C – Expressions and Equations, Lesson 5, Transforming Formulas (r. 2018)

EXPRESSIONS AND EQUATIONS
Transforming Formulas

Common Core Standard	Next Generation Standard
A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V=IR to highlight resistance R.*	**AI-A.CED.4** Rewrite formulas to highlight a quantity of interest, using the same reasoning as in solving equations. e.g., Rearrange Ohm's law *V* = *IR* to highlight resistance *R*.

LEARNING OBJECTIVES

Students will be able to:

1) rewrite (transform) formulas to isolate specific variables.

Overview of Lesson

Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

formula transform transformation isolate

BIG IDEAS

Properties and operations can be used to transform **formulas** to isolate different variables in the same ways that equations are manipulated to isolate a variable.

Example: The **formula** $P = 2l + 2w$ can be used to find the perimeter of a rectangle. In English, $P = 2l + 2w$ translates as "The *perimeter equals two times the length plus two times the width*." In the **formula** $P = 2l + 2w$, the *P* variable is already isolated. You can isolate the *l* variable or the *w* variables, as follows. (*Note that the steps and operations are the same as with regular equations.*)

To isolate the *l* variable:	To isolate the *w* variable:
Start with the formula: $P = 2l + 2w$ Move the term 2w to the left expression. $P - 2w = 2l$ Divide both sides of the equation by 2.	Start with the formula: $P = 2l + 2w$ Move the term 2*l* to the left expression. $P - 2l = 2w$ Divide both sides of the equation by 2.

$\frac{P-2w}{2}=l$ You now have a formula for *l* in terms of *P* and *w*.	$\frac{P-2l}{2}=w$ You now have a formula for *l* in terms of *P* and *w*.

DEVELOPING ESSENTIAL SKILLS

Isolate each variable in the Volume formula for a rectangular prism $V = lwh$.

$$V = lwh$$
$$\frac{V}{wh} = l$$
$$\frac{V}{lh} = w$$
$$\frac{V}{lw} = h$$

Isolate each variable in the slope intercept formula of a line $y = mx + b$.

$$y = mx + b$$
$$\frac{y-b}{x} = m$$
$$\frac{y-b}{m} = x$$
$$y - mx = b$$

REGENTS EXAM QUESTIONS

A.CED.A.4: Transforming Formulas

69) The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. The radius, *r*, of the cone may be expressed as

1) $\sqrt{\frac{3V}{\pi h}}$
2) $\sqrt{\frac{V}{3\pi h}}$
3) $3\sqrt{\frac{V}{\pi h}}$
4) $\frac{1}{3}\sqrt{\frac{V}{\pi h}}$

70) The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$. Express b_1 in terms of *A*, *h*, and b_2. The area of a trapezoid is 60 square feet, its height is 6 ft, and one base is 12 ft. Find the number of feet in the other base.

71) The equation for the volume of a cylinder is $V = \pi r^2 h$. The positive value of *r*, in terms of *h* and *V*, is

1) $r = \sqrt{\frac{V}{\pi h}}$
3) $r = 2V\pi h$

2) $r = \sqrt{V\pi h}$

4) $r = \dfrac{V}{2\pi}$

72) The distance a free falling object has traveled can be modeled by the equation $d = \frac{1}{2}at^2$, where *a* is acceleration due to gravity and *t* is the amount of time the object has fallen. What is *t* in terms of *a* and *d*?

1) $t = \sqrt{\dfrac{da}{2}}$

3) $t = \left(\dfrac{da}{d}\right)^2$

2) $t = \sqrt{\dfrac{2d}{a}}$

4) $t = \left(\dfrac{2d}{a}\right)^2$

73) The volume of a large can of tuna fish can be calculated using the formula $V = \pi r^2 h$. Write an equation to find the radius, *r*, in terms of *V* and *h*. Determine the diameter, to the nearest inch, of a large can of tuna fish that has a volume of 66 cubic inches and a height of 3.3 inches.

74) Michael borrows money from his uncle, who is charging him simple interest using the formula $I = \mathrm{Pr}t$. To figure out what the interest rate, *r*, is, Michael rearranges the formula to find *r*. His new formula is *r* equals

1) $\dfrac{I - P}{t}$

3) $\dfrac{I}{Pt}$

2) $\dfrac{P - I}{t}$

4) $\dfrac{Pt}{I}$

75) The formula for the sum of the degree measures of the interior angles of a polygon is $S = 180(n - 2)$. Solve for *n*, the number of sides of the polygon, in terms of *S*.

76) Solve the equation below for *x* in terms of *a*.

$$4(ax + 3) - 3ax = 25 + 3a$$

77) Boyle's Law involves the pressure and volume of gas in a container. It can be represented by the formula $P_1V_1 = P_2V_2$. When the formula is solved for P_2, the result is

1) $P_1V_1V_2$

3) $\dfrac{P_1V_1}{V_2}$

2) $\dfrac{V_2}{P_1V_1}$

4) $\dfrac{P_1V_2}{V_1}$

78) The formula for blood flow rate is given by $F = \dfrac{p_1 - p_2}{r}$, where *F* is the flow rate, p_1 the initial pressure, p_2 the final pressure, and *r* the resistance created by blood vessel size. Which formula can *not* be derived from the given formula?

1) $p_1 = Fr + p_2$

3) $r = F(p_2 - p_1)$

2) $p_2 = p_1 - Fr$

4) $r = \dfrac{p_1 - p_2}{F}$

79) Using the formula for the volume of a cone, express *r* in terms of *V*, *h*, and π.

80) The formula $F_g = \frac{GM_1M_2}{r^2}$ calculates the gravitational force between two objects where *G* is the gravitational constant, M_1 is the mass of one object, M_2 is the mass of the other object, and *r* is the distance between them. Solve for the positive value of *r* in terms of F_g, *G*, M_1, and M_2.

81) Students were asked to write a formula for the length of a rectangle by using the formula for its perimeter, $p = 2\ell + 2w$. Three of their responses are shown below.

I. $\ell = \frac{1}{2}p - w$

II. $\ell = \frac{1}{2}(p - 2w)$

III. $\ell = \frac{p - 2w}{2}$

Which responses are correct?

1) I and II, only
2) II and III, only
3) I and III, only
4) I, II, and III

SOLUTIONS

69) ANS: 1

Strategy: Use the four column method.

Notes	Left Expression	Sign	Right Expression
Given	V	=	$\frac{1}{3}\pi r^2 h$
Multiply both expressions by 3	$3V$	=	$\pi r^2 h$
Divide both expressions by πh	$\frac{3V}{\pi h}$	=	$\frac{\pi r^2 h}{\pi h}$
Simplify	$\frac{3V}{\pi h}$	=	r^2
Take square root of both sides.	$\sqrt{\frac{3V}{\pi h}}$	=	r

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

70) ANS:

a) $b_1 = \frac{2A}{h} - b_2$

b) The other base is 8 feet.

Strategy: Use the four column method to isolate b_1 and create a new formula, then use it to find the length of the other base.

Notes	Left Expression	Sign	Right Expression
Given	A	=	$\frac{1}{2}h(b_1 + b_2)$
Multiply both expressions by 2	$2A$	=	$h(b_1 + b_2)$

Divide both expressions by *h*	$\frac{2A}{h}$	=	$\frac{h(b_1 + b_2)}{h}$
Simplify	$\frac{2A}{h}$	=	$b_1 + b_2$
Subtract b_2 from both expressions	$\frac{2A}{h} - b_2$	=	b_1

Substitute the values stated in the problem in the formula.

$$A = 60,\ h = 6,\ b_2 = 12$$

$$b_1 = \frac{2A}{h} - b_2$$

$$b_1 = \frac{2(60)}{6} - 12$$

$$b_1 = \frac{120}{6} - 12$$

$$b_1 = 20 - 12$$

$$b_1 = 8 \text{ feet}$$

PTS: 4 NAT: A.CED.A.4 TOP: Transforming Formulas

71) ANS: 1

Strategy: Use the four column method to isolate *r*.

Notes	Left Expression	Sign	Right Expression
Given	V	=	$\pi r^2 h$
Divide both expressions by πh	$\frac{V}{\pi h}$	=	$\frac{\pi r^2 h}{\pi h}$
Simplify	$\frac{V}{\pi h}$	=	r^2
Take square root of both expressions.	$\sqrt{\frac{V}{\pi h}}$	=	r

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

72) ANS: 2

Strategy: Use the four column method. Isolate *t*.

Notes	Left Expression	Sign	Right Expression
Given	d	=	$\frac{1}{2}at^2$
Multiply both expressions by 2	$2d$	=	at^2
Divide both expressions by *a*	$\frac{2d}{a}$	=	$\frac{at^2}{a}$
Simplify	$\frac{2d}{a}$	=	t^2

Take square root of both expressions	$\sqrt{\frac{2d}{a}}$	=	t

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

73) ANS:

a) $r = \sqrt{\frac{V}{\pi h}}$

b) 5 inches

Strategy: Use the four column method to isolate *r* and create a new formula, then use the new formula to answer the problem.

Notes	Left Expression	Sign	Right Expression
Given	V	=	$\pi r^2 h$
Divide both expressions by πh	$\frac{V}{\pi h}$	=	$\frac{\pi r^2 h}{\pi h}$
Simplify	$\frac{V}{\pi h}$	=	r^2
Take square root of both expressions.	$\sqrt{\frac{V}{\pi h}}$	=	r

Substitute the values from the problem into the new equation.

$$V = 66,\ h = 3.3$$

$$r = \sqrt{\frac{V}{\pi h}}$$

$$r = \sqrt{\frac{66}{\pi(3.3)}}$$

$$r = \sqrt{\frac{20}{\pi}}$$

$$r \approx \sqrt{6.4}$$

$$r \approx 2.52$$

If the radius is approximately 2.5 inches, the diameter is approximately 5 inches.

PTS: 4 NAT: A.CED.A.4 TOP: Transforming Formulas

74) ANS: 3

Strategy: Isolate r, as follows:

$$I = \mathrm{Pr}t$$

$$I = Pt(r)$$

$$\frac{I}{Pt} = r$$

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

75) ANS:

$$S = 180(n-2)$$
$$S = 180n - 360$$
$$S + 360 = 180n$$
$$\frac{S+360}{180} = n$$

or

$$\frac{S}{180} + 2 = n$$

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

76) ANS:

$x = \frac{13}{a} + 3$

$$4(ax+3) - 3ax = 25 + 3a$$
$$4ax + 12 - 3ax = 25 + 3a$$
$$ax + 12 = 25 + 3a$$
$$ax = 13 + 3a$$
$$ax - 3a = 13$$
$$a(x-3) = 13$$
$$x - 3 = \frac{13}{a}$$
$$x = \frac{13}{a} + 3$$

PTS: 2 NAT: A.CED.A.4

77) ANS: 3

Given	P_1V_1	=	P_2V_2
Divide by V_2	$\frac{P_1V_1}{V_2}$	=	$\frac{P_1\cancel{V_2}}{\cancel{V_2}}$
Simplify	$\frac{P_1V_1}{V_2}$	=	P_1

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

78) ANS: 3

$$F = \frac{p_1 - p_2}{r}$$
$$rF = p_1 - p_2$$
$$r = \frac{p_1 - p_2}{F}$$

If $r = \frac{p_1 - p_2}{F}$, then $r = F(p_2 - p_1)$ cannot be true.

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

79) ANS:

$$V = \frac{1}{3}\pi r^2 h.$$

$$3V = \pi r^2 h$$

$$\frac{3V}{\pi h} = \frac{\pi r^2 h}{\pi h}$$

$$\frac{3V}{\pi h} = r^2$$

$$\sqrt{\frac{3V}{\pi h}} = r$$

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

80) ANS:

$$F_g = \frac{GM_1M_2}{r^2}$$

$$r^2F_g = GM_1M_2$$

$$r^2 = \frac{GM_1M_2}{F_g}$$

$$r = \sqrt{\frac{GM_1M_2}{F_g}}$$

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas

81) ANS: 4

Strategy: Transform the formula to isolate the l variable.

$$p = 2\ell + 2w$$

$$p - 2w = 2l$$

$$\frac{p - 2w}{2} = l$$

This is solution III.

NOTE that solution III can also be expressed as:

$$\frac{1}{2}(p - 2w) = l$$

This is solution II.

NOTE also that the distributive property of multiplication can transform solution II into:

$$\frac{1}{2}p - w = l$$

This is solution I.

The correct answer choice is I, II, and III.

PTS: 2 NAT: A.CED.A.4 TOP: Transforming Formulas