

C – Expressions and Equations, Lesson 2, Modeling Expressions (r. 2018)

EXPRESSIONS AND EQUATIONS

Modeling Expressions

Common Core Standards	Next Generation Standards
<p>A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.</p> <p>A-SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients. <small>NYS ED: The “such as” listed are not the only parts of an expression students are expected to know; others include, but are not limited to, degree of a polynomial, leading coefficient, constant term, and the standard form of a polynomial (descending exponents).</small></p> <p>A-SSE.A.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret $P(1+r)_n$ as the product of P and a factor not depending on P.</i></p>	<p>AI-A.SSE.1 Interpret expressions that represent a quantity in terms of its context.</p> <p>AI-A.SSE.1a Write the standard form of a given polynomial and identify the terms, coefficients, degree, leading coefficient, and constant term.</p> <p>AI-A.SSE.1b Interpret expressions by viewing one or more of their parts as a single entity. e.g., Interpret $P(1+r)_n$ as the product of P and a factor not depending on P. Note: This standard is a fluency expectation for Algebra I. Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square, and other mindful algebraic calculations.</p>

LEARNING OBJECTIVES

Students will be able to:

- 1) Use academic language to identify the terms, coefficients, degree, leading coefficient, and constant term of a mathematical statement.
- 2) Relate parts of equations and expressions to real world contexts.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
<p>Overview of Lesson</p> <ul style="list-style-type: none"> - activate students' prior knowledge - vocabulary - learning objective(s) - big ideas: direct instruction - modeling 	<p>guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work</p> <ul style="list-style-type: none"> - developing essential skills - Regents exam questions - formative assessment assignment (exit slip, explain the math, or journal entry)

VOCABULARY

coefficient	expression	standard form
constant	leading coefficient	term
degree	leading term	variable
degree of an equation	monomial	variable expression
equation	polynomial	

BIG IDEAS

Important skills in mathematics involve recognizing and using academic vocabulary to:

- 1) communicate the structure of mathematics and;
- 2) relate parts of mathematical equations and expressions to real world contexts.

Equation An equation consists of two *expressions* connected by an equal sign. The equal sign indicates that both *expressions* have the same (equal) value. The two expressions in an equation are typically called the *left expression* and the *right expression*.

Expression An expression is a mathematical statement or phrase consisting of one or more *terms*. *Terms* are the building blocks of expressions, similar to the way that letters are the building blocks of words. An expression will always be either a monomial or a polynomial.

- Monomial expressions have only one term.
- Polynomial expressions have two or more terms.

Term A term is a *number*, a *variable*, or the *product* of numbers and variables.

- Terms in an expression are always separated by a plus sign or minus sign.
- Terms in an expression are always either positive or negative.
- Numbers and variables connected by the operations of division and multiplication are parts of the same term.
- Terms, together with their signs, can be moved around within the same expression without changing the value of the expression. If you move a term from the left expression to the right expression, or from the right expression to the left expression (across the equal sign), the plus or minus sign associated with the term must be changed.

Leading Term: The leading term in a polynomial expression is the highest degree term.

Variable A variable is a quantity whose value can change or vary. In algebra, a letter is typically used to represent a variable. The value of the letter can change. The letter x is commonly used to represent a variable, but other letters can also be used. The letters s , o , and sometimes l are avoided by some students because they are easily confused in equations with numbers.

- **Independent Variable:** Always shown on the x -axis, the independent variable is the input for an equation.
- **Dependent Variable:** Always shown on the y -axis, the dependent variable is the output of the equation.
- **Variable Term:** A term that contains at least one variable.
- **Variable Expression:** A mathematical phrase that contains at least one variable.

Example: The equation $2x+3 = 5$ contains a left expression and a right expression. The two expressions are connected by an equal sign. The expression on the left is a polynomial variable expression containing two terms, which are $+2x$ and $+3$. The expression on the right is monomial that contains only one term, which is the constant $+5$.

Coefficient: A coefficient is the numerical factor of a term in a polynomial. It is typically thought of as the number in front of a variable.

Example: 14 is the coefficient in the term $14x^3y$.

- **Leading Coefficient:** The leading coefficient of a polynomial is the coefficient of the leading term.

Constant: A constant is a number with a constant value (ie. not a variable).

Standard Form of a Polynomial: A polynomial is in standard form when the degrees of its terms are in descending order.

Examples: $3x^3 + 5x^2 + 7x$ is in standard form.

$5x^2 + 3x^3 + 7x$ is *not* in standard..

DEVELOPING ESSENTIAL SKILLS

Answer each question about the following mathematical statements:

Mathematical Statements	$y = x + 3 - 2x^2$	$4x^4 - 6x^2 + 3x^3 + 2x - 2$	$0 = x + 4$
Is this mathematical statement an expression or an equation?	Equation	Expression	Equation
How many terms are in this mathematical statement?	4	5	3
What is the leading term?	$-2x^2$	$4x^4$	$-2x^2$
What is the degree of this mathematical statement?	Second	Fourth	First
What is the coefficient of the lowest variable term?	1	2	1
What is the constant?	3	-2	0 and 4
Write this mathematical statement in standard form.	$y = -2x^2 + x + 3$	$4x^4 + 3x^3 - 6x^2 + 2x - 2$	$x = -4$

REGENTS EXAM QUESTIONS (through June 2018)

A.SSE.A.1: Modeling Expressions

- 49) To watch a varsity basketball game, spectators must buy a ticket at the door. The cost of an adult ticket is \$3.00 and the cost of a student ticket is \$1.50. If the number of adult tickets sold is represented by a and student tickets sold by s , which expression represents the amount of money collected at the door from the ticket sales?
- 1) $4.50as$
 - 2) $4.50(a + s)$
 - 3) $(3.00a)(1.50s)$
 - 4) $3.00a + 1.50s$

A leading coefficient is the coefficient of the first term of a polynomial written in descending order of exponents. Since the leading coefficient is seven, choices a and c can be eliminated, leaving choice d as the only possible answer.

Does it make sense? Yes. $7x^5 + 2x^2 + 6$ has a leading coefficient of seven, is a fifth degree polynomial because 5 is the highest exponent, and a constant term of six.

PTS: 2 NAT: A.SSE.A.1 TOP: Modeling Expressions

51) ANS:

I disagree. The leading coefficient of a polynomial is the coefficient of the term with the highest exponent when all of the terms are arranged in descending order by exponents.

$$-2x^3 + 8x^2 - 4x + 5$$

Pat should have written that the leading coefficient is -2 .

PTS: 2 NAT: A.SSE.A.1

52) ANS: 4

Strategy:

Step 1. Create a table of values by starting at week 0 with \$310 dollars.

Week	\$\$\$
x	y
0	310
1	280
2	250
3	220
4	190
5	160
6	130
7	100
8	70

Step 2. Use a graphing calculator to determine which expression reproduces the table of values.

NORMAL FLOAT AUTO REAL RADIAN MP		NORMAL FLOAT AUTO REAL RADIAN MP				
		PRESS + FOR Δ b1				
	X	Y1	Y2	Y3	Y4	
Plot1 Plot2 Plot3	0	310	250	-30	310	
$\backslash Y_1 \equiv 310 - 8X$	1	302	280	280	280	
$\backslash Y_2 \equiv 280 + 30(X - 1)$	2	294	310	590	250	
$\backslash Y_3 \equiv 310X - 30$	3	286	340	900	220	
$\backslash Y_4 \equiv 280 - 30(X - 1)$	4	278	370	1210	190	
$\backslash Y_5 =$	5	270	400	1520	160	
$\backslash Y_6 =$	6	262	430	1830	130	
$\backslash Y_7 =$	7	254	460	2140	100	
$\backslash Y_8 =$	8	246	490	2450	70	
$\backslash Y_9 =$	9	238	520	2760	40	
	10	230	550	3070	10	
	X=0					

PTS: 2 NAT: A.SSE.A.1 TOP: Modeling Expressions

53) ANS: 2

On the stationary bike, Konnor can burn 5 Cal/min.

b represents the number of minutes Konnor spends on the stationary bike.

5 times b represents the number of Calories that Konnor can burn on the stationary bike.

PTS: 2 NAT: A.SSE.A.1 TOP: Modeling Expressions

54) ANS: 3

Strategy: Find the polynomials that have the exponents decreasing from left to right. This is the definition of standard form.

I. $15x^4 - 6x + 3x^2 - 1$ is not in standard form because the exponent of the middle term is less than the exponent of the third term.

II. $12x^3 + 8x + 4$ is in standard form because the exponents decrease from left to right.

III. $2x^5 + 8x^2 + 10x$ is in standard form because the exponents decrease from left to right.

Fred is correct. II and III are in standard form.

PTS: 2

NAT: A.SSE.A.1

TOP: Modeling Expressions