

M – Functions, Lesson 1, Defining Functions (r. 2018)

FUNCTIONS

Defining Functions

Common Core Standard	Next Generation Standard
<p>F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.</p>	<p>AI-F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. Note: Domain and range can be expressed using inequalities, set builder notation, verbal description, and interval notations for functions of subsets of real numbers to the real numbers.</p>

LEARNING OBJECTIVES

Students will be able to:

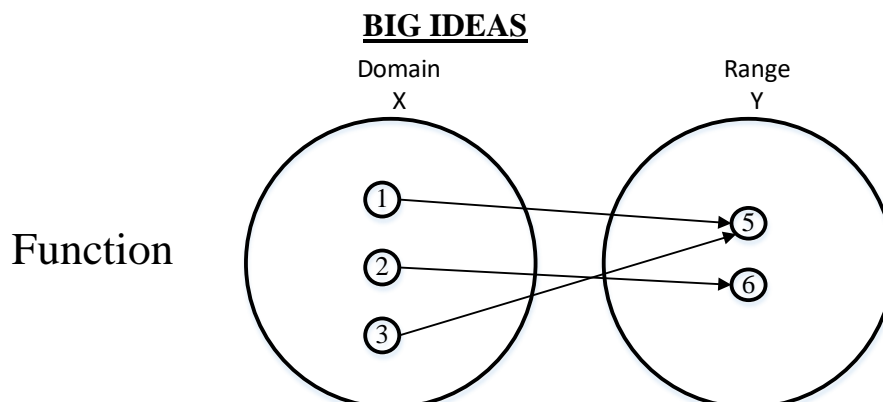
- 1) Define and identify functions.

Overview of Lesson

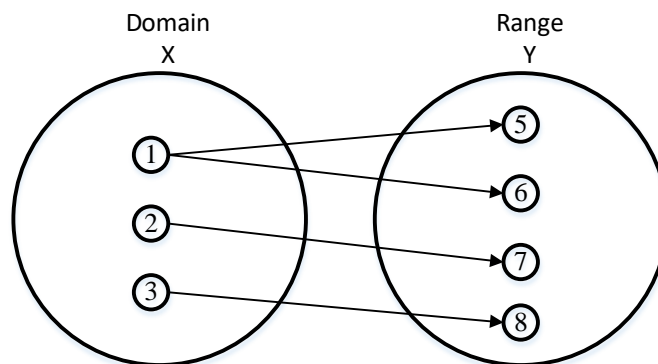
Teacher Centered Introduction	Student Centered Activities
<p>Overview of Lesson</p> <ul style="list-style-type: none"> - activate students' prior knowledge - vocabulary - learning objective(s) - big ideas: direct instruction - modeling 	<p>guided practice ← Teacher: anticipates, monitors, selects, sequences, and connects student work</p> <ul style="list-style-type: none"> - developing essential skills - Regents exam questions - formative assessment assignment (exit slip, explain the math, or journal entry)

VOCABULARY

Function: A rule that assigns to each number x in the **function's domain** (x -axis) a unique number $f(x)$ in the function's **range** (y -axis). A function takes the input value of an independent variable and pairs it with one and only one output value of a dependent variable.



Not a
Function



Expressed as ordered Pairs:

Function: (1,5) (2,6) (3,5)

Not a Function: (1,5) (2,7) (3,8) (1,6)

	<p>Function: A function is a relation that assigns exactly one value of the dependent variable to each value of the independent variable. A function is always a relation. Example: $y=2x$</p>
	<p>Relation: A relation may produce more than one output for a given input. A relation may or may not be a function. Example: $y^2 = x$ $y = \sqrt{x}$ This is not a function, because when $x=16$, there is more than one y-value. $\sqrt{16} = \pm 4$.</p>

A function can be represented four ways. These are:

- a context (verbal description)
- a function rule (equation)
- a table of values
- a graph.

Function Rules show the relationship between dependent and independent variables in the form of an equation with two variables.

- The *independent* variable is the *input* of the function and is typically denoted by the x -variable.
- The *dependent* variable is the *output* of the function and is typically denoted by the y -variable.

All linear equations in the form $y = mx + b$ are functions except vertical lines.

2nd degree and higher equations may or may not be functions.

Tables of Values show the relationship between dependent and independent variables in the form of a table with columns and rows:

- The *independent variable* is the *input* of the function and is typically shown in the left column of a vertical table or the top row of a horizontal table.
- The *dependent variable* is the *output* of the function and is typically shown in the right column of a vertical table or the bottom row of a horizontal table.

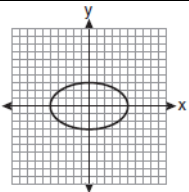
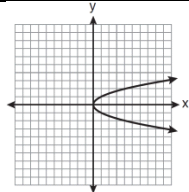
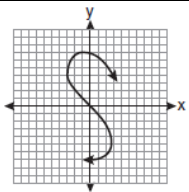
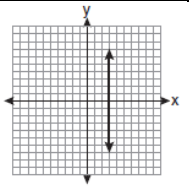
Function		Not A Function	
x	y	x	y
1	5	1	5
2	6	2	6
3	7	3	7
4	8	4	8
5	9	2	9

Graphs show the relationship between dependent and independent variables in the form of line or curve on a coordinate plane:

- The value of independent variable is input of the function and is typically shown on the x-axis (horizontal axis) of the coordinate plane.
- The value of the dependent variable is the output of the function and is typically shown on the y-axis (vertical axis) of the coordinate plane.

Vertical Line Test: If a vertical line passes through a graph of an equation more than once, the graph is *not* a graph of a function.

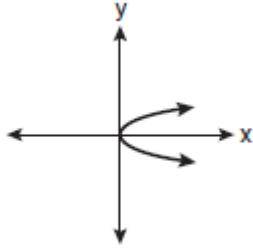
If you can draw a vertical line through any value of x in a relation, and the vertical line intersects the graph in two or more places, the relation is not a function.

			
Circles and Ellipses ...are <u>not functions</u> .	Parabola-like graphs that open to the side ...are <u>not functions</u> .	S-Curves ...are <u>not functions</u>	Vertical lines ...are <u>not functions</u> .

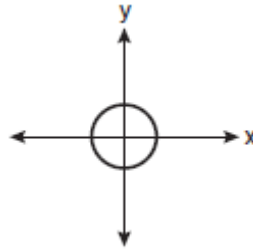
DEVELOPING ESSENTIAL SKILLS

1. Which graph represents a function?

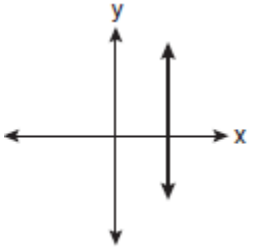
a.



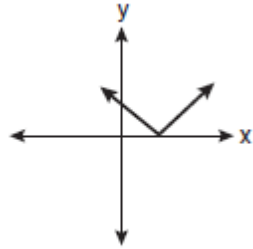
c.



b.



d.



2. Which relation is *not* a function?

a. $\{(1, 5), (2, 6), (3, 6), (4, 7)\}$

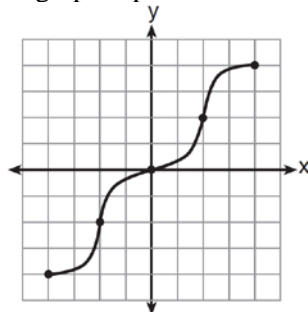
c. $\{(-1, 6), (1, 3), (2, 5), (1, 7)\}$

b. $\{(4, 7), (2, 1), (-3, 6), (3, 4)\}$

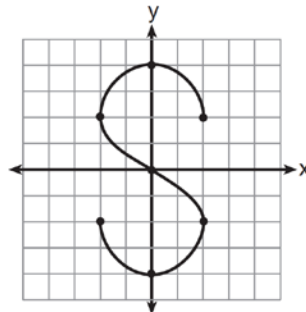
d. $\{(-1, 2), (0, 5), (5, 0), (2, -1)\}$

3. Which graph represents a function?

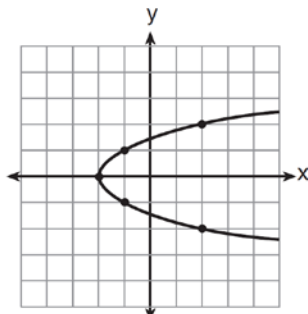
a.



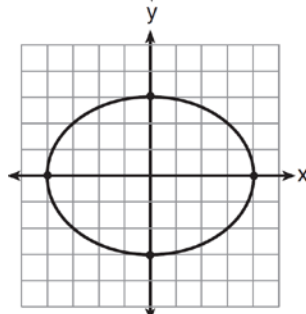
c.



b.



d.



4. Which relation is *not* a function?

a. $\{(2, 4), (1, 2), (0, 0), (-1, 2), (-2, 4)\}$

c. $\{(2, 2), (1, 1), (0, 0), (-1, 1), (-2, 2)\}$

b. $\{(2, 4), (1, 1), (0, 0), (-1, 1), (-2, 4)\}$

d. $\{(2, 2), (1, 1), (0, 0), (1, -1), (2, -2)\}$

5. Which relation is a function?

a. $\{(2, 1), (3, 1), (4, 1), (5, 1)\}$

c. $\{(2, 3), (3, 2), (4, 2), (2, 4)\}$

b. $\{(1, 2), (1, 3), (1, 4), (1, 5)\}$

d. $\{(1, 6), (2, 8), (3, 9), (3, 12)\}$

6. Which set is a function?

a. $\{(3, 4), (3, 5), (3, 6), (3, 7)\}$

c. $\{(6, 7), (7, 8), (8, 9), (6, 5)\}$

b. $\{(1, 2), (3, 4), (4, 3), (2, 1)\}$

d. $\{(0, 2), (3, 4), (0, 8), (5, 6)\}$

ANSWERS

1. ANS: D
2. ANS: C
3. ANS: A
4. ANS: D
5. ANS: A
6. ANS: B

REGENTS EXAM QUESTIONS (through June 2018)

F.IF.A.1: Defining Functions

399) Which table represents a function?

1)

x	2	4	2	4
f(x)	3	5	7	9

2)

x	0	-1	0	1
f(x)	0	1	-1	0

3)

x	3	5	7	9
f(x)	2	4	2	4

4)

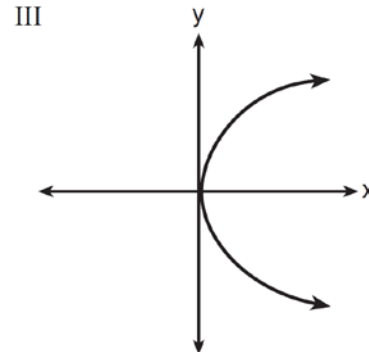
x	0	1	-1	0
f(x)	0	-1	0	1

400) The function f has a domain of $\{1, 3, 5, 7\}$ and a range of $\{2, 4, 6\}$. Could f be represented by $\{(1, 2), (3, 4), (5, 6), (7, 2)\}$? Justify your answer.

401) Which representations are functions?

I

x	y
2	6
3	-12
4	7
5	5
2	-6



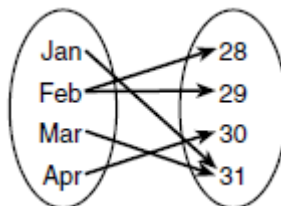
II $\{(1,1), (2,1), (3,2), (4,3), (5,5), (6,8), (7,13)\}$

IV $y = 2x + 1$

- 1) I and II
- 2) II and IV

- 3) III, only
- 4) IV, only

402) A mapping is shown in the diagram below.



This mapping is

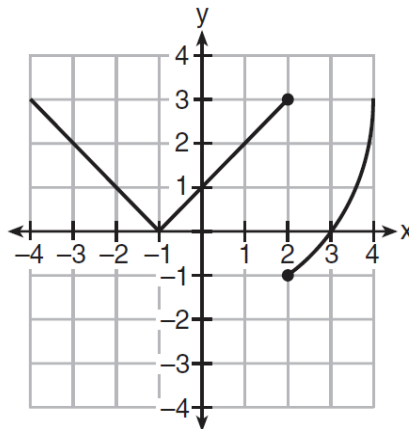
- 1) a function, because Feb has two outputs, 28 and 29
- 2) a function, because two inputs, Jan and Mar, result in the output 31
- 3) not a function, because Feb has two outputs, 28 and 29
- 4) not a function, because two inputs, Jan and Mar, result in the output 31

403) A function is shown in the table below.

x	f(x)
-4	2
-1	-4
0	-2
3	16

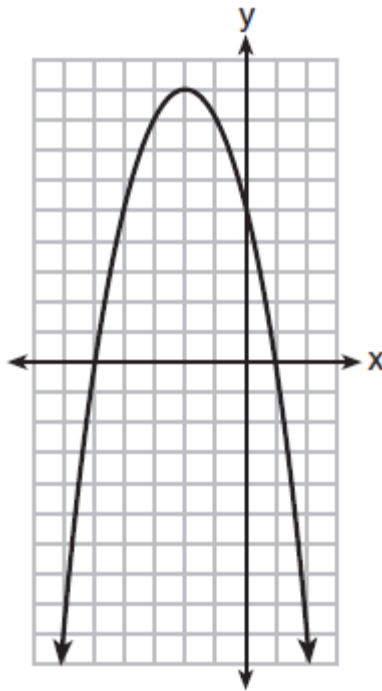
If included in the table, which ordered pair, $(-4, 1)$ or $(1, -4)$, would result in a relation that is no longer a function? Explain your answer.

404) Marcel claims that the graph below represents a function.



State whether Marcel is correct. Justify your answer.

- 405) Nora says that the graph of a circle is a function because she can trace the whole graph without picking up her pencil. Mia says that a circle graph is *not* a function because multiple values of x map to the same y -value. Determine if either one is correct, and justify your answer completely.
- 406) A relation is graphed on the set of axes below.



Based on this graph, the relation is

- | | |
|--|---|
| 1) a function because it passes the horizontal line test | 3) not a function because it fails the horizontal line test |
| 2) a function because it passes the vertical line test | 4) not a function because it fails the vertical line test |

407) A function is defined as $\{(0, 1), (2, 3), (5, 8), (7, 2)\}$. Isaac is asked to create one more ordered pair for the function. Which ordered pair can he add to the set to keep it a function?

- | | |
|-------------|-------------|
| 1) $(0, 2)$ | 3) $(7, 0)$ |
| 2) $(5, 3)$ | 4) $(1, 3)$ |

SOLUTIONS

399) ANS: 3

Strategy: Eliminate wrong answers. A function is a relation that assigns exactly one value of the dependent variable to each value of the independent variable.

Answer choice a **is not** a function because there are two values of y when $x = 2$.

Answer choice b **is not** a function because there are two values of y when $x = 0$.

Answer choice c **is** a function because only one value of y is paired with each value of x .

Answer choice d **is not** a function because there are two values of y when $x = 0$.

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions

400) ANS:

Yes, because every element of the domain is assigned one unique element in the range.

Strategy: Determine if any value of x has more than one associated value of y . A function has one and only one value of y for every value of x .

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions

401) ANS: 2

Strategy: Determine if each of the for views are functions, then select from the answer choices. A function is a relation that assigns exactly one value of the dependent variable to each value of the independent variable.

I *is not* a function because when $x = 2$, y can equal both 6 and -6.

II *is* a function because there are no values of x that have more than one value of y .

III *is not* a function because it fails the vertical line test, which means there are values of x that have more than one value of y .

IV *is* a function because it is a straight line that is not vertical.

Answer choice b is the correct answer.

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions

402) ANS: 3

A function has one and only one output for each input. The diagram shows that February maps to two different output numbers, so the diagram cannot represent a function.

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions

KEY: ordered pairs

403) ANS:

$(-4, 1)$, because then every element of the domain is not assigned one unique element in the range.

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions

404) ANS:

Marcel is not correct, because the relation does not pass the vertical line test. If you draw the vertical line $x = 2$, there will be more than one value of y . A function can have one and only one value of y for every value of x .

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions

KEY: graphs

405) ANS:

Neither is correct.

Nora's reason is wrong since a circle is not a function because it fails the vertical line test.

Although Mia correctly states that a circle is not a function, her reasoning is wrong. She confuses the variables in the definition of a function, which states that a function has one and only one value of y for each value of x . It is okay for a y to be associated with multiple values of x in a function. It is not okay for an x to be associated with multiple values of y .

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions

KEY: graphs

406) ANS: 2

A function has one and only one value of y for each value of x . A graph represents a function if there are no vertical lines that intersect the graph at more than one point.

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions

KEY: graphs

407) ANS: 4

Strategy. Use the definition of a function to eliminate wrong answers. (i.e. for each value of x in a function, there can be one and only one value of y).

Choice 1: $(0, 2)$ Wrong, because 0 is already paired with $y = 1$.

Choice 2: $(5, 3)$ Wrong, because 5 is already paired with $y = 8$.

Choice 3: $(7, 0)$ Wrong, because 7 is already paired with $y = 2$.

Choice 4: $(1, 3)$ Correct, because 1 is not paired with any other value of y .

PTS: 2 NAT: F.IF.A.1 TOP: Defining Functions

KEY: ordered pairs