M – Functions, Lesson 6, Transformations with Functions (r. 2018)

FUNCTIONS

Transformations with Functions

Common Core Standard	Next Generation Standard
F-BF.3 Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, ~~*f(kx)*~~, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing ~~even and odd~~ functions from their graphs and algebraic expressions for them. ~~PARCC: Identifying the effect on the graph of replacing *f(x)* by *f(x) +k*, *kf(x)*, and *f(x+k)* for specific values of *k* (both positive and negative) is limited to linear and quadratic functions. Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. Tasks do not involve recognizing even and odd functions.~~	**AI-F.BF.3a** Using *f(x) + k*, *k f(x)*, and *f(x + k)*: i) identify the effect on the graph when replacing *f(x)* by *f(x) + k*, *k f(x)*, and *f(x + k)* for specific values of *k* (both positive and negative); ii) find the value of *k* given the graphs; **iii) write a new function using the value of *k*;** and iv) use technology to experiment with cases and explore the effects on the graph. (Shared standard with Algebra II) **Note: Tasks are limited to linear, quadratic, square root, and absolute value functions; and exponential functions of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).**

NOTE: This lesson is related to Polynomials, Lesson 6, Graphing Polynomial Functions

LEARNING OBJECTIVES

Students will be able to:

1)

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ← Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

down	left	transform
function	right	up

BIG IDEAS

Transforming Any Function

The graph of any function is changed when either $f(x)$ or x is multiplied by a scalar, or when a constant is added to or subtracted from either $f(x)$ or x. A graphing calculator can be used to explore the translations of graph views of functions.

Up and Down

The addition or subtraction of a constant **outside the parentheses** moves the graph up or down by the value of the constant.

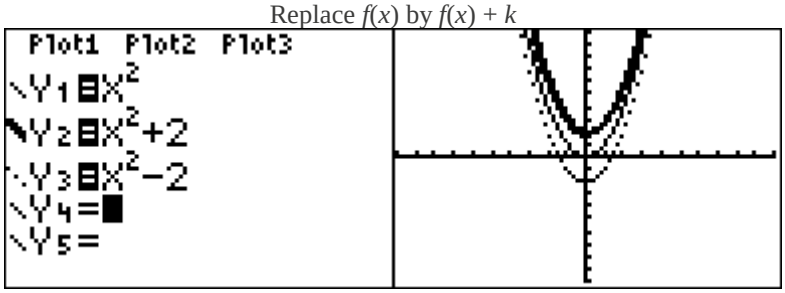
$f(x) \Leftrightarrow f(x) \pm k$ moves the graph up or down k units $\updownarrow$.

+k moves the graph up.

-k moves the graph down.

Examples:

Replace $f(x)$ by $f(x) + k$



Left and Right

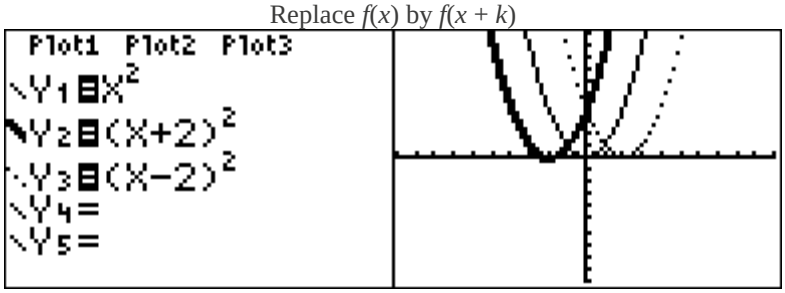
The addition or subtraction of a constant **inside the parentheses** moves the graph left or right by the value of the constant.

$f(x) \Leftrightarrow f(x \pm k)$ moves the graph left or right k units $\updownarrow$.

+k moves the graph leftg k units.

-k moves the graph right k units.

Replace $f(x)$ by $f(x + k)$



Width and Direction of a Parabola

Changing the value of *a* in a quadratic affects the width and direction of a parabola. The bigger the absolute value of *a*, the narrower the parabola.

$f(x) \Leftrightarrow f(kx)$ changes the direction and width of a parabola.

+k opens the parabola upward.

-k opens the parabola downward.

If k is a fraction less than 1, the parabola will get wider.

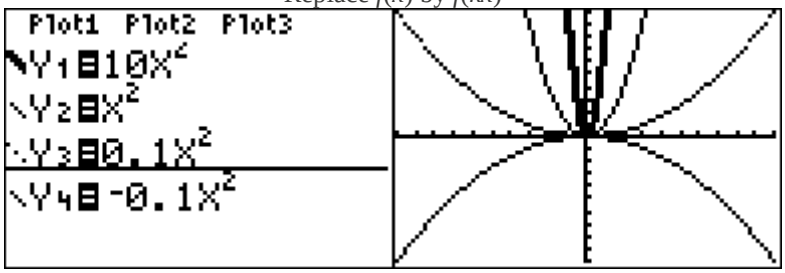
As k approaches zero, the parabola approaches a straight horizontal line.

If k is a number greater than 1, the parabola will get narrower.

As k approaches infinity, the parabola approaches a straight vertical line.

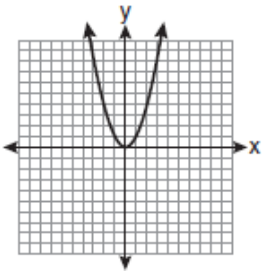
Examples:

Replace *f*(*x*) by *f*(*kx*)



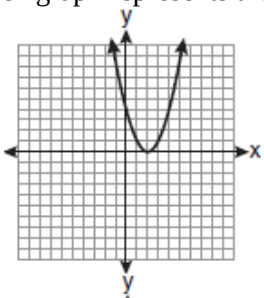
DEVELOPING ESSENTIAL SKILLS

1. The graph below shows the function $f(x)$.

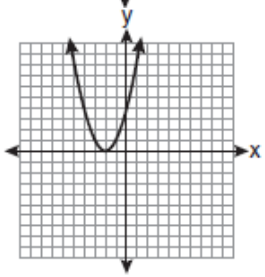


Which graph represents the function $f(x+2)$?

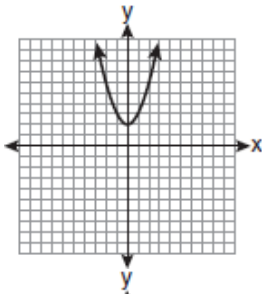
a.



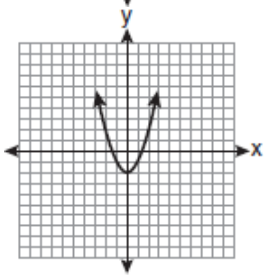
b.



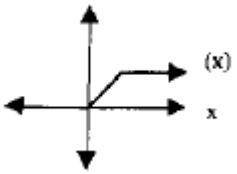
c.



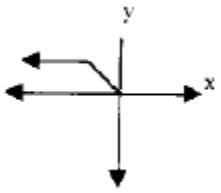
d.

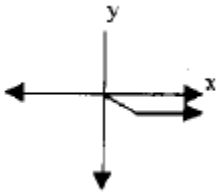


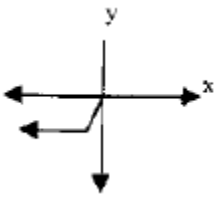
2.The graph below represents $f(x)$.

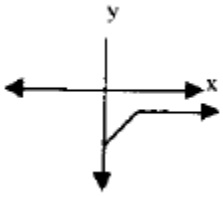


Which of the following is the graph of $-f(x)$?

a. 

c. 

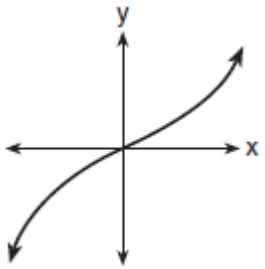
b. 

d. 

3. The minimum point on the graph of the equation $y = f(x)$ is $(-1,-3)$. What is the minimum point on the graph of the equation $y = f(x) + 5$?

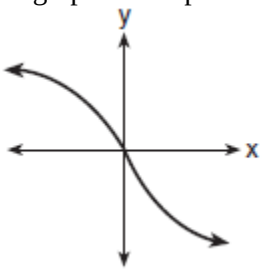
a. $(-1,2)$
b. $(-1,-8)$
c. $(4,-3)$
d. $(-6,-3)$

4. The graph below represents $f(x)$.

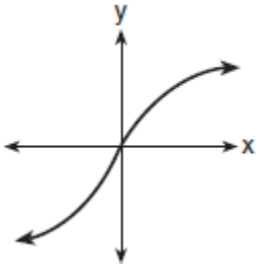


Which graph best represents $f(-x)$?

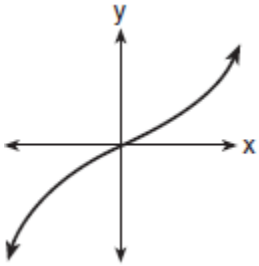
a.



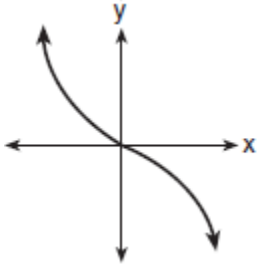
b.



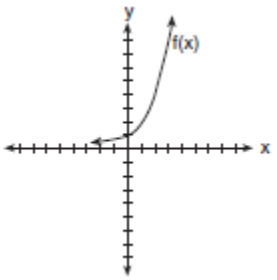
c.



d.

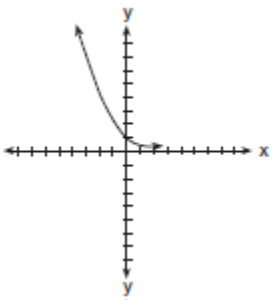


5. The graph of $f(x)$ is shown in the accompanying diagram

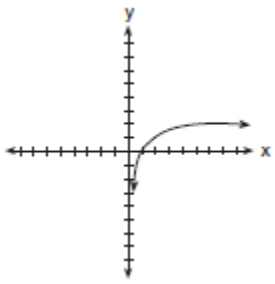


Which graph represents $f(x)_{r_{x-axis} \circ r_{y-axis}}$?

a.



c.



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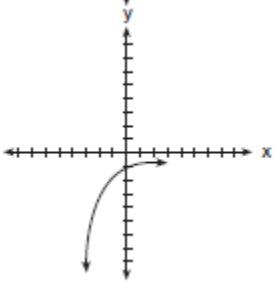
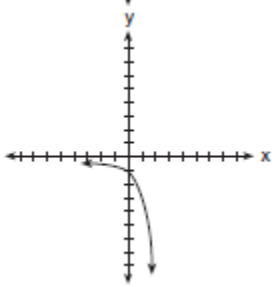


d.



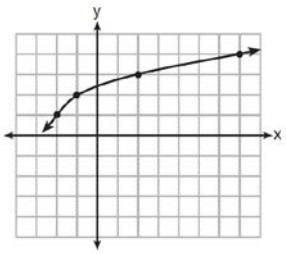
ANSWERS

1. ANS: B
2. ANS: C
3. ANS: A
4. ANS: D
5. ANS: B

REGENTS EXAM QUESTIONS (through June 2018)

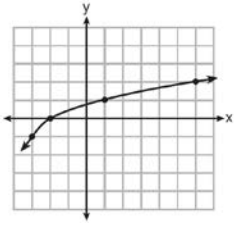
F.BF.B.3: Transformations with Functions

462) The graph of $y = f(x)$ is shown below.

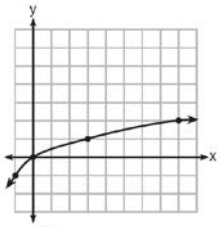


What is the graph of $y = f(x+1) - 2$?

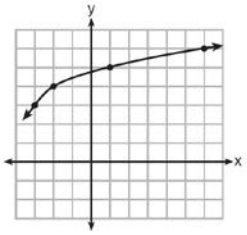
1)



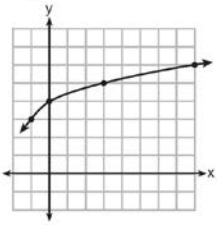
3)



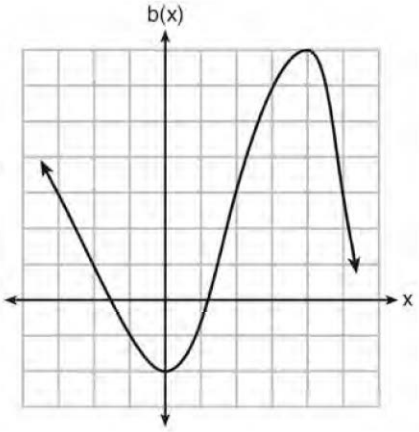
2)



4)



463) Richard is asked to transform the graph of *b(x)* below.



The graph of *b(x)* is transformed using the equation $h(x) = b(x-2) - 3$. Describe how the graph of *b(x)* changed to form the graph of *h(x)*.

SOLUTIONS

462) ANS: 1

Strategy: Identify the differences between the two function rules, then verify using the four points shown in the answer choices.

Function rules:

Difference #1: The term $f(x)$ becomes $f(x+1)$. This means the graph will move to the left 1 unit. The mapping of each x value can be expressed as $(x) \rightarrow (x-1)$
Difference #2: The term -2 is added to the function rule. This means the graph will move 2 units down. The mapping of each y value can be expressed as $(y) \rightarrow (y-2)$.

The 2 differences in the function rules mean that each point on the graph will move left 1 unit and down 2 units. Answer choice (a) shows this:

$y = f(x)$	(-2, 1)	(-1, 2)	(2, 3)	(7, 7)
$y = f(x+1) - 2$	(-3, -1)	(-2, 0)	(1, 1)	(6, 2)

PTS: 2 NAT: F.BF.B.3 TOP: Graphing Radical Functions

463) ANS:
Every point moves down 3 units.
Every point moves right 2 units.

PTS: 2 NAT: F.BF.B.3