E – Linear Equations, Lesson 3, Writing Linear Equations (r. 2018)

LINEAR EQUATIONS
Writing Linear Equations

Common Core Standard	**Next Generation Standard**
A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, ~~often forming a curve (which could be a line)~~.	**AI-A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. **Note: Graphing linear equations is a fluency recommendation for Algebra I. Students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity; as well as modeling linear phenomena.**

LEARNING OBJECTIVES

Students will be able to:

1) Determine if different equations represent the same mathematical relationship between two variables.
2) Write the equation of a line of a line given two points on the line or one point and the slope of the line.

Overview of Lesson

Teacher Centered Introduction	**Student Centered Activities**
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice 🡨Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

transform	equivalent	relationship
isolate	y = mx +b form	

BIG IDEAS

Three Facts About Graphs and Their Equations

1. The graph of an equation represents the set of all points that satisfy the equation (make the equation balance).

2. Each and every point on the graph of an equation represents a coordinate pair that can be substituted into the equation to make the equation true.

3. If a point is on the graph of the equation, the point is a solution to the equation.

Equivalent Forms of Equations

An equation represents a mathematical relationship between variables. The same relationship between the variables can be represented in many different ways. For example, y = 2x, 2y = 4x, and 3y = 6x all represent the same idea that y is 2 times x.

To determine if different equations represent the same mathematical relationship between variables, use one or more of the following strategies.

- transform the different equations into equivalent forms. If the equations can be transformed into identical forms, the equations represent the same mathematical relationship between the variables.
- isolate the same variable in all the equations and input the equations in a graphing calculator. If the tables of values and graphs are identical, the equations represent the same mathematical relationship between the variables.

Given Two Points on a Line, or One Point and the Slope of a Line, How to Write the Equation of the Line

STEP 1. First, find the slope. If not given, use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

STEP 2. Set up and label three columns, as follows:

Write what you are given in this column.	**y = mx + b**	**y = mx + b**
y = **m =** **x =** **b =**	*Substitute the values from the first column into the formula and solve for the unknown b value in this column.*	*Use this column to write the final equation by substituting m and b in the slope-intercept form.*

STEP 3. Complete each column, left to right. The last column will be the equation of the line.

Example:

Write the equation of the line that passes through (-5, 6) and (7, 2).

Step 1. Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2-6}{7-(-5)}$$

$$m = \frac{-4}{12}$$

$$m = -\frac{1}{3}$$

Write what you are given in this column.	**y = mx + b**	**y = mx + b**

y = 2 **m** = $-\frac{1}{3}$ **x** = 7 **b** = b	$2=-\frac{1}{3}(7)+b$ $2=-\frac{7}{3}+b$ $2+\frac{7}{3}=b$ $4\frac{1}{3}=b$	$y=\frac{1}{3}x+4\frac{1}{3}$

DEVELOPING ESSENTIAL SKILLS

Which of the following equations represent the same mathematical relationship between the variables? Justify your answer.

$$y=3x+6$$

$$y=3(x+2)$$

$$y-4=3x+2$$

$$\frac{1}{3}y=x+2$$

All of the equations represent the same mathematical relationship.

$$y=3x+6$$

Use distributive property $\quad y=3(x+2)$

$$\boxed{y=3x+6}$$

Add 4 to both expressions $\quad y-4=3x+2$

$$\boxed{y=3x+6}$$

Multiply both expressions by 3 $\quad \frac{1}{3}y=x+2$

$$\boxed{y=3x+6}$$

Write the equation of the line that passes through the points (-2, -8) and (6, 16).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{16 - (-8)}{6 - (-2)}$$
$$m = \frac{24}{8}$$
$$m = 3$$

Write what you are given in this column.	**y = mx + b**	**y = mx + b**
y = 16 **m** = 3 **x** = 6 **b** = b	$16 = 3(6) + b$ $16 = 18 + b$ $-2 = b$	$y = 3x - 2$

REGENTS EXAM QUESTIONS (through June 2018)

A.REI.D.10: Writing Linear Equations

135) The graph of a linear equation contains the points $(3, 11)$ and $(-2, 1)$. Which point also lies on the graph?

1) $(2, 1)$
2) $(2, 4)$
3) $(2, 6)$
4) $(2, 9)$

136) Sue and Kathy were doing their algebra homework. They were asked to write the equation of the line that passes through the points $(-3, 4)$ and $(6, 1)$. Sue wrote $y - 4 = -\frac{1}{3}(x + 3)$ and Kathy wrote $y = -\frac{1}{3}x + 3$. Justify why both students are correct.

137) How many of the equations listed below represent the line passing through the points $(2, 3)$ and $(4, -7)$?

$$5x + y = 13$$
$$y + 7 = -5(x - 4)$$
$$y = -5x + 13$$
$$y - 7 = 5(x - 4)$$

1) 1
2) 2
3) 3
4) 4

SOLUTIONS

135) ANS: 4
Strategy: Find the slope of the line between the two points, then use $y - mx + b$ to find the y-intercept, then write the equation of the line and determine which answer choice is also on the line.

STEP 1. Find the slope of the line that passes through the points $(3, 11)$ and $(-2, 1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 11}{-2 - 3} = \frac{-10}{-5} = 2$$

Write $y = 2x + b$

STEP 2. Use either given point and the equation $y = 2x + b$ to solve for *b*, the y-intercept. The following calculation uses the point (3,11).

$$y = 2x + b$$

$$11 = 2(3) + b$$

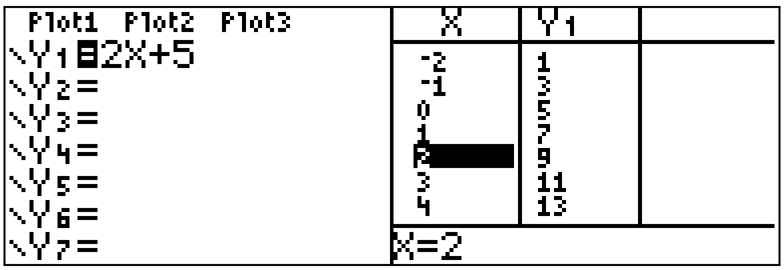
$$11 = 6 + b$$

$$5 = b$$

Write $y = 2x + 5$

STEP 3 Determine which answer choice balances the equation $y = 2x + 5$.

Use a graphing calculator



or simply solve the equation $y = 2x + 5$ for *y* when $x = 2$.

$$y = 2x + 5$$

$$y = 2(2) + 5$$

$$y = 4 + 5$$

$$y = 9$$

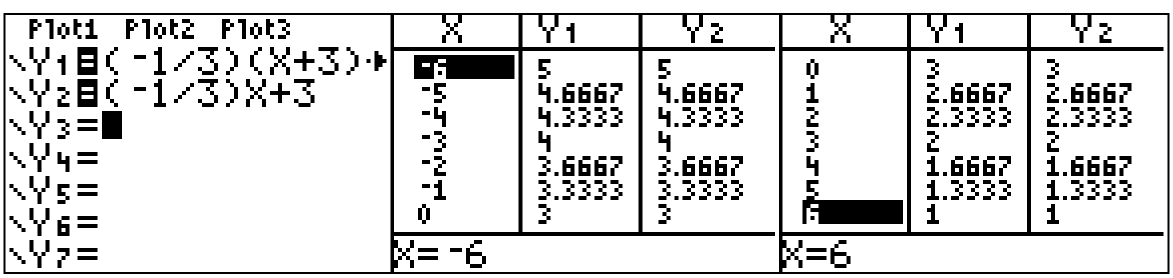
The point (2, 9) is also on the line.

PTS: 2 NAT: A.REI.D.10 TOP: Graphing Linear Functions

136) ANS:

Strategy: Input both equations in a graphing calculator and see if they produce the same outputs.

Sue's Equation y_1	Kathy's Equation y_2
$y_1 - 4 = -\frac{1}{3}(x+3)$ $y_1 = -\frac{1}{3}(x+3) + 4$	$y_2 = -\frac{1}{3}x + 3$



Both students are correct because both equations pass through the points $(-3, 4)$ and $(6, 1)$.

Alternate justification: Show that the points $(-3, 4)$ and $(6, 1)$ satisfy both equations.

Sue's Equation y_1		Kathy's Equation y_2	
$y-4=-\frac{1}{3}(x+3)$		$y=-\frac{1}{3}x+3$	
$(-3,4)$	$(6,1)$	$(-3,4)$	$(6,1)$
$4-4=-\frac{1}{3}(-3+3)$	$1-4=-\frac{1}{3}(6+3)$	$y=-\frac{1}{3}x+3$	$y=-\frac{1}{3}x+3$
$0=-\frac{1}{3}(0)$	$-3=-\frac{1}{3}(9)$	$4=-\frac{1}{3}(-3)+3$	$1=-\frac{1}{3}(6)+3$
$0=0$	$-3=-3$	$4=1+3$	$1=-2+3$
		$4=4$	$1=1$

Both students are correct because the points $(-3,4)$ and $(6,1)$ satisfy both equations.

PTS: 2 NAT: A.REI.D.10 TOP: Writing Linear Equations
KEY: other forms

137) ANS: 3

Step 1. Transform each equation for input into a graphing calculator.

Original	Input in Calculator
$5x+y=13$	$y=13-5x$
$y+7=-5(x-4)$	$y=-5(x-4)-7$
$y=-5x+13$	$y=-5x+13$
$y-7=5(x-4)$	$y=5(x-4)+7$

Step 2. Input each equation in a graphing calculator and inspect the tables of values for the points $(2,3)$ and $(4,-7)$.