M – Functions, Lesson 9, Graphing Piecewise-Defined Functions (r. 2018)

FUNCTIONS

Graphing Piecewise-Defined Functions

Common Core Standard	Next Generation Standard
F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	**AI-F.IF.7** Graph functions and show key features of the graph by hand and by using technology where appropriate. (Shared standard with Algebra II)

LEARNING OBJECTIVES

Students will be able to:

1) Graph and interpret piecewise functions.
2) Input piecewise functions in a graphing calculator.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡐Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

closed dot
continuous
function
interval
open dot
piece
piecewise function
sub function

BIG IDEAS

PIECEWISE FUNCTIONS

A **piecewise function** is a function that is defined by two or more *sub* functions, with each sub function applying to a certain interval on the x-axis. Each *sub* function may also be referred to as a *piece* of the overall **piecewise function**, hence the name piecewise.

Example. The following is a piecewise function:

$$f(x) = \begin{cases} 2x+1, & x<1 \\ 4, & x \geq 1 \end{cases}$$

This example of a piecewise function has two "pieces," or sub functions.

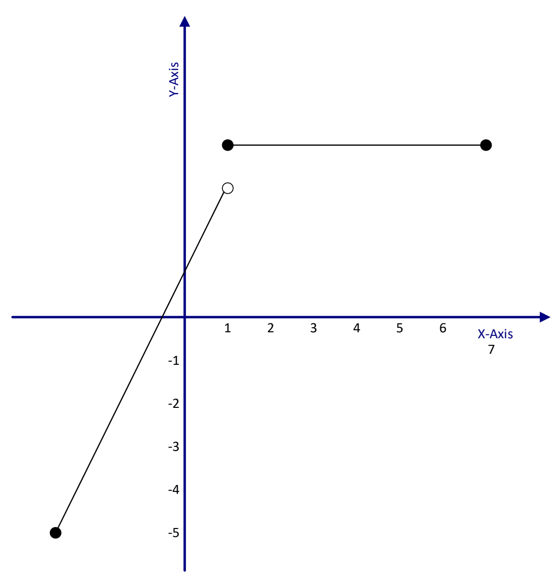
a. Over the interval $x<1$, the sub function is $f(x) = 2x+1$.

b. Over the interval $x \geq 1$, the sub function is $f(x) = 4$..

A table of values for this function.

x	$f(x) = 2x + 1$	$f(x) = 4$
-3	-5	na
-2	-3	na
-1	-1	na
0	1	na
1	na	4
2	na	4
3	na	4
4	na	4
5	na	4
6	na	4
7	na	4

A graph for this function.



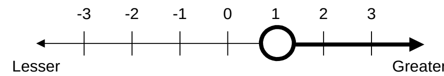
Continuity

Piecewise functions are often discontinuous, which means that the graph will not appear as a single line. In the above table, the piecewise function is discontinuous when $x = 3$. This is because $x = 3$ is not included in the sub function. Because piecewise functions are often discontinuous, care must be taken to use proper inequalities notation when graphing.

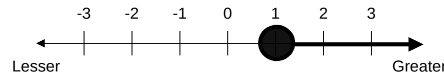
Using Line Segments to Define Pieces

If the circle at the beginning or end of a solution set (graph) is empty, that value *is not included* in the solution set. If the circle is filled in, that value *is included* in the solution set.

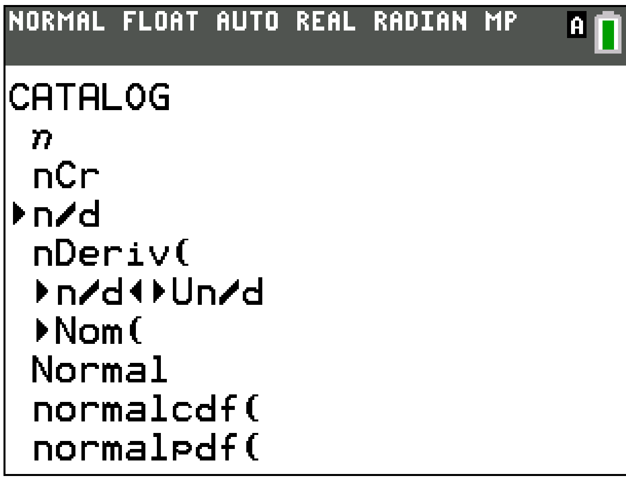
The number 1 is not included in the this solution set:

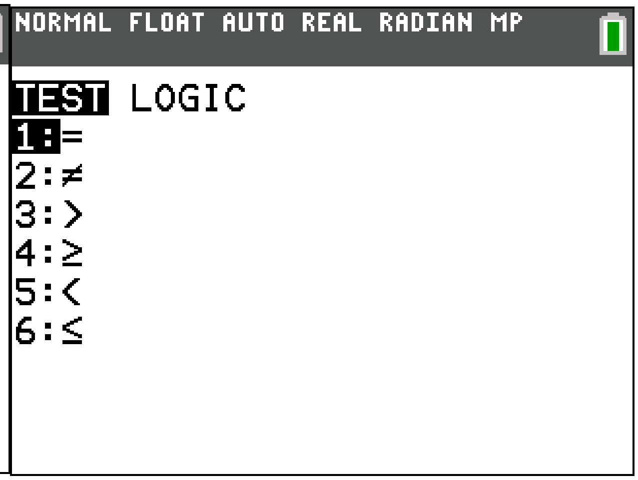


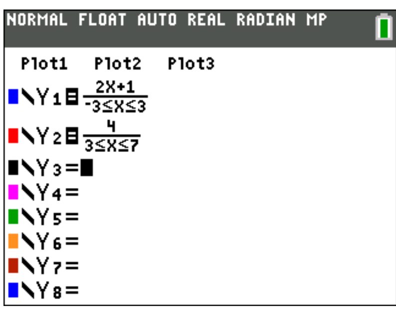
The number 1 is included in this following solution set:

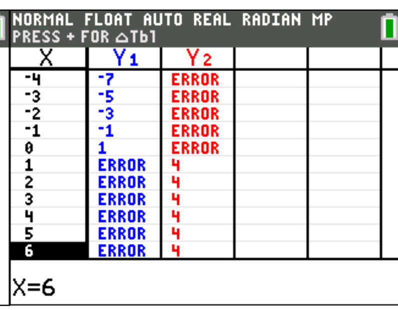


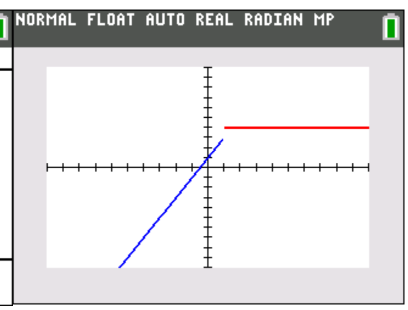
NOTE: The TI83/84 family of graphing calculators can graph piecewise functions using the n-d function in the catalog and the test (second-math) function, as shown in the following screenshots.











DEVELOPING ESSENTIAL SKILLS

Use technology to graph the following piecewise functions.

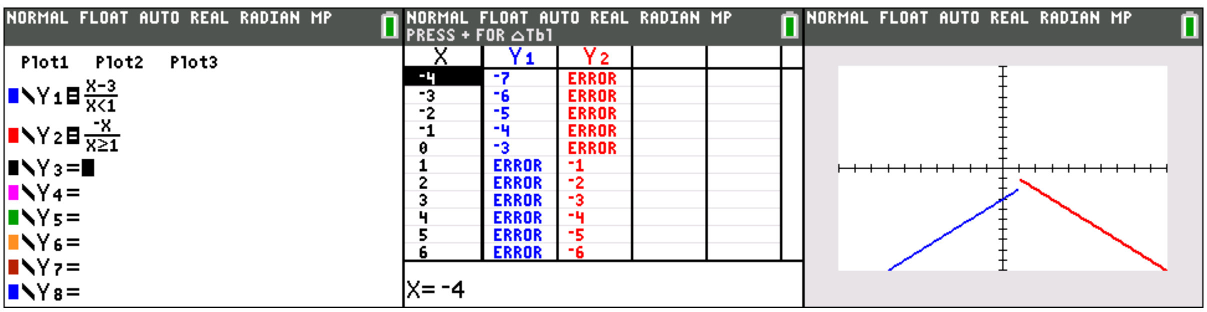
$$f\left(x\right)=\begin{cases} x-3, & x<1 \\ -x, & x\geq 1 \end{cases}$$

$$g\left(x\right)=\begin{cases} x-5, & x<1 \\ -x+2, & x\geq 1 \end{cases}$$

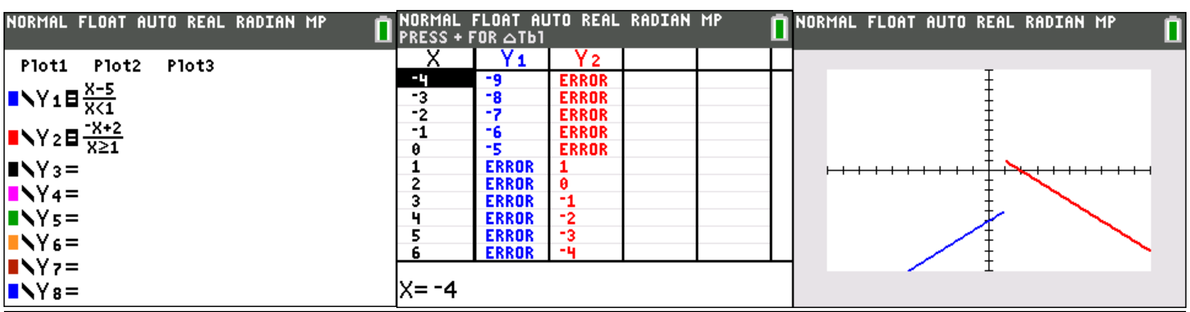
$$h\left(x\right)=\begin{cases} x+3, & x<1 \\ 2x-1, & x\geq 1 \end{cases}$$

ANSWERS

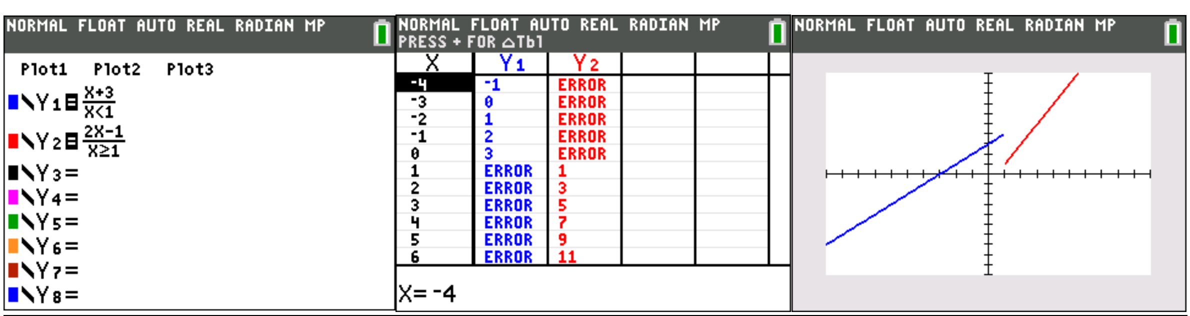
$f\left(x\right)$



$g\left(x\right)$



$h\left(x\right)$

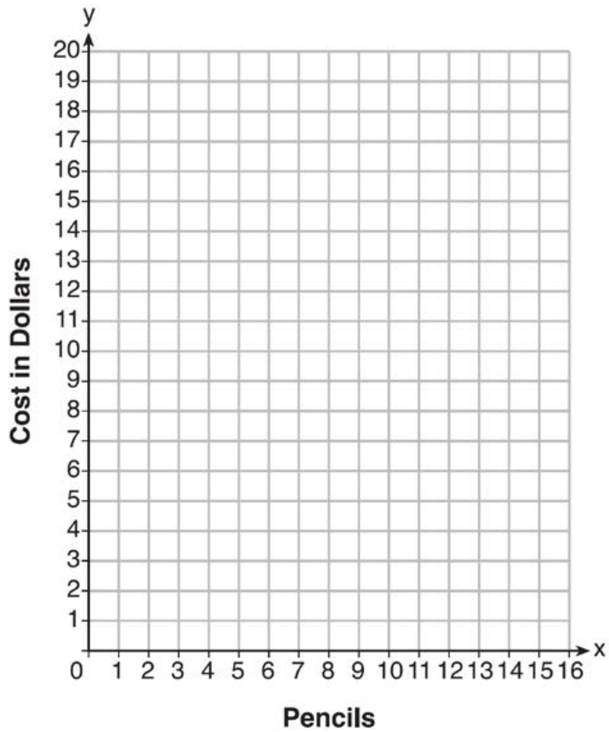


F.IF.C.7: Graphing Piecewise-Defined Functions

479) At an office supply store, if a customer purchases fewer than 10 pencils, the cost of each pencil is $1.75. If a customer purchases 10 or more pencils, the cost of each pencil is $1.25. Let c be a function for which $c(x)$ is the cost of purchasing x pencils, where x is a whole number.

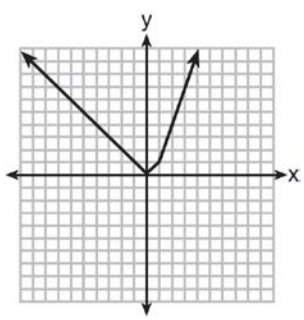
$$c(x) = \begin{cases} 1.75x, \text{ if } 0 \leq x \leq 9 \\ 1.25x, \text{ if } x \geq 10 \end{cases}$$

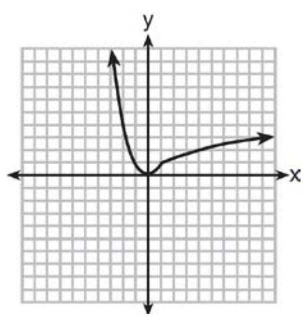
Create a graph of c on the axes below.



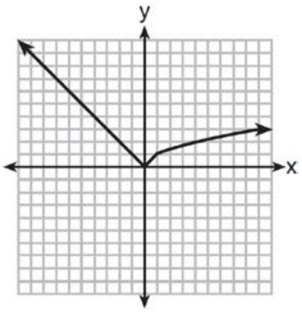
A customer brings 8 pencils to the cashier. The cashier suggests that the total cost to purchase 10 pencils would be less expensive. State whether the cashier is correct or incorrect. Justify your answer.

480) Which graph represents $f(x) = \begin{cases} |x| & x < 1 \\ \sqrt{x} & x \geq 1 \end{cases}$?

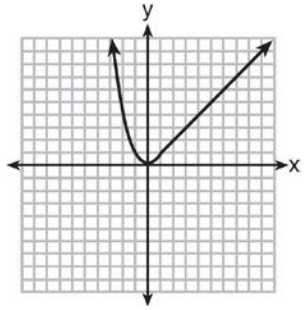
1)


3)


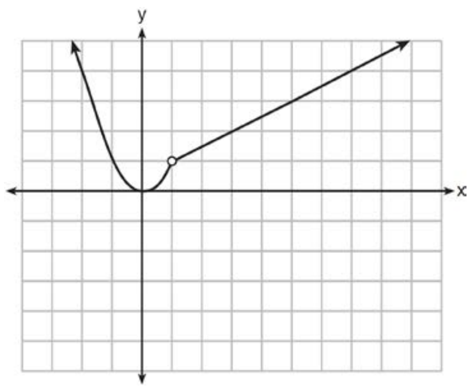
2)



4)



481) A function is graphed on the set of axes below.



Which function is related to the graph?

1) $f(x) = \begin{cases} x^2, \ x < 1 \\ x - 2, \ x > 1 \end{cases}$

3) $f(x) = \begin{cases} x^2, \ x < 1 \\ 2x - 7, \ x > 1 \end{cases}$

2) $f(x) = \begin{cases} x^2, \ x < 1 \\ \frac{1}{2}x + \frac{1}{2}, \ x > 1 \end{cases}$

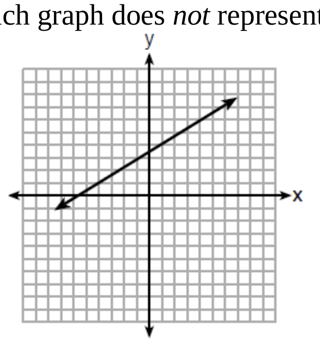
4) $f(x) = \begin{cases} x^2, \ x < 1 \\ \frac{3}{2}x - \frac{9}{2}, \ x > 1 \end{cases}$

482) Graph the following function on the set of axes below.

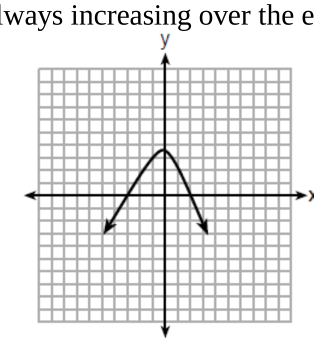
$$f(x) = \begin{cases} |x|, & -3 \le x < 1 \\ 4, & 1 \le x \le 8 \end{cases}$$

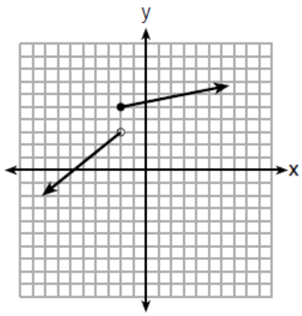
483) Which graph does *not* represent a function that is always increasing over the entire interval $-2 < x < 2$?

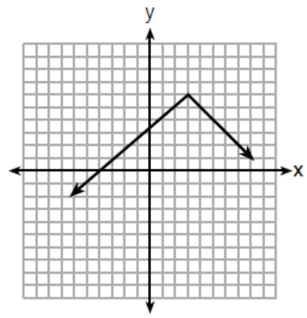
1)



3)

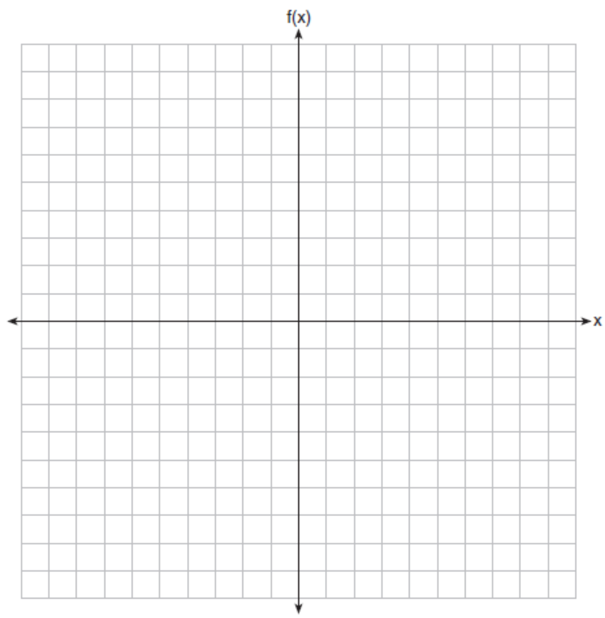


2) 

4) 

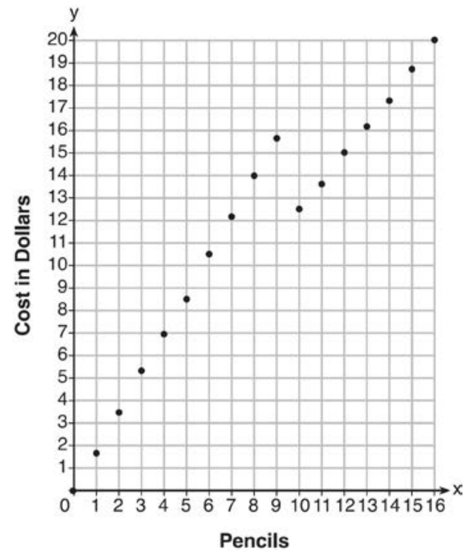
484) On the set of axes below, graph the piecewise function:

$$f(x) = \begin{cases} -\frac{1}{2}x, & x < 2 \\ x, & x \geq 2 \end{cases}$$



SOLUTIONS

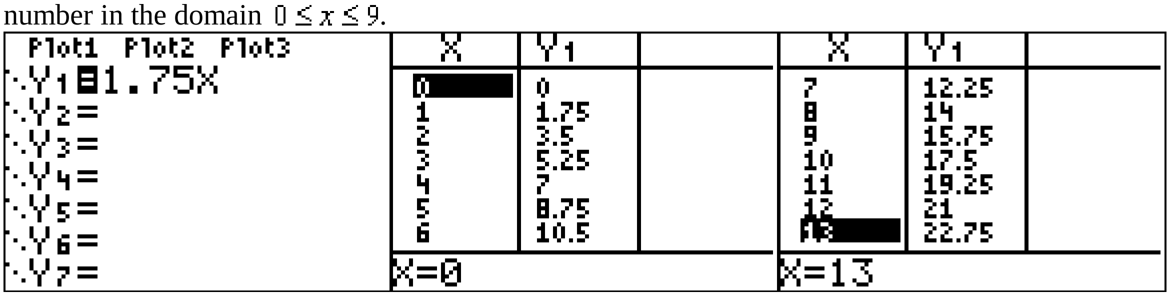
479) ANS:



The cashier is correct. 8 pencils cost \$14 and 10 pencils cost \$12.50.

Strategy: Use a graphing calculator and graph the function in two sections. Note that the domain of the function is whole numbers. You cannot buy a part of a pencil. This means that the graph of the function will consist of points and not lines. After completing the graph, answer the questions presented in the problem.

STEP 1: Graph the section of the function represented by $c(x) = 1.75x$. Plot closed dots for each whole number in the domain $0 \le x \le 9$.



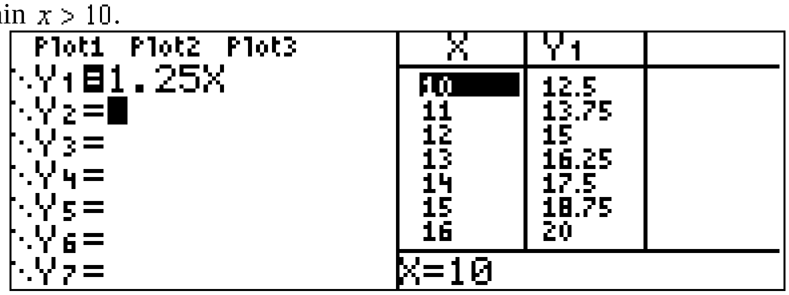
X	Y1
0	0
1	1.75
2	3.5
3	5.25
4	7
5	8.75
6	10.5

X=0

X	Y1
7	12.25
8	14
9	15.75
10	17.5
11	19.25
12	21
13	22.75

X=13

STEP 2: Graph the section of the function represented by $c(x) = 1.25x$. Plot closed dots for each whole number in the domain $x > 10$.



X	Y1
10	12.5
11	13.75
12	15
13	16.25
14	17.5
15	18.75
16	20

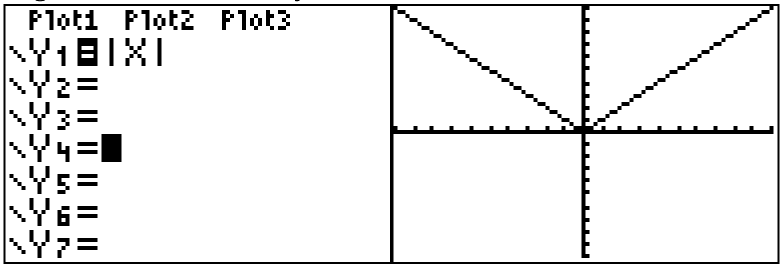
X=10

STEP 3: Answer the questions presented in the problem.
The data tables and the graph show that it would be cheaper to purchase 10 pencils that to purchase 8 pencils.

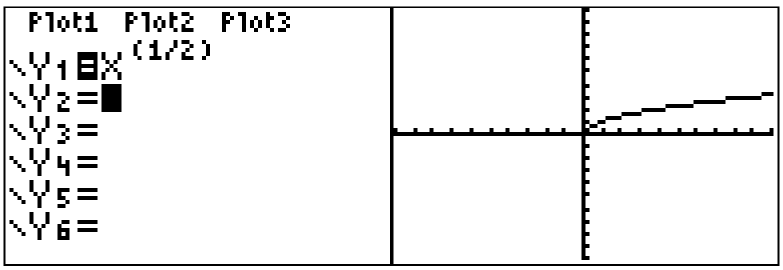
PTS: 4 NAT: F.IF.C.7 TOP: Graphing Piecewise-Defined Functions

480) ANS: 2
Strategy: Eliminate wrong answers.

The left half of each graph corresonds to $f(x) = |x|$ over the domain $x < 1$. The graph of $f(x) = |x|$ should not curve because *x* is of the first degree. Answer choices *c* and *d* should be eliminated because they have curves over the domain $x < 1$. A quick look at the graph of $f(x) = |x|$ in a graphing calculator shows why answer choices *c* and *d* should be eliminated.



The graph of $f(x) = \sqrt{x}$ over the domain $x \geq 1$ should be not be a straight line because the degree of x is not 1. A quick look at the graph of $f(x) = \sqrt{x}$ in a graphing calculator shows that answer choice *b* is correct.



PTS: 2 NAT: F.IF.C.7 TOP: Graphing Piecewise-Defined Functions
KEY: bimodalgraph

481) ANS: 2

Strategy: Since $f(x) = x^2, x < 1$ is included in every answer choice, concentrate on the linear functions for $x > 1$.

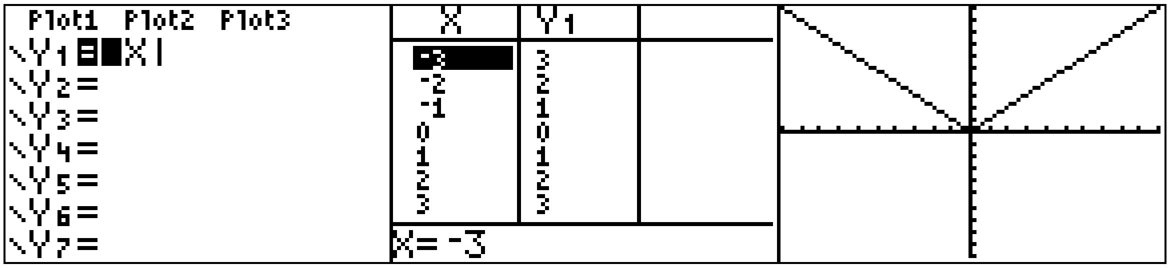
The linear equation has a slope of $\frac{rise}{run} = \frac{1}{2}$. The only linear function that has a slope of $\frac{1}{2}$ is $f(x) = \frac{1}{2}x + \frac{1}{2}$, which is answer choice b.

PTS: 2 NAT: F.IF.C.7 TOP: Graphing Piecewise-Defined Functions

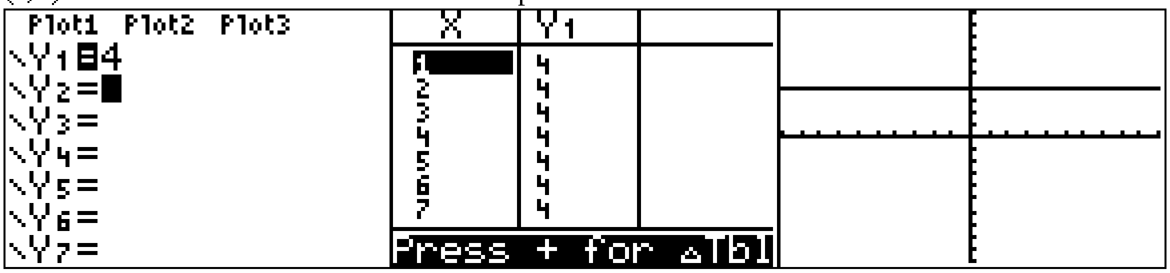
482) ANS:

Strategy: Use a graphing calculator and graph the function in sections, paying careful attention to open and closed circles at the end of each function segment.

STEP 1. Graph $f(x) = |x|$ over the interval $-3 \leq x < 1$. Use a closed dot for $(-3, 3)$ and an open dot for $(1, 1)$. Use data from the table of values to plot the interval $-3 \leq x < 1$.



STEP 2: Graph $f(x) = 4$ over the interval $1 \le x \le 8$. Use a closed dot for $(1,4)$ and a closed dot for $(8,4)$. Use data from the table of values to plot the interval $1 \le x \le 8$.

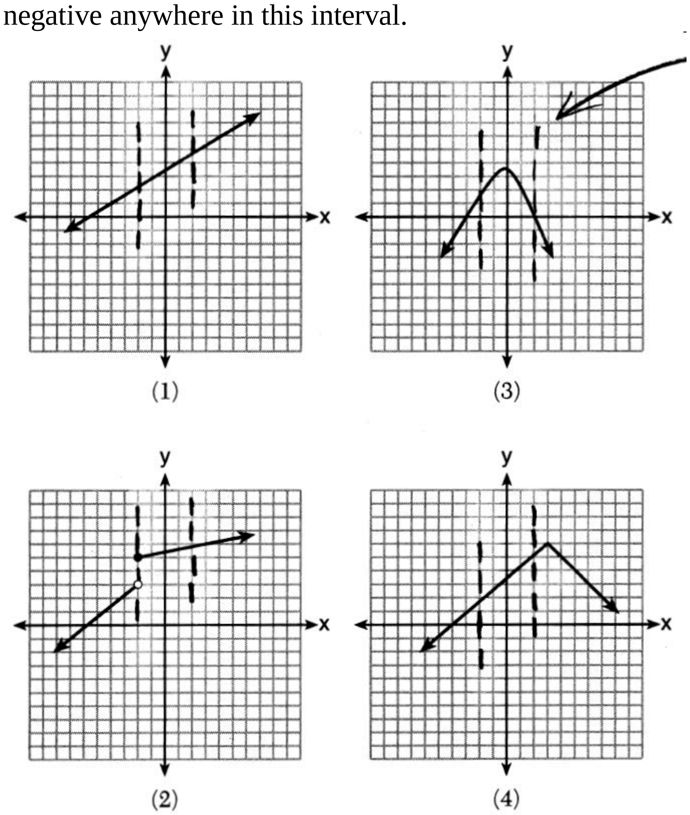


Do not connect the two graph segments.

PTS: 2 NAT: F.IF.C.7 TOP: Graphing Piecewise-Defined Functions

483) ANS: 3

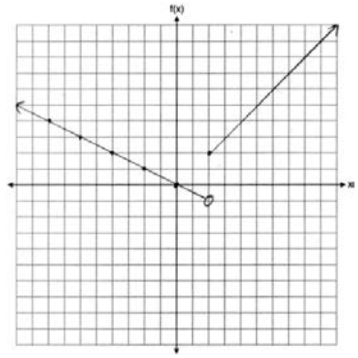
Strategy: Looks at the slope of the graph over the interval $-2 < x < 2$. Select the answer choice where the slope of the graph is negative anywhere in this interval.



Answer choice (3) is the only graph that has a negative slope over the interval $-2 < x < 2$.

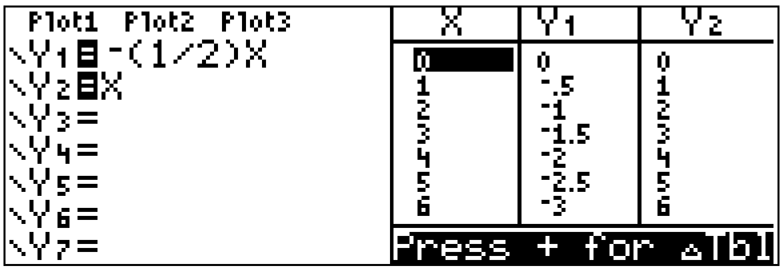
PTS: 2 NAT: F.IF.C.7 TOP: Graphing Piecewise-Defined Functions

484) ANS:



Strategy: Use a graphing calculator to help find and plot two points that define the lines for each part of this piecewise function.

STEP 1. Input the piecewise function as two separate equations in a graphing calculator and inspect the table of values for both functions.



STEP 2. Plot the points for both functions when $x = 2$, which is the x-value where the function changes from the first piece to the second piece.

(2, -1) is plotted for the first part of the function $y_1 = -(1/2)x$ with an open circle, because the domain for this piece of the function is $x < 2$.

(2, 2) is plotted for the second part of the function $y_2 = x$ with a closed circle, because the domain for this piece of the function is $x \geq 2$.

STEP 3. Pick a second point in the domain $x < 2$ to plot for the first piece (y_1) of the function.

(0, 0) is an easy ordered pair to plot.

STEP 4. Draw a directed line that starts at (2,-1) and passes through (0,0).

STEP 5. Pick a second point in the domain $x \geq 2$ to plot for the second piece (y_2) of the function.

(6, 6) is an easy ordered pair to plot.

STEP 6. Draw a directed line that starts at (2,2) and passes through (6,6).

PTS: 2 NAT: F.IF.C.7 TOP: Graphing Piecewise-Defined Functions