I – Systems, Lesson 7, Other Systems (r. 2018)

SYSTEMS
Other Systems

Common Core Standard	**Next Generation Standard**
A-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, ~~or find successive approximations~~. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential~~, and logarithmic functions~~. ~~PARCC: Tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. Finding the solutions approximately is limited to cases where f(x) and g(x) are polynomial functions.~~	**AI-A.REI.11** Given the equations $y = f(x)$ and $y = g(x)$: i) **recognize** that each x-coordinate of the intersection(s) is the solution to the equation $f(x) = g(x)$; ii)find the solutions approximately using technology to graph the functions or make tables of values; and iii) **interpret the solution in context**. (Shared standard with Algebra II) **Notes: Algebra I tasks are limited to cases where $f(x)$ and $g(x)$ are linear, polynomial, absolute value, and exponential functions of the form** $f(x) = a(b)^x$ **where $a > 0$ and $b > 0$ ($b \neq 1$).** **Students should be taught to find the solutions approximately by using technology to graph the functions *and* by making tables of values. When solving any problem, students can choose either strategy.**

LEARNING OBJECTIVES

Students will be able to:

1) use technology to create tables and graphs to find solutions of systems of equations involving linear and non-linear functions.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ← Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

absolute value equation	linear equation	slope intercept form
exponential equation	piecewise equation	solution of a system
families of functions	quadratic equations	

BIG IDEAS

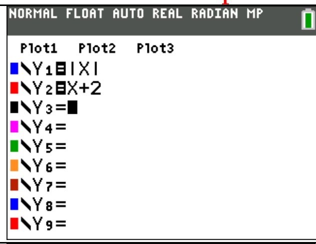
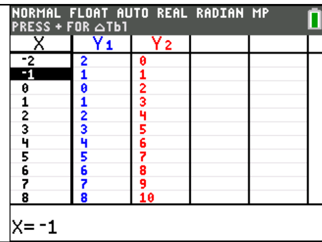
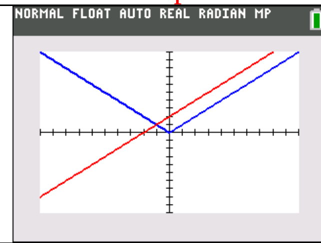
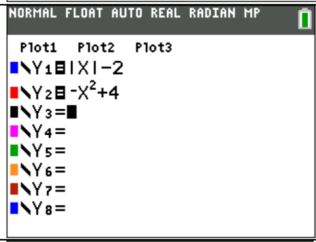
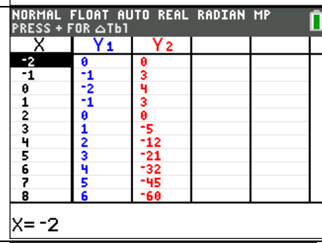
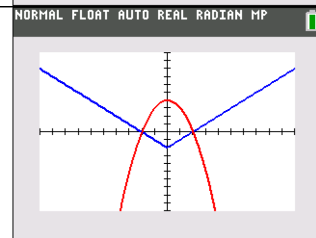
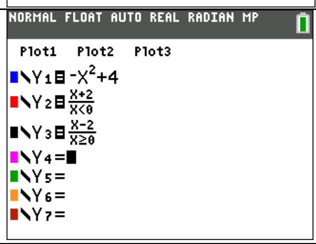
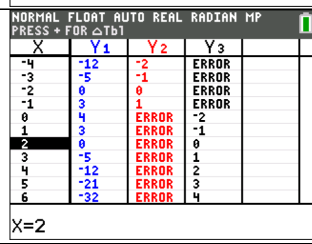
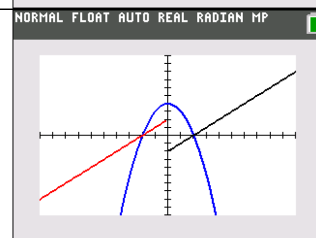
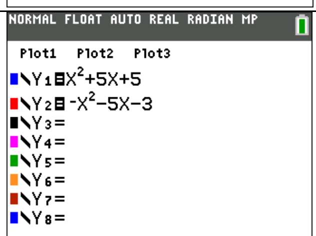
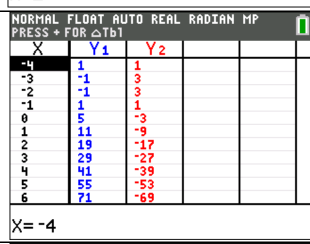
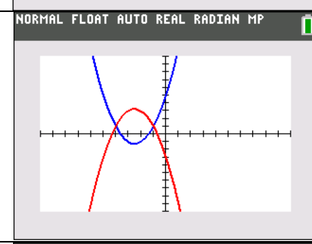
A **solution of a system** of equations makes each equation in the system true. This rule can be applied to systems involving different families of functions. Typically, graphing is the easiest way to solve systems of equations involving different families of functions. Algebraic solutions may also be used.

DEVELOPING ESSENTIAL SKILLS

Use technology to create a table of value and graph for each system, then state the solution(s) for each system.

1.	2.	3.	4.	5.
absolute linear $y=\lvert x\rvert$ $y=x+2$	absolute quadratic $y=\lvert x\rvert-2$ $y=-x^2+4$	quadratic piecewise $y=-x^2+4$ $y=\begin{cases} x+2 & x<0 \\ x-2 & x\geq 0 \end{cases}$	quadratic quadratic $y=x^2+5x+5$ $y=-x^2-5x-3$	absolute quadratic $y=-\lvert x\rvert+2$ $y=x^2$

Answers

<table>
<tr><th>Solutions</th><th>Calculator Input</th><th>Table</th><th>Graph</th></tr>
<tr>
<td>1.
$y=|x|$
$y=x+2$
(-1,1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.
$y=|x|-2$
$y=-x^2+4$
(-2,0) and (2,0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.
$y=-x^2+4$
$y=\begin{cases} x+2 & x<0 \\ x-2 & x\geq 0 \end{cases}$
(-2,0) and (2,0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.
$y=x^2+5x+5$
$y=-x^2-5x-3$
(-4,1) and (-1,1)</td>
<td></td>
<td></td>
<td></td>
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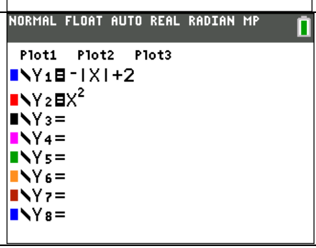
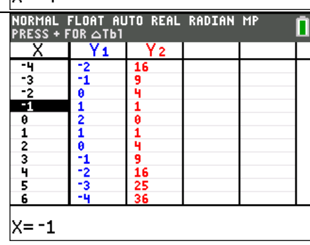
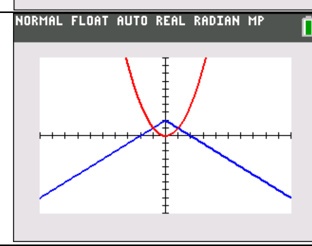
<td>5.
$y=-|x|+2$
$y=x^2$
(-1,1) and (1,1)</td>
<td></td>
<td></td>
<td></td>
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A.REI.D.11: Other Systems

294) Two functions, $y = |x-3|$ and $3x+3y=27$, are graphed on the same set of axes. Which statement is true about the solution to the system of equations?

1) $(3,0)$ is the solution to the system because it satisfies the equation $y = |x-3|$.
2) $(9,0)$ is the solution to the system because it satisfies the equation $3x+3y=27$.
3) $(6,3)$ is the solution to the system because it satisfies both equations.
4) $(3,0)$, $(9,0)$, and $(6,3)$ are the solutions to the system of equations because they all satisfy at least one of the equations.

295) On the set of axes below, graph

$$g(x) = \frac{1}{2}x + 1$$

and

$$f(x) = \begin{cases} 2x+1, & x \le -1 \\ 2 - x^2, & x > -1 \end{cases}$$

How many values of *x* satisfy the equation $f(x) = g(x)$? Explain your answer, using evidence from your graphs.

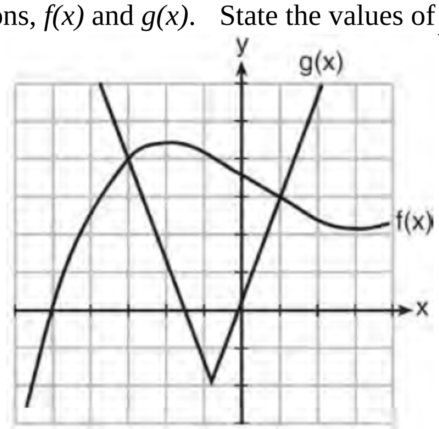
296) Given the functions $h(x) = \frac{1}{2}x + 3$ and $j(x) = |x|$, which value of *x* makes $h(x) = j(x)$?

1) -2
2) 2
3) 3
4) -6

297) The graphs of the functions $f(x) = |x-3| + 1$ and $g(x) = 2x+1$ are drawn. Which statement about these functions is true?

1) The solution to $f(x) = g(x)$ is 3.
2) The solution to $f(x) = g(x)$ is 1.
3) The graphs intersect when $y = 1$.
4) The graphs intersect when $x = 3$.

298) The graph below shows two functions, *f(x)* and *g(x)*. State the values of *x* for which $f(x) = g(x)$.



299) Which value of *x* results in equal outputs for $j(x) = 3x - 2$ and $b(x) = |x+2|$?

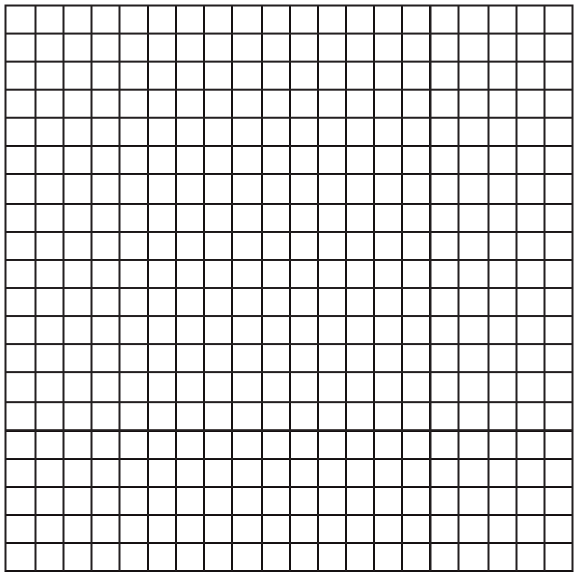
1) -2

2) 2

3) $\frac{2}{3}$

4) 4

300) Graph $f(x) = |x|$ and $g(x) = -x^2 + 6$ on the grid below. Does $f(-2) = g(-2)$? Use your graph to explain why or why not.



SOLUTIONS

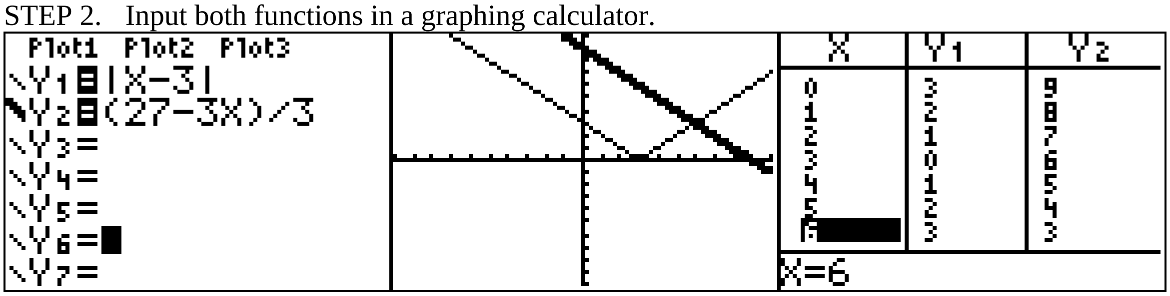
294) ANS: 3
Strategy: Input both functions in a graphing calculator, then use the table and graph views of the function to select the correct answer.
STEP 1. Transpose the second function for input into a graphing calculator.

$$3x + 3y = 27$$

$$3y = 27 - 3x$$

$$y = \frac{27 - 3x}{3}$$

STEP 2. Input both functions in a graphing calculator.



When $x = 6$, the value of y in both equations is 3. (6, 3) is the solution to this system.

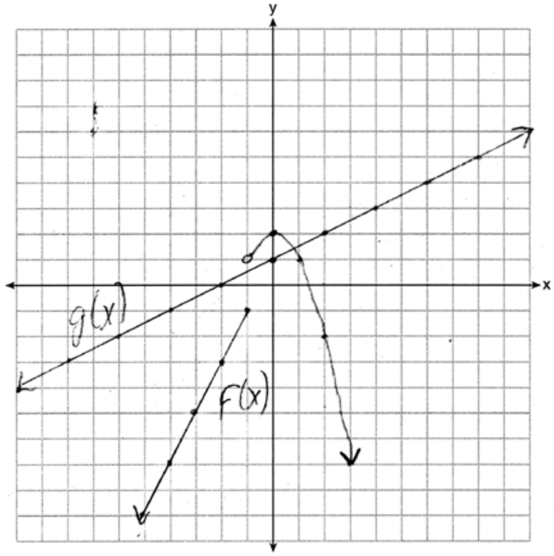
PTS: 2 NAT: A.REI.D.11 TOP: Nonlinear Systems

295) ANS:

Step 1. Plot $g(x) = \frac{1}{2}x + 1$

Step 2. Plot $f(x) = 2x + 1$ over the interval $x \le -1$

Step 3. Plot $f(x) = 2 - x^2$ over the interval $x > -1$

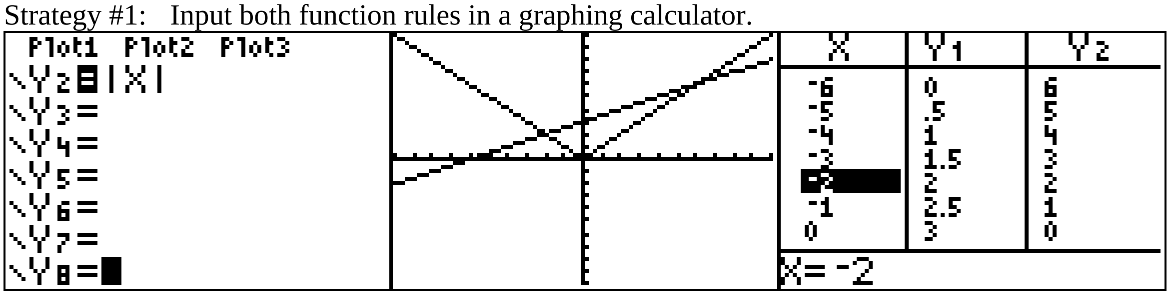


Only 1 value of x satisfies the equation $f(x) = g(x)$, because the graphs only intersect once.

PTS: 4 NAT: F.IF.C.7 TOP: Other Systems

296) ANS: 1

Strategy #1: Input both function rules in a graphing calculator.



Strategy #2: Set the right expressions of both functions equal to one another. Then solve for the positive and negative values of $|x|$.

$$\frac{1}{2}x + 3 = |x|$$

$\frac{1}{2}x + 3 = x$ $x + 6 = 2x$ $6 = x$	$-\left(\frac{1}{2}x = 3\right) = x$ $-\frac{1}{2}x - 3 = x$ $-x - 6 = 2x$ $-6 = 3x$ $-2 = x$

Check:

$h(x) = \frac{1}{2}x + 3$ $h(-2) = \frac{1}{2}(-2) + 3$ $h(-2) = -1 + 3$ $h(-2) = 2$	$j(x) = \lvert x \rvert$ $j(-2) = \lvert -2 \rvert$ $j(x) = 2$

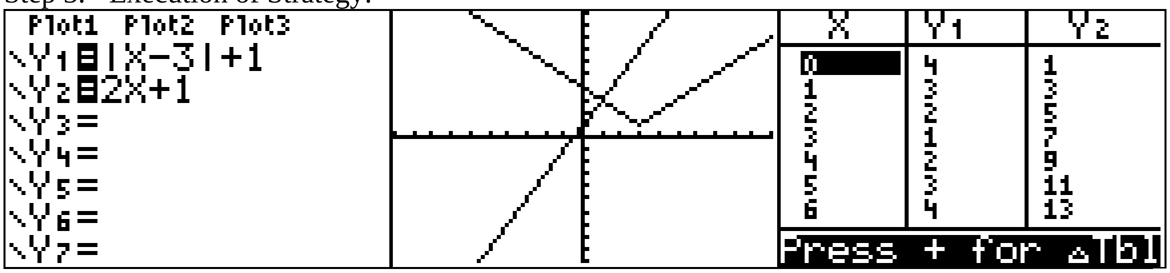
PTS: 2 NAT: A.REI.D.11 TOP: Other Systems

297) ANS: 2

Step 1. Understand that only of the answer choices is true.

Step 2. Strategy. Input both functions in a graphing calculator and explore the truth of each answer choice.

Step 3. Execution of Strategy.



The graph and table show that the solution for this system of equations is (1,3). This means that $f(1) = 3$ and $g(1) = 3$. Accordingly, when x is 1, $f(x) = g(x)$. The correct answer is choice b).

Step 4. Does is make sense? Yes. All of the other answer choices can be eliminated as wrong. The problem can be checked algebraically as follows:

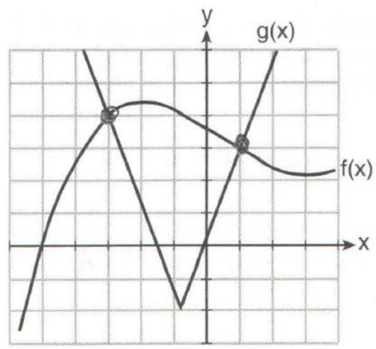
Given: $f(x) = \lvert x - 3 \rvert + 1$ and $g(x) = 2x + 1$, find $f(x) = g(x)$

$\lvert x - 3 \rvert + 1 = 2x + 1$ $\lvert x - 3 \rvert = 2x$ $x - 3 = 2x$ $-3 = x$ This is an extraneous solution. $\lvert -3 - 3 \rvert + 1 = 2(-3) + 1$ $\lvert -6 \rvert + 1 = -6 + 1$ $6 + 1 = -6 + 1$ $7 \neq -5$	$\lvert x - 3 \rvert + 1 = 2x + 1$ $\lvert x - 3 \rvert = 2x$ $-x + 3 = 2x$ $3 = 3x$ $1 = x$ This solution checks. $\lvert 1 - 3 \rvert + 1 = 2(1) + 1$ $\lvert -2 \rvert + 1 = 2 + 1$ $2 + 1 = 2 + 1$ $3 = 3$

PTS: 2 NAT: A.REI.D.11 TOP: Other Systems

298) ANS:

30 The graph below shows two functions, $f(x)$ and $g(x)$. State all the values of x for which $f(x) = g(x)$.



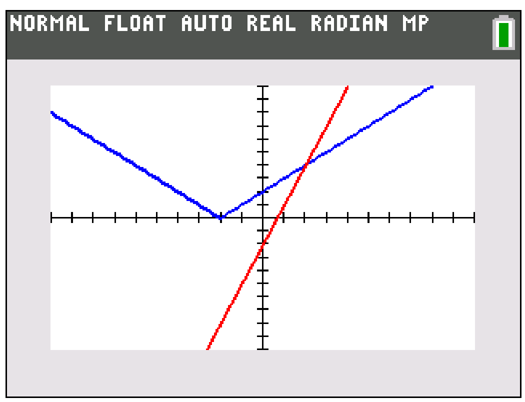
-3 and 1

PTS: 2 NAT: A.REI.D.11

299) ANS: 2

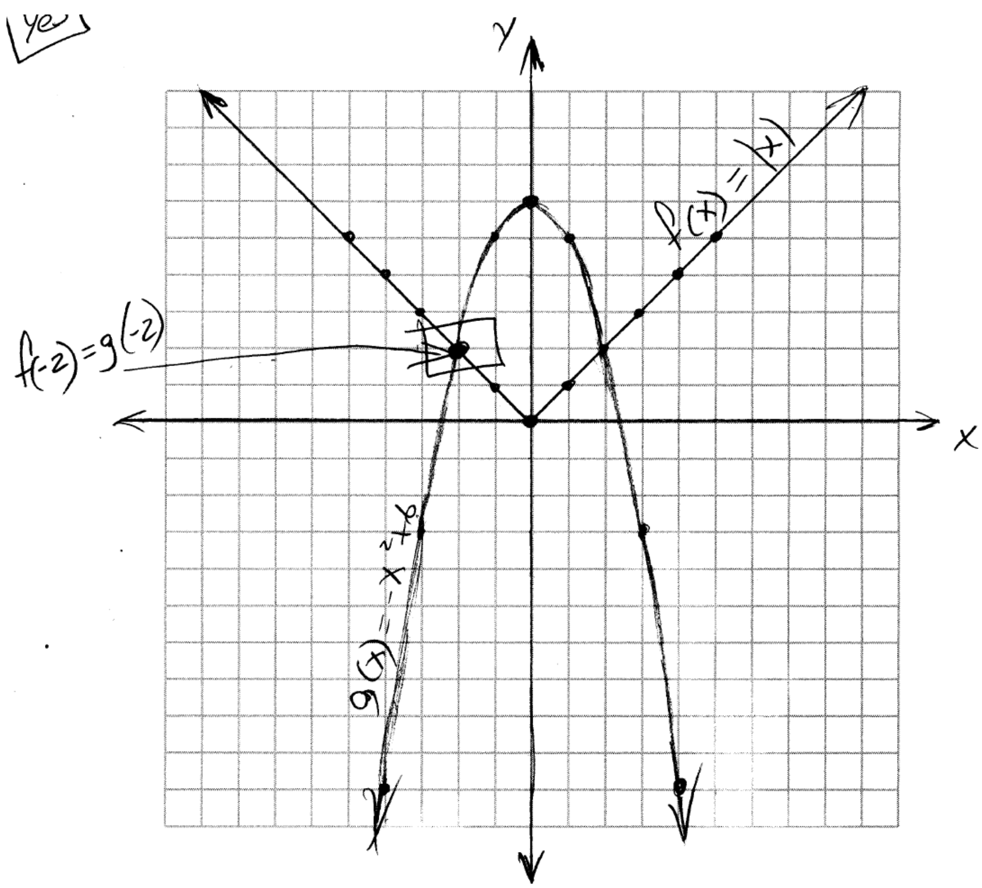
When $x = 2$, $f(x) = h(x)$.

$$f(x) = h(x)$$
$$3x - 2 = |x + 2|$$
$$3x - 2 = x + 2$$
$$2x = 4$$
$$x = 2$$
$$f(2) = 3(2) - 2$$
$$f(2) = 4$$
$$h(2) = |2 + 2|$$
$$h(2) = 4$$



PTS: 2 NAT: A.REI.D.11 TOP: Other Systems
KEY: AI

300) ANS:



Yes, because the graph of $f(x)$ intersects the graph of $g(x)$ at $x=-2$.

PTS: 4 NAT: A.REI.D.11 TOP: Other Systems
KEY: AI