POLYNOMIALS
Graphing Polynomial Functions

Common Core Standard	**Next Generation Standard**
F-BF.3 Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, ~~*f(kx)*~~, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing ~~even and odd~~ functions from their graphs and algebraic expressions for them. ~~PARCC: Identifying the effect on the graph of replacing *f(x)* by *f(x) +k*, *kf(x)*, and *f(x+k)* for specific values of *k* (both positive and negative) is limited to linear and quadratic functions. Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. Tasks do not involve recognizing even and odd functions.~~	**AI-F.BF.3a** Using *f(x) + k*, *k f(x)*, and *f(x + k)*: i) identify the effect on the graph when replacing *f(x)* by *f(x) + k*, *k f(x)*, and *f(x + k)* for specific values of *k* (both positive and negative); ii) find the value of *k* given the graphs; **iii) write a new function using the value of *k*;** and iv) use technology to experiment with cases and explore the effects on the graph. (Shared standard with Algebra II) **Note: Tasks are limited to linear, quadratic, square root, and absolute value functions; and exponential functions of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ ($b \neq 1$).**

LEARNING OBJECTIVES

Students will be able to:

1) Use a constant *k* in the equation of the parabola to move the graph of parabolas up, down, left, and/or right.
2) Use a constant *k* in the equation of the parabola to make the parabola open upward or downward.
3) Use a constant *k* in the equation of the parabola to make the parabola narrower or wider.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ← Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

constant	scalar	vertex
narrower	translation	wider

BIG IDEAS

The graph of a function is changed when either $f(x)$ or x is multiplied by a scalar, or when a constant is added to or subtracted from either $f(x)$ or x. A graphing calculator can be used to explore the translations of graph views of functions.

Up and Down

The addition or subtraction of a constant **outside the parentheses** moves the graph up or down by the value of the constant.

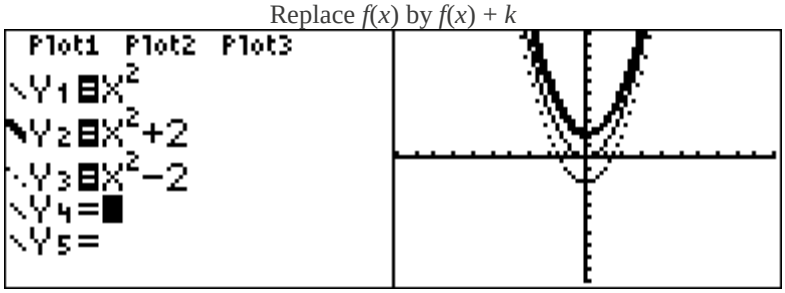
$f(x) \Leftrightarrow f(x) \pm k$ moves the graph up or down k units $\updownarrow$.

+k moves the graph up.

-k moves the graph down.

Examples:

Replace $f(x)$ by $f(x) + k$



Left and Right

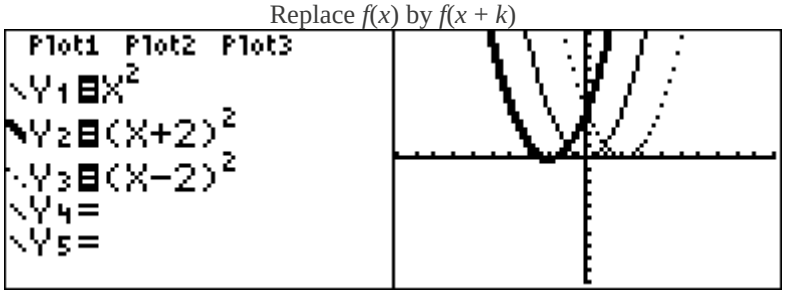
The addition or subtraction of a constant **inside the parentheses** moves the graph left or right by the value of the constant.

$f(x) \Leftrightarrow f(x \pm k)$ moves the graph left or right k units $\updownarrow$.

+k moves the graph leftg k units.

-k moves the graph right k units.

Replace $f(x)$ by $f(x + k)$



Width and Direction of a Parabola

Changing the value of *a* in a quadratic affects the width and direction of a parabola. The bigger the absolute value of *a*, the narrower the parabola.

$f(x) \Leftrightarrow f(kx)$ changes the direction and width of a parabola.

+k opens the parabola upward.

-k opens the parabola downward.

If k is a fraction less than 1, the parabola will get wider.

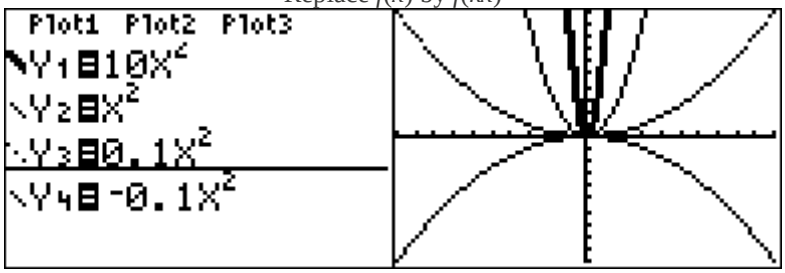
As k approaches zero, the parabola approaches a straight horizontal line.

If k is a number greater than 1, the parabola will get narrower.

As k approaches infinity, the parabola approaches a straight vertical line.

Examples:

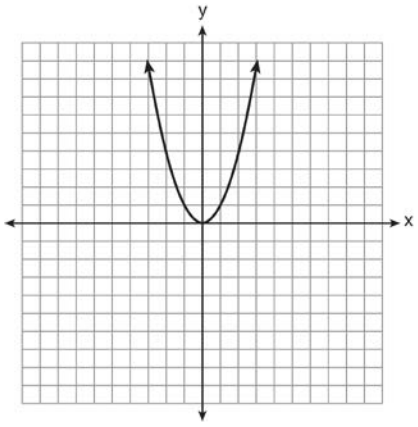
Replace *f*(*x*) by *f*(*kx*)



DEVELOPING ESSENTIAL SKILLS

1. Consider the graph of the equation $y = ax^2 + bx + c$, when $a \neq 0$. If *a* is multiplied by 3, what is true of the graph of the resulting parabola?
   a. The vertex is 3 units above the vertex of the original parabola.
   b. The new parabola is 3 units to the right of the original parabola.
   c. The new parabola is wider than the original parabola.
   d. The new parabola is narrower than the original parabola.
2. Melissa graphed the equation $y = x^2$ and Dave graphed the equation $y = -3x^2$ on the same coordinate grid. What is the relationship between the graphs that Melissa and Dave drew?
   a. Dave's graph is wider and opens in the opposite direction from Melissa's graph.
   b. Dave's graph is narrower and opens in the opposite direction from Melissa's graph.
   c. Dave's graph is wider and is three units below Melissa's graph.
   d. Dave's graph is narrower and is three units to the left of Melissa's graph.
3. The graph of a parabola is represented by the equation $y = ax^2$ where *a* is a positive integer. If *a* is multiplied by 2, the new parabola will become
   a. narrower and open downward
   b. narrower and open upward
   c. wider and open downward
   d. wider and open upward
4. How is the graph of $y = x^2 + 4x + 3$ affected when the coefficient of x^2 is changed to a smaller positive number?
   a. The graph becomes wider, and the *y*-intercept changes.
   b. The graph becomes wider, and the *y*-intercept stays the same.
   c. The graph becomes narrower, and the *y*-intercept changes.
   d. The graph becomes narrower, and the *y*-intercept stays the same.
5. Which is the equation of a parabola that has the same vertex as the parabola represented by $y = x^2$, but is wider?
   a. $y = x^2 + 2$
   b. $y = x^2 - 2$
   c. $y = 2x^2$
   d. $y = \frac{1}{2}x^2$

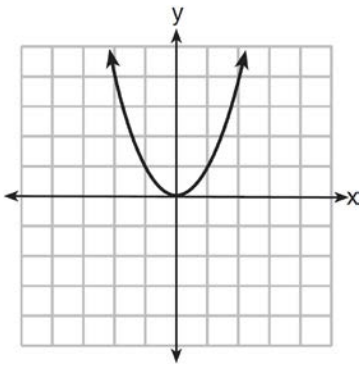
6.The graph of the equation $y = x^2$ is shown below.



Which statement best describes the change in this graph when the coefficient of x^2 is multiplied by 4?

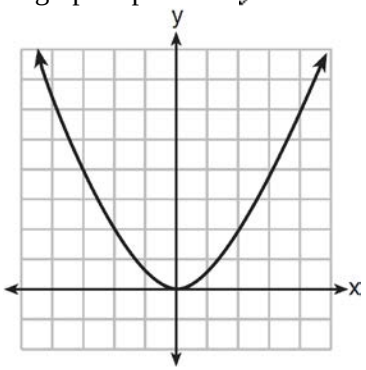
a. The parabola becomes wider.
b. The parabola becomes narrower.
c. The parabola will shift up four units.
d. The parabola will shift right four units.

7. The graph of $y = x^2$ is shown below.

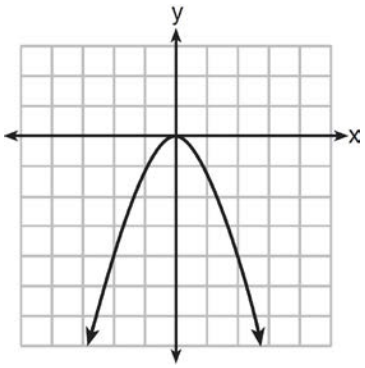


Which graph represents $y = 2x^2$?

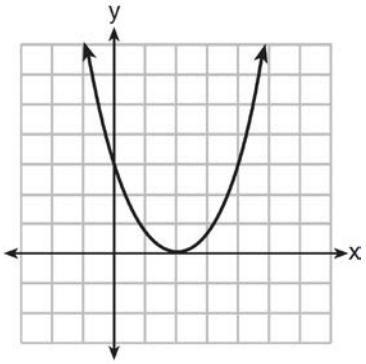
a.



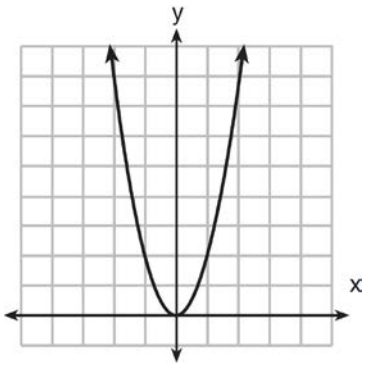
c.



b.



d.



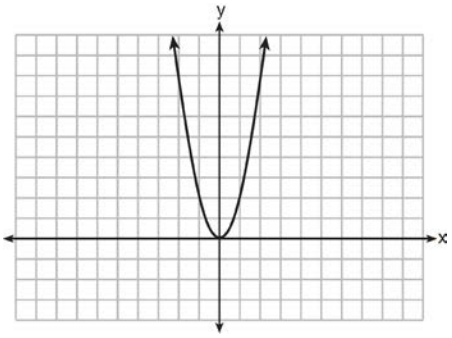
ANSWERS

1. ANS: D
2. ANS: B
3. ANS: B
4. ANS: B
5. ANS: D
6. ANS: B
7. ANS: D

REGENTS EXAM QUESTIONS (through June 2018)

F.BF.B.3: Graphing Polynomial Functions

372) The graph of the equation $y = ax^2$ is shown below.



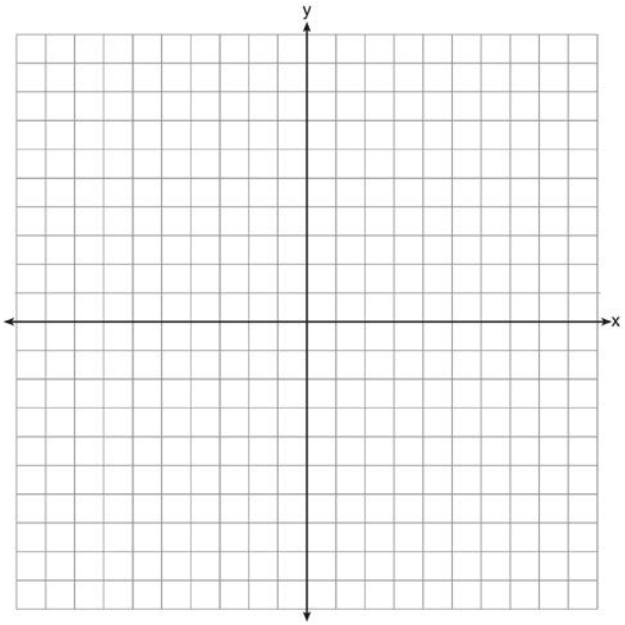
If *a* is multiplied by $-\frac{1}{2}$, the graph of the new equation is

1) wider and opens downward
2) wider and opens upward
3) narrower and opens downward
4) narrower and opens upward

373) How does the graph of $f(x) = 3(x-2)^2 + 1$ compare to the graph of $g(x) = x^2$?

1) The graph of $f(x)$ is wider than the graph of $g(x)$, and its vertex is moved to the left 2 units and up 1 unit.
2) The graph of $f(x)$ is narrower than the graph of $g(x)$, and its vertex is moved to the right 2 units and up 1 unit.
3) The graph of $f(x)$ is narrower than the graph of $g(x)$, and its vertex is moved to the left 2 units and up 1 unit.
4) The graph of $f(x)$ is wider than the graph of $g(x)$, and its vertex is moved to the right 2 units and up 1 unit.

374) The vertex of the parabola represented by $f(x) = x^2 - 4x + 3$ has coordinates $(2,-1)$. Find the coordinates of the vertex of the parabola defined by $g(x) = f(x-2)$. Explain how you arrived at your answer. [The use of the set of axes below is optional.]



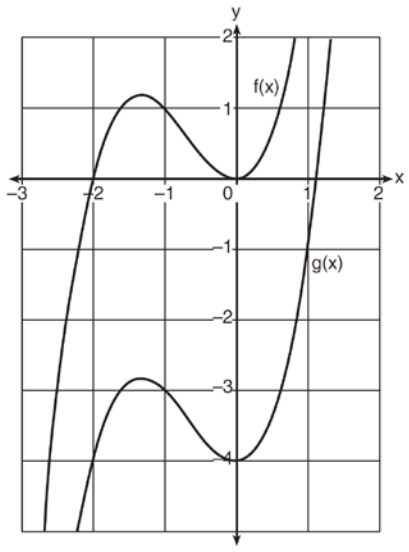
375) Given the graph of the line represented by the equation $f(x) = -2x + b$, if b is increased by 4 units, the graph of the new line would be shifted 4 units

1) right
2) up
3) left
4) down

376) When the function $f(x) = x^2$ is multiplied by the value a, where $a > 1$, the graph of the new function, $g(x) = ax^2$

1) opens upward and is wider
2) opens upward and is narrower
3) opens downward and is wider
4) opens downward and is narrower

377) In the diagram below, $f(x) = x^3 + 2x^2$ is graphed. Also graphed is $g(x)$, the result of a translation of $f(x)$.



Determine an equation of $g(x)$. Explain your reasoning.

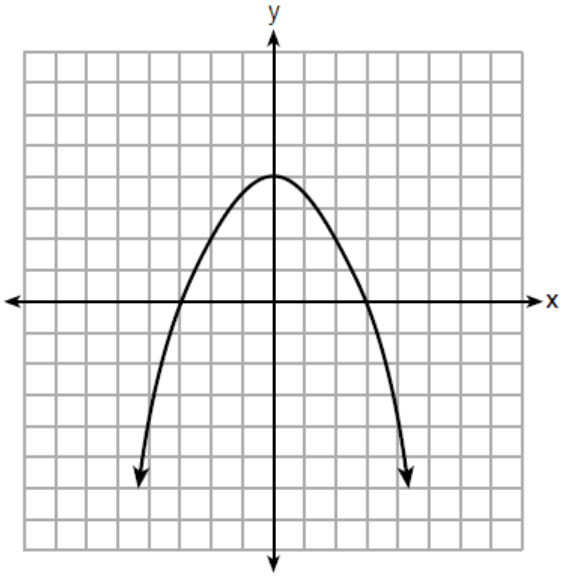
378) In the functions $f(x) = kx^2$ and $g(x) = |kx|$, k is a positive integer. If k is replaced by $\frac{1}{2}$, which statement about these new functions is true?

1) The graphs of both $f(x)$ and $g(x)$ become wider.
2) The graph of $f(x)$ becomes narrower and the graph of $g(x)$ shifts left.
3) The graphs of both $f(x)$ and $g(x)$ shift vertically.
4) The graph of $f(x)$ shifts left and the graph of $g(x)$ becomes wider.

379) If the original function $f(x) = 2x^2 - 1$ is shifted to the left 3 units to make the function $g(x)$, which expression would represent $g(x)$?

1) $2(x-3)^2 - 1$
2) $2(x+3)^2 - 1$
3) $2x^2 + 2$
4) $2x^2 - 4$

380) The graph of the function $p(x)$ is represented below. On the same set of axes, sketch the function $p(x+2)$.



SOLUTIONS

372) ANS: 1

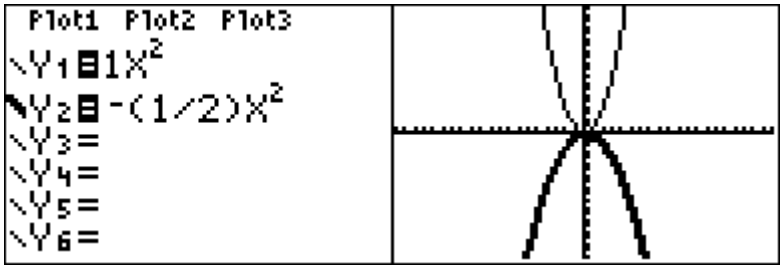
Strategy: Use the following general rules for quadratics, then check with a graphiong calculator.
As the value of a approaches 0, the parabola gets wider.
A positive value of a opens upward.
A negative value of a opens downward.

Check with graphing calculator:

Assume $a = 1$, then $y_1 = 1x^2$

If a is multiplied by $-\frac{1}{2}$, then $y_2 = -\frac{1}{2}x^2$.

Input both equations in a graphing calculator, as follows:



PTS: 2 NAT: F.BF.B.3 TOP: Transformations with Functions and Relations

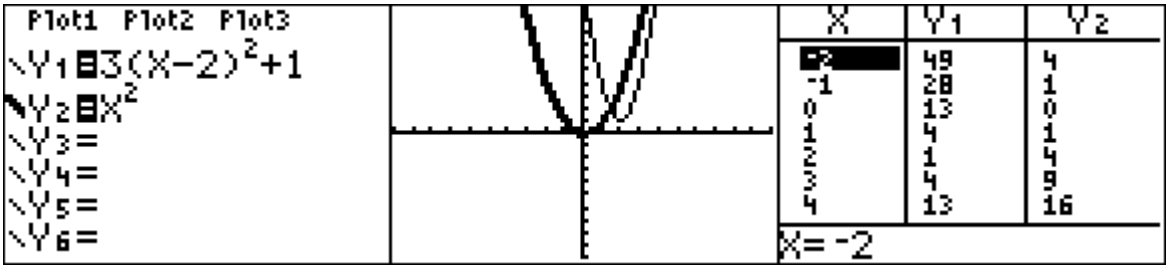
373) ANS: 2

Strategy: Input both functions in a graphing calculator and compare them.

Let the graph of Y_1 be the graph of $f(x) = 3(x-2)^2 + 1$

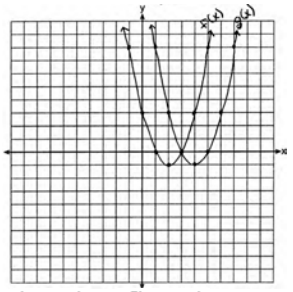
Let the graph of Y_2 be the graph of $g(x) = x^2$

Input both functions in a graphing calculator.
g(x) is the thick line and *f(x)* is the thin line.



PTS: 2 NAT: F.BF.B.3 TOP: Transformations with Functions and Relations

374) ANS:



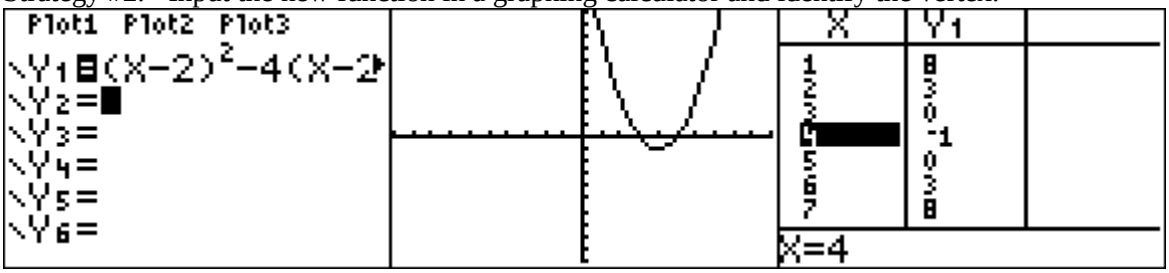
$(4,-1)$. $f(x-2)$ is a horizontal shift two units to the right

Strategy 1: Compose a new function, find the axis of symmetry, solve for g(x) at axis of symmetry, as follows:

$f(x) = x^2 - 4x + 3$ and $g(x) = f(x-2)$ Therefore: $g(x) = (x-2)^2 - 4(x-2) + 3$ $g(x) = x^2 - 4x + 4 - 4x + 8 + 3$ $g(x) = x^2 - 8x + 15$	$axis\ of\ symmetry = \frac{-b}{2a} = \frac{-(-8)}{2(1)} = \frac{8}{2} = 4$ $g(x) = x^2 - 8x + 15$ $g(4) = (4)^2 - 8(4) + 15$ $g(4) = 16 - 32 + 15$ $g(4) = -1$

The coordinates of the vertex of *g(x)* are (4,-1)

Strategy #2. Input the new function in a graphing calculator and identify the vertex.



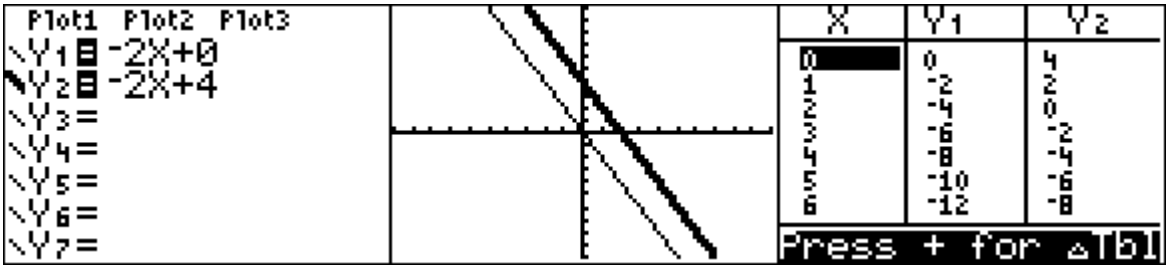
PTS: 2 NAT: F.BF.B.3 TOP: Transformations with Functions and Relations

375) ANS: 2

Strategy: Use the characteristics of the slope intercept form of a line, which is $y = mx + b$, where y is the dependent variable, m is the slope, x is the dependent variable, and b is the y-inctercept.

If b (the y-intercept) is increased by four, the slope remains the same and the new line is shifted up 4 units.

Check using a graphing calculator.



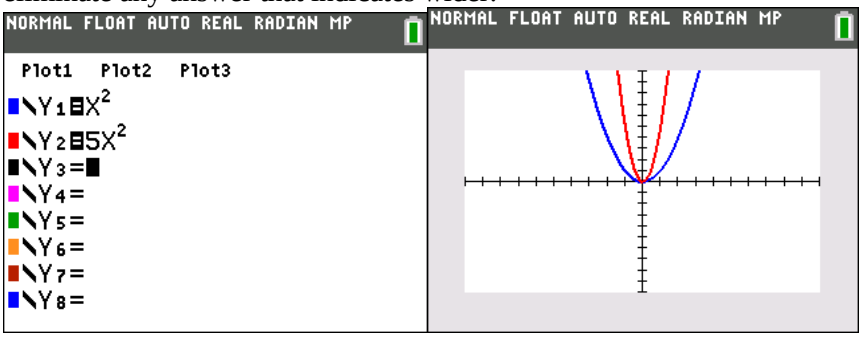
PTS: 2 NAT: F.BF.B.3 TOP: Transformations with Functions and Relations

376) ANS: 2

Strategy: Eliminate wrong answers.

Step 1. Since $a > 1$, a must be positive and the graph of $g(x) = ax^2$ must open upward. Eliminate any choice that opens downward.

Step 2. Determine if the graph gets wider or narrower by selecting a number larger than 1 for a, then input both functions in a graphing calculator and compare their graphs. The graph gets narrower, so eliminate any answer that indicates wider.



~~a)~~ opens upward and is ~~wider~~

b) opens upward and is narrower

~~c)~~ ~~opens downward~~ and is ~~wider~~

~~d)~~ ~~opens downward~~ and is narrower

PTS: 2 NAT: F.BF.B.3 STA: A.G.5 TOP: Graphing Polynomial Functions

377) ANS:

$$g(x) = x^3 + 2x^2 - 4$$

f(x) has a y-intercept of 0.

g(x) has a y-intercept of -4.

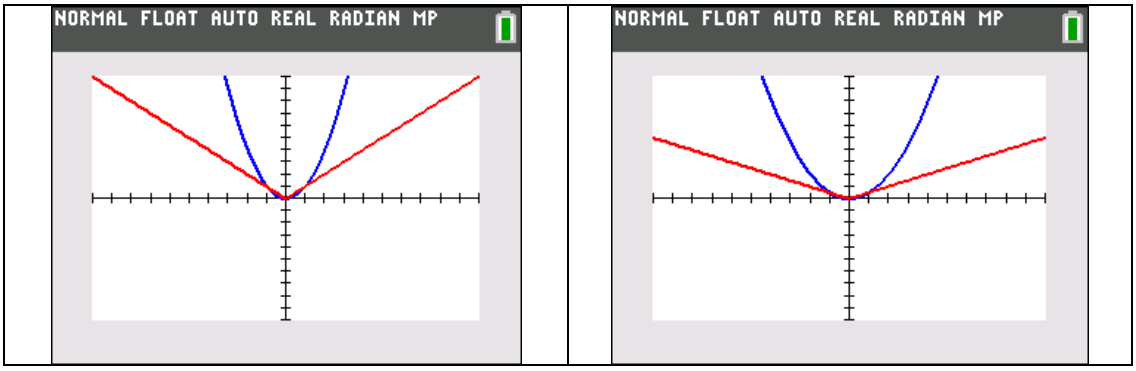
Every point on f(x) is a translation down 4 units to create g(x).

PTS: 2 NAT: F.BF.B.3 TOP: Graphing Polynomial Functions

378) ANS: 1

Since k is a positive integer, the lowest possible value for k is 1. If k is replaced by $\frac{1}{2}$, the graphs of both $f(x)$ and $g(x)$ will become wider.

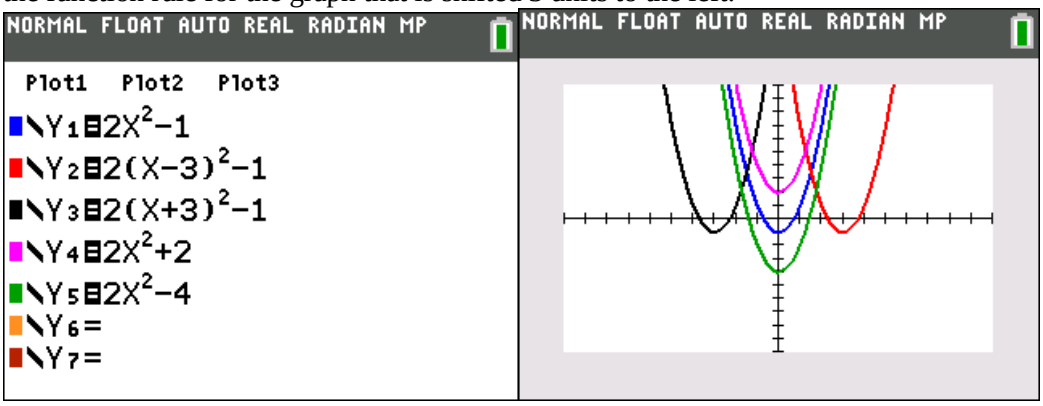
Let k = 1	Let $k = 1$ be replaced by $k = \frac{1}{2}$



PTS: 2 NAT: F.BF.B.3 TOP: Graphing Polynomial Functions

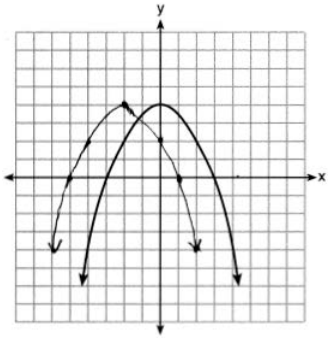
379) ANS: 2

Strategy: Input the orginal function and the four answer choices in a graphing calculator. Then, select the function rule for the graph that is shifted 3 units to the left.



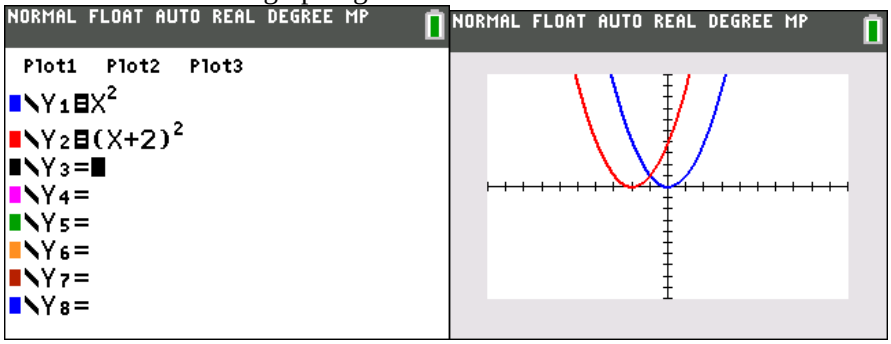
PTS: 2 NAT: F.BF.B.3 TOP: Graphing Polynomial Functions

380) ANS:



Strategy: Solve a simpler problem - pick a simple quadratic function, such as $y = x^2$ and see what happens to the graph when the function is changed to $y = (x + 2)^2$.

STEP 1. Input both in functions in a graphing calcualor.



STEP 2. Observe that the graph moves two units to the left.
STEP 3. Move every point of the original function two units to the left.

PTS: 2 NAT: F.BF.B.3 TOP: Graphing Polynomial Functions