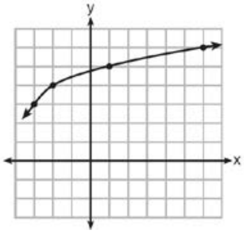


What is the graph of $y = f(x + 1) - 2$?

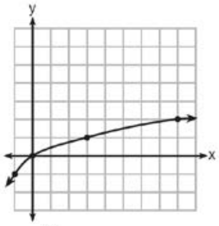
1)



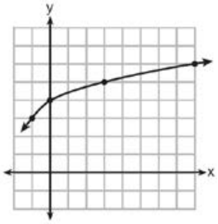
2)



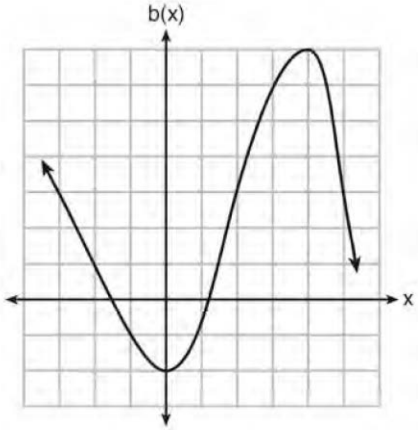
3)



4)



463) Richard is asked to transform the graph of *b(x)* below.



The graph of *b(x)* is transformed using the equation $h(x) = b(x - 2) - 3$. Describe how the graph of *b(x)* changed to form the graph of *h(x)*.

SOLUTIONS

462) ANS: 1

Strategy: Identify the differences between the two function rules, then verify using the four points shown in the answer choices.

Function rules:

Difference #1: The term $f(x)$ becomes $f(x+1)$. This means the graph will move to the left 1 unit. The mapping of each x value can be expressed as $(x) \rightarrow (x-1)$

Difference #2: The term -2 is added to the function rule. This means the graph will move 2 units down. The mapping of each y value can be expressed as $(y) \rightarrow (y-2)$.

The 2 differences in the function rules mean that each point on the graph will move left 1 unit and down 2 units. Answer choice (a) shows this:

$y = f(x)$	(-2, 1)	(-1, 2)	(2, 3)	(7, 7)
$y = f(x+1) - 2$	(-3, -1)	(-2, 0)	(1, 1)	(6, 2)

PTS: 2 NAT: F.BF.B.3 TOP: Graphing Radical Functions

463) ANS:

Every point moves down 3 units.
Every point moves right 2 units.

PTS: 2 NAT: F.BF.B.3

M – Functions, Lesson 7, Comparing Functions (r. 2018)

FUNCTIONS

Comparing Functions

CC Standard	NG Standard
F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* PARCC: Tasks are limited to linear functions, quadratic functions, square root, cube root, piecewise defined (including step functions and absolute value functions), and exponential functions with domains in the integers.	**AI-F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (Shared standard with Algebra II) Note: Algebra I tasks are limited to the following functions: linear, quadratic, square root, piecewise defined (including step and absolute value), and **exponential functions of the form** $f(x) = a(b)^x$ **where** $a > 0$ **and** $b > 0$ $(b \neq 1)$.

LEARNING OBJECTIVES

Students will be able to:

1) Compare properties of two functions each represented in a different way.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

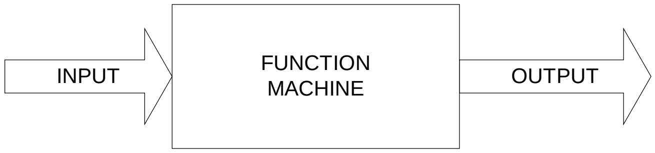
context
equation
four views of a function
function rule

graph
maximum
minimum
table of values

vertex
x-intercept
y-intercept

BIG IDEAS

Definition of a Function: a function takes the input value of an independent variable and pairs it with one and only one output value of a dependent variable.



Function: A function is a relation that assigns exactly one value of the dependent variable to each value of the independent variable. A function is always a relation.

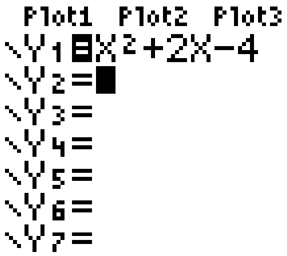
Example: y=2x

A function can be represented mathematically through four inter-related views. These are:

#1 a function rule (equation)
#2 a table of values
#3 a graph.
#4 context (words)

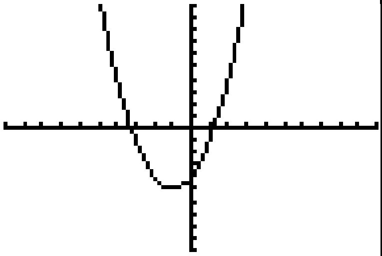
The TI-83+ graphing calculator allows you to input the function rule and access the graph and table of values, as shown below:

Y= Function Rule View

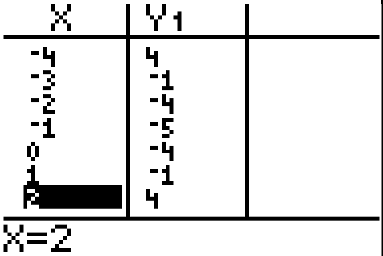


```
Plot1 Plot2 Plot3
\Y1=X²+2X-4
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

GRAPH View



2nd TABLE View



```
X     Y1
-4    4
-3    -1
-2    -4
-1    -5
0     -4
1     -1
2     4
X=2
```

Function Rules show the relationship between dependent and independent variables in the form of an equation with two variables.

- § The **independent** variable is the **input** of the function and is typically denoted by the x-variable.
- § The **dependent** variable is the **output** of the function and is typically denoted by the y-variable.

When inputting function rules in a TI 83+ graphing calculator, the y-value (dependent variable) must be isolated as the left expression of the equation.

Tables of Values show the relationship between dependent and independent variables in the form of a table with columns and rows:

- § The **independent** variable is the **input** of the function and is typically shown in the left column of a vertical table or the top row of a horizontal table.
- § The **dependent** variable is the **output** of the function and is typically shown in the right column of a vertical table or the bottom row of a horizontal table.

Graphs show the relationship between dependent and independent variables in the form of line or curve on a coordinate plane:

- § The value of **independent** variable is the **input** of the function and is typically shown on the **x-axis** (horizontal axis) of the coordinate plane.
- § The value of the **dependent** variable is the **output** of the function and is typically shown on the **y-axis** (vertical axis) of the coordinate plane.

Name: ________________________________

DEVELOPING ESSENTIAL SKILLS

1 The *x*-value of which function's *x*-intercept is larger, *f* or *h*? Justify your answer.

$$f(x) = x - 5$$

x	**h(x)**
−1	6
0	4
1	2
2	0
3	−2

2 Consider the function $p(x) = x^2 - 2x - 4$ and the function *q* represented in the table below.

x	***q*(*x*)**
−2	−8
−1	0
0	0
1	−2
2	0

Determine which function has the *smaller* minimum value for the domain $[-2, 2]$. Justify your answer.

Name: ________________________________

3 Which function shown below has a greater average rate of change on the interval $[-2,4]$? Justify your answer.

$$g(x) = 4x^3 - 5x^2 + 3$$

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

1 ANS: *f(x)* The graph of *f(x)* crosses the x-axis when x = 5. The graph of *h(x)* crosses the x-axis when x = 2.

2 ANS: *q* has the smaller minimum value for the domain $[-2,2]$. *p*'s minimum is -5 *q*'s minimum is -8.

3 ANS: $g(x)$ has a greater rate of change

$$\frac{f(4)-f(-2)}{4--2} = \frac{80-1.25}{6} = 13.125$$

$$\frac{g(4)-g(-2)}{4--2} = \frac{179--49}{6} = 38$$

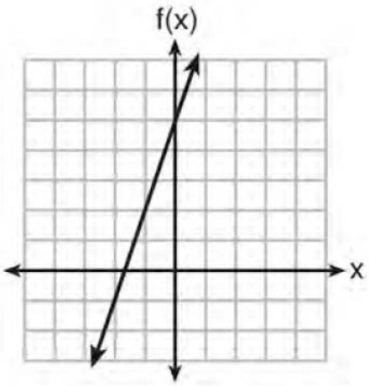
Name: ___________________________________

REGENTS EXAM QUESTIONS (through June 2018)

F.IF.C.9: Comparing Functions

464) Which function has the greatest y-intercept?

1) $f(x) = 3x$
2) $2x + 3y = 12$
3) the line that has a slope of 2 and passes through (1, -4).
4)



465) Given the following quadratic functions:

$$g(x) = -x^2 - x + 6$$

and

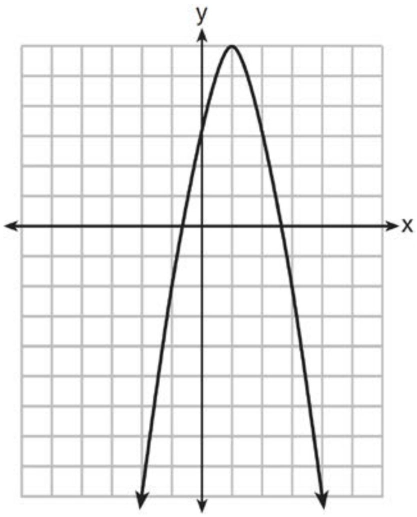
x	–3	–2	–1	0	1	2	3	4	5
n(x)	–7	0	5	8	9	8	5	0	–7

Which statement about these functions is true?

1) Over the interval $-1 \le x \le 1$, the average rate of change for $n(x)$ is less than that for $g(x)$.
2) The y-intercept of $g(x)$ is greater than the y-intercept for $n(x)$.
3) The function $g(x)$ has a greater maximum value than $n(x)$.
4) The sum of the roots of $n(x) = 0$ is greater than the sum of the roots of $g(x) = 0$.

466) Let f be the function represented by the graph below.

Name: ________________________________



Let g be a function such that $g(x) = -\frac{1}{2}x^2 + 4x + 3$. Determine which function has the larger maximum value. Justify your answer.

467) Which quadratic function has the largest maximum?

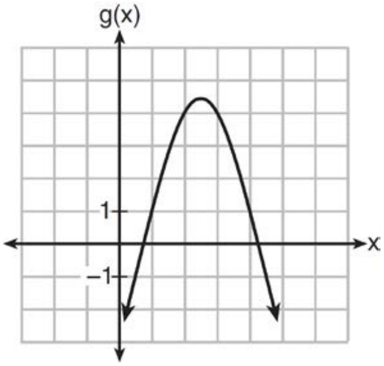
1) $h(x) = (3 - x)(2 + x)$

2)

x	f(x)
−1	−3
0	5
1	9
2	9
3	5
4	−3

3) $k(x) = -5x^2 - 12x + 4$

4)

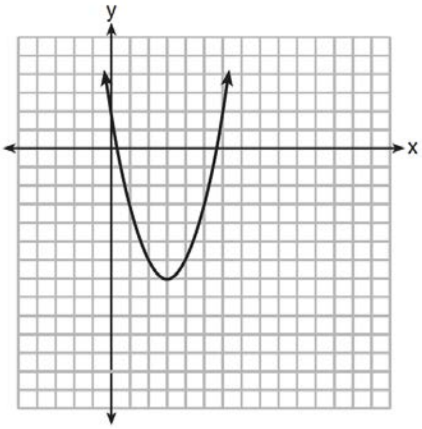


468) Which statement is true about the quadratic functions $g(x)$, shown in the table below, and $f(x) = (x - 3)^2 + 2$?

x	g(x)
0	4
1	−1
2	−4
3	−5
4	−4
5	−1
6	4

1) They have the same vertex.
2) They have the same zeros.
3) They have the same axis of symmetry.
4) They intersect at two points.

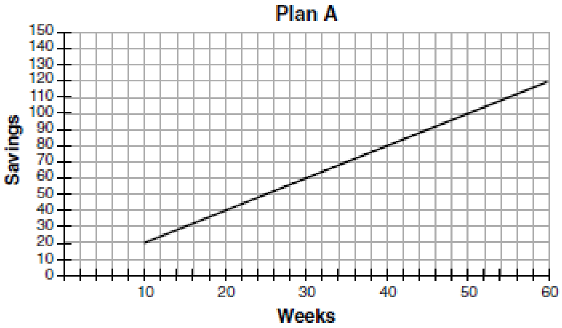
469) The graph representing a function is shown below.



Which function has a minimum that is *less* than the one shown in the graph?

1) $y = x^2 - 6x + 7$
2) $y = |x + 3| - 6$
3) $y = x^2 - 2x - 10$
4) $y = |x - 8| + 2$

470) Nancy works for a company that offers two types of savings plans. Plan *A* is represented on the graph below.

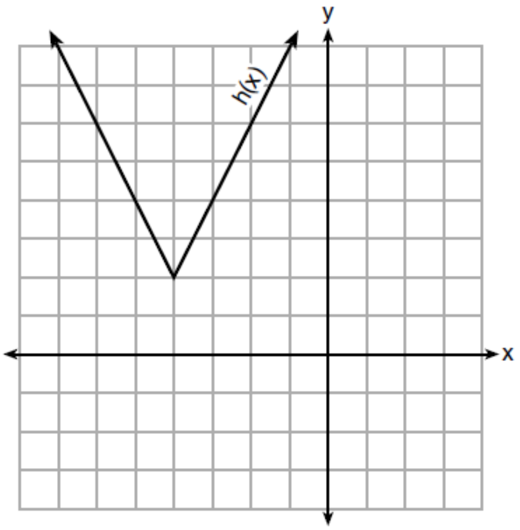


Plan *B* is represented by the function $f(x) = 0.01 + 0.05x^2$, where *x* is the number of weeks. Nancy wants to have the highest savings possible after a year. Nancy picks Plan *B*. Her decision is

1) correct, because Plan *B* is an exponential function and will increase at a faster rate
2) correct, because Plan *B* is a quadratic function and will increase at a faster rate
3) incorrect, because Plan *A* will have a higher value after 1 year
4) incorrect, because Plan *B* is a quadratic function and will increase at a slower rate

471) The function $h(x)$, which is graphed below, and the function $g(x) = 2|x + 4| - 3$ are given.

Name: ___________________________________



Which statements about these functions are true?

I. $g(x)$ has a lower minimum value than $h(x)$.

II. For all values of x, $h(x) < g(x)$.

III. For any value of x, $g(x) \neq h(x)$.

1) I and II, only
2) I and III, only
3) II and III, only
4) I, II, and III

472) Which quadratic function has the largest maximum over the set of real numbers?

1) $f(x) = -x^2 + 2x + 4$

2)

x	k(x)
–1	–1
0	3
1	5
2	5
3	3
4	–1

3) $g(x) = -(x-5)^2 + 5$

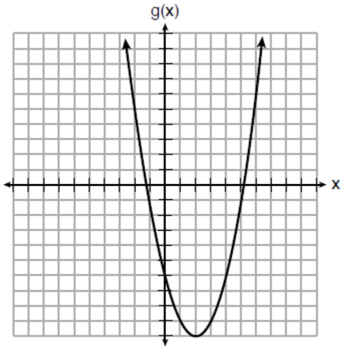
4)

x	h(x)
–2	–9
–1	–3
0	1
1	3
2	3
3	1

473) Which of the quadratic functions below has the *smallest* minimum value?

1) $h(x) = x^2 + 2x - 6$

2)



(2)

3) $k(x) = (x+5)(x+2)$

4)

x	f(x)
–1	–2
0	–5
1	–6
2	–5
3	–2

Name: ___________________________________

SOLUTIONS

464) ANS: 4

Strategy: Find y-intercept for each answer choice, then eliminate wrong answers.

Eliminate $f(x) = 3x$ because $f(0) = 3(0) = 0$.

Eliminate $2x + 3y = 12$ because
$$\begin{aligned} 2(0) + 3y &= 12 \\ 3y &= 12 \\ y &= 4 \end{aligned}$$

Eliminate the line that has slope of 2 and passes through (1, -4) because it has a positive slope and it's y-intercept must be less than -4.

Choose the graph because the y-intecept is 5, which is greater than the y-intercepts of the other three choices.

PTS: 2 NAT: F.IF.C.9

465) ANS: 4

Strategy: Each answer choice must be evaluated using a different strategy.

a. Use the slope formula to find the rate of change for

$$m_{g(x)} = \frac{[g(1)] - [g(-1)]}{[1] - [-1]} = \frac{4-6}{2} = \frac{-2}{2} = -1$$

$$m_{n(x)} = \frac{[n(1)] - [n(-1)]}{[1] - [-1]} = \frac{9-5}{2} = \frac{4}{2} = 2$$

Statement a is false. The average rate of change for $n(x)$ is *more* than that for $g(x)$.

b. Compare the y-intercepts for both functions. The y-intercepts occur when $x = 0$.

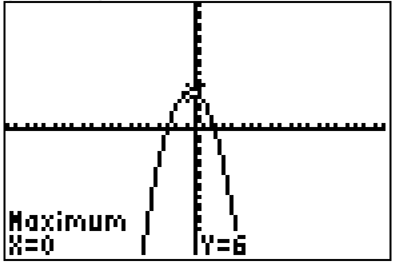
The y-intercept for g(x) = 6. $g(0) = -0^2 - 0 + 6 = 6$

The y-intercept for n(x) = 8 from the table.

Statement b is false. The *y*-intercept of $g(x)$ is *less* than the *y*-intercept for $n(x)$.

c. Compare the maxima of both functions.

The maxima of $g(x) = -x^2 - x + 6$ is 6. This can be found manually or with a graphing calculator.

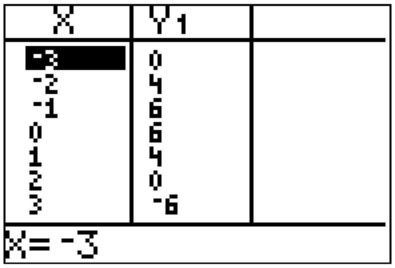


The maxima of n(x) = 9, which can be seen in the table.

Statement c is false. The function $g(x)$ has a *smaller* maximum value than $n(x)$.

d. Compare the sum of the roots for both functions.

The sum of the roots for g(x) = $-3 + 2 = -1$ from a graphing calculator.



X	Y1
-3	0
-2	4
-1	6
0	6
1	4
2	0
3	-6

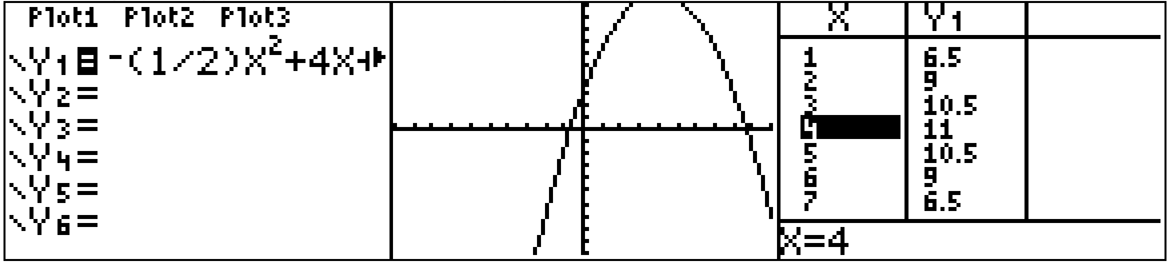
X= -3

The sum of the roots for n(x) = $-2+4=2$ from the table.
Statement d is true. The sum of the roots of $n(x)=0$ is greater than the sum of the roots of $g(x)=0$.

PTS: 2 NAT: F.IF.C.9 TOP: Graphing Quadratic Functions

466) ANS:
Function g has the larger maximum value. The maximum of function *g* is 11. The maximum of function *f* is 6.

Strategy: Determine the maximum for *f* from the graph. Determine the maximum for g by inputting the function rule in a graphing calculator and inspecting the graph.



```
 Plot1  Plot2  Plot3
\Y1■-(1/2)X²+4X+▸
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

X	Y1
1	6.5
2	9
3	10.5
4	11
5	10.5
6	9
7	6.5

X=4

The table of values shows the maximum for *g* is 11.

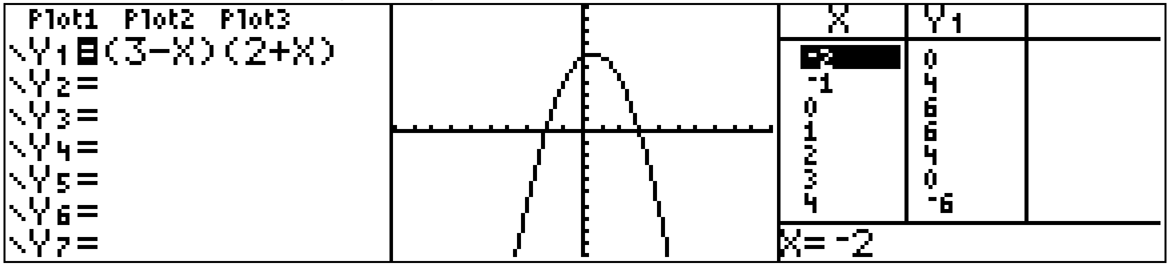
Another way of finding the maximum for g is to use the axis of symmetry formula and the function rule, as follows: $x=\frac{-b}{2a}=\frac{-4}{2\left(-\frac{1}{2}\right)}=\frac{-4}{-1}=4$

$$y=-\frac{1}{2}(4)^2+4(4)+3=-8+16+3=11$$

PTS: 2 NAT: F.IF.C.9 TOP: Graphing Quadratic Functions

467) ANS: 3
Strategy: Each answer choice needs to be evaluated for the largest maximum using a different strategy..

a) Input $h(x)=(3-x)(2+x)$ in a graphing calculator and find the maximum.



```
 Plot1  Plot2  Plot3
\Y1■(3-X)(2+X)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

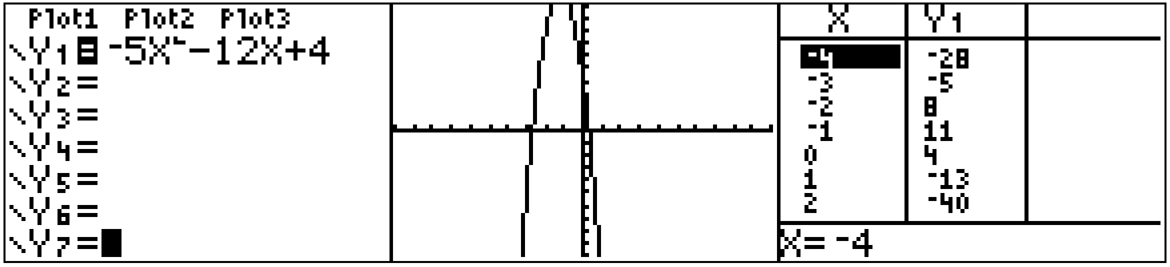
X	Y1
-2	0
-1	4
0	6
1	6
2	4
3	0
4	-6

X= -2

The maximum for answer choice *a* is a little more than 6.

b) The table shows that the maximum is a little more than 9.

c) Input $k(x) = -5x^2 - 12x + 4$ in a graphing calculator and find the maximum.



```
Plot1 Plot2 Plot3
\Y1=-5X²-12X+4
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

X	Y1
-4	-28
-3	-5
-2	8
-1	11
0	4
1	-13
2	-40

X= -4

The table of values shows that the maximum is 11 or more.

d) The graph shows that the maximum is a little more than 4.

Answer choice *c* is the best choice.

PTS: 2 NAT: F.IF.C.9 TOP: Graphing Quadratic Functions

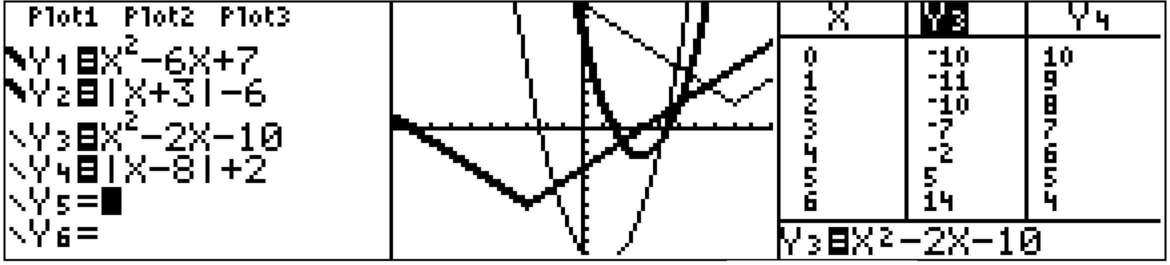
468) ANS: 3

The first function $f(x) = (x-3)^2 + 2$ is in vertex form $y = a(x-h)^2 + k$ and has its vertex at (3,2).
The second function is in table form and has its vertex at (3, -5).
Therefore, the axis of symmetry for both functions is x=3..

PTS: 2 NAT: F.IF.C.9 TOP: Comparing Functions
KEY: AI

469) ANS: 3

Strategy: The graph shows a parabola with a vertex at (3, -7), so the minima is at -7. Identify the lowest y-value of each function rule. Then, select the function rule that has a lowest y value that is less than -7.



```
Plot1 Plot2 Plot3
\Y1=X²-6X+7
\Y2=|X+3|-6
\Y3=X²-2X-10
\Y4=|X-8|+2
\Y5=
\Y6=
```

X	Y3	Y4
0	-10	10
1	-11	9
2	-10	8
3	-7	7
4	-2	6
5	5	5
6	14	4

Y3=X²-2X-10

The graph view of the four functions shows that the function $y = x^2 - 2x - 10$ has a y-value less than -7.

PTS: 2 NAT: F.IF.C.9 TOP: Comparing Functions

470) ANS: 2

Observe: The function $f(x) = 0.01 + 0.05x^2$ is a second degree equation, so it must be a quadratic function. One year equals 52 weeks.
Strategy:
Step 1.: Solve the Plan *B* function for $x = 52$

$$f(x) = 0.01 + 0.05x^2$$

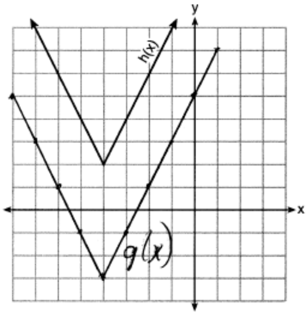
$$f(52) = 0.01 + 0.05(52)^2$$

$$f(52) = 135.21$$

Step 2. Compare the Plan A (52, 105) and Plan B (52, 135.21) coordinates for 52 weeks and observe that B has higher savings.
Step 3. Eliminate wrong answers.
~~a)~~ correct, because Plan *B* is an ~~exponential~~ function and will increase at a faster rate b) correct, because Plan *B* is a quadratic function and will increase at a faster rate
~~c)~~ ~~incorrect~~, because Plan *A* will have a higher value after 1 year
~~d)~~ ~~incorrect~~, because Plan *B* is a quadratic function and will increase at a slower rate

PTS: 2 NAT: F.IF.C.9 TOP: Comparing Functions

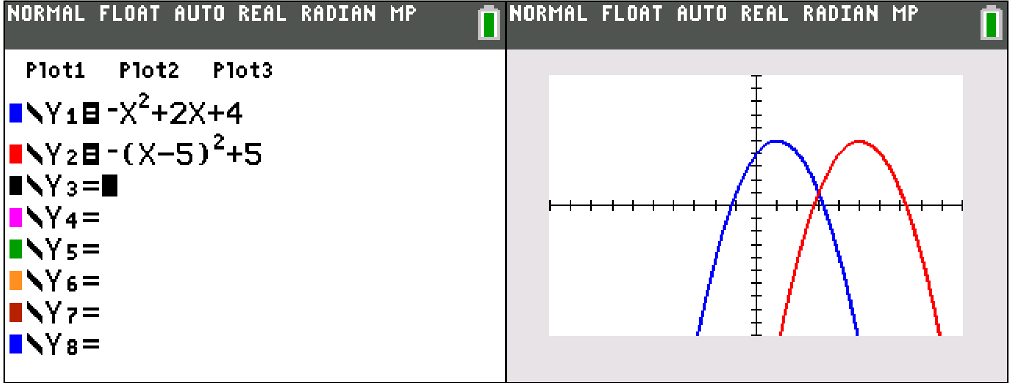
471) ANS: 2
Strategy: Graph $g(x) = 2|x+4| - 3$, then examine the truth value of the answer choices.



I. $g(x)$ has a lower minimum value than $h(x)$. True: $-3 < 2$
II. For all values of *x*, $h(x) < g(x)$. False. $h(x)$ is always > than $g(x)$.
III. For any value of *x*, $g(x) \neq h(x)$. True. $h(x)$ is always 5 more than $g(x)$.

PTS: 2 NAT: F.IF.C.9 TOP: Comparing Functions

472) ANS: 2
Strategy: Find the maximum y-value for each function rule using a graphing calculator. Estimate the maximum y-value for each table.



```
NORMAL FLOAT AUTO REAL RADIAN MP
 Plot1  Plot2  Plot3
■\Y1=-X²+2X+4
■\Y2=-(X-5)²+5
■\Y3=■
■\Y4=
■\Y5=
■\Y6=
■\Y7=
■\Y8=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
```

Both function rules have maximum values of 5.
The maximum value of $k(x)$ is estimated as greater that 5.
The maximum value of $h(x)$ is estimated as less than 5.

PTS: 2 NAT: F.IF.C.9 TOP: Comparing Functions

473) ANS: 2
Strategy: Determine the minimum y-value for each function, then choose the smallest y-value.
STEP 1. Evaluate each answer choice.
Answer Choice 1. The minimum can be found by transforming the function from standard form to vertex form.

$$h(x) = x^2 + 2x - 6$$

$$x^2 + 2x - 6 = 0$$

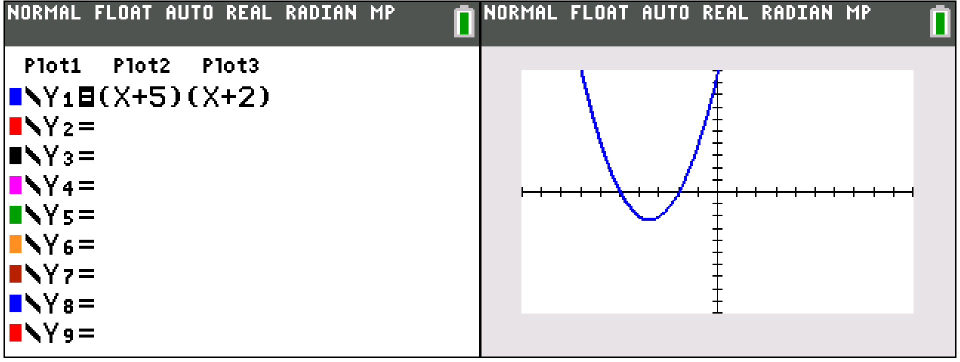
$$x^2 + 2x = 6$$

$$x^2 + 2x + (1)^2 = 6 + (1)^2$$

$$(x+1)^2 = 7$$

$$(x+1)^2 - 7 = 0$$

The vertex occurs at (-1, -7), so the minimum y-value is -7.
Answer Choice 2. The minimum can be found by inspection of the graph. The minimum y-value is -10.
Answer Choice 3. The minimum can be found using the graph or table views of the function in a graphing calculator. The minimum y-value is between -2 and -3.



Plot1 Plot2 Plot3
Y1=(X+5)(X+2)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
Y8=
Y9=

Answer Choice 4. The miinimum can be found by inspection of the table of values. The minim y-value is -6.
STEP 2. Pick the lowest y-value of all the answer choices.

PTS: 2 NAT: F.IF.C.9 TOP: Comparing Functions

FUNCTIONS

Relating Graphs to Events

CC Standard	**NG Standard**
F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; ~~end behavior~~; ~~and periodicity~~.* PARCC: Tasks have a real-world context. Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piece-wise defined functions (including step functions and absolute value functions) and exponential functions with domains in the integers.	**AI-F.IF.4** For a function that models a relationship between two quantities: i) interpret key features of graphs and tables in terms of the quantities; and ii) sketch graphs showing key features given a verbal description of the relationship. (Shared standard with Algebra II) Notes: • Algebra I key features include the following: intercepts, **zeros**; intervals where the function is increasing, decreasing, positive, or negative; maxima, minima; and symmetries. • Tasks have a real-world context and are limited to the following functions: linear, quadratic, square root, piece-wise defined (including step and absolute value), and exponential functions of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).

LEARNING OBJECTIVES

Students will be able to:

1) relate graphs to real-world contexts, and
2) relate real-world contexts to graphs.

Overview of Lesson

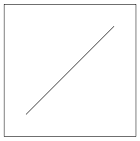
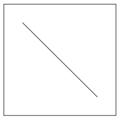
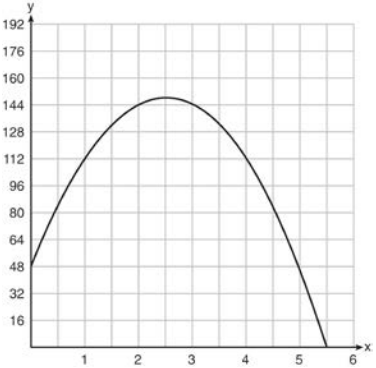
Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

speed
rate of change
increasing
decreasing
interval

BIG IDEAS

Increasing or Decreasing Intervals and Slope

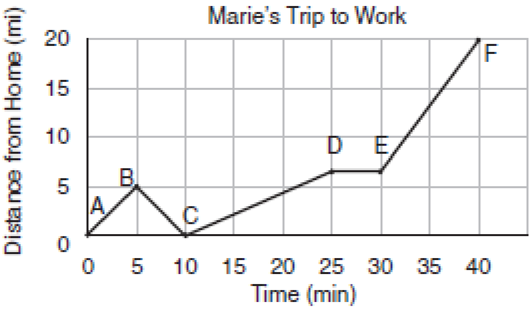
 **A Function is Increasing Over an Interval that has Positive Slope**	 **A Function is Decreasing Over an Interval that has Negative Slope.**
Example: **The function at right is** - **increasing over the interval** **$0 < x < 2.5$** - **decreasing over the interval** **$2.5 < x < 5.5$**	

NOTE: Graphs involving time and distance variables are about speed.

$$Speed = \frac{\text{distance}}{\text{time}}$$

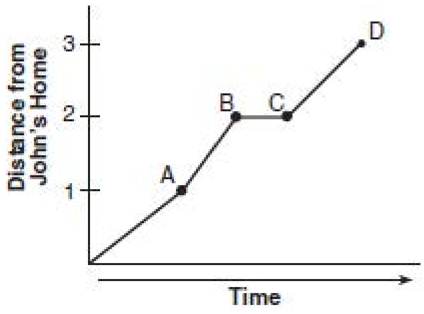
DEVELOPING ESSENTIAL SKILLS

1. The accompanying graph shows Marie's distance from home (A) to work (F) at various times during her drive.



Marie left her briefcase at home and had to return to get it. State which point represents when she turned back around to go home and explain how you arrived at that conclusion. Marie also had to wait at the railroad tracks for a train to pass. How long did she wait?

2. John left his home and walked 3 blocks to his school, as shown in the accompanying graph.



What is one possible interpretation of the section of the graph from point *B* to point *C*?

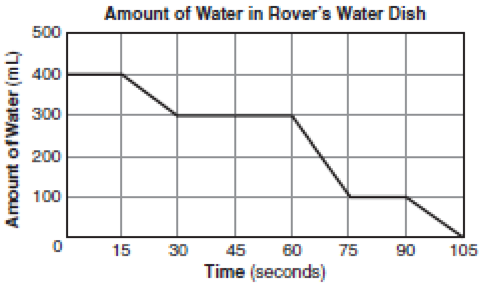
a. John arrived at school and stayed throughout the day.

b. John waited before crossing a busy street.

c. John returned home to get his mathematics homework.

d. John reached the top of a hill and began walking on level ground.

3. The accompanying graph show the amount of water left in Rover's water dish over a period of time.



How long did Rover wait from the end of his first drink to the start of his second drink of water?

a. 10 sec

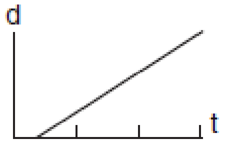
b. 30 sec

c. 60 sec

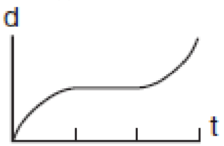
d. 75 sec

4. A bug travels up a tree, from the ground, over a 30-second interval. It travels fast at first and then slows down. It stops for 10 seconds, then proceeds slowly, speeding up as it goes. Which sketch best illustrates the bug's distance (*d*) from the ground over the 30-second interval (*t*)?

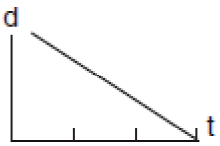
a.



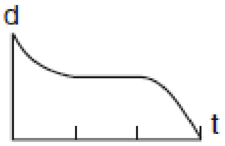
c.



b.



d.



ANSWERS

1. ANS:
B, 5 minutes. At point B, Mary's distance from home begins to decrease, representing the point where she turned back around to go home. The interval between points D and E is the only portion of the graph where Mary's distance from home remains constant. It lasts for 5 mins.

2. ANS: B

   Between points *B* and *C*, John's distance from home remains constant. (2) represents an interpretation in which John's distance remains constant, waiting before crossing a busy street. (1) also represents an interpretation in which John's distance remains constant, but at points *B* and *C*, John had not yet arrived at school. In both (3) and (4), John's distance from school is changing.

3. ANS: B

   When Rover is drinking, the amount of water in his dish decreases over time. The first decrease ends at 30 seconds and the second decrease begins at 60 seconds. The difference between these points is 30 seconds.

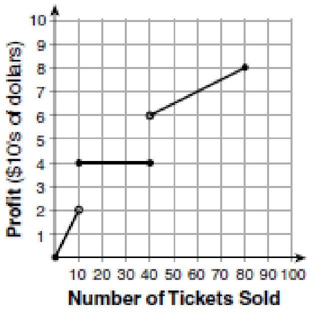
4. ANS: C

   In this sketch, the bug's speed is decreasing during the first third of time, equals 0 during the second third of time and is increasing the last third of time. In (4), the bug is traveling down the tree. In (1) and (2), the bug's speed remains constant.

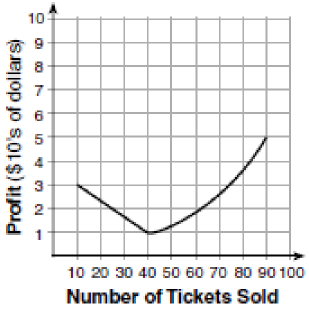
F.IF.B.4: Relating Graphs to Events

474) To keep track of his profits, the owner of a carnival booth decided to model his ticket sales on a graph. He found that his profits only declined when he sold between 10 and 40 tickets. Which graph could represent his profits?

1)

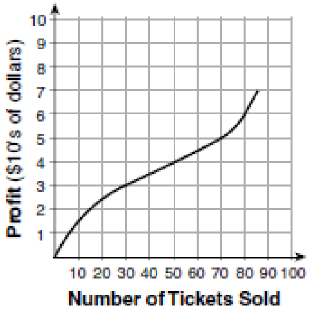


3)



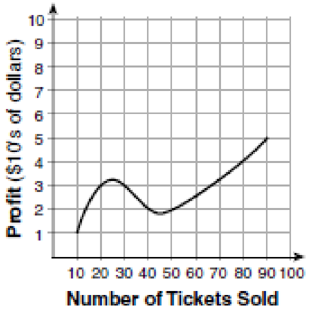
Profits in this graph decline between 10 and 40 tickets, so

2)



The slope in this graph is always positive, so the profit never declines.

4)

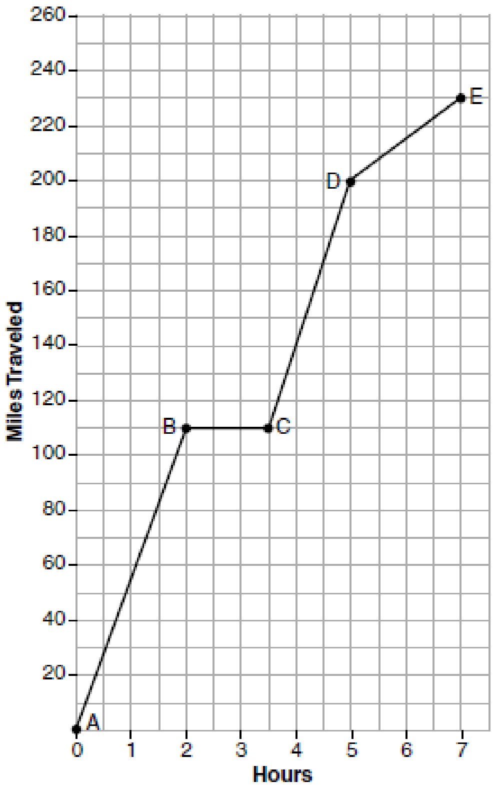


475) During a snowstorm, a meteorologist tracks the amount of accumulating snow. For the first three hours of the storm, the snow fell at a constant rate of one inch per hour. The storm then stopped for two hours and then started again at a constant rate of one-half inch per hour for the next four hours.

a) On the grid below, draw and label a graph that models the accumulation of snow over time using the data the meteorologist collected.

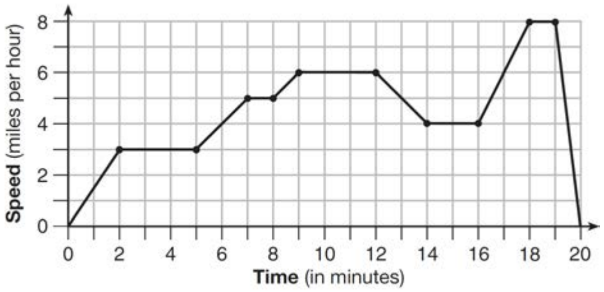
b) If the snowstorm started at 6 p.m., how much snow had accumulated by midnight?

476) The graph below models Craig's trip to visit his friend in another state. In the course of his travels, he encountered both highway and city driving.



Based on the graph, during which interval did Craig most likely drive in the city? Explain your reasoning. Explain what might have happened in the interval between *B* and *C*. Determine Craig's average speed, to the *nearest tenth of a mile per hour*, for his entire trip.

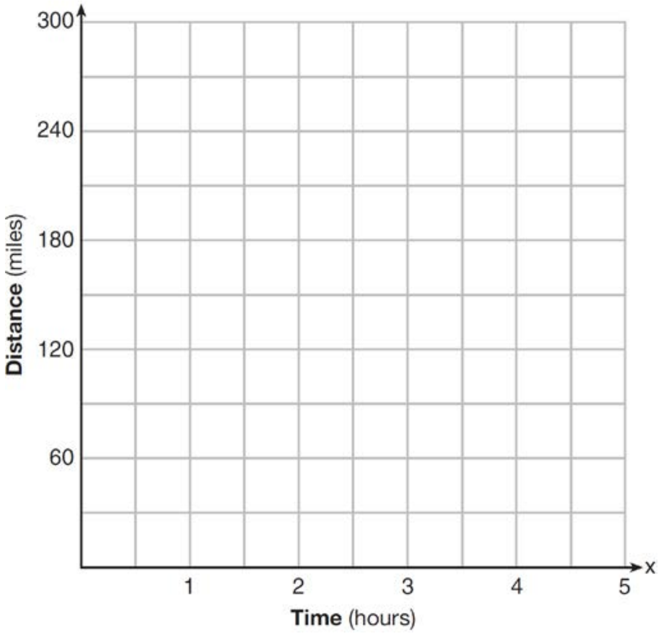
477) The graph below represents a jogger's speed during her 20-minute jog around her neighborhood.



Which statement best describes what the jogger was doing during the $9-12$ minute interval of her jog?

1) She was standing still.
2) She was increasing her speed.
3) She was decreasing her speed
4) She was jogging at a constant rate.

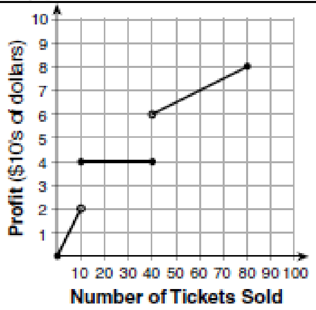
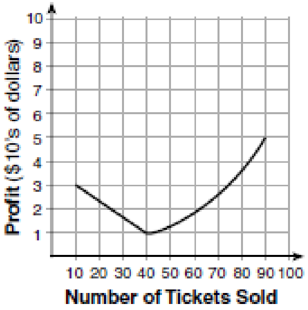
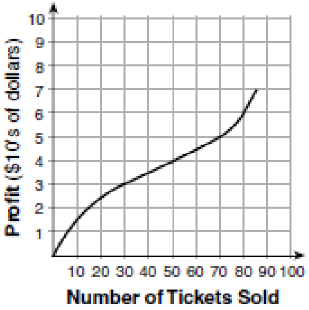
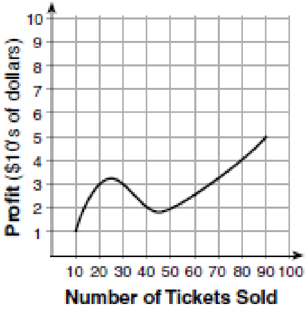
478) A driver leaves home for a business trip and drives at a constant speed of 60 miles per hour for 2 hours. Her car gets a flat tire, and she spends 30 minutes changing the tire. She resumes driving and drives at 30 miles per hour for the remaining one hour until she reaches her destination. On the set of axes below, draw a graph that models the driver’s distance from home.



SOLUTIONS

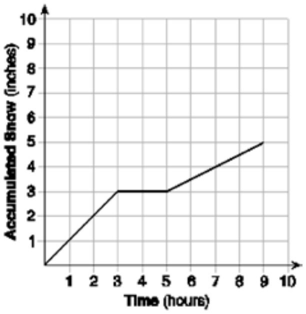
474) ANS: 3

Declining profits over the interval 10 to 40 tickets sold will be shown as a negative slope.

 a) Over the interval between 10 and 40 tickets sold, there is no negative slope, so this is a wrong choice.	 c) Over the interval between 10 and 40 tickets sold, the slope is negative, so this is the correct choice.
 b) Over the interval between 10 and 40 tickets sold, the slope is always positive, so this is a wrong choice.	 d) Over the interval between 10 and 40 tickets sold, there is first a positive slope and then a negative, so this is a wrong choice.

PTS: 2 NAT: F.IF.B.4 TOP: Relating Graphs to Events

475) ANS:



At midnight, 6 hours after the storm began, $3\frac{1}{2}$ inches of snow have fallen.

Strategy - Part a). Label the x and y axes and the corresponding intervals, then use the rates of change from the problem to complete the graph.

STEP 1. Plot the rate of change for the first three hours of the storm. The rate of change during this time is 1 inch per 1 hour.

STEP 2. Plot no change in accumulation for the two hours when the storm is stopped.

STEP 3. Plot the rate of change for the next four hours. During this interval, the rate of change is $\frac{1}{2}$ inch per 1 hour.

Strategy: Part b). Determine which point on the graph corresponds to midnight.

Midnight it six hours after 6 p.m., so the coordinate $\left(6, 3\frac{1}{2}\right)$ can be used to determine the amount of accumulation at midnight. The amount of snow accumulation at midnight is $3\frac{1}{2}$ inches.

PTS: 4 NAT: F.IF.B.4 TOP: Relating Graphs to Events

476) ANS:

Craig most likely was driving in the city during the interval $\overline{DE}$. The slope of $\overline{DE}$ is less steep than the slopes of $\overline{AB}$ and $\overline{CD}$, indicating a lower speed. Speed limits are usually lower in the city than on the highway.

During the interval $\overline{BC}$, Craig stopped.

Craig's average speed for the entire trip was 32.9 miles per hour.

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{230\text{ miles}}{7\text{ hours}}$$

$$\frac{230}{7} = 32.857 \approx 32.9$$

PTS: 4 NAT: F.IF.B.4 TOP: Relating Graphs to Events

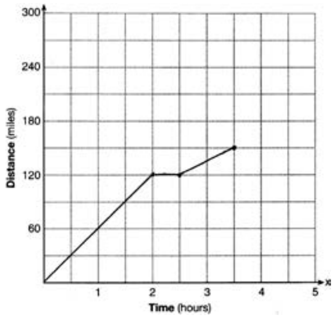
477) ANS: 4

Strategy: Pay close attention to the labels on the x-axis and the y-axis, then eliminate wrong answers.
NOTE: A horizontal line (no slope) means that speed is not changing.

Answer a can be eliminated because she would have a speed of 0 if she were standing still. She was only standing still at the start and end of her jog.
Answer b can be eliminated because the speed does not change during the $9-12$ minute interval of her jog.
Answer c can be eliminated because the speed does not change during the $9-12$ minute interval of her jog.
Answer d is the correct choice because a horizontal line (no slope) means that speed is not changing.

PTS: 2 NAT: F.IF.B.4 TOP: Relating Graphs to Events

478) ANS:



Strategy - Use the speed of the car as the rate of change to complete the graph.

STEP 1. Plot 2 hours at 60 miles per hour slope, based on the language “... a constant speed of 60 miles per hour for 2 hours.”

STEP 2. Plot $\frac{1}{2}$ hour at 0 slope based on the language “...she spends 30 minutes changing the tire.”

STEP 3. Plot 1 hour at 30 miles per hour slope based on the language “...drives at 30 miles per hour for the remaining one hour...”

PTS: 2 NAT: F.IF.B.4 TOP: Relating Graphs to Events

FUNCTIONS
Graphing Piecewise-Defined Functions

Common Core Standard	**Next Generation Standard**
F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	**AI-F.IF.7** Graph functions and show key features of the graph by hand and by using technology where appropriate. (Shared standard with Algebra II)

LEARNING OBJECTIVES

Students will be able to:

1) Graph and interpret piecewise functions.
2) Input piecewise functions in a graphing calculator.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

closed dot
continuous
function
interval
open dot
piece
piecewise function
sub function

BIG IDEAS

PIECEWISE FUNCTIONS

A **piecewise function** is a function that is defined by two or more *sub* functions, with each sub function applying to a certain interval on the x-axis. Each *sub* function may also be referred to as a *piece* of the overall **piecewise function**, hence the name piecewise.

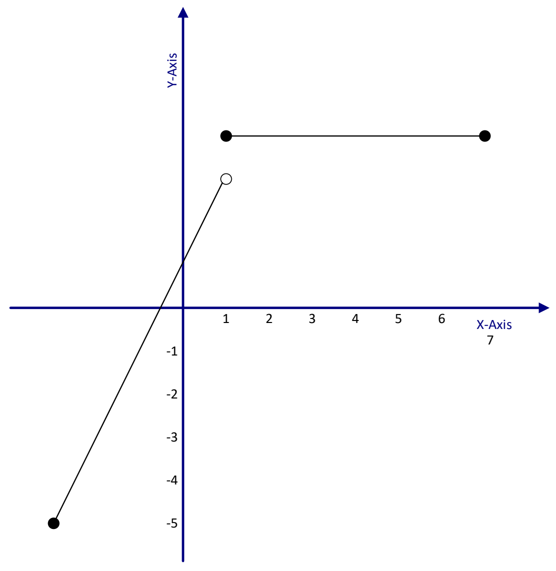
Example. The following is a piecewise function:

$$f\left(x\right)=\begin{cases}2x+1, & x<1\\ 4, & x\geq 1\end{cases}$$

This example of a piecewise function has two “pieces,” or sub functions.

a. Over the interval $x<1$, the sub function is $f\left(x\right)=2x+1$.

b. Over the interval $x\geq 1$, the sub function is $f\left(x\right)=4$..

A table of values for this function.			A graph for this function.
x	$f(x) = 2x+1$	$f(x) = 4$	
-3	-5	na	
-2	-3	na	
-1	-1	na	
0	1	na	
1	na	4	
2	na	4	
3	na	4	
4	na	4	
5	na	4	
6	na	4	
7	na	4	

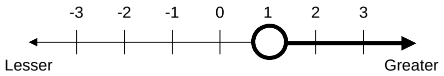
Continuity

Piecewise functions are often discontinuous, which means that the graph will not appear as a single line. In the above table, the piecewise function is discontinuous when $x = 3$. This is because $x = 3$ is not included in the sub function. Because piecewise functions are often discontinuous, care must be taken to use proper inequalities notation when graphing.

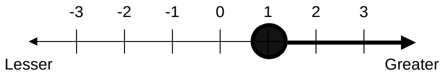
Using Line Segments to Define Pieces

If the circle at the beginning or end of a solution set (graph) is empty, that value *is not included* in the solution set. If the circle is filled in, that value *is included* in the solution set.

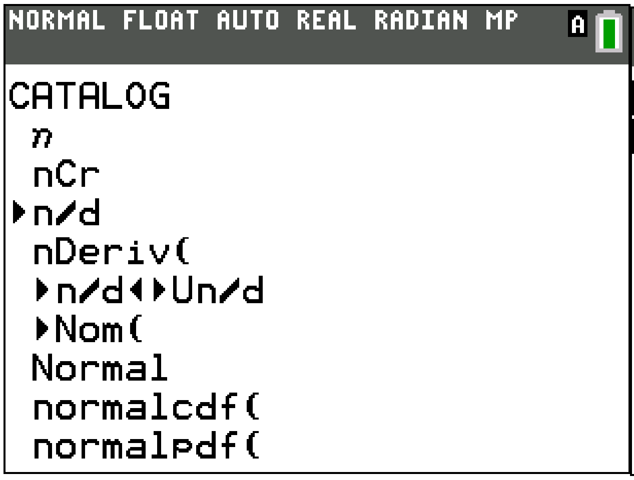
The number 1 is not included in the this solution set:

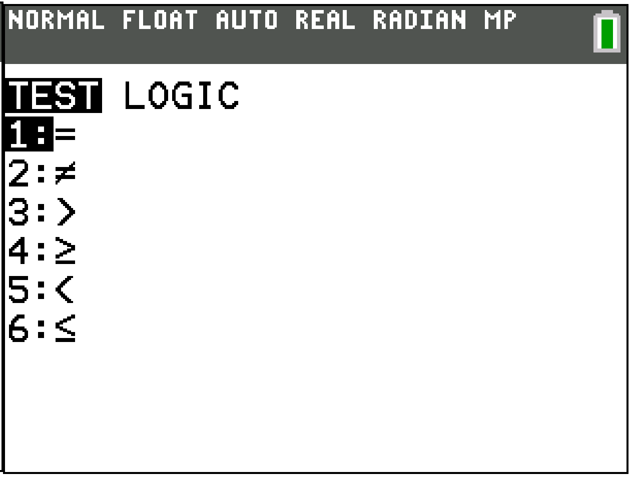


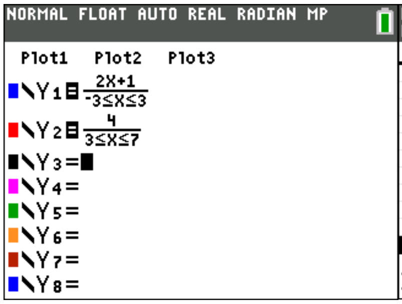
The number 1 is included in this following solution set:



NOTE: The TI83/84 family of graphing calculators can graph piecewise functions using the n-d function in the catalog and the test (second-math) function, as shown in the following screenshots.







$Y_1 = \frac{2X+1}{-3 \le X \le 3}$

$Y_2 = \frac{4}{3 \le X \le 7}$

$Y_3 =$

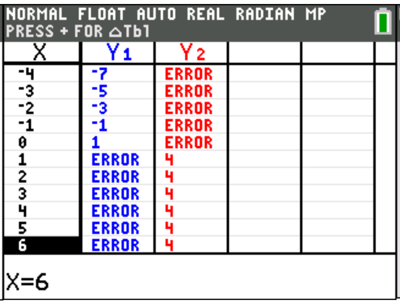
$Y_4 =$

$Y_5 =$

$Y_6 =$

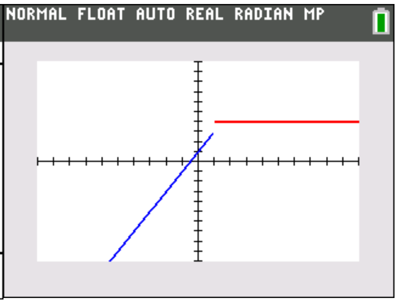
$Y_7 =$

$Y_8 =$



X	Y1	Y2
-4	-7	ERROR
-3	-5	ERROR
-2	-3	ERROR
-1	-1	ERROR
0	1	ERROR
1	ERROR	4
2	ERROR	4
3	ERROR	4
4	ERROR	4
5	ERROR	4
6	ERROR	4

X=6



DEVELOPING ESSENTIAL SKILLS

Use technology to graph the following piecewise functions.

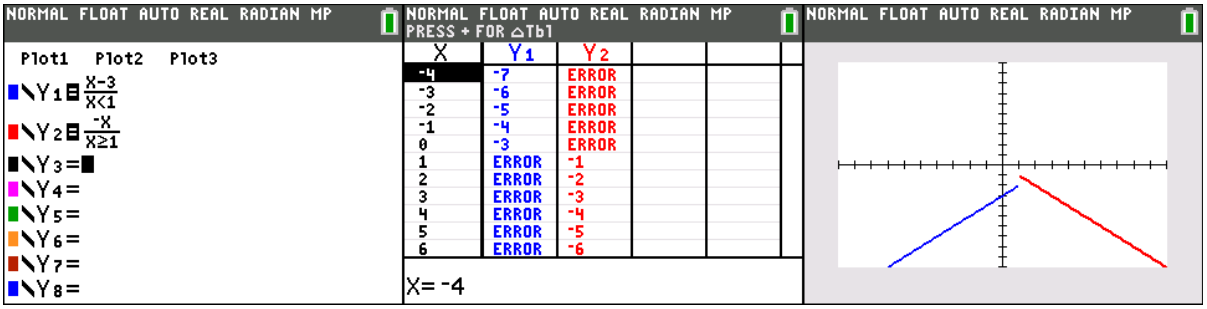
$$f(x)=\begin{cases} x-3, & x<1 \\ -x, & x\geq 1 \end{cases}$$

$$g(x)=\begin{cases} x-5, & x<1 \\ -x+2, & x\geq 1 \end{cases}$$

$$h(x)=\begin{cases} x+3, & x<1 \\ 2x-1, & x\geq 1 \end{cases}$$

ANSWERS

$f(x)$



NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

■\Y1 $\frac{X-3}{X<1}$

■\Y2 $\frac{-X}{X\geq 1}$

■\Y3=

■\Y4=

■\Y5=

■\Y6=

■\Y7=

■\Y8=

NORMAL FLOAT AUTO REAL RADIAN MP

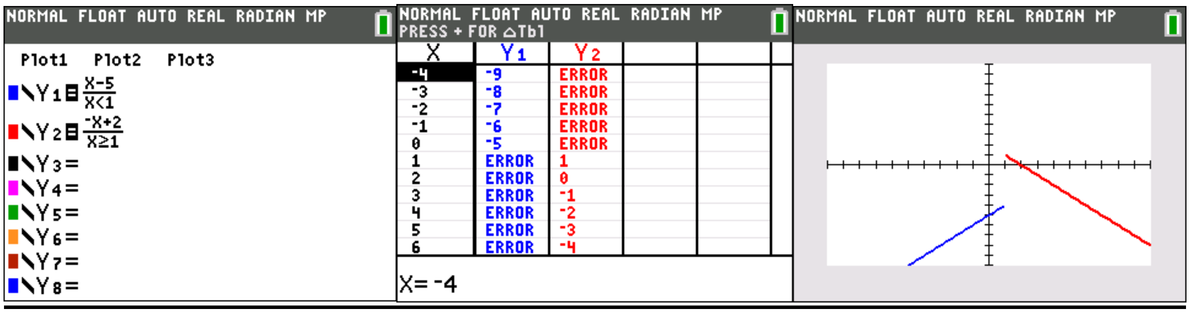
PRESS + FOR ΔTbl

X	Y1	Y2
-4	-7	ERROR
-3	-6	ERROR
-2	-5	ERROR
-1	-4	ERROR
0	-3	ERROR
1	ERROR	-1
2	ERROR	-2
3	ERROR	-3
4	ERROR	-4
5	ERROR	-5
6	ERROR	-6

X= -4

NORMAL FLOAT AUTO REAL RADIAN MP

$g(x)$



NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

■\Y1 $\frac{X-5}{X<1}$

■\Y2 $\frac{-X+2}{X\geq 1}$

■\Y3=

■\Y4=

■\Y5=

■\Y6=

■\Y7=

■\Y8=

NORMAL FLOAT AUTO REAL RADIAN MP

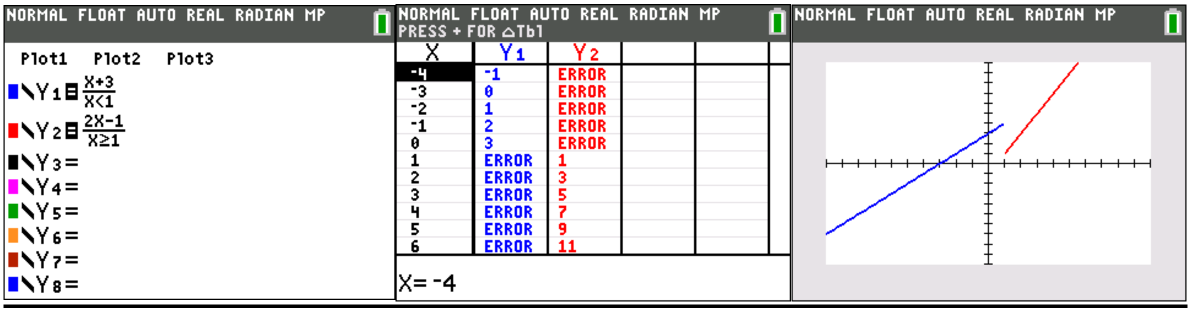
PRESS + FOR ΔTbl

X	Y1	Y2
-4	-9	ERROR
-3	-8	ERROR
-2	-7	ERROR
-1	-6	ERROR
0	-5	ERROR
1	ERROR	1
2	ERROR	0
3	ERROR	-1
4	ERROR	-2
5	ERROR	-3
6	ERROR	-4

X= -4

NORMAL FLOAT AUTO REAL RADIAN MP

$h(x)$



NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

■\Y1 $\frac{X+3}{X<1}$

■\Y2 $\frac{2X-1}{X\geq 1}$

■\Y3=

■\Y4=

■\Y5=

■\Y6=

■\Y7=

■\Y8=

NORMAL FLOAT AUTO REAL RADIAN MP

PRESS + FOR ΔTbl

X	Y1	Y2
-4	-1	ERROR
-3	0	ERROR
-2	1	ERROR
-1	2	ERROR
0	3	ERROR
1	ERROR	1
2	ERROR	3
3	ERROR	5
4	ERROR	7
5	ERROR	9
6	ERROR	11

X= -4

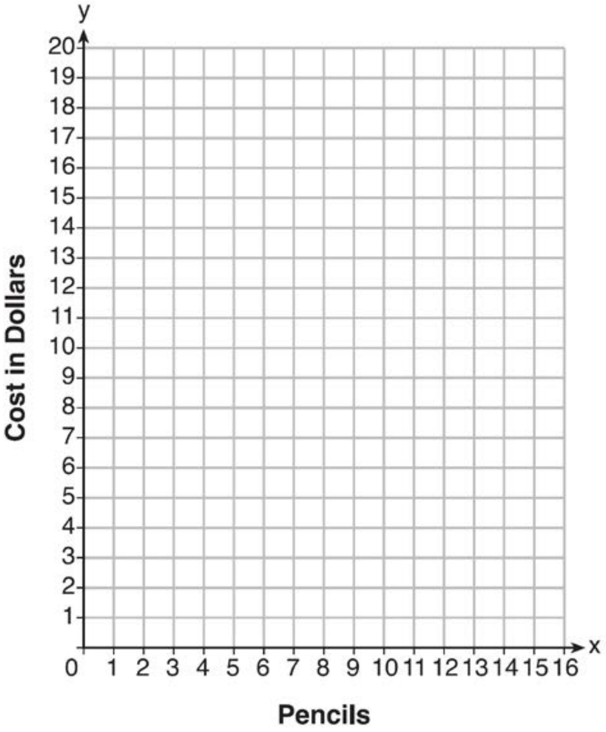
NORMAL FLOAT AUTO REAL RADIAN MP

F.IF.C.7: Graphing Piecewise-Defined Functions

479) At an office supply store, if a customer purchases fewer than 10 pencils, the cost of each pencil is $1.75. If a customer purchases 10 or more pencils, the cost of each pencil is $1.25. Let *c* be a function for which $c(x)$ is the cost of purchasing *x* pencils, where *x* is a whole number.

$$c(x) = \begin{cases} 1.75x, \text{ if } 0 \le x \le 9 \\ 1.25x, \text{ if } x \ge 10 \end{cases}$$

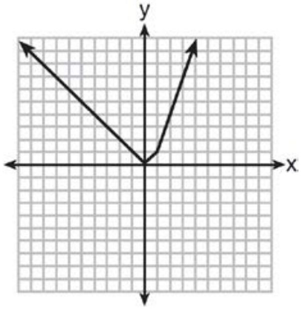
Create a graph of *c* on the axes below.



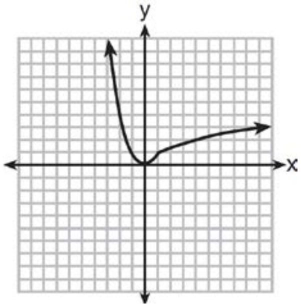
A customer brings 8 pencils to the cashier. The cashier suggests that the total cost to purchase 10 pencils would be less expensive. State whether the cashier is correct or incorrect. Justify your answer.

480) Which graph represents $f(x) = \begin{cases} |x| & x < 1 \\ \sqrt{x} & x \ge 1 \end{cases}$?

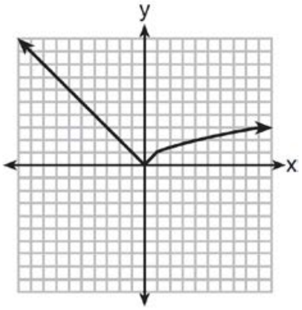
1)



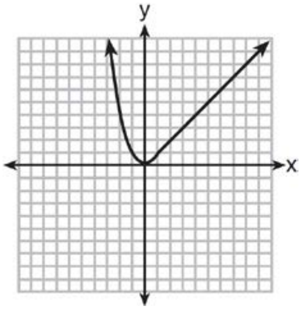
3)



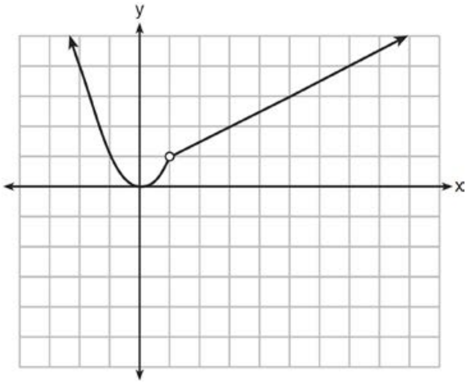
2)



4)



481) A function is graphed on the set of axes below.



Which function is related to the graph?

1) $f(x) = \begin{cases} x^2, x < 1 \\ x - 2, x > 1 \end{cases}$

3) $f(x) = \begin{cases} x^2, x < 1 \\ 2x - 7, x > 1 \end{cases}$

2) $f(x) = \begin{cases} x^2, x < 1 \\ \frac{1}{2}x + \frac{1}{2}, x > 1 \end{cases}$

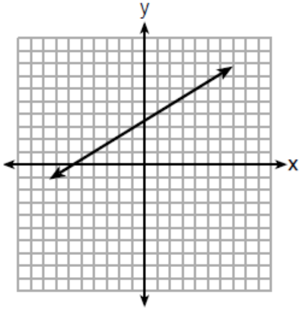
4) $f(x) = \begin{cases} x^2, x < 1 \\ \frac{3}{2}x - \frac{9}{2}, x > 1 \end{cases}$

482) Graph the following function on the set of axes below.

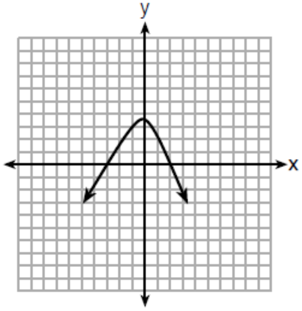
$$f(x) = \begin{cases} |x|, & -3 \le x < 1 \\ 4, & 1 \le x \le 8 \end{cases}$$

483) Which graph does *not* represent a function that is always increasing over the entire interval $-2 < x < 2$?

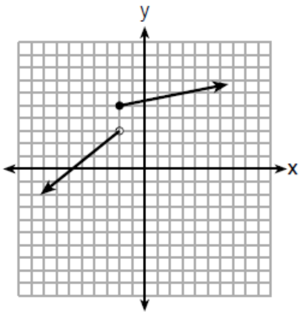
1)



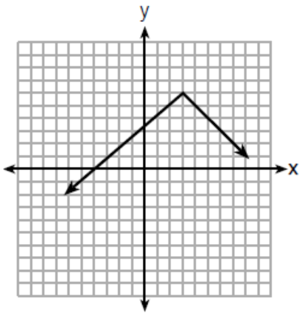
3)



2)

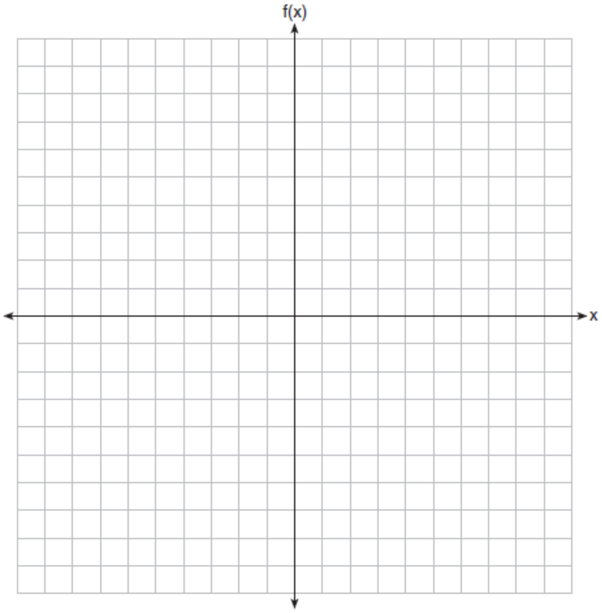


4)



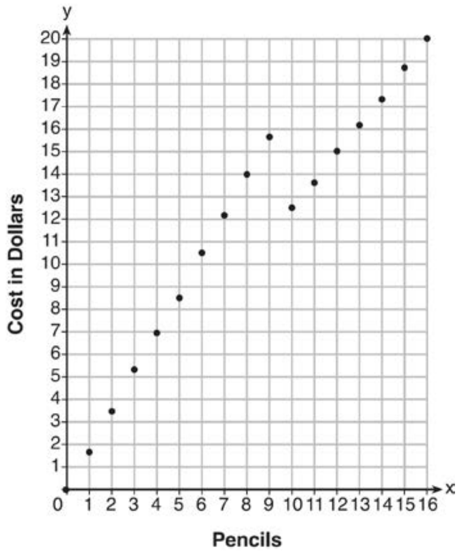
484) On the set of axes below, graph the piecewise function:

$$f(x) = \begin{cases} -\frac{1}{2}x, & x < 2 \\ x, & x \geq 2 \end{cases}$$



SOLUTIONS

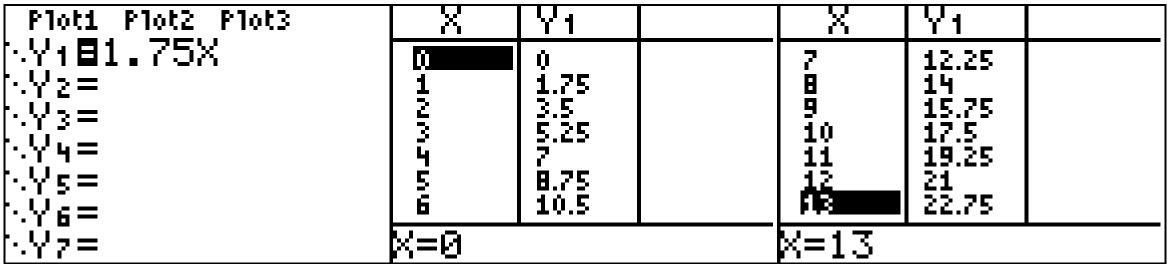
479) ANS:



The cashier is correct. 8 pencils cost $14 and 10 pencils cost $12.50.

Strategy: Use a graphing calculator and graph the function in two sections. Note that the domain of the function is whole numbers. You cannot buy a part of a pencil. This means that the graph of the function will consist of points and not lines. After completing the graph, answer the questions presented in the problem.

STEP 1: Graph the section of the function represented by $c(x) = 1.75x$. Plot closed dots for each whole number in the domain $0 \le x \le 9$.

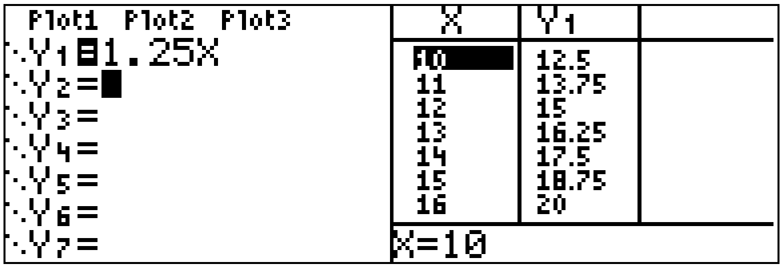


```
Plot1 Plot2 Plot3
\Y1=1.75X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
X     Y1
0     0
1     1.75
2     3.5
3     5.25
4     7
5     8.75
6     10.5
X=0
```

```
X     Y1
7     12.25
8     14
9     15.75
10    17.5
11    19.25
12    21
13    22.75
X=13
```

STEP 2: Graph the section of the function represented by $c(x) = 1.25x$. Plot closed dots for each whole number in the domain $x > 10$.



```
Plot1 Plot2 Plot3
\Y1=1.25X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

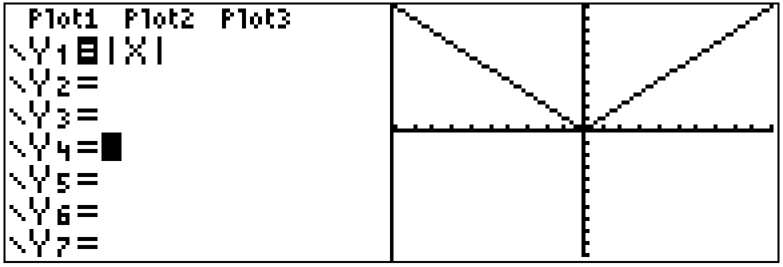
```
X     Y1
10    12.5
11    13.75
12    15
13    16.25
14    17.5
15    18.75
16    20
X=10
```

STEP 3: Answer the questions presented in the problem.
The data tables and the graph show that it would be cheaper to purchase 10 pencils that to purchase 8 pencils.

PTS: 4 NAT: F.IF.C.7 TOP: Graphing Piecewise-Defined Functions

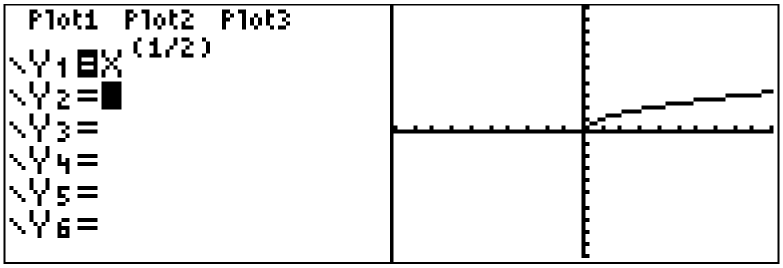
480) ANS: 2
Strategy: Eliminate wrong answers.

The left half of each graph corresonds to $f(x) = |x|$ over the domain $x < 1$. The graph of $f(x) = |x|$ should not curve because *x* is of the first degree. Answer choices *c* and *d* should be eliminated because they have curves over the domain $x < 1$. A quick look at the graph of $f(x) = |x|$ in a graphing calculator shows why answer choices *c* and *d* should be eliminated.



```
Plot1  Plot2  Plot3
\Y1=|X|
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

The graph of $f(x) = \sqrt{x}$ over the domain $x \geq 1$ should be not be a straight line because the degree of x is not 1. A quick look at the graph of $f(x) = \sqrt{x}$ in a graphing calculator shows that answer choice *b* is correct.



```
Plot1  Plot2  Plot3
\Y1=X^(1/2)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

PTS: 2 NAT: F.IF.C.7 TOP: Graphing Piecewise-Defined Functions
KEY: bimodalgraph

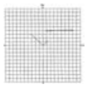
481) ANS: 2

Strategy: Since $f(x) = x^2, x < 1$ is included in every answer choice, concentrate on the linear functions for $x > 1$.

The linear equation has a slope of $\frac{rise}{run} = \frac{1}{2}$. The only linear function that has a slope of $\frac{1}{2}$ is $f(x) = \frac{1}{2}x + \frac{1}{2}$, which is answer choice b.

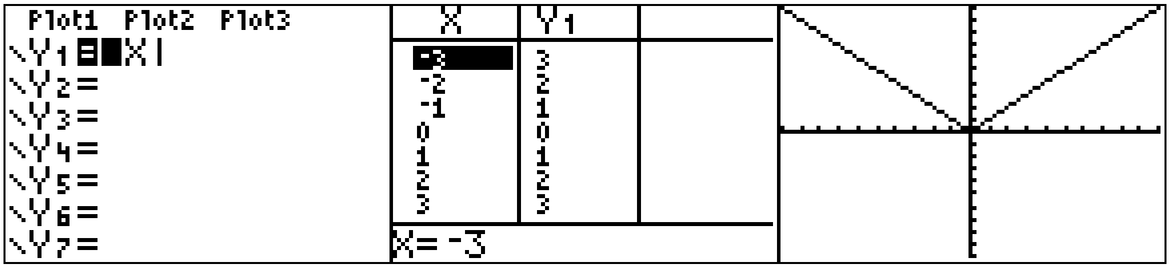
PTS: 2 NAT: F.IF.C.7 TOP: Graphing Piecewise-Defined Functions

482) ANS:



Strategy: Use a graphing calculator and graph the function in sections, paying careful attention to open and closed circles at the end of each function segment.

STEP 1. Graph $f(x) = |x|$ over the interval $-3 \leq x < 1$. Use a closed dot for $(-3, 3)$ and an open dot for $(1, 1)$. Use data from the table of values to plot the interval $-3 \leq x < 1$.

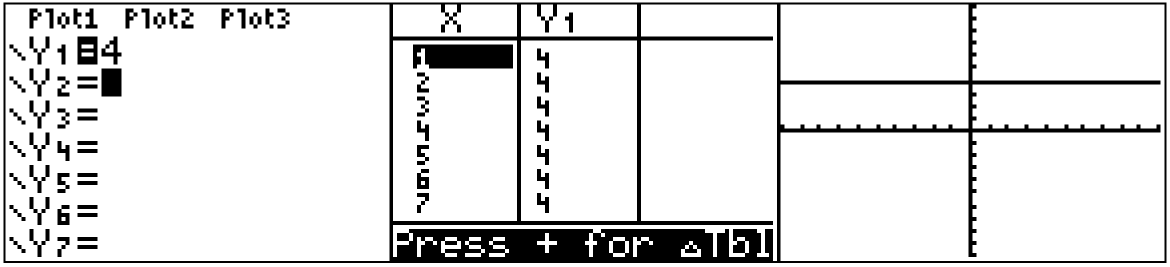


```
Plot1  Plot2  Plot3
\Y1=|X|
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

X	Y1	
-3	3	
-2	2	
-1	1	
0	0	
1	1	
2	2	
3	3	

X=-3

STEP 2: Graph $f(x) = 4$ over the interval $1 \le x \le 8$. Use a closed dot for $(1,4)$ and a closed dot for $(8,4)$. Use data from the table of values to plot the interval $1 \le x \le 8$.



```
Plot1  Plot2  Plot3
\Y1=4
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

X	Y1	
1	4	
2	4	
3	4	
4	4	
5	4	
6	4	
7	4	

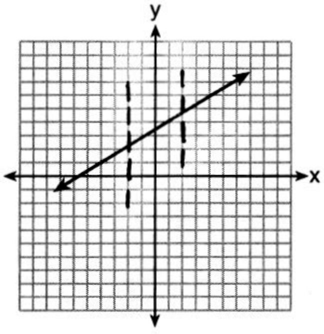
Press + for ΔTbl

Do not connect the two graph segments.

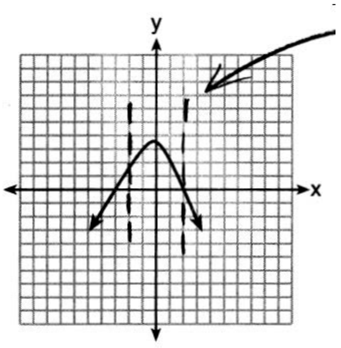
PTS: 2 NAT: F.IF.C.7 TOP: Graphing Piecewise-Defined Functions

483) ANS: 3

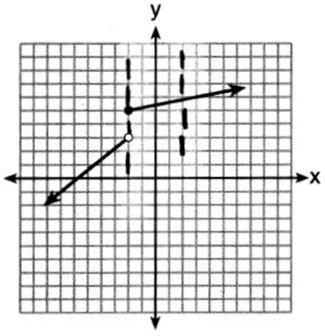
Strategy: Looks at the slope of the graph over the interval $-2 < x < 2$. Select the answer choice where the slope of the graph is negative anywhere in this interval.



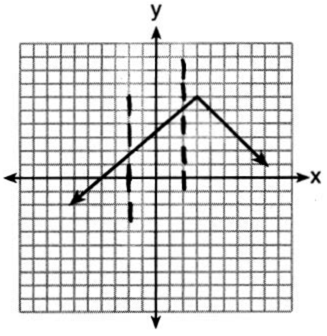
(1)



(3)



(2)

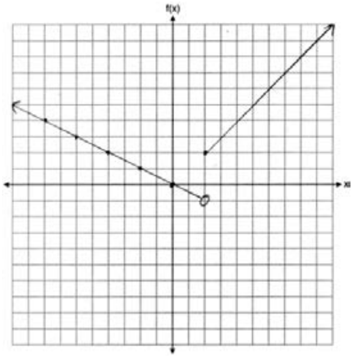


(4)

Answer choice (3) is the only graph that has a negative slope over the interval $-2 < x < 2$.

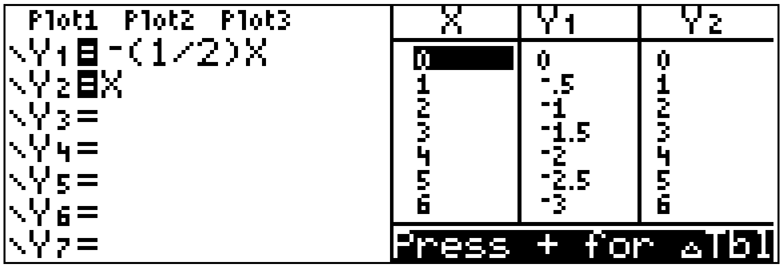
PTS: 2 NAT: F.IF.C.7 TOP: Graphing Piecewise-Defined Functions

484) ANS:



Strategy: Use a graphing calculator to help find and plot two points that define the lines for each part of this piecewise function.

STEP 1. Input the piecewise function as two separate equations in a graphing calculator and inspect the table of values for both functions.



```
 Plot1  Plot2  Plot3          X      Y1     Y2
\Y1=-(1/2)X                   0      0      0
\Y2=X                         1      -.5    1
\Y3=                          2      -1     2
\Y4=                          3      -1.5   3
\Y5=                          4      -2     4
\Y6=                          5      -2.5   5
\Y7=                          6      -3     6
                              Press + for ΔTbl
```

STEP 2. Plot the points for both functions when $x = 2$, which is the x-value where the function changes from the first piece to the second piece.

(2, -1) is plotted for the first part of the function $y_1 = -(1/2)x$ with an open circle, because the domain for this piece of the function is $x < 2$.

(2, 2) is plotted for the second part of the function $y_2 = x$ with a closed circle, because the domain for this piece of the function is $x \geq 2$.

STEP 3. Pick a second point in the domain $x < 2$ to plot for the first piece (y_1) of the function.

(0, 0) is an easy ordered pair to plot.

STEP 4. Draw a directed line that starts at (2,-1) and passes through (0,0).

STEP 5. Pick a second point in the domain $x \geq 2$ to plot for the second piece (y_2) of the function.

(6, 6) is an easy ordered pair to plot.

STEP 6. Draw a directed line that starts at (2,2) and passes through (6,6).

PTS: 2 NAT: F.IF.C.7 TOP: Graphing Piecewise-Defined Functions

M – Functions, Lesson 10, Graphing Step Functions (r. 2018)

FUNCTIONS

Graphing Step Functions

Common Core Standard	Next Generation Standard
F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	**AI-F.IF.7** Graph functions and show key features of the graph by hand and by using technology where appropriate. (Shared standard with Algebra II)

NOTE: This lesson is related to Functions, Lesson 9, Graphing Piecewise Functions

LEARNING OBJECTIVES

Students will be able to:

1) Graph and interpret step functions.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

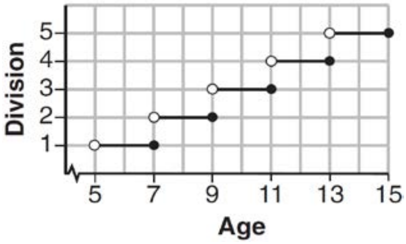
VOCABULARY

closed dot
continuous
function
interval
open dot
piece
piecewise function
sub function

BIG IDEAS

STEP FUNCTIONS

A step function is typically a piecewise function with many sub functions that resemble stair steps.



Each step corresponds to a specific domain. The function rule for the graph above is:

$$f(x) = \begin{cases} 1, & 5 < x \le 7 \\ 2, & 7 < x \le 9 \\ 3, & 9 < x \le 11 \\ 4, & 11 < x \le 13 \\ 5, & 13 < x \le 15 \end{cases}$$

DEVELOPING ESSENTIAL SKILLS

Model each context with a step function.

1. You want to bring cupcakes to math club to celebrate your birthday. Each box of cupcakes contains 6 cupcakes and costs $4.00. You expect as many as 30 students to be at math club. Create a function rule that models the cost in terms of the number of students in math club.

$$C(s) = \begin{cases} 4, & 0 < s \le 6 \\ 8, & 7 < s \le 12 \\ 12, & 13 < s \le 18 \\ 16, & 19 < s \le 24 \end{cases}$$

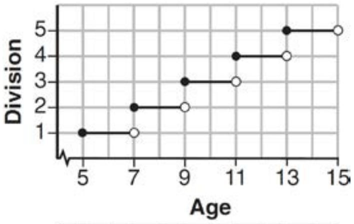
2. You're ordering pizza for your math teacher's birthday party. You estimate that each pizza will serve 4 people and that up to 26 people may attend. Create a function rule that models the number of pizzas you need to order in terms of the number of people attending.

$$P(s) = \begin{cases} 1, & 0 < s \le 4 \\ 2, & 5 < s \le 8 \\ 3, & 9 < s \le 12 \\ 4, & 13 < s \le 16 \\ 5, & 17 < s \le 20 \\ 6, & 21 < s \le 24 \\ 7, & 25 < s \le 28 \end{cases}$$

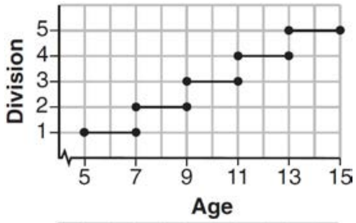
F.IF.C.7: Graphing Step Functions

485) Morgan can start wrestling at age 5 in Division 1. He remains in that division until his next odd birthday when he is required to move up to the next division level. Which graph correctly represents this information?

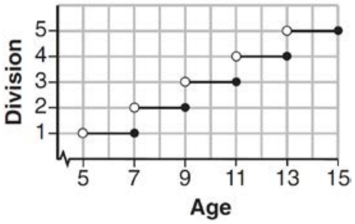
1)



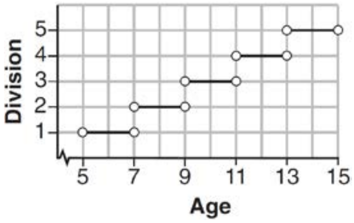
3)



2)



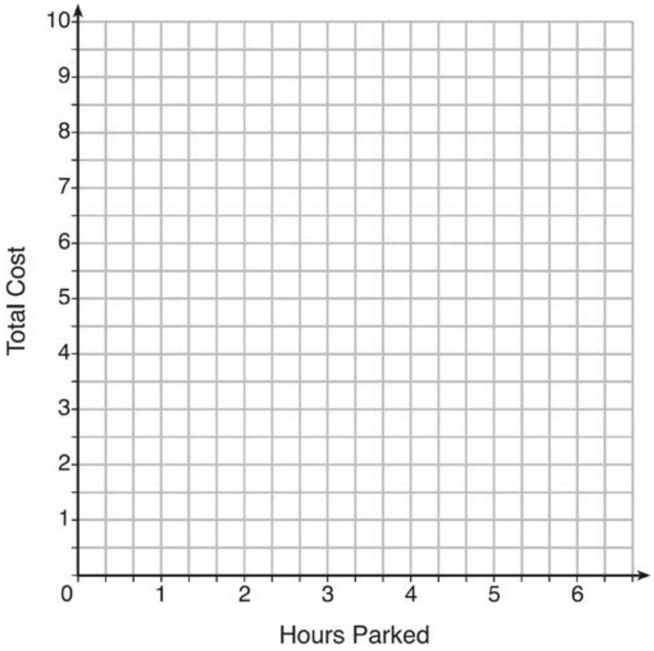
4)



486) The table below lists the total cost for parking for a period of time on a street in Albany, N.Y. The total cost is for any length of time up to and including the hours parked. For example, parking for up to and including 1 hour would cost $1.25; parking for 3.5 hours would cost $5.75.

Hours Parked	Total Cost
1	1.25
2	2.50
3	4.00
4	5.75
5	7.75
6	10.00

Graph the step function that represents the cost for the number of hours parked.



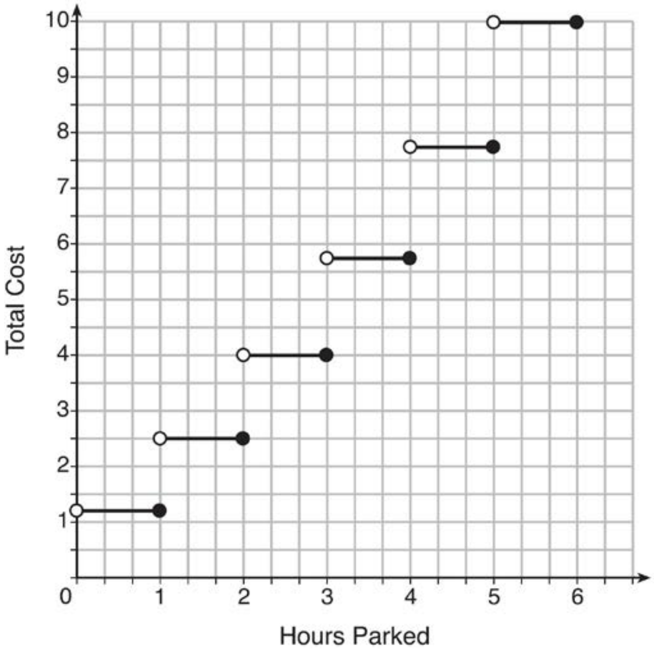
Explain how the cost per hour to park changes over the six-hour period.

SOLUTIONS

485) ANS: 1
Strategy: Focus on whether the line segments should begin and end with closed or open circles. A closed circle is included. An open circle is not included.

PTS: 2 NAT: F.IF.C.7 TOP: Graphing Step Functions
KEY: bimodalgraph

486) ANS:



The cost per houir to park gets bigger over the six hour period.

Strategy: Graph this step function by hand using information from the table. This function has too many sections to easily input into a graphing calculator.

STEP 1. Graph the section for the domain $0 < x \leq 1$. The table shows that this interval corresponds to a cost of $1.25 on the y-axis. Use an open dot at (0, 1.25) and a closed dot at (1, 1.25). Connect the two dots with a solid line.
STEP 2. Graph the section for the domain $1 < x \leq 2$. The table shows that this interval corresponds to a cost of $2.50 on the y-axis. Use an open dot at (1, 2.50) and a closed dot at (2, 2.50). Connect the two dots with a solid line.
STEP 3. Graph the section for the domain $2 < x \leq 3$. The table shows that this interval corresponds to a cost of $4.00 on the y-axis. Use an open dot at (2, 4.00) and a closed dot at (3, 4.00). Connect the two dots with a solid line.
STEP 4. Graph the section for the domain $3 < x \leq 4$. The table shows that this interval corresponds to a cost of $5.75 on the y-axis. Use an open dot at (3, 4.75) and a closed dot at (4, 4.75). Connect the two dots with a solid line.
STEP 5. Graph the section for the domain $4 < x \leq 5$. The table shows that this interval corresponds to a cost of $7.75 on the y-axis. Use an open dot at (4, 7.75) and a closed dot at (5, 7.75). Connect the two dots with a solid line.
STEP 6. Graph the section for the domain $5 < x \leq 6$. The table shows that this interval corresponds to a cost of $10.00 on the y-axis. Use an open dot at (5, 10.00) and a closed dot at (6, 10.00). Connect the two dots with a solid line.

STEP 7: Answer the question based on the graph and the table.

PTS: 4 NAT: F.IF.C.7 TOP: Graphing Step Functions

SEQUENCES AND SERIES

Sequences

CC Standard	NG Standard
F-IF.A.3 Recognize that sequences are functions, ~~sometimes defined recursively~~, whose domain is a subset of the integers. ~~*For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*~~ ~~PARCC: This standard is part of the Major work in Algebra I and will be assessed accordingly.~~	**AI-F.IF.3** Recognize that a sequence is a function whose domain is a subset of the integers. (Shared standard with Algebra II) Notes: • Sequences (arithmetic and geometric) will be written explicitly and only in subscript notation. • Work with geometric sequences may involve an exponential equation/formula of the form $a_n = ar_{n-1}$, where a is the first term and r is the common ratio.
F-LE.2 Construct linear and exponential functions, ~~including arithmetic and geometric sequences~~, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). PARCC: Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).	**AI-F.LE.2** Construct a linear or exponential function **symbolically** given: i) a graph; ii) a description of the relationship; iii) two input-output pairs (include reading these from a table). (Shared standard with Algebra II) Note: Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).

LEARNING OBJECTIVES

Students will be able to:

1) Define sequences as recursive functions.
2) Evaluate recursive functions for the nth term.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

arithmetic progression
explicit formula
geometric progression
pattern
recursive formula

sequence
series
set
term

BIG IDEAS

An **explicit formula** is one where you do not need to know the value of the term in front of the term that you are seeking. For example, if you want to know the 55th term in a series, an explicit formula could be used without knowing the value of the 54th term.

Example: The sequence 3, 11, 19, 27, ... begins with 3, and 8 is added each time to form the pattern. The sequence can be shown in a table as follows:

Term # (n)	1	2	3	4
$f(n)$	3	11	19	27

Explicit formulas for the sequence 3, 11, 19, 27, ... can be written as:

$f(n) = 8n - 5$ or $f(n) = 3 + 8(n-1)$

Using these **explicit formulas**, we can find the following values for any term, and we do not need to know the value of any other term, as shown below:

$f(n) = 8n - 5$	$f(n) = 3 + 8(n-1)$
$f(1) = 8(1) - 5 = 3$	$f(1) = 3 + 8(1-1) = 3 + 0 = 3$
$f(2) = 8(2) - 5 = 16 - 5 = 11$	$f(2) = 3 + 8(2-1) = 3 + 8 = 11$
$f(3) = 8(3) - 5 = 24 - 5 = 19$	$f(3) = 3 + 8(3-1) = 3 + 16 = 19$
$f(4) = 8(4) - 5 = 32 - 5 = 27$	$f(4) = 3 + 8(4-1) = 3 + 24 = 27$
$f(5) = 8(5) - 5 = 40 - 5 = 35$	$f(5) = 3 + 8(5-1) = 3 + 32 = 35$
$f(10) = 8(10) - 5 = 80 - 5 = 75$	$f(10) = 3 + 8(10-1) = 3 + 72 = 75$
$f(100) = 8(100) - 5 = 800 - 5 = 795$	$f(100) = 3 + 8(100-1) = 3 + 792 = 795$

Recursive formulas requires you to know the value of another term, usually the preceding term, to find the value of a specific term.

Example: Using the same sequence 3, 11, 19, 27, ... as above, a **recursive formula** for the sequence 3, 11, 19, 27, ... can be written as:

$$f(n+1) = f(n) + 8$$

This **recursive formula** tells us that the value of any term in the sequence is equal to the value of the term before it plus 8. A recursive formula must usually be anchored to a specific term in the sequence (usually the first term), so the recursive formula for the sequence 3, 11, 19, 27, ... could be anchored with the statement

$$f(1) = 3$$

Using this **recursive formula**, we can reconstruct the sequence as follows:

$$f(1) = 3$$
$$f(2) = f(1) + 8 = 3 + 8 = 11$$
$$f(3) = f(2) + 8 = 11 + 8 = 19$$
$$f(4) = f(3) + 8 = 19 + 8 = 27$$
$$f(5) = f(4) + 8 = 27 + 8 = 35$$
$$f(10) = f(9) + 8 = ? + 8 = ?$$

Observe that the recursive formula $f(n+1) = f(n) + 8$ includes two different values of the dependent variable, which in this example are $f(n)$ and $f(n+1)$, and we can only reconstruct our original sequence using this **recursive formula** if we know the term immediately preceding the term we are seeking.

Two Kinds of Sequences

arithmetic sequence *(A2T)* A set of numbers in which the common difference between each term and the preceding term is constant.

Example: In the **arithmetic sequence** 2, 5, 8, 11, 14, … the common difference between each term and the preceding term is 3. A table of values for this sequence is:

Term # (n)	1	2	3	4	5
$f(n)$	2	5	8	11	14

An **explicit formula** for this sequence is $f(n) = 3n - 1$

A **recursive formula** for this sequence is: $f(n+1) = f(n) + 3, \quad f(1) = 2$

geometric sequence *(A2T)* A set of terms in which each term is formed by multiplying the preceding term by a common nonzero constant.

Example: In the geometric sequence $2, 4, 8, 16, 32...$ the common ratio is 2. Each term is 2 times the preceding term. A table of values for this sequence is:

Term (n)	1	2	3	4	5
$f(n)$	2	4	8	16	32

An **explicit formula** for this sequence is $f(n) = 2^n$

A **recursive formula** for this sequence is: $f(n+1) = 2f(n), \quad f(1) = 2$

DEVELOPING ESSENTIAL SKILLS

1) If $f(1)=5$ and $f(n)=-3f(n-1)$, then $f(4)=$

1) -15	3) 45
2) 20	4) -135

2) If a sequence is defined recursively by $f(0)=6$ and $f(n+1)=-3f(n)+4$ for all $n \geq 0$, then $f(2)$ is equal to

1) 22	3) 46
2) -27	4) -14

3) In a sequence, the first term is 3 and the common difference is 4. The fifth term of this sequence is

1) -11	3) 16
2) -8	4) 19

4) Given the function $f(n)$ defined by the following:

$$f(1)=7$$

$$f(n)=-3f(n-1)+4$$

Which set could represent the range of the function?

1) $\{7,-17,55,-111,...\}$	3) $\{1,7,17,55,...\}$
2) $\{7,25,79,321,...\}$	4) $\{1,7,25,79,...\}$

ANSWERS

1) 4
2) 3
3) 4
4) 1

REGENTS EXAM QUESTIONS (through June 2018)

F.IF.A.3, F.LE.A.2: Sequences and Series

487) If $f(1) = 3$ and $f(n) = -2f(n-1) + 1$, then $f(5) =$

1) -5	3) 21
2) 11	4) 43

488) If a sequence is defined recursively by $f(0) = 2$ and $f(n+1) = -2f(n) + 3$ for $n \geq 0$, then $f(2)$ is equal to

1) 1	3) 5
2) -11	4) 17

489) In a sequence, the first term is 4 and the common difference is 3. The fifth term of this sequence is

1) -11	3) 16
2) -8	4) 19

490) Given the function $f(n)$ defined by the following:

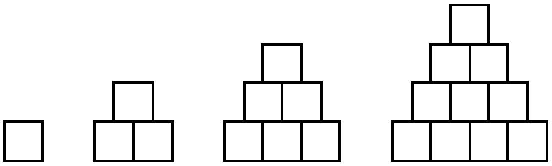
$$f(1) = 2$$

$$f(n) = -5f(n-1) + 2$$

Which set could represent the range of the function?

1) $\{2, 4, 6, 8, \dots\}$	3) $\{-8, -42, -208, 1042, \dots\}$
2) $\{2, -8, 42, -208, \dots\}$	4) $\{-10, 50, -250, 1250, \dots\}$

491) A sequence of blocks is shown in the diagram below.



This sequence can be defined by the recursive function $a_1 = 1$ and $a_n = a_{n-1} + n$. Assuming the pattern continues, how many blocks will there be when $n = 7$?

1) 13	3) 28
2) 21	4) 36

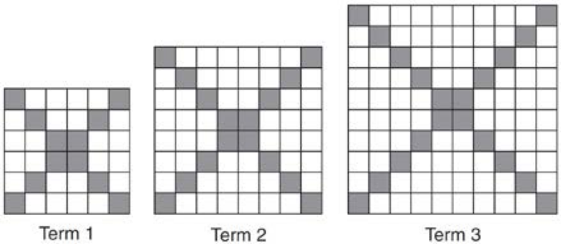
492) Determine and state whether the sequence $1, 3, 9, 27, \dots$ displays exponential behavior. Explain how you arrived at your decision.

493) A sunflower is 3 inches tall at week 0 and grows 2 inches each week. Which function(s) shown below can be used to determine the height, $f(n)$, of the sunflower in *n* weeks?

I. $f(n) = 2n + 3$
II. $f(n) = 2n + 3(n-1)$
III. $f(n) = f(n-1) + 2 \text{ where } f(0) = 3$

1) I and II	3) III, only
2) II, only	4) I and III

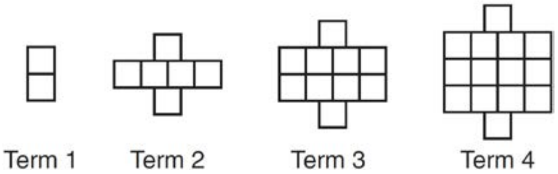
494) The diagrams below represent the first three terms of a sequence.



Assuming the pattern continues, which formula determines a_n, the number of shaded squares in the *n*th term?

1) $a_n = 4n + 12$
3) $a_n = 4n + 4$
2) $a_n = 4n + 8$
4) $a_n = 4n + 2$

495) A pattern of blocks is shown below.



If the pattern of blocks continues, which formula(s) could be used to determine the number of blocks in the *n*th term?

I	II	III
$a_n = n + 4$	$a_1 = 2$ $a_n = a_{n-1} + 4$	$a_n = 4n - 2$

1) I and II
3) II and III
2) I and III
4) III, only

496) The third term in an arithmetic sequence is 10 and the fifth term is 26. If the first term is a_1, which is an equation for the *n*th term of this sequence?

1) $a_n = 8n + 10$
3) $a_n = 16n + 10$
2) $a_n = 8n - 14$
4) $a_n = 16n - 38$

497) Which recursively defined function has a first term equal to 10 and a common difference of 4?

1) $f(1) = 10$
$f(x) = f(x - 1) + 4$
3) $f(1) = 10$
$f(x) = 4f(x - 1)$
2) $f(1) = 4$
$f(x) = f(x - 1) + 10$
4) $f(1) = 4$
$f(x) = 10f(x - 1)$

498) Which recursively defined function represents the sequence $3, 7, 15, 31, \ldots$?

1) $f(1) = 3,\ f(n + 1) = 2^{f(n)} + 3$
3) $f(1) = 3,\ f(n + 1) = 2f(n) + 1$
2) $f(1) = 3,\ f(n + 1) = 2^{f(n)} - 1$
4) $f(1) = 3,\ f(n + 1) = 3f(n) - 2$

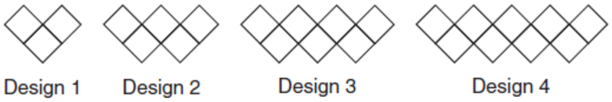
499) Which function defines the sequence -6, -10, -14, -18, ..., where $f(6) = -26$?

1) $f(x) = -4x - 2$
3) $f(x) = -x + 32$
2) $f(x) = 4x - 2$
4) $f(x) = -x - 26$

500) In 2014, the cost to mail a letter was 49¢ for up to one ounce. Every additional ounce cost 21¢. Which recursive function could be used to determine the cost of a 3-ounce letter, in cents?

1) $a_1 = 49;\ a_n = a_{n-1} + 21$
2) $a_1 = 0;\ a_n = 49a_{n-1} + 21$
3) $a_1 = 21;\ a_n = a_{n-1} + 49$
4) $a_1 = 0;\ a_n = 21a_{n-1} + 49$

501) If the pattern below continues, which equation(s) is a recursive formula that represents the number of squares in this sequence?



1) $y = 2x + 1$
2) $y = 2x + 3$
3) $a_1 = 3$
$a_n = a_{n-1} + 2$
4) $a_1 = 1$
$a_n = a_{n-1} + 2$

502) On the main floor of the Kodak Hall at the Eastman Theater, the number of seats per row increases at a constant rate. Steven counts 31 seats in row 3 and 37 seats in row 6. How many seats are there in row 20?

1) 65
2) 67
3) 69
4) 71

503) If $a_n = n\left(a_{n-1}\right)$ and $a_1 = 1$, what is the value of a_5?

1) 5
2) 20
3) 120
4) 720

504) The expression $3(x^2 + 2x - 3) - 4(4x^2 - 7x + 5)$ is equivalent to

1) $-13x - 22x + 11$
2) $-13x^2 + 34x - 29$
3) $19x^2 - 22x + 11$
4) $19x^2 + 34x - 29$

SOLUTIONS

487) ANS: 4

Strategy: Use the recursive formula: $f(1) = 3$ and $f(n) = -2f(n-1) + 1$ to find each term in the sequence.

$f(1) = 3$

$f(n) = -2f(n-1) + 1$

$f(2) = -2f(2-1) + 1 = -2f(1) + 1 = -2(3) + 1 = -6 + 1 = -5$

$f(3) = -2f(3-1) + 1 = -2f(2) + 1 = -2(-5) + 1 = 10 + 1 = 11$

$f(4) = -2f(4-1) + 1 = -2f(3) + 1 = -2(11) + 1 = -22 + 1 = -21$

$f(5) = -2f(5-1) + 1 = -2f(4) + 1 = -2(-21) + 1 = 42 + 1 = 43$

PTS: 2 NAT: F.IF.A.3 TOP: Sequences

488) ANS: 3

Strategy: Use the recursive formula: $f(0) = 2$ and $f(n+1) = -2f(n) + 3$ to find each term in the sequence.

$$f(0) = 2$$

$$f(1) = f(0 + 1) = -2f(n) + 3 = -2(2) + 3 = -4 + 3 = -1$$

$$f(2) = f(1 + 1) = -2f(n) + 3 = -2(-1) + 3 = 2 + 3 = 5$$

Answer choice *c* corresponds to $f(2) = 5$.

PTS: 2 NAT: F.IF.A.3 TOP: Sequences

489) ANS: 3

Step 1. Understand that the problem wants to know the fifth term in a sequence when the first term is 4 and the common difference is 3.

Step 2. Strategy. Build a table.

Step 3. Execute the strategy.	Term #	1	2	3	4	5
	Value	4	7	10	13	16

Step 4. Does it make sense? Yes. You can check it by writing the following formula based on the table and using it to find any term in this arithmetic sequence.

$$a_n = 3n + 1$$

$$a_5 = 3(5) + 1$$

$$a_5 = 16$$

PTS: 2 NAT: F.IF.A.3 TOP: Sequences KEY: term

490) ANS: 2

The first value in the function must be 2. Therefore, $\{-8, -42, -208, 1042, \ldots\}$ and $\{-10, 50, -250, 1250, \ldots\}$ must be wrong choices.

$$f(n) = -5f(n-1) + 2$$

$$f(1) = 2$$

$$f(2) = -5f(1) + 2$$

$$f(2) = -5(2) + 2$$

$$f(2) = -10 + 2$$

$$f(2) = -8$$

Since -8 is the second number, the correct answer choice is $\{2, -8, 42, -208, \ldots\}$.

PTS: 2 NAT: F.IF.A.3 TOP: Sequences KEY: term

491) ANS: 3

$$a_n = a_{n-1} + n$$

$$a_1 = 1$$

$$a_2 = 1 + 2 = 3$$

$$a_3 = 3 + 3 = 6$$

$$a_4 = 6 + 4 = 10$$

$$a_5 = 10 + 5 = 15$$

$$a_6 = 16 + 6 = 21$$

$$a_7 = 21 + 7 = 28$$

PTS: 2 NAT: F.IF.A.3 TOP: Sequences KEY: term

492) ANS:

Yes. Each number in the sequence is three times bigger than the previous number, so the sequence has a common ratio, which is 3.

PTS: 2 NAT: F.LE.A.1 TOP: Families of Functions

493) ANS: 4

Strategy: If sunflower's height is modelled using a table, then the three formulas can be tested to see which one(s) produce results that agree with the table.

Weeks (n)	Height $f(n)$	$f(n) = 2n + 3$	$f(n) = 2n + 3(n - 1)$	$f(n) = f(n-1) + 2$ where $f(0) = 3$
0	3	$f(0) = 2(0) + 3 = 3$	$f(0) = 2(0) + 3(0 - 1) = -3$	$f(0) = 3$
1	5	$f(1) = 2(1) + 3 = 5$		$f(1) = f(0) + 2 = 3 + 2 = 5$
2	7	$f(2) = 2(2) + 3 = 7$		$f(2) = f(1) + 2 = 5 + 2 = 7$
3	9	$f(3) = 2(3) + 3 = 9$		$f(3) = f(2) + 2 = 7 + 2 = 9$

Formula I, $f(n) = 2n + 3$, is an explicit formula that *agrees* with the table.

Formula II is an explicit formula that *does not agree* with the table.

Formula III, $f(n) = f(n-1) + 2$ where $f(0) = 3$, is a recursive formula that agrees with the table.

PTS: 2 NAT: F.IF.A.3 TOP: Sequences

494) ANS: 2

Strategy: Examine the pattern, then test each formula and eliminate wrong choices.

Term 1 has 12 shaded squares.
Term 2 has 16 shaded squares.
Term 3 has 20 shaded squares.

Choice	Equation	Term 1 = 12	Term 2 = 16	Term 3 = 20
a	$a_n = 4n + 12$	= 16 (eliminate)		
b	$a_n = 4n + 8$	= 12 (correct)	= 16 (correct)	= 20 (correct)
c	$a_n = 4n + 4$	= 8 (eliminate)		
d	$a_n = 4n + 2$	= 6 (eliminate)		

PTS: 2 NAT: F.LE.A.2 TOP: Sequences

495) ANS: 3

Strategy: Examine the pattern, then test each formula and eliminate wrong choices.

Term 1 has 2 squares.
Term 2 has 6 squares.
Term 3 has 10 squares.
Term 4 has 14 squares

n	1	2	3	4
a_n	2	6	10	14

Formula	Equation	Term 1 = 2	Term 2 = 6	Term 3 = 10	Term 4 = 14
I	$a_n = n + 4$	$a_n = n + 4$ $a_1 = 1 + 4$ $a_1 = 5$			

		This is wrong, so eliminate choices a and b..			
II	$a_1 = 2$ $a_n = a_{n-1} + 4$	$a_1 = 2$ correct	$a_n = a_{n-1} + 4$ $a_2 = a_1 + 4$ $a_2 = 2 + 4$ $a_2 = 6$ correct	$a_n = a_{n-1} + 4$ $a_3 = a_2 + 4$ $a_3 = 6 + 4$ $a_3 = 10$ correct	$a_n = a_{n-1} + 4$ $a_4 = a_3 + 4$ $a_4 = 10 + 4$ $a_3 = 14$ correct
III	$a_n = 4n - 2$	$a_n = 4n - 2$ $a_1 = 4(1) - 2$ $a_1 = 4 - 2$ $a_1 = 2$ correct	$a_n = 4n - 2$ $a_1 = 4(1) - 2$ $a_1 = 4 - 2$ $a_1 = 2$ correct	$a_n = 4n - 2$ $a_1 = 4(1) - 2$ $a_1 = 4 - 2$ $a_1 = 2$ correct	$a_n = 4n - 2$ $a_1 = 4(1) - 2$ $a_1 = 4 - 2$ $a_1 = 2$ correct

Choose answer choice *c* because Formulas II and III are both correct.

PTS: 2 NAT: F.BF.A.1 TOP: Sequences

496) ANS: 2

Strategy: Build the sequence in a table, then test each equation choice and eliminate wrong answers.

a_1	a_2	a_3	a_4	a_5
		10		26

The a_4 term must be half way between 10 and 26, so it must be 18.

The common difference is 8, so we can fill in the rest of the table as follows:

a_1	a_2	a_3	a_4	a_5
-6	2	10	18	26

The first term in the sequence is -6.

Choice	Equation	Term a_1 = -6	Term a_3 = 10	Term a_5 = 26
a	$a_n = 8n + 10$	= 18 (eliminate)		
b	$a_n = 8n - 14$	= -6 (correct)	= 10 (correct)	= 26 (correct)
c	$a_n = 16n + 10$	= 26 (eliminate)		
d	$a_n = 16n - 38$	= -12 (eliminate)		

PTS: 2 NAT: F.LE.A.2 TOP: Sequences

497) ANS: 1

Strategy: Eliminate wrong answers.

Choices *b* and *d* have first terms equal to 4, but the problem states that the first term is equal to 10. Therefore, eliminate choices *b* and *d*.

A common difference of 4 requires the addition or subtraction of 4 to find the next term in the sequence. Eliminate choice *c* because choice *c multiplies* the preceding term by 4.

Choice *a* is correct because the first term is 10 and 4 is added to each preceding term.

PTS: 2 NAT: F.IF.A.3 TOP: Sequences

498) ANS: 3

Each choice has a first term equal to 3.
Each additional term is twice its preceding term plus 1.
Strategy: Eliminate wrong answers and check.
All choices have show the the first term equals three: $f(1) = 3$.
Eliminate $f(1) = 3,\ f(n+1) = 2^{f(n)} + 3$ and $f(1) = 3,\ f(n+1) = 2^{f(n)} - 1$ because they are exponential.
Eliminate $f(1) = 3,\ f(n+1) = 3f(n) - 2$ because each term is not three times its preceding term minus two.
Check $f(1) = 3,\ f(n+1) = 2f(n) + 1$ as follows:

$$f(1) = 3,\ f(n+1) = 2f(n) + 1$$

$$f(2) = 2(3) + 1 = 7$$

$$f(3) = 2(7) + 1 = 15$$

$$f(4) = 2(15) + 1 = 31$$

$f(1) = 3,\ f(n+1) = 2f(n) + 1$ produces the sequence 3, 7, 15, 31,.....

PTS: 2 NAT: F..IF.A.3 TOP: Sequences

499) ANS: 1
Strategy #1
Construct the following table from the problem:

x	1	2	3	4	5	6
f(x)	-6	-10	-14	-18		

Then, input the four answer choices in a graphing calculator and inspect the table view to determine which answer choice reproduces the table.

Strategy #2
Use a graphing calculator to find a regression equation for the data in the above table.

PTS: 2 NAT: F.IF.A.2

500) ANS: 1
Strategy: Eliminate wrong answers.
The first ounce costs 49 cents, so eliminate any answer choice where a_1 does not equal 49.

PTS: 2 NAT: F.LE.A.2 TOP: Sequences

501) ANS: 3
STEP 1: Count the number of squares in Designs, 1, 2, 3, and 4.

Design 1 = 3
Design 2 = 5
Design 3 = 7
Design 4 = 9

STEP 2: Eliminate answer choices $y = 2x + 1$ and $y = 2x + 3$because they are not written as recursive formulas.
STEP 3: Eliminate $a_1 = 1$ because the first value in the sequence is three, so $a_1 \neq 1$.

$$a_n = a_{n-1} + 2$$

STEP 4: Choose $a_1 = 3$

$$a_n = a_{n-1} + 2$$

PTS: 2 NAT: F.LE.A.2 TOP: Sequences

502) ANS: 1
Strategy: Find the constant rate of change, then write an equation to solve for the number of seats in row 20.

STEP 1. Find the constant rate of change.

Δx	x values increase by 1			
row # (x)	3	4	5	6
# seats (y)	31	33	35	37
Δy	y values increase by 2			

$$\text{constant rate of change} = m = \frac{\Delta y}{\Delta x} = \frac{2}{1} = 2$$

STEP 2. Write the slope-intercept form of the line having a constant rate of change of 2 and any pair of known x and y values.

Given	Solve for b	Write the Entire Equation
$x = 3$	$y = mx + b$	$y = mx + b$
$y = 31$	$31 = 2(3) + b$	$y = 2x + 25$
$m = 2$	$31 = 6 + b$	
$b = ???$	$25 = b$	

STEP 3. Use the linear equation to solve for $x = 20$.

Notes	Left Expression	Sign	Right Expression
Given	y	=	$2x + 25$
Let x equal 20	y	=	$2(20) + 25$
Remove Parentheses	y	=	40+25
Simplify	y	=	65

PTS: 2 NAT: F.IF.A.3 TOP: Sequences KEY: term

503) ANS: 3

Strategy: Build a table.

n	Calculations	a_n
1	$a_1 = 1$, Given	1
2	$a_2 = n\left(a_{2-1}\right) = 2 \cdot 1 = 2$	2
3	$a_3 = n\left(a_{3-1}\right) = 3 \cdot 2 = 6$	6
4	$a_4 = n\left(a_{4-1}\right) = 4 \cdot 6 = 24$	24
5	$a_5 = n\left(a_{2-1}\right) = 5 \cdot 24 = 120$	120

The correct answer is $a_5 = 120$.

PTS: 2 NAT: F.IF.A.3 TOP: Sequences KEY: term

504) ANS: 2

Strategy: Use the distributive property to clear parentheses, then combine like terms.

Notes	Expression
Given	$3(x^2 + 2x - 3) - 4(4x^2 - 7x + 5)$
Distributive Property	$3x^2 + 6x - 9 - 16x^2 + 28x - 20$
Reorder by Like Terms	$3x^2 - 16x^2 + 6x + 28x - 9 - 20$
Combine Like Terms	$-13x^2 + 34x - 29$

PTS: 2 NAT: A.APR.A.1 TOP: Operations with Polynomials
KEY: subtraction