H – Quadratics, Lesson 1, Solving Quadratics (r. 2018)

QUADRATICS

Solving Quadratics

Common Core Standards	Next Generation Standards
A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	**AI-A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (Shared standard with Algebra II)
A-REI.B.4a Solve quadratic equations in one variable. NYSED: Solutions may include simplifying radicals.	**AI-A.REI.4** Solve quadratic equations in one variable. Note: Solutions may include simplifying radicals.

NOTE: This lesson is in four parts and typically requires four or more days to complete.

LEARNING OBJECTIVES

Students will be able to:

1) Transform a quadratic equation into standard form and identify the values of a, b, and c.
2) Convert factors of quadratics to solutions.
3) Convert solutions of quadratics to factors.
4) Solve quadratics using the quadratic formula.
5) Solve quadratics using the completing the square method.
6) Solve quadratics using the factoring by grouping method.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

box method of factoring
completing the square
constant
factoring by grouping
factors
forms of a quadratic
linear term
multiplication property of zero

quadratic equation
quadratic formula
quadratic term
roots
solutions
standard form of a quadratic
x-axis intercepts
zeros

Part 1 – Overview of Quadratics

BIG IDEAS

The **standard form** of a quadratic is: $ax^2 + bx + c = 0$.

- ax^2 is the quadratic term
- bx is the linear term
- c is the constant term

Note: If the quadratic terms is removed, the remaining terms are a linear equation.

The definition of a **quadratic equation** is: an equation of the second degree.
Examples of quadratics in different **forms**:

Forms	Examples
standard form	$6x^2 + 11x - 35 = 0$ $2x^2 - 4x - 2 = 0$ $-4x^2 - 7x + 12 = 0$ $20x^2 - 15x - 10 = 0$ $x^2 - x - 3 = 0$ $5x^2 - 2x - 9 = 0$ $3x^2 + 4x + 2 = 0$ $-x^2 + 6x + 18 = 0$
without the *bx* term (the linear term)	$2x^2 - 64 = 0$ $x^2 - 16 = 0$ $9x^2 + 49 = 0$ $-2x^2 - 4 = 0$ $4x^2 + 81 = 0$ $-x^2 - 9 = 0$ $3x^2 - 36 = 0$ $6x^2 + 144 = 0$
without the *c* term (the constant term)	$x^2 - 7x = 0$ $2x^2 + 8x = 0$ $-x^2 - 9x = 0$ $x^2 + 2x = 0$ $-6x^2 - 3x = 0$ $-5x^2 + x = 0$ $-12x^2 + 13x = 0$ $11x^2 - 27x = 0$

factored forms	$(x+2)(x-3)=0$ $(x+1)(x+6)=0$ $(x-6)(x+1)=0$ $(x-5)(x+3)=0$ $(x-5)(x+2)=0$ $(x-4)(x+2)=0$ $(2x+3)(3x-2)=0$ $-3(x-4)(2x+3)=0$
other forms	$x(x-2)=4$ $x(2x+3)=12$ $3x(x+8)=-2$ $5x^2=9-x$ $-6x^2=-2+x$ $x^2=27x-14$ $x^2+2x=1$ $4x^2-7x=15$ $-8x^2+3x=-100$ $25x+6=99x^2$

(Source: your dictionary.com)

Multiplication Property of Zero: The **multiplication property of zero** says that if the product of two numbers or expressions is zero, then one or both of the numbers or expressions must equal zero. More simply, if $x \cdot y = 0$, then either $x = 0$ or $y = 0$, or, both x and y equal zero.

Example: The quadratic equation $(x+2)(x-4)=0$ has two factors: $(x+2)$ and $(x-4)$. The multiplication property of zero says that one or both of these factors must equal zero, because the product of these two factors is zero. Therefore, write two equations, as follows:

Eq #1 $(x+2)=0$ Therefore, $x=-2$

Eq #2 $(x-4)=0$ Therefore, $x=4$

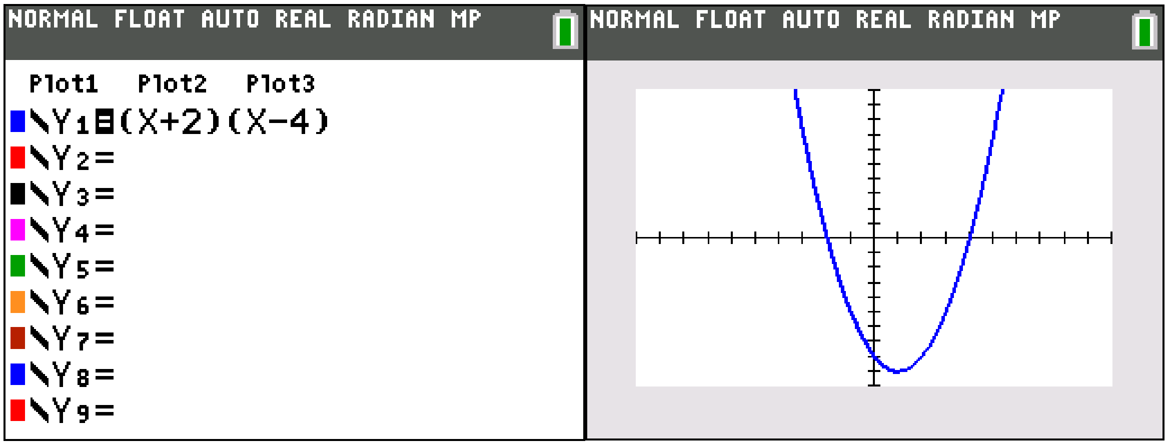
By the multiplication property of zero, $x=\{-2,+4\}$.

Zeros: A **zero** of a quadratic equation is a **solution** or **root** of the equation. The words **zero**, **solution**, and **root** all mean the same thing. The zeros of a quadratic equation are the value(s) of x when $y = 0$. A quadratic equation can have one, two, or no zeros. There are four general strategies for finding the zeros of a quadratic equation:

1) Solve the quadratic equation using the quadratic formula.
2) Solve the quadratic equation using the completing the square method.
3) Solve the quadratic equation using the factoring by grouping method.
4) Input the quadratic equation into a graphing calculator and find the x-axis intercepts.

x-axis intercepts: The zeros of a quadratic can be found by inspecting the graph view of the equation. In graph form, the zeros of a quadratic equation are the x-values of the coordinates of the **x-axis intercepts** of the graph of the equation. The graph of a quadratic equation is called a parabola and can intercept the x-axis in one, two, or no places.

Example: Find the x-axis intercepts of the quadratic equation $(x+2)(x-4)=0$ by inspecting the x-axis intercepts of its graph.



```
Y1=(X+2)(X-4)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
Y8=
Y9=
```

The coordinates of the x-axis intercepts are are $(-2,0)$ and $(4,0)$. These x-axis intercepts show that the values of x when y=0 are -2 and 4, so the solutions of the quadratic equation are $x=\{-2,+4\}$.

The Difference Between Zeros and Factors

Factor: A **factor** is:

1) a whole number that is a **divisor** of another number, or
2) an algebraic expression that is a **divisor** of another algebraic expression.

Examples:

- 1, 2, 3, 4, 6, and 12 all divide the number 12, so 1, 2, 3, 4, 6, and 12 are all factors of 12.
- $(x-3)$ and $(x+2)$ will divide the trinomial expression x^2-x-6, so $(x-3)$ and $(x+2)$ are both factors of the x^2-x-6.

Start with Factors and Find Zeros

Remember that the **factors** of an expression are *related to* the **zeros** of the expression by the **multiplication property of zero**. Thus, if you know the **factors**, it is easy to find the **zeros**.

Example: If the factors of the quadratic equation $2x^2+5x+6=0$ are $(2x+2)$ and $(x+3)$, then by the multiplication property of zero: either $(2x+2)=0$, or $(x+3)=0$, or both equal zero. Solving each equation for *x* results in the zeros of the equation, as follows:

$$\begin{aligned}(2x+2)&=0\\2x&=-2\\x&=-1\end{aligned}\qquad\begin{aligned}(x+3)&=0\\x&=-3\end{aligned}$$

Start with Zeros and Find Factors

If you know the **zeros** of an expression, you can work backwards using the **multiplication property of zero** to find the **factors** of the expression. For example, if you inspect the graph of an equation and find that it has **x-intercepts** at $(3,0)$ and $(-2,0)$, then you know that the solutions are $x=3$ and $x=-2$. You can use these two equations to find the factors of the quadratic expression, as follows:

$$\begin{aligned}x&=3\\(x-3)&=0\end{aligned}$$

$$\begin{aligned}x&=-2\\(x+2)&=0\end{aligned}$$

The factors of a quadratic equation with zeros of 3 and -2 are $(x-3)$ and $(x+2)$.

With practice, you can probably move back and forth between the **zeros** of an expression and the **factors** of an expression with ease.

Part 1 – Overview of Quadratics

DEVELOPING ESSENTIAL SKILLS

Convert the following quadratic equations to standard form and identify the values of a, b, and c:

$x(x-2)=4$	$x^2-2x-4=0$	a= 1 , b=-2 , c=-4
$x(2x+3)=12$	$2x^2+6x-12=0$	a= 2, b=6 , c= -12
$3x(x+8)=-2$	$3x^2+24x+2=0$	a= 3 , b= 24, c= 2
$5x^2=9-x$	$5x^2+x-9=0$	a= 5 , b= 1, c= –9
$-6x^2=-2+x$	$-6x^2-x+2=0$	a= -6, b= -1, c= 2
$x^2=27x-14$	$x^2-27x+14=0$	a= 1 , b=-27 , c= 14
$x^2+2x=1$	$x^2+2x-1=0$	a= 1 , b=2 , c=-1
$4x^2-7x=15$	$4x^2-7x-15=0$	a= 4 , b=-7 , c=-1
$-8x^2+3x=-100$	$-8x^2+3x+100=0$	a= -8 , b=3 , c=100
$25x+6=99x^2$	$-99x^2+25x+6=0$	a= -99 , b=25 , c=6
$2x^2=64$	$2x^2-64=0$	a=2 , b= 0, c=
$0=-16+x^2$	$x^2-16=0$	a=1 , b= 0, c= -16
$49=-9x^2$	$9x^2+49=0$	a= 9 , b=0 , c=49
$x^2=7x$	$x^2-7x=0$	a= 1 , b=-7 , c=0
$2x^2=-+8x$	$2x^2+8x=0$	a= 2 , b=8 , c= 0
$0=-9x-x^2$	$-x^2-9x=0$	a=-1 , b=-9 , c= 0

Find the zeros of the following quadratic equations:

$a.\quad (x+2)(x-3)=0$	$a.\quad x=\{-2,3\}$
$b.\quad (x+1)(x+6)=0$	$b.\quad x=\{-6,1\}$
$c.\quad (x-6)(x+1)=0$	$c.\quad x=\{-1,6\}$
$d.\quad (x-5)(x+3)=0$	$d.\quad x=\{3,5\}$
$e.\quad (x-5)(x+2)=0$	$e.\quad x=\{2,5\}$
$f.\quad (x-4)(x+2)=0$	$f.\quad x=\{-2,4\}$
$g.\quad (2x+3)(3x-2)=0$	$g.\quad x=\left\{-\frac{3}{2},\frac{2}{3}\right\}$
$h.\quad -3(x-4)(2x+3)=0$	$h.\quad x=\left\{-\frac{3}{2},4\right\}$

Part 2 – The Quadratic Formula

The **quadratic formula** is: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Quadratic Formula Song

SOLVING QUADRATIC EQUATIONS STRATEGY #1: Use the Quadratic Formula

Start with any quadratic equation in the form of $ax^2 + bx + c = 0$	$x^2 + 2x - 24 = 0$ The right expression *must* be zero.
Identify the values of a, b, and c.	$a = 1$, $b = 2$, and $c = -24$
Substitute the values of a, b, and c into the quadratic formula, which is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \dfrac{-(2) \pm \sqrt{(2)^2 - 4(1)(-24)c}}{2(1)}$
Solve for x	$x = \dfrac{-(2) \pm \sqrt{100}}{2}$ $x = \dfrac{-(2) \pm 10}{2}$ $x = \dfrac{-(2) + 10}{2} \Rightarrow x = \dfrac{8}{2} \Rightarrow x = 4$ $x = \dfrac{-(2) - 10}{2} \Rightarrow x = \dfrac{-12}{2} = -6$

The quadratic formula can be used to solve any quadratic equation.

Part 2 – The Quadratic Formula

DEVELOPING ESSENTIAL SKILLS

Solve the following quadratic equations using the quadratic formula. Leave answers in simplest radical form.

$x^2 - x - 3 = 0$	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ a = 1, b = -1, c = -3 $x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)}$ $x = \dfrac{1 \pm \sqrt{1+12}}{2}$ $x = \dfrac{1 \pm \sqrt{13}}{2}$

$20x^2 - 15x - 10 = 0$	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ a = 20, b = -15, c = -10 $x=\frac{-(-15)\pm\sqrt{(-15)^2-4(20)(-10)}}{2(20)}$ $x=\frac{15\pm\sqrt{225+800}}{40}$ $x=\frac{15\pm\sqrt{1025}}{40}$ $x=\frac{15\pm\sqrt{25}\times\sqrt{41}}{40}$ $x=\frac{15\pm5\sqrt{41}}{40}$ $x=\frac{3\pm\sqrt{41}}{8}$
$2x^2 - 4x - 2 = 0$	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ a=2, b=-4,c=-2 $x=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(-2)}}{2(2)}$ $x=\frac{4\pm\sqrt{16+16}}{4}$ $x=\frac{4\pm\sqrt{32}}{4}$ $x=\frac{4\pm\sqrt{16}\times\sqrt{2}}{4}$ $x=\frac{4\pm4\sqrt{2}}{4}$ $x=1\pm\sqrt{2}$

$6x^2 + 11x = 35$	$6x^2 + 11x -35 = 0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ a = 6, b = 11, c =-35 $x=\frac{-(11)\pm\sqrt{(11)^2-4(6)(-35)}}{2(6)}$ $x=\frac{-11\pm\sqrt{121+840}}{12}$ $x=\frac{-11\pm\sqrt{961}}{12}$ $x=\frac{-11\pm 31}{12}$ $x=\frac{20}{12}$ and $x=\frac{-42}{12}$ $x=\left\{\frac{5}{3},-\frac{7}{2}\right\}$
$-7x + 12 = 4x^2$	$-4x^2 - 7x + 12 = 0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ a = -4, b = -7, c = 12 $x=\frac{-(-7)\pm\sqrt{(-7)^2-4(-4)(12)}}{2(-4)}$ $x=\frac{7\pm\sqrt{49+192}}{-8}$ $x=\frac{7\pm\sqrt{241}}{-8}$

Part 3 – The Box Method of Factoring

	gcf	*gcf*
gcf	ax^2	mx
gcf	nx	c

The Box Method
for
Factoring a Trinomial

$$ax^2 + bx + c = 0$$

$$bx = mx + nx$$

INSTRUCTIONS	EXAMPLE
STEP 1 Start with a factorable quadratic in standard form: $ax^2 + bx + c = 0$ and a 2-row by 2-column table.	Solve by factoring: $6x^2 - x - 12 = 0$
STEP 2 Copy the quadratic term into the upper left box and the constant term into the lower right box.	$\begin{array}{\|c\|c\|}\hline 6x^2 & \\ \hline & -12 \\ \hline\end{array}$
STEP 3 Multiply the quadratic term by the constant term and write the product to the right of the table.	$\begin{array}{\|c\|c\|}\hline 6x^2 & \\ \hline & -12 \\ \hline\end{array}$ $6x^2 \times -12 = \boxed{-72x^2}$
STEP 4 Factor the product from STEP 3 until you obtain two factors that *sum* to the linear term (*bx*).	$1x \times -72x$ $-1x \times 72x$ $2x \times -36x$ $-2x \times 36x$ $3x \times -24x$ $-3x \times 24x$ $4x \times -18x$ $-4x \times 18x$ $6x \times -12x$ $-6x \times 12x$ $8x \times -9x$ These two factors sum to *bx* $-8x \times 9x$

STEP 5 Write one of the two factors found in STEP 4 in the upper right box and the other in the lower left box. Order does not matter.	$\begin{array}{\|c\|c\|}\hline 6x^2 & -9x \\ \hline 8x & -12 \\ \hline\end{array}$
STEP 6 Find the greatest common factor of each row and each column and record these factors to the left of each row and above each column. Give each factor the same plus or minus value as the nearest term in a box. NOTE: If all four of the greatest common factors share a common factor, reduce each factor by the common factor and add the common factor as a third factor. Eg. $(3x-9)(3x-15) \Rightarrow 3(x-3)(x-5)$	$\begin{array}{c\|c\|c\|} & 2x & -3 \\ \cline{2-3} 3x & 6x^2 & -9x \\ \cline{2-3} 4 & 8x & -12 \\ \cline{2-3}\end{array}$
STEP 7 Write the expressions above and beside the box as binomial factors of the original trinomial.	$(2x-3)(3x+4)=0$
STEP 8 Check to see that the factored quadratic is the same as the original quadratic.	$(2x-3)(3x+4)=0$ $6x^2+8x-9x-12=0$ $6x^2-9x-12=0$ check
STEP 9 Convert the factors to zeros.	$(2x-3)=0$ $2x=3$ $x=\boxed{\frac{3}{2}}$ $(3x+4)=0$ $3x=-4$ $x=\boxed{-\frac{4}{3}}$

Part 3 – The Box Method of Factoring

DEVELOPING ESSENTIAL SKILLS

Solve each quadratic by factoring.

$x^2-2x-8 = 0$	$\begin{array}{c\|c\|c\|} & x & -4 \\ \cline{2-3} x & x^2 & -4x \\ \cline{2-3} 2 & 2x & -8 \\ \cline{2-3}\end{array}$

<table>
<tr><td></td><td>$(x-4)(x+2)=0$
$\boxed{x=\{-2,4\}}$</td></tr>
<tr><td>$x^2-3x-10=0$</td><td><table><tr><td></td><td>x</td><td>-5</td></tr><tr><td>x</td><td>x^2</td><td>-5x</td></tr><tr><td>2</td><td>2x</td><td>-10</td></tr></table>$(x-5)(x+2)=0$
$\boxed{x=\{5,-2\}}$</td></tr>
<tr><td>$x^2-2x-15=0$</td><td><table><tr><td></td><td>x</td><td>-5</td></tr><tr><td>x</td><td>x^2</td><td>-5x</td></tr><tr><td>3</td><td>3x</td><td>-15</td></tr></table>$(x-5)(x+3)=0$
$\boxed{\text{x}=\{-3,5\}}$</td></tr>
<tr><td>$6x^2+5x-6$
$6x^2-4x+9x-6$
$(2x+3)(3x-2)=0$</td><td><table><tr><td></td><td>3x</td><td>-2</td></tr><tr><td>2x</td><td>$6x^2$</td><td>-4x</td></tr><tr><td>3</td><td>9x</td><td>-6</td></tr></table>$(2x+3)(3x-2)=0$
$\boxed{x=\left\{-\frac{3}{2},\frac{2}{3}\right\}}$</td></tr>
<tr><td>$10x^2+4x-6=0$
$10x^2-6x+10x-6=0$
$(2x+2)(5x-3)=0$</td><td><table><tr><td></td><td>10x</td><td>-6</td></tr><tr><td>2x</td><td>$10x^2$</td><td>-6x</td></tr><tr><td>2</td><td>10x</td><td>-6</td></tr></table>$(10x-6)(2x+2)=0$
$2(5x-3)(x+1)=0$
$\boxed{\text{x}=\left\{-1,\frac{3}{5}\right\}}$</td></tr>
</table>

Part 4 – Completing the Square

SOLVING QUADRATIC EQUATIONS STRATEGY #3: Completing the Square

completing the square algorithm
A process used to change an expression of the form $ax^2 + bx + c$ into a perfect square binomial by adding a suitable constant.

Source: NYSED Mathematics Glossary

PROCEDURE TO FIND THE ZEROS AND EXTREMES OF A QUADRATIC	
Start with any quadratic equation of the general form $ax^2 + bx + c = 0$	
STEP 1 **Isolate all terms with** x^2 **and** x **on one side of the equation. If** $a \neq 1$**, divide every term in the equation by** a **to get one expression in the form of** $x^2 + bx$	
STEP 2 **Complete the Square by adding** $\left(\frac{b}{2}\right)^2$ **to both sides of the equation.**	
STEP 3 **Factor the side containing** $x^2 + bx + \left(\frac{b}{2}\right)^2$ **into a binomial expression of the form** $\left(x + \frac{b}{2}\right)^2$	
STEP 4a **(solving for roots and zeros only)** **Take the square root of both sides of the equation and simplify,**	**STEP 4b** **(solving for maxima and minima only)** **Multiply both sides of the equation by** a**.** **Move all terms to left side of equation.** **Solve the factor in parenthesis for axis of symmety and x-value of the vertex..** **The number not in parentheses is the y-value of the vertes.**

STEPS:	EXAMPLE A	EXAMPLE B
Start with any quadratic equation of the general form $ax^2 + bx + c = n$	$x^2 + 2x + 3 = 4$	$5x^2 + 2x + 3 = 4$

STEP 1) **Isolate all terms with x^2 and x on one side of the equation.** **If $a \neq 1$, divide every term in the equation by a to get one expression in the form of x^2+bx**	$x^2+2x=1$	$5x^2+2x=1$ $\frac{5x^2}{5}+\frac{2x}{5}=\frac{1}{5}$ $x^2+\frac{2}{5}x=\frac{1}{5}$
STEP 2) **Complete the Square by adding $\left(\frac{b}{2}\right)^2$ to both sides of the equation.**	$b=2,\ \frac{b}{2}=\frac{2}{2}=1,\ \left(\frac{b}{2}\right)^2=(1)^2$ $x^2+2x+(1)^2=1+(1)^2$ $x^2+2x+(1)^2=2$	$b=\frac{2}{5},\ \frac{b}{2}=\frac{1}{5},\ \left(\frac{b}{2}\right)^2=\left(\frac{1}{5}\right)^2$ $x^2+\frac{2}{5}x+\left(\frac{1}{5}\right)^2=\frac{1}{5}+\left(\frac{1}{5}\right)^2$
STEP 3) **Factor the side containing $x^2+bx+\left(\frac{b}{2}\right)^2$ into a binomial expression of the form $\left(x+\frac{b}{2}\right)^2$**	$(x+1)^2=2$	$\left(x+\frac{1}{5}\right)^2=\frac{1}{5}+\left(\frac{1}{5}\right)^2$ $\left(x+\frac{1}{5}\right)^2=\frac{5}{25}+\frac{1}{25}$ $\left(x+\frac{1}{5}\right)^2=\frac{6}{25}$
STEP 4a) **Take the square roots of both sides of the equation and simplify.**	$\sqrt{(x+1)^2}=\sqrt{2}$ $x+1=\pm\sqrt{2}$ $x=\boxed{-1\pm\sqrt{2}}$	$\sqrt{\left(x+\frac{1}{5}\right)^2}=\sqrt{\frac{6}{25}}$ $x+\frac{1}{5}=\pm\frac{\sqrt{6}}{5}$ $x=-\frac{1}{5}\pm\frac{\sqrt{6}}{5}=\boxed{\frac{1\pm\sqrt{6}}{5}}$

STEP 4b **Multiply both sides of the equation by *a*. Move all terms to left side of equation. Solve the factor in parenthesis for axis of symmety and x-value of the vertex. The number not in parentheses is the y-value of the vertes.**	$1(x+1)^2 = 1(2)$ $(x+1)^2 = 2$ $(x+1)^2 - 2 = 0$ vertex form. -1 is the axis of symmetry -2 is the y value of the vertex The vertex is at $(-1,-2)$ $(x+1)^2 = 2$	$5\left(x+\frac{1}{5}\right)^2 = 5\left(\frac{6}{25}\right)$ $5\left(x+\frac{1}{5}\right)^2 = \frac{6}{5}$ $5\left(x+\frac{1}{5}\right)^2 - \frac{6}{5} = 0$ vertex form. $-\frac{1}{5}$ is the axis of symmetry $-\frac{6}{5}$ is the y value of the vertex The vertex is at $\left(-\frac{1}{5},-\frac{6}{5}\right)$

Part 4 – Completing the Square

DEVELOPING ESSENTIAL SKILLS

Solve the following quadratic equations by completing the square.

$x^2 - x - 3 = 0$	$x^2 - x - 3 = 0$ $x^2 - x = 3$ $x^2 - x + \left(\frac{1}{2}\right)^2 = 3 + \left(\frac{1}{2}\right)^2$ $\left(x - \frac{1}{2}\right)^2 = 3 + \frac{1}{4}$ $x - \frac{1}{2} = \pm\sqrt{\frac{13}{4}}$ $x = \frac{1}{2} \pm \frac{\sqrt{13}}{2}$ $x = \frac{1 \pm \sqrt{13}}{2}$

$20x^2 - 15x - 10 = 0$	$20x^2 - 15x - 10 = 0$ $20x^2 - 15x = 10$ $\frac{20x^2}{20} - \frac{15x}{20} = \frac{10}{20}$ $x^2 - \frac{3x}{4} = \frac{1}{2}$ $x^2 - \frac{3x}{4} + \left(-\frac{3}{8}\right)^2 = \frac{1}{2} + \left(-\frac{3}{8}\right)^2$ $\left(x - \frac{3}{8}\right)^2 = \frac{32}{64} + \frac{9}{64}$ $\left(x - \frac{3}{8}\right)^2 = \frac{41}{64}$ $x - \frac{3}{8} = \pm\sqrt{\frac{41}{64}}$ $x = \frac{3}{8} \pm \frac{\sqrt{41}}{8}$ $x = \frac{3+\sqrt{41}}{8}$
$2x^2 - 4x - 2 = 0$	$2x^2 - 4x - 2 = 0$ $2x^2 - 4x = 2$ $\frac{2x^2}{2} - \frac{4x}{2} = \frac{2}{2}$ $x^2 - 2x = 1$ $x^2 - 2x + \left(-\frac{2}{2}\right)^2 = 1 + \left(-\frac{2}{2}\right)^2$ $(x-1)^2 = 1+1$ $x - 1 = \pm\sqrt{2}$ $x = 1 \pm \sqrt{2}$

$6x^2 + 11x = 35$	$6x^2+11x-35=0$ $6x^2+11x=35$ $\frac{6x^2}{6}+\frac{11x}{6}=\frac{35}{6}$ $x^2+\frac{11x}{6}=\frac{35}{6}$ $x^2+\frac{11x}{6}+\left(\frac{11}{12}\right)^2=\frac{35}{6}+\left(\frac{11}{12}\right)^2$ $\left(x+\frac{11}{12}\right)^2=\frac{35}{6}+\frac{121}{144}$ $\left(x+\frac{11}{12}\right)^2=\frac{840}{144}+\frac{121}{144}$ $\left(x+\frac{11}{12}\right)^2=\frac{961}{144}$ $x+\frac{11}{12}=\pm\sqrt{\frac{961}{144}}$ $x+\frac{11}{12}=\pm\frac{31}{12}$ $x=-\frac{11}{12}\pm\frac{31}{12}$ $x=\frac{42}{12}$ and $\frac{-20}{12}$ $x=\frac{7}{2}$ and $-\frac{5}{3}$ $x=\left\{\frac{5}{3},-\frac{7}{2}\right\}$

$-7x + 12 = 4x^2$	$-4x^2 - 7x = -12$ $\frac{-4x^2}{-4} - \frac{7x}{-4} = \frac{-12}{-4}$ $x^2 + \frac{7}{4}x = 3$ $x^2 + \frac{7}{4}x + \left(\frac{7}{8}\right)^2 = 3 + \left(\frac{7}{8}\right)^2$ $x^2 + \frac{7}{4}x + \left(\frac{7}{8}\right)^2 = \frac{192}{64} + \frac{49}{64}$ $\left(x + \frac{7}{8}\right)^2 = \frac{241}{64}$ $x + \frac{7}{8} = \pm\frac{\sqrt{241}}{8}$ $x = \frac{7}{8} \pm \frac{\sqrt{241}}{8}$ $x = \frac{7 \pm \sqrt{241}}{8}$

A.APR.B.3, A.REI.B.4: Solving Quadratics

169) Solve $8m^2 + 20m = 12$ for *m* by factoring.

170) Keith determines the zeros of the function $f(x)$ to be -6 and 5. What could be Keith's function?
1) $f(x) = (x+5)(x+6)$
2) $f(x) = (x+5)(x-6)$
3) $f(x) = (x-5)(x+6)$
4) $f(x) = (x-5)(x-6)$

171) In the equation $x^2 + 10x + 24 = (x+a)(x+b)$, *b* is an integer. Find algebraically *all* possible values of *b*.

172) Which equation has the same solutions as $2x^2 + x - 3 = 0$
1) $(2x-1)(x+3) = 0$
2) $(2x+1)(x-3) = 0$
3) $(2x-3)(x+1) = 0$
4) $(2x+3)(x-1) = 0$

173) The zeros of the function $f(x) = 3x^2 - 3x - 6$ are
1) -1 and -2
2) 1 and -2
3) 1 and 2
4) -1 and 2

174) The zeros of the function $f(x) = 2x^2 - 4x - 6$ are
1) 3 and -1
2) 3 and 1
3) -3 and 1
4) -3 and -1

175) Janice is asked to solve $0 = 64x^2 + 16x - 3$. She begins the problem by writing the following steps:

Line 1 $0 = 64x^2 + 16x - 3$
Line 2 $0 = B^2 + 2B - 3$
Line 3 $0 = (B+3)(B-1)$

Use Janice’s procedure to solve the equation for x.

Explain the method Janice used to solve the quadratic equation.

176) What is the solution set of the equation $(x-2)(x-a) = 0$?
1) -2 and *a*
2) -2 and -*a*
3) 2 and *a*
4) 2 and -*a*

177) The function $r(x)$ is defined by the expression $x^2 + 3x - 18$. Use factoring to determine the zeros of $r(x)$. Explain what the zeros represent on the graph of $r(x)$.

178) If the quadratic formula is used to find the roots of the equation $x^2 - 6x - 19 = 0$, the correct roots are
1) $3 \pm 2\sqrt{7}$
2) $-3 \pm 2\sqrt{7}$
3) $3 \pm 4\sqrt{14}$
4) $-3 \pm 4\sqrt{14}$

179) Which equation has the same solution as $x^2 - 6x - 12 = 0$?
1) $(x+3)^2 = 21$
2) $(x-3)^2 = 21$
3) $(x+3)^2 = 3$
4) $(x-3)^2 = 3$

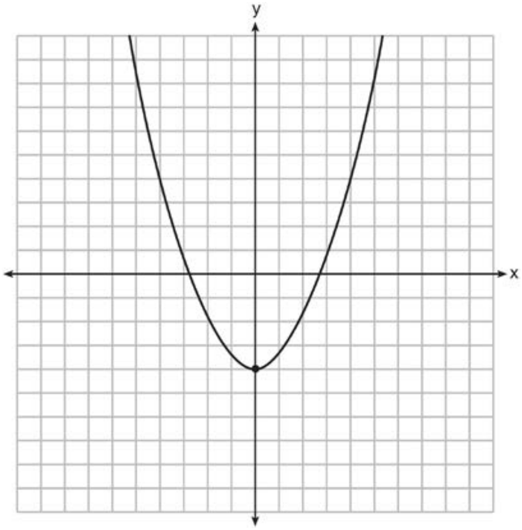
180) What are the roots of the equation $x^2 + 4x - 16 = 0$?

1) $2 \pm 2\sqrt{5}$ 3) $2 \pm 4\sqrt{5}$
2) $-2 \pm 2\sqrt{5}$ 4) $-2 \pm 4\sqrt{5}$

181) Write an equation that defines $m(x)$ as a trinomial where $m(x) = (3x - 1)(3 - x) + 4x^2 + 19$. Solve for *x* when $m(x) = 0$.

182) If $4x^2 - 100 = 0$, the roots of the equation are
1) -25 and 25 3) -5 and 5
2) -25, only 4) -5, only

183) Ryker is given the graph of the function $y = \frac{1}{2}x^2 - 4$. He wants to find the zeros of the function, but is unable to read them exactly from the graph.



Find the zeros in simplest radical form.

184) A student was given the equation $x^2 + 6x - 13 = 0$ to solve by completing the square. The first step that was written is shown below.

$$x^2 + 6x = 13$$

The next step in the student’s process was $x^2 + 6x + c = 13 + c$. State the value of *c* that creates a perfect square trinomial. Explain how the value of *c* is determined.

185) Which equation has the same solutions as $x^2 + 6x - 7 = 0$?
1) $(x + 3)^2 = 2$ 3) $(x - 3)^2 = 16$
2) $(x - 3)^2 = 2$ 4) $(x + 3)^2 = 16$

186) Solve the equation $4x^2 - 12x = 7$ algebraically for *x*.

187) When directed to solve a quadratic equation by completing the square, Sam arrived at the equation $\left(x - \frac{5}{2}\right)^2 = \frac{13}{4}$. Which equation could have been the original equation given to Sam?
1) $x^2 + 5x + 7 = 0$ 3) $x^2 - 5x + 7 = 0$

2) $x^2 + 5x + 3 = 0$ 4) $x^2 - 5x + 3 = 0$

188) A student is asked to solve the equation $4(3x - 1)^2 - 17 = 83$. The student's solution to the problem starts as $4(3x - 1)^2 = 100$

$(3x - 1)^2 = 25$

A correct next step in the solution of the problem is

1) $3x - 1 = \pm 5$ 3) $9x^2 - 1 = 25$

2) $3x - 1 = \pm 25$ 4) $9x^2 - 6x + 1 = 5$

189) What are the solutions to the equation $x^2 - 8x = 10$?

1) $4 \pm \sqrt{10}$ 3) $-4 \pm \sqrt{10}$

2) $4 \pm \sqrt{26}$ 4) $-4 \pm \sqrt{26}$

190) The solution of the equation $(x + 3)^2 = 7$ is

1) $3 \pm \sqrt{7}$ 3) $-3 \pm \sqrt{7}$

2) $7 \pm \sqrt{3}$ 4) $-7 \pm \sqrt{3}$

191) When solving the equation $x^2 - 8x - 7 = 0$ by completing the square, which equation is a step in the process?

1) $(x - 4)^2 = 9$ 3) $(x - 8)^2 = 9$

2) $(x - 4)^2 = 23$ 4) $(x - 8)^2 = 23$

192) Solve the equation for *y*: $(y - 3)^2 = 4y - 12$

193) Fred's teacher gave the class the quadratic function $f(x) = 4x^2 + 16x + 9$.

a) State two different methods Fred could use to solve the equation $f(x) = 0$.

b) Using one of the methods stated in part *a*, solve $f(x) = 0$ for *x*, to the *nearest tenth.*

194) What is the solution of the equation $2(x + 2)^2 - 4 = 28$?

1) 6, only 3) 2 and -6

2) 2, only 4) 6 and -2

195) Amy solved the equation $2x^2 + 5x - 42 = 0$. She stated that the solutions to the equation were $\frac{7}{2}$ and -6.

Do you agree with Amy's solutions? Explain why or why not.

196) The height, *H*, in feet, of an object dropped from the top of a building after *t* seconds is given by $H(t) = -16t^2 + 144$. How many feet did the object fall between one and two seconds after it was dropped? Determine, algebraically, how many seconds it will take for the object to reach the ground.

197) What are the solutions to the equation $3x^2 + 10x = 8$?

1) $\frac{2}{3}$ and -4 3) $\frac{4}{3}$ and -2

2) $-\frac{2}{3}$ and 4 4) $-\frac{4}{3}$ and 2

198) Find the zeros of $f(x) = (x-3)^2 - 49$, algebraically.

199) Which value of *x* is a solution to the equation $13 - 36x^2 = -12$?

1) $\frac{36}{25}$
2) $\frac{25}{36}$
3) $-\frac{6}{5}$
4) $-\frac{5}{6}$

200) The method of completing the square was used to solve the equation $2x^2 - 12x + 6 = 0$. Which equation is a correct step when using this method?

1) $(x-3)^2 = 6$
2) $(x-3)^2 = -6$
3) $(x-3)^2 = 3$
4) $(x-3)^2 = -3$

201) What are the solutions to the equation $x^2 - 8x = 24$?

1) $x = 4 \pm 2\sqrt{10}$
2) $x = -4 \pm 2\sqrt{10}$
3) $x = 4 \pm 2\sqrt{2}$
4) $x = -4 \pm 2\sqrt{2}$

202) Solve the equation $x^2 - 6x = 15$ by completing the square.

203) What are the solutions to the equation $3(x-4)^2 = 27$?

1) 1 and 7
2) -1 and -7
3) $4 \pm \sqrt{24}$
4) $-4 \pm \sqrt{24}$

204) The quadratic equation $x^2 - 6x = 12$ is rewritten in the form $(x+p)^2 = q$, where *q* is a constant. What is the value of *p*?

1) -12
2) -9
3) -3
4) 9

205) Solve for *x* to the *nearest tenth*: $x^2 + x - 5 = 0$.

SOLUTIONS

169) ANS:

$m = \frac{1}{2}$ and $m = -3$

Strategy: Factor by grouping.

$$8m^2 + 20m = 12$$

$$8m^2 + 20m - 12 = 0$$

$$|ac| = 96$$

The factors of 96 are:

1 and 96

2 and 48

3 and 32

4 and 24 (use these)

$$8m^2 + 24m - 4m - 12 = 0$$

$$\left(8m^2 + 24m\right) - (4m + 12) = 0$$

$$8m(m + 3) - 4(m + 3) = 0$$

$$(8m - 4)(m + 3) = 0$$

Use the multiplication property of zero to solve for m.

$8m - 4 = 0$	$m + 3 = 0$
$8m = 4$	$m = -3$
$m = \frac{4}{8}$	
$m = \frac{1}{2}$	

PTS: 2 NAT: A.SSE.B.3 TOP: Solving Quadratics

170) ANS: 3

Strategy: Convert the zeros to factors.

If the zeros of $f(x)$ are -6 and 5, then the factors of $f(x)$ are $(x + 6)$ and $(x - 5)$.
Therefore, the function can be written as $f(x) = (x + 6)(x - 5)$.
The correct answer choice is *c*.

PTS: 2 NAT: A.SSE.B.3 TOP: Solving Quadratics

171) ANS:
6 and 4

Strategy: Factor the trinomial $x^2 + 10x + 24$ into two binomials.

$$x^2 + 10x + 24$$

$$(x + \_\_\_)(x + \_\_\_)$$

The factors of 24 are:

1 and 24

2 and 12

3 and 8

4 and 6 (use these)

$$(x + 4)(x + 6)$$

Possible values for *a* and *c* are 4 and 6.

PTS: 2 NAT: A.SSE.B.3 TOP: Solving Quadratics

172) ANS: 4

Strategy 1: Factor by grouping.

$$2x^2 + x - 3 = 0$$

$$|ac| = 6$$

Factors of 6 are

1 and 6

2 and 3 (use these)

$$2x^2 + 3x - 2x - 3 = 0$$

$$\left(2x^2 + 3x\right) - (2x + 3) = 0$$

$$x(2x - 3) - 1(2x + 3) = 0$$

$$(x - 1)(2x + 3) = 0$$

Answer choice *d* is correct

Strategy 2: Work backwards by using the disrtibutive property to expand all answer choices and match the expanded trinomials to the function $2x^2 + x - 3 = 0$.

a. $(2x - 1)(x + 3) = 0$ $2x^2 + 6x - x - 3$ $2x^2 + 5x - 3$ (Wrong Choice)	c. $(2x - 3)(x + 1) = 0$ $2x^2 + 2x - 3x - 3$ $2x^2 - x - 3$ (Wrong Choice)
b. $(2x + 1)(x - 3) = 0$ $2x^2 - 6x + x - 3 = 0$ $2x^2 - 5x - 3 = 0$ (Wrong Choice)	d. $(2x + 3)(x - 1) = 0$ $2x^2 - 2x + 3x - 3 = 0$ $2x^2 + x - 3 = 0$ (Correct Choice)

PTS: 2 NAT: A.SSE.B.3 TOP: Solving Quadratics

173) ANS: 4

Strategy 1. Factor, then use the multiplication property of zero to find zeros.

$$3x^2 - 3x - 6 = 0$$

$$3(x^2 - x - 2) = 0$$

$$3(x-2)(x+1) = 0$$

$$x = 2, -1$$

Strategy 2. Use the quadratic formula.

$a = 3, b = -3,$ and $c = -6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-6)}}{2(3)}$$

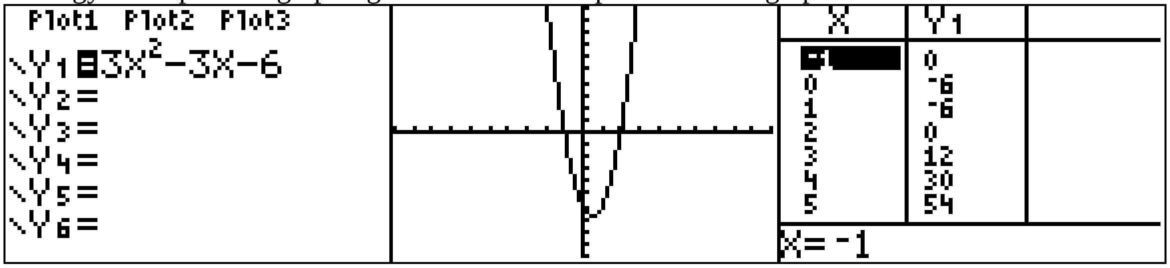
$$x = \frac{3 \pm \sqrt{9 + 72}}{6}$$

$$x = \frac{3 \pm \sqrt{81}}{6}$$

$$x = \frac{3 \pm 9}{6}$$

$$x = \frac{12}{6} = 2 \text{ and } x = \frac{-6}{6} = -1$$

Strategy 3. Input into graphing calculator and inspect table and graph.



Plot1 Plot2 Plot3

\Y1=3X²−3X−6

\Y2=

\Y3=

\Y4=

\Y5=

\Y6=

X	Y1
	0
0	⁻6
1	⁻6
2	0
3	12
4	30
5	54

X= ⁻1

PTS: 2 NAT: A.SSE.B.3 TOP: Solving Quadratics

174) ANS: 1

Strategy #1: Solve by factoring:

$$f(x) = 2x^2 - 4x - 6$$

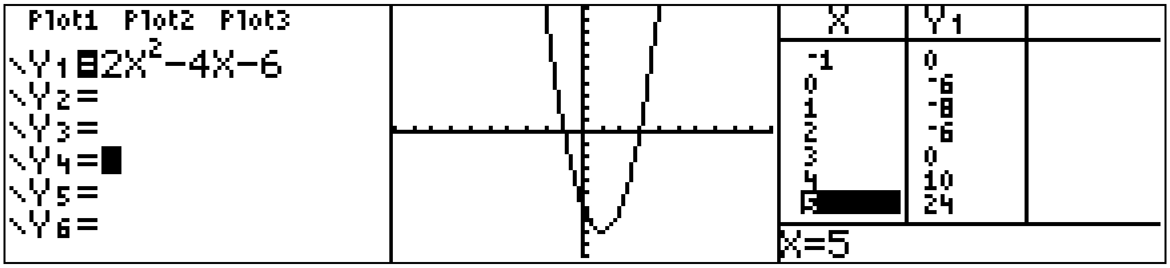
$$0 = 2x^2 - 4x - 6$$

$$0 = 2(x^2 - 2x - 3)$$

$$0 = 2(x-3)(x+1)$$

$$x = 3 \textit{ and } x = -1$$

Strategy #2: Solve by inputing equation into graphing calculator, the use the graph and table views to identify the zeros of the function.



```
Plot1 Plot2 Plot3
\Y1=2X²-4X-6
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

X	Y1
-1	0
0	-6
1	-8
2	-6
3	0
4	10
5	24

X=5

The graph and table views show the zeros to be at -1 and 3.

PTS: 2 NAT: A.SSE.B.3 TOP: Solving Quadratics
KEY: zeros of polynomials

175) ANS:
Use Janice’s procedure to solve for X.
Line 4 $B = -3$ and $B = 1$
Line 5 Therefore:
$8x = -3$ and $8x = 1$

$x = -\frac{3}{8}$ $x = \frac{1}{8}$

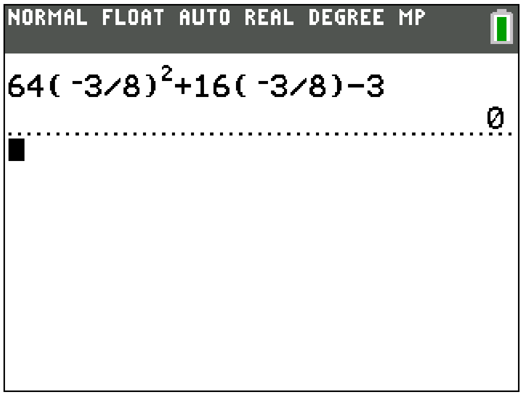
Explain the method Janice used to solve the quadratic formula.

Janice made the problem easier by substituting B for *8x*, then solving for B. After solving for B, she reversed her substitution and solved for *x*.

Check:

$x = -\frac{3}{8}$

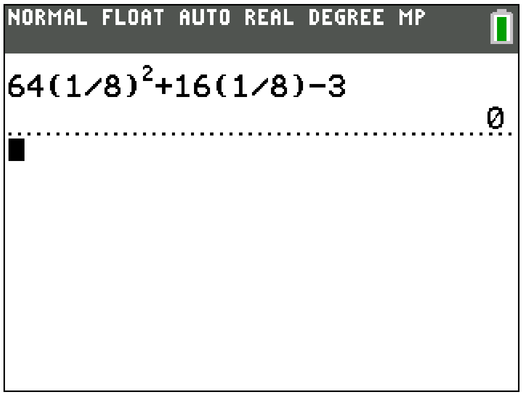
$0 = 64x^2 + 16x - 3$



```
NORMAL FLOAT AUTO REAL DEGREE MP
64(-3/8)²+16(-3/8)-3
                         0
```

$x = \frac{1}{8}$

$0 = 64x^2 + 16x - 3$



```
NORMAL FLOAT AUTO REAL DEGREE MP
64(1/8)²+16(1/8)-3
                         0
```

PTS: 4 NAT: A.SSE.B.3a

176) ANS: 3
The solution set of a quadratic equation includes all values of x when y equals zero. In the equation $(x-2)(x-a)=0$, the value of y is zero and $(x-2)$ and $(x-a)$ are factors whose product is zero.

The multiplication property of zero says, if the product of two factors is zero, then one or both of the factors must be zero.

Therefore, we can write: $x - 2 = 0$ and $x - a = 0$.

$$x = 2 \qquad x = a$$

PTS: 2 NAT: A.SSE.B.3 TOP: Solving Quadratics

177) ANS:

$x = \{-6, 3\}$

Factor $x^2 + 3x - 18$ as follows:

$$x^2 + 3x - 18 = 0$$

$$(x + 6)(x - 3) = 0$$

Then, use the multiplication property of zero to find the zeros, as follows:

$$x + 6 = 0 \quad \text{and} \quad x - 3 = 0$$

$$x = -6 \qquad x = 3$$

The zeros of a function are the *x*-values when $y = 0$. On a graph, the zeros are the values of *x* at the *x*-axis intercepts.

PTS: 4 NAT: A.SSE.B.3 TOP: Solving Quadratics

178) ANS: 1

Strategy: Use the quadratic equation to solve $x^2 - 6x - 19 = 0$, where $a = 1$, $b = -6$, and $c = -19$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-19)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{112}}{2}$$

$$x = \frac{6 \pm \sqrt{16} \cdot \sqrt{7}}{2}$$

$$x = \frac{6 \pm 4\sqrt{7}}{2}$$

$$x = 3 \pm 2\sqrt{7}$$

Answer choice *a* is correct.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: quadratic formula

179) ANS: 2

Strategy: Use the distributive property to expand each answer choice, the compare the expanded trinomial to the given equation $x^2 - 6x - 12 = 0$. Equivalent equations expressed in different terms will have the same solutions.

a.	c.

$(x+3)^2 = 21$ $(x+3)(x+3) = 21$ $x^2 + 6x + 9 = 21$ $x^2 + 6x - 12 = 0$ (Wrong Choice)	$(x+3)^2 = 3$ $(x+3)(x+3) = 3$ $x^2 + 6x + 9 = 3$ $x^2 + 6x + 6 = 0$ (Wrong Choice)
b. $(x-3)^2 = 21$ $(x-3)(x-3) = 21$ $x^2 - 6x + 9 = 21$ $x^2 - 6x - 12 = 0$ (Correct Choice)	d. $(x-3)^2 = 3$ $(x-3)(x-3) = 3$ $x^2 - 6x + 9 = 3$ $x^2 - 6x + 6 = 0$ (Wrong Choice)

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

180) ANS: 2

Strategy 1: Use the quadratic equation to solve $x^2 + 4x - 16 = 0$, where $a = 1$, $b = 4$, and $c = -16$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-16)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{80}}{2}$$

$$x = \frac{-4 \pm \sqrt{16}\sqrt{5}}{2}$$

$$x = \frac{-4 \pm 4\sqrt{5}}{2}$$

$$x = -2 \pm 2\sqrt{5}$$

Answer choice *b* is correct.

Strategy 2: Solve by completing the square:

$$x^2+4x-16=0$$

$$x^2+4x=16$$

$$(x+2)^2=16+2^2$$

$$(x+2)^2=20$$

$$\sqrt{(x+2)^2}=\sqrt{20}$$

$$x+2=\pm2\sqrt{5}$$

$$x=-2\pm2\sqrt{5}$$

Answer choice *b* is correct.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: quadratic formula

181) ANS:
$x=-8$ and $x=-2$

Strategy: Transform the expression $(3x-1)(3-x)+4x^2+19$ to a trinomial, then set the expression equal to 0 and solve it.

STEP 1. Transform $(3x-1)(3-x)+4x^2+19$ into a trinomial.

$$(3x-1)(3-x)+4x^2+19$$

$$9x-3x^2-3+x+4x^2+19$$

$$x^2+10x+16$$

STEP 2. Set the trinomial expression equal to 0 and solve.

$$x^2+10x+16=0$$

$$(x+8)(x+2)=0$$

$$x=-8\text{ and}-2$$

PTS: 4 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: factoring

182) ANS: 3
Strategy: Solve using root operations.

$$4x^2-100=0$$

$$4x^2=100$$

$$x^2=25$$

$$\sqrt{x^2}=\sqrt{25}$$

$$x=\pm5$$

Answer choice *c* is correct.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: taking square roots

183) ANS:
$x=\pm2\sqrt{2}$

Strategy: Use root operations to solve for x in the equeation $y = \frac{1}{2}x^2 - 4$.

$$\frac{1}{2}x^2 - 4 = 0$$
$$x^2 - 8 = 0$$
$$x^2 = 8$$
$$\sqrt{x^2} = \sqrt{8}$$
$$x = \pm\sqrt{8}$$
$$x = \pm\sqrt{4}\sqrt{2}$$
$$x = \pm 2\sqrt{2}$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: taking square roots

184) ANS:

The value of c that creates a perfect square trinomial is $\left(\frac{6}{2}\right)^2$, which is equal to 9.

The value of c is determined by taking half the value of *b*, when $a = 1$, and squaring it. In this problem, $b = 6$, so $\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

185) ANS: 4

Strategy: Use the distributive property to expand each answer choice, the compare the expanded trinomial to the given equation $x^2 + 6x - 7 = 0$. Equivalent equations expressed in different terms will have the same solutions.

a. $(x+3)^2 = 2$ $(x+3)(x+3) = 2$ $x^2 + 6x + 9 = 2$ $x^2 + 6x + 7 = 0$ (Wrong Choice)	c. $(x-3)^2 = 16$ $(x-3)(x-3) = 16$ $x^2 - 6x + 9 = 16$ $x^2 - 6x - 7 = 0$ (Wrong Choice)
b. $(x-3)^2 = 2$ $(x-3)(x-3) = 2$ $x^2 - 6x + 9 = 2$ $x^2 - 6x + 7 = 0$ (Wrong Choice)	d. $(x+3)^2 = 16$ $(x+3)(x+3) = 16$ $x^2 + 6x + 9 = 16$ $x^2 + 6x - 7 = 0$ (Correct Choice)

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

186) ANS:

Strategy 1: Solve using factoring by grouping.

$$4x^2 - 12x = 7$$

$$4x^2 - 12x - 7 = 0$$

$$|ac| = 28$$

The factors of 28 are

1 and 28

2 and 14 (use these)

$$4x^2 - 14x + 2x - 7 = 0$$

$$\left(4x^2 - 14x\right) + (2x - 7) = 0$$

$$2x(2x - 7) + 1(2x - 7) = 0$$

$$(2x + 1)(2x - 7) = 0$$

$$x = -\frac{1}{2}$$

$$x = \frac{7}{2}$$

Strategy 2: Solve by completing the square.

$$4x^2 - 12x = 7$$

$$\frac{4x^2}{4} - \frac{12x}{4} = \frac{7}{4}$$

$$x^2 - 3x = \frac{7}{4}$$

$$x^2 - 3x + \left(\frac{-3}{2}\right)^2 = \frac{7}{4} + \left(\frac{-3}{2}\right)^2$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{7}{4} + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{16}{4}$$

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{4}$$

$$x - \frac{3}{2} = \pm 2$$

$$x = \frac{3}{2} \pm 2$$

$$x = -\frac{1}{2} \text{ and } \frac{7}{2}$$

Strategy 3. Solve using the quadratic formula, where $a = 4$, $b = -12$, and $c = -7$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(-7)}}{2(4)}$$

$$x = \frac{12 \pm \sqrt{144 + 112}}{8}$$

$$x = \frac{12 \pm \sqrt{256}}{8}$$

$$x = \frac{12 \pm 16}{8}$$

$$x = \frac{3 \pm 4}{2}$$

$$x = -\frac{1}{2} \text{ and } \frac{7}{2}$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: factoring

187) ANS: 4

Strategy: Assume that Sam’s equation is correct, then expand the square in his equation and simplify.

$$x^2 - 5x + 3 = 0$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{13}{4}$$

$$\left(x - \frac{5}{2}\right)\left(x - \frac{5}{2}\right) = \frac{13}{4}$$

$$x^2 - 5x + \frac{25}{4} = \frac{13}{4}$$

$$x^2 - 5x = \frac{13}{4} - \frac{25}{4}$$

$$x^2 - 5x = \frac{-12}{4}$$

$$x^2 - 5x = -3$$

$$x^2 - 5x + 3 = 0$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: completing the square

188) ANS: 1

Strategy: The next step should be to take the square roots of both expressions.

$$(3x - 1)^2 = 25$$

$$\sqrt{(3x-1)^2} = \sqrt{25}$$

$$3x - 1 = \pm 5$$

The correct answer choice is *a*.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: completing the square

189) ANS: 2

$$x^2 - 8x = 10$$

$$x^2 - 8x + (4)^2 = 10 + (4)^2$$

$$(x-4)^2 = 10 + 16$$

$$(x-4)^2 = 26$$

$$\sqrt{(x-4)^2} = \sqrt{26}$$

$$x - 4 = \pm\sqrt{26}$$

$$x = 4 \pm \sqrt{26}$$

$$(x-4)^2 = 26$$

$$x - 4 = \pm\sqrt{26}$$

$$x = 4 \pm \sqrt{26}$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

190) ANS: 3
Strategy 1: Solve using root operations.

$$(x+3)^2 = 7$$

$$\sqrt{(x+3)^2} = \sqrt{7}$$

$$x + 3 = \pm\sqrt{7}$$

$$x = -3 \pm \sqrt{7}$$

Strategy 2. Solve using the quadratic equation.

$$(x+3)^2 = 7$$

$$x^2 + 6x + 9 = 7$$

$$x^2 + 6x + 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1,\ \ b = 6,\ \ c = 2$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 8}}{2}$$

$$x = \frac{-6 \pm \sqrt{28}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{7}}{2}$$

$$x = -3 \pm \sqrt{7}$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

191) ANS: 2

$$x^2 - 8x - 7 = 0$$

$$x^2 - 8x = 7$$

$$x^2 - 8x + (-4)^2 = 7 + (-4)^2$$

$$x^2 - 8x + 16 = 7 + 16$$

$$(x-4)^2 = 23$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

192) ANS:
The solutions are $y = 3$ and $y = 7$.

$$(y-3)^2 = 4y-12$$
$$y^2-6y+9 = 4y-12$$
$$y^2-10y+21 = 0$$
$$(y-7)(y-3) = 0$$
$$y-7 = 0$$
$$y = 7$$
$$y-3 = 0$$
$$y = 3$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: factoring

193) ANS:

a) Quadratic formula and completing the square.

b) -0.7 and -3.3

Complete the Square Method	Quadratic Formula Method
	$f(x) = 4x^2+16x+9$ a=4, b=16, c=9 $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $x = \frac{-16 \pm \sqrt{(16)^2-4(4)(9)}}{2(4)}$ $x = \frac{-16 \pm \sqrt{112}}{8}$ $x = \frac{-16+\sqrt{112}}{8} = \frac{-5.416}{8} = -.677 = -0.7$ $x = \frac{-16-\sqrt{112}}{8} = \frac{-26.583}{8} = -3.322 = -3.3$

$$f(x)=4x^2+16x+9$$

$$4x^2+16x+9=0$$

$$4x^2+16x=-9$$

$$\frac{4x^2}{4}+\frac{16x}{4}=\frac{-9}{4}$$

$$x^2+4x=-\frac{9}{4}$$

$$x^2+4x+(2)^2=-\frac{9}{4}+(2)^2$$

$$(x+2)^2=-\frac{9}{4}+4$$

$$(x+2)^2=-\frac{9}{4}+\frac{16}{4}$$

$$(x+2)^2=\frac{7}{4}$$

$$x+2=\pm\sqrt{\frac{7}{4}}$$

$$x+2=\pm\frac{\sqrt{7}}{2}$$

$$x=-2\pm\frac{\sqrt{7}}{2}$$

$$x=-2+\frac{\sqrt{7}}{2}=-0.677=-0.7$$

$$x=-2-\frac{\sqrt{7}}{2}=-3.322=-3.3$$

PTS: 1 NAT: A.REI.A.1

194) ANS: 3

Step 1. Understand that solving the equation means isolating the value of x.

Step 2. Strategy. Isolate x.

Step 3. Execution of strategy.

$$2(x+2)^2-4=28$$

$$2(x+2)^2=28+4$$

$$2(x+2)^2=32$$

$$\frac{2(x+2)^2}{2}=\frac{32}{2}$$

$$(x+2)^2=16$$

$$x+2=\sqrt{16}$$

$$x+2=\pm 4$$

$$x=-2\pm 4$$

$$x=2$$

$$x=-6$$

Step 4. Does it make sense? Yes. The values 2 and -6 satisfy the equation $2(x+2)^2-4=28$.

x=2	x=-6
$2(x+2)^2-4=28$	$2(x+2)^2-4=28$
$2(2+2)^2-4=28$	$2(-6+2)^2-4=28$
$2(4)^2-4=28$	$2(-4)^2-4=28$
$2(16)-4=28$	$2(16)-4=28$
$32-4=28$	$32-4=28$
$28=28$	$28=28$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: taking square roots

195) ANS:
Yes. I agree with Amy’s solution. I get the same solutions by using the quadratic formula.

$$2x^2 + 5x - 42 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-42)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{25 + 336}}{4}$$

$$x = \frac{-5 \pm \sqrt{361}}{4}$$

$$x = \frac{-5 \pm 19}{4}$$

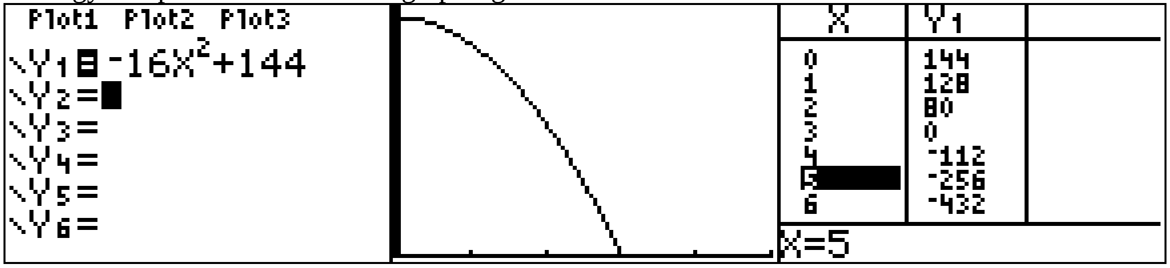
$$x = \frac{14}{4} = \frac{7}{2}$$

$$x = \frac{-24}{4} = -6$$

NOTE: Acceptable explanations could also be made by: 1) substituting Amy's solutions into the original equation and showing that both solutions make the equation balance; 2) solving the quadratic by completing the square and getting Amy's solutions; or 3) solving the quadratic by factoring and getting Amy's solutions.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: factoring NOT: NYSED classifies this as A.REI.A

196) ANS:
How many feet did the object fall between one and two seconds after it was dropped?
Strategy: Input the function in a graphing calculator.



```
 Plot1 Plot2 Plot3
\Y1=-16X²+144
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

X	Y1
0	144
1	128
2	80
3	0
4	-112
5	-256
6	-432

X=5

After one second, the object is 128 feet above the ground.
After two seconds, the object is 80 feet above the ground.
The object fell $128 - 80 = 48$ feet between one and two seconds after it was dropped.

Determine algebraically how many seonds it will take for the object to reach the ground.

$$\begin{aligned} H(t) &= -16t^2 + 144 \\ 0 &= -16t^2 + 144 \\ 16t^2 &= 144 \\ t^2 &= \frac{144}{16} \\ t^2 &= 9 \\ t &= 3 \end{aligned}$$

The object will hit the ground after 3 seconds.

PTS: 4 NAT: A.SSE.B.3 TOP: Solving Quadratics

197) ANS: 1

$$\begin{aligned} &3x^2 + 10x = 8 \\ &3x^2 + 10x - 8 = 0 \\ &a = 3 \quad b = 10 \quad c = -8 \\ &x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &x = \frac{-10 \pm \sqrt{10^2 - 4(3)(-8)}}{2(3)} \\ &x = \frac{-10 \pm \sqrt{196}}{6} \\ &x = \frac{-10 \pm 14}{6} \\ &x = \frac{4}{6} = \frac{2}{3} \text{ and } x = \frac{-24}{6} = -4 \end{aligned}$$

PTS: 2 NAT: A.REI.B.4

198) ANS:

$\{10, -4\}$

$$\begin{aligned} f(x) &= (x-3)^2 - 49 \\ 0 &= (x-3)^2 - 49 \\ 49 &= (x-3)^2 \\ \pm 7 &= x - 3 \\ 3 \pm 7 &= x \\ x = 10 &\text{ and } x = -4 \end{aligned}$$

PTS: 2 NAT: A.REI.B.4

199) ANS: 4

Given	$13 - 36x^2$	=	-12

Add (12)	+12	=	+12
Simplify	$25 - 36x^2$	=	0
Add ($36x^2$)	$+36x^2$	=	$+36x^2$
Simplify	25	=	$+36x^2$
Divide (36)	$\frac{25}{36}$	=	$\frac{36x^2}{36}$
Simplify	$\frac{25}{36}$	=	x^2
Square Root	$\pm\frac{5}{6}$	=	x

The only correct answer choice is $-\frac{5}{6}$.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: taking square roots

200) ANS: 1

Given	$2x^2 - 12x + 6$	=	0
Divide by 2	$\frac{2x^2 - 12x + 6}{2}$	=	$\frac{0}{2}$
Simplify	$x^2 - 6x + 3$	=	0
Subtract 3	-3	=	-3
Simplify	$x^2 - 6x$	=	-3
Complete the Square	$x^2 - 6x + \left(\frac{-6}{2}\right)^2$	=	$-3 + \left(\frac{-6}{2}\right)^2$
Simplify	$x^2 - 6x + (-3)^2$	=	$-3 + (-3)^2$
Factor and Simplify	$(x-3)^2$	=	-3 + 9
Simplify	$(x-3)^2$	=	6

$$2(x^2 - 6x + 3) = 0$$

$$x^2 - 6x = -3$$

$$x^2 - 6x + 9 = -3 + 9$$

$$(x-3)^2 = 6$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

201) ANS: 1

Strategy 1: Use the quadratic equation to solve $x^2 - 8x = 24$, where $a = 1$, $b = -8$, and $c = -24$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-24)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{160}}{2}$$

$$x = \frac{8 \pm \sqrt{16}\sqrt{10}}{2}$$

$$x = \frac{8 \pm 4\sqrt{10}}{2}$$

$$x = 4 \pm 2\sqrt{10}$$

Answer choice *a* is correct.

Strategy 2. Solve by completing the square.

$$x^2 - 8x = 24$$

$$(x-4)^2 = 24 + (-4)^2$$

$$(x-4)^2 = 24 + 16$$

$$(x-4)^2 = 40$$

$$\sqrt{(x-4)^2} = \sqrt{40}$$

$$x - 4 = \pm 2\sqrt{10}$$

$$x = 4 \pm 2\sqrt{10}$$

Answer choice *a* is correct.

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics

KEY: completing the square

202) ANS:

$$x^2 - 6x = 15$$

$$x^2 - 6x + \left(\frac{-6}{2}\right)^2 = 15 + \left(\frac{-6}{2}\right)^2$$

$$x^2 - 6x + (-3)^2 = 15 + (-3)^2$$

$$(x-3)^2 = 15 + 9$$

$$(x-3)^2 = 24$$

$$\sqrt{(x-3)^2} = \sqrt{24}$$

$$x - 3 = \pm\sqrt{24}$$

$$x = 3 \pm \sqrt{24}$$

$$x = 3 \pm \sqrt{4}\sqrt{6}$$

$$x = 3 \pm 2\sqrt{6} \quad \text{Answer}$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

203) ANS: 1

$$3(x-4)^2 = 27$$

$$\frac{3(x-4)^2}{3} = \frac{27}{3}$$

$$(x-4)^2 = 9$$

$$\sqrt{(x-4)^2} = \sqrt{9}$$

$$x - 4 = \pm 3$$

$$x = 1, 7$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: taking square roots

204) ANS: 3

Strategy: Rewrite $x^2 - 6x = 12$ in the form of $(x+p)^2 = q$ and find the value of p.

Notes	Left Epression	Sign	Right Expression
Given	$x^2 - 6x$	=	12
Complete the Square	$x^2 - 6x + (-3)^2$	=	$12 + (-3)^2$
Exponents and Parentheses	$x^2 - 6x + 9$	=	$12 + 9$
Factor left expression and simplify right expression	$(x-3)^2$	=	21
Compare to form given in the question.	$(x+p)^2$	=	q

$$p = -3$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

205) ANS:
Answer: -2.8, 1.8

Strategy: Use the quadratic formula

STEP 1. Identify the values of a, b, and c in $x^2 + x - 5 = 0$.

$$a = 1$$

$$b = 1$$

$$c = -5$$

STEP 2. Substitute these values in the quadratic formula and solve.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 + 20}}{2}$$

$$x = \frac{-1 \pm \sqrt{21}}{2}$$

$$x = \frac{-1 \pm 4.58}{2}$$

$$x = \frac{-1 + 4.58}{2} \qquad x = \frac{-1 - 4.58}{2}$$

$$x = \frac{4.58}{2} \qquad x = \frac{-5.58}{2}$$

$$x = 1.79 \approx 1.8 \qquad x = -2.79 \approx -2.8$$

PTS: 2 NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: quadratic formula