**K – Polynomials, Lesson 3, Factoring Polynomials (r. 2018)**

# POLYNOMIALS
## Factoring Polynomials

| **Common Core Standard** | **Next Generation Standard** |
|---|---|
| **A-SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$ .<br>PARCC: Tasks limited to numerical and polynomial expressions in one variable. Recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to -evaluate form (53+47)(53-47). See an opportunity to rewrite $a^2 + 9a + 14$ as (a+7)(a+2).<br>NYSED: Does not include factoring by grouping and factoring the sum and difference of cubes. | **AI-A.SSE.2 Recognize** and use the structure of an expression to identify ways to rewrite it.<br>(Shared standard with Algebra II)<br>**e.g.,**<br>$\mathbf{x^3 - x^2 - x = x(x^2 - x - 1)}$<br>$\mathbf{53^2 - 47^2 = (53 + 47)(53 - 47)}$<br>$\mathbf{16x^2 - 36 = (4x)^2 - (6)^2 = (4x + 6)(4x - 6) = 4(2x + 3)(2x - 3)}$ **or**<br>$\mathbf{16x^2 - 36 = 4(4x^2 - 9) = 4(2x + 3)(2x - 3)}$<br>$\mathbf{-2x^2 + 8x + 10 = -2(x^2 - 4x - 5) = -2(x - 5)(x + 1)}$<br>$\mathbf{x^4 + 6x^2 - 7 = (x^2 + 7)(x^2 - 1) = (x^2 + 7)(x + 1)(x - 1)}$<br>Note: Algebra I expressions are limited to numerical and polynomial expressions in one variable. **Use factoring techniques such as factoring out a greatest common factor, factoring the difference of two perfect squares, factoring trinomials of the form $ax_2+bx+c$ with a lead coefficient of 1, or a combination of methods to factor completely.** Factoring will not involve factoring by grouping and factoring the sum and difference of cubes. |

## LEARNING OBJECTIVES

Students will be able to:

1) factor monomials
2) factor binomials, and
3) factor trinomials

### Overview of Lesson

| **Teacher Centered Introduction** | **Student Centered Activities** |
|---|---|
| **Overview of Lesson**<br>**- activate students' prior knowledge**<br>**- vocabulary**<br>**- learning objective(s)**<br>**- big ideas: direct instruction**<br>**- modeling** | **guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work**<br>**- developing essential skills**<br>**- Regents exam questions**<br>**- formative assessment assignment (exit slip, explain the math, or journal entry)** |

## VOCABULARY

binomial
factor completely
greatest common factor
monomial
perfect square
term
trinomial

# BIG IDEAS

Factoring polynomials is one of four general methods taught in the Regents mathematics curriculum for finding the roots of a quadratic equation. The other three methods are the quadratic formula, completing the square and graphing.

- The roots of a quadratic equation can found using the **factoring** method when the discriminant's value is equal to either zero or a perfect square.

**Factoring Monomials**:

$$204x^2 = 2\left(102x^2\right) = 2 \bullet 2\left(51x^2\right) = 2 \bullet 2 \bullet 3\left(17x^2\right) = 2^2 \bullet 3 \bullet 17 \bullet x^2$$

**Factoring Binomials**: *NOTE: This is the inverse of the distributive property.*

$$3(x+2) = 3x+6$$

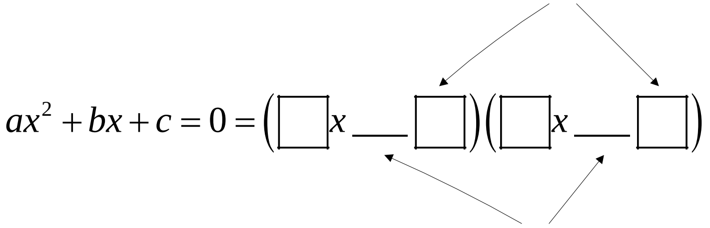
$$2x^2 + 6x = 2x(x+3)$$

## Factoring Trinomials

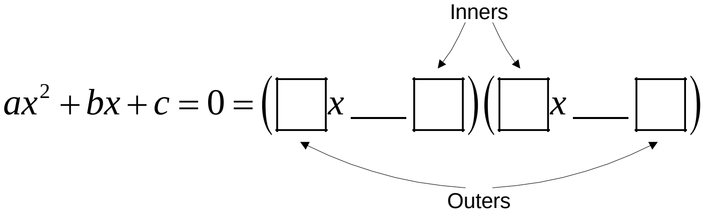
Standard Approach

Given a trinomial in the form $ax^2 + bx + c = 0$ whose discriminant equals zero or a perfect square, it may be factored as follows:

STEP 1. The product of these two numbers must equal c.



STEP 2. The signs of these two numbers are determined by the signs of b and c.



STEP 3. The product of the outer numbers plus the product of the inner numbers must sum to b.

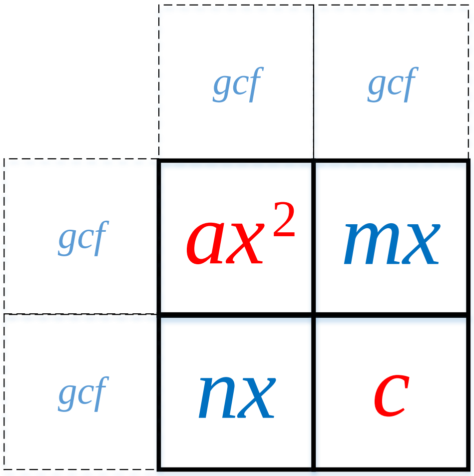
**Modeling:**

$$x^2 - 5x + 6 = (x-2)(x-3)$$

$$2x^2 - 8x + 6 = (2x-2)(x-3)$$

$$4x^2 - 10x + 6 = (2x-2)(2x-3)$$

Box Method



# The Box Method for Factoring a Trinomial

$$ax^2 + bx + c = 0$$

$$bx = mx + nx$$

| INSTRUCTIONS | EXAMPLE |
|---|---|
| STEP 1 Start with a factorable quadratic in standard form: $ax^2 + bx + c = 0$ and a 2-row by 2-column table. | Solve by factoring: $6x^2 - x - 12 = 0$ |
| STEP 2 Copy the quadratic term into the upper left box and the constant term into the lower right box. | $\begin{array}{\|c\|c\|}\hline 6x^2 & \\ \hline & -12 \\ \hline\end{array}$ |
| STEP 3 Multiply the quadratic term by the constant term and write the product to the right of the table. | $\begin{array}{\|c\|c\|}\hline 6x^2 & \\ \hline & -12 \\ \hline\end{array}$ $6x^2 \times -12 = \boxed{-72x^2}$ |
| STEP 4 Factor the product from STEP 3 until you obtain two factors that *sum* to the linear term (*bx*). | $1x \times -72x$<br>$-1x \times 72x$<br>$2x \times -36x$<br>$-2x \times 36x$<br>$3x \times -24x$<br>$-3x \times 24x$<br>$4x \times -18x$<br>$-4x \times 18x$<br>$6x \times -12x$<br>$-6x \times 12x$<br>$8x \times -9x$ These two factors sum to *bx*<br>$-8x \times 9x$ |

<table>
<tr><td>STEP 5 Write one of the two factors found in STEP 4 in the upper right box and the other in the lower left box. Order does not matter.</td><td>$$\begin{array}{|c|c|}\hline 6x^2 & -9x \\ \hline 8x & -12 \\ \hline \end{array}$$</td></tr>
<tr><td>STEP 6 Find the greatest common factor of each row and each column and record these factors to the left of each row and above each column. Give each factor the same plus or minus value as the nearest term in a box.<br>NOTE: If all four of the greatest common factors share a common factor, reduce each factor by the common factor and add the common factor as a third factor. Eg. $(3x-9)(3x-15) \Rightarrow 3(x-3)(x-5)$</td><td>$$\begin{array}{c|c|c|}  & 2x & -3 \\ \hline 3x & 6x^2 & -9x \\ \hline 4 & 8x & -12 \\ \hline \end{array}$$</td></tr>
<tr><td>STEP 7 Write the expressions above and beside the box as binomial factors of the original trinomial.</td><td>$(2x-3)(3x+4)=0$</td></tr>
<tr><td>STEP 8 Check to see that the factored quadratic is the same as the original quadratic.</td><td>$(2x-3)(3x+4)=0$<br>$6x^2+8x-9x-12=0$<br>$6x^2-9x-12=0$ check</td></tr>
<tr><td>STEP 9 Convert the factors to zeros.</td><td>$(2x-3)=0$<br>$2x=3$<br>$x=\boxed{\frac{3}{2}}$<br>$(3x+4)=0$<br>$3x=-4$<br>$x=\boxed{-\frac{4}{3}}$</td></tr>
</table>

## DEVELOPING ESSENTIAL SKILLS

1.Factored completely, the expression $2x^2+10x-12$ is equivalent to

a. $2(x-6)(x+1)$ c. $2(x+2)(x+3)$
b. $2(x+6)(x-1)$ d. $2(x-2)(x-3)$

2. Factored completely, the expression $3x^2-3x-18$ is equivalent to

a. $3(x^2-x-6)$ c. $(3x-9)(x+2)$
b. $3(x-3)(x+2)$ d. $(3x+6)(x-3)$

3. What are the factors of the expression $x^2+x-20$?

a. $(x+5)$ and $(x+4)$ c. $(x-5)$ and $(x+4)$
b. $(x+5)$ and $(x-4)$ d. $(x-5)$ and $(x-4)$

4. Factored completely, the expression $3x^3 - 33x^2 + 90x$ is equivalent to
   a. $3x(x^2 - 33x + 90)$
   b. $3x(x^2 - 11x + 30)$
   c. $3x(x + 5)(x + 6)$
   d. $3x(x - 5)(x - 6)$
5. Factor completely: $5x^3 - 20x^2 - 60x$
6. The greatest common factor of $3m^2n + 12mn^2$ is?
   a. $3n$
   b. $3m$
   c. $3mn$
   d. $3mn^2$
7. When factored completely, the expression $3x^2 - 9x + 6$ is equivalent to
   a. $(3x - 3)(x - 2)$
   b. $(3x + 3)(x - 2)$
   c. $3(x + 1)(x - 2)$
   d. $3(x - 1)(x - 2)$
8. Which is a factor of $x^2 + 5x - 24$?
   a. $(x + 4)$
   b. $(x - 4)$
   c. $(x + 3)$
   d. $(x - 3)$
9. Which expression is a factor of $x^2 + 2x - 15$?
   a. $(x - 3)$
   b. $(x + 3)$
   c. $(x + 15)$
   d. $(x - 5)$
10. Which expression is a factor of $n^2 + 3n - 54$?
    a. $n + 6$
    b. $n^2 + 9$
    c. $n - 9$
    d. $n + 9$
11. What are the factors of $x^2 - 10x - 24$?
    a. $(x - 4)(x + 6)$
    b. $(x - 4)(x - 6)$
    c. $(x - 12)(x + 2)$
    d. $(x + 12)(x - 2)$
12. If one factor of $56x^4y^3 - 42x^2y^6$ is $14x^2y^3$, what is the other factor?
    a. $4x^2 - 3y^3$
    b. $4x^2 - 3y^2$
    c. $4x^2y - 3xy^3$
    d. $4x^2y - 3xy^2$
13. If $3x$ is one factor of $3x^2 - 9x$, what is the other factor?
    a. $3x$
    b. $x^2 - 6x$
    c. $x - 3$
    d. $x + 3$
14. Factor completely: $3x^2 + 15x - 42$
15. Factored completely, the expression $2y^2 + 12y - 54$ is equivalent to
    a. $2(y + 9)(y - 3)$
    b. $2(y - 3)(y - 9)$
    c. $(y + 6)(2y - 9)$
    d. $(2y + 6)(y - 9)$
16. What are the factors of $x^2 - 5x + 6$?
    a. $(x + 2)$ and $(x + 3)$
    b. $(x - 2)$ and $(x - 3)$
    c. $(x + 6)$ and $(x - 1)$
    d. $(x - 6)$ and $(x + 1)$
17. The greatest common factor of $4a^2b$ and $6ab^3$ is
    a. $2ab$
    b. $2ab^2$
    c. $12ab$
    d. $24a^3b^4$

## Answers

1. ANS: B
2. ANS: B

3. ANS: B
4. ANS: D

$3x^3 - 33x^2 + 90x = 3x(x^2 - 11x + 30) = 3x(x-5)(x-6)$

5. ANS:

$5x^3 - 20x^2 - 60x$

$5x(x^2 - 4x - 12)$

$5x(x+2)(x-6)$

6. ANS: C
7. ANS: D
8. ANS: D
9. ANS: A
10. ANS: D
11. ANS: C
12. ANS: A
13. ANS: C
14. ANS:

$3(x+7)(x-2)$. $3x^2 + 15x - 42 = 3(x^2 + 5x - 14) = 3(x+7)(x-2)$

15. ANS: A
16. ANS: B
17. ANS: A

## REGENTS EXAM QUESTIONS (through June 2016)

# A.SSE.A.2: Factoring Polynomials

343) Which expression is equivalent to $x^4 - 12x^2 + 36$?

1) $(x^2 - 6)(x^2 - 6)$
2) $(x^2 + 6)(x^2 + 6)$
3) $(6 - x^2)(6 + x^2)$
4) $(x^2 + 6)(x^2 - 6)$

344) Four expressions are shown below.

I $2(2x^2 - 2x - 60)$
II $4(x^2 - x - 30)$
III $4(x + 6)(x - 5)$
IV $4x(x - 1) - 120$

The expression $4x^2 - 4x - 120$ is equivalent to

1) I and II, only
2) II and IV, only
3) I, II, and IV
4) II, III, and IV

345) When factored completely, $x^3 - 13x^2 - 30x$ is

1) $x(x + 3)(x - 10)$
2) $x(x - 3)(x - 10)$
3) $x(x + 2)(x - 15)$
4) $x(x - 2)(x + 15)$

346) Factor the expression $x^4 + 6x^2 - 7$ completely.

347) The trinomial $x^2 - 14x + 49$ can be expressed as

1) $(x - 7)^2$
2) $(x + 7)^2$
3) $(x - 7)(x + 7)$
4) $(x - 7)(x + 2)$

## SOLUTIONS

343) ANS: 1

Strategy 1. Factor $x^4 - 12x^2 + 36$

$$x^4 - 12x^2 + 36$$

$$\left(x^2 - \_\_\_\right)\left(x^2 - \_\_\_\right)$$

The factors of 36 are:

1 and 36,

2 and 18,

3 and 12,

4 and 9,

6 and 6 (use these because they sum to 12)

$$\left(x^2 - 6\right)\left(x^2 - 6\right)$$

Strategy 2. Work backwards using the distributive property to check each answer choice.

| | |
|---|---|
| a<br>$(x^2-6)(x^2-6)$<br>$x^4-6x^2-6x^2+36$<br>$x^4-12x^2+36$<br>(correct) | c<br>$(6-x^2)(6+x^2)$<br>$36+6x^2-6x^2-x^4$<br>$36-x^4$<br>(wrong) |
| b<br>$(x^2+6)(x^2+6)$<br>$x^4+6x^2+6x^2+36$<br>$x^4+12x^2+36$<br>(wrong) | d<br>$(x^2+6)(x^2-6)$<br>$x^4-6x^2+6x^2-36$<br>$x^4-36$<br>(wrong) |

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

344) ANS: 3

Strategy: Use the distributive property to expand each expression, then match the expanded expressions to the answer choices.

| | |
|---|---|
| I<br>$2(2x^2-2x-60)$<br>$4x^2-4x-120$<br>*yes* | III<br>$4(x+6)(x-5)$<br>$(4x+24)(x-5)$<br>$4x^2-20x+24x-120$<br>$4x^2+4x-120$<br>*no* |
| II<br>$4(x^2-x-30)$<br>$4x^2-4x-120$<br>*yes* | IV<br>$4x(x-1)-120$<br>$4x^2-4x-120$<br>*yes* |

Answer choice *c* is correct.

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

345) ANS: 3

$$x^3-13x^2-30x$$
$$x(x^2-13x-30)$$
$$x(x+2)(x-15)$$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

346) ANS:

$\left(x^2+7\right)(x+1)(x-1)$

Strategy: Factor the trinomial, then factor the perfect square.

STEP 1. Factor the trinomial $x^4+6x^2-7$.

$x^4 + 6x^2 - 7$

$\left(x^2 + \_\_\_\right)\left(x^2 - \_\_\_\right)$

The factors of 7 are 1 and 7.

$\left(x^2 + 7\right)\left(x^2 - 1\right)$

STEP 2. Factor the perfect square.

$\left(x^2 + 7\right)\left(x^2 - 1\right)$

$\left(x^2 + 7\right)(x + 1)(x - 1)$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

347) ANS: 1

Strategy. Multiply binomials and eliminate wrong answers.

Choice 1: $(x - 7)^2$ Correct

$(x - 7)(x - 7)$

$x^2 - 7x - 7x + 49$

$x^2 - 14x + 49$

Choice 2: $(x + 7)^2$ Wrong: middle term has wrong sign.

$(x + 7)(x + 7)$

$x^2 + 7x + 7x + 49$

$x^2 + 14x + 49$

Choice 3: $(x - 7)(x + 7)$ Wrong: no middle term and second term has wrong sign.

$x^2 + 7x - 7x - 49$

$x^2 - 49$

Choice 4: $(x - 7)(x + 2)$ Wrong: middle term and third term have wrong coefficients.

$x^2 + 2x - 7x - 14$

$x^2 - 5x - 14$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

KEY: quadratic