The University of the State of New York
The State Education Department

EXAMINATION IN EXPERIMENTAL TWELFTH YEAR MATHEMATICS

June 1961

Name of pupil..... Name of school.....

Part I

Answer twenty-five of the thirty questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answer on the line at the right.

Questions 1-11: Write the number preceding the expression that best completes the statement or answers the question.

1 "I will go to the game only if it doesn't rain." Let p stand for "I will go to the game" and let q stand for "it doesn't rain." Symbolically, the above statement can be expressed as

(1) $p \longrightarrow q$
(2) $q \longrightarrow p$
(3) $p \longrightarrow \sim q$
(4) $\sim q \longrightarrow p$
(5) $\sim p \longrightarrow q$

1.....

2 Which of the following is a tautology?

(1) $\sim(p \wedge q) \longleftrightarrow (\sim p \wedge \sim q)$
(2) $(p \longrightarrow q) \longleftrightarrow (\sim p \wedge q)$
(3) $(p \vee q) \longrightarrow (p \wedge q)$
(4) $(p \vee q) \wedge \sim p \longleftrightarrow (q \wedge \sim p)$
(5) $(p \longrightarrow q) \longleftrightarrow (\sim p \longrightarrow \sim q)$

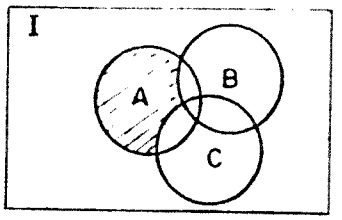
2.....

3 The negation of $\forall_x (p \vee \sim q)$ is represented by

(1) $\forall_x (p \wedge \sim q)$
(2) $\forall_x (\sim p \wedge q)$
(3) $\exists_x (p \wedge \sim q)$
(4) $\exists_x (\sim p \wedge q)$
(5) $\exists_x (\sim p \vee q)$

3.....

4 In the Venn diagram A, B and C are the interiors of the circles lying within the rectangle I. The shaded area is represented by



(1) A'

(2) $A \cup (B \cap C)$

(3) $A' \cap (B \cup C)$

(4) $(B \cap C)'$

(5) $A \cap (B \cup C)'$

4.....

5 $[(A \cup B') \cap B]' =$

(1) $A' \cup B'$
(2) $A \cap B$
(3) $A \cup B$
(4) $A' \cap B'$
(5) A

5.....

6 If the functions f and g are defined respectively as $f(x) = x^2 + 1$ and $g(x) = 2x$, then the composite function $f[g(x)]$ is

(1) $x^2 + 2x + 1$
(2) $2x^2 + 1$
(3) $2x^3 + 2x$
(4) $4x^2 + 1$
(5) $2x^2 + 2$

6.....

7 A function f is defined by $\{(x, y)\}$ where x and y are real numbers. Which of the following statements is true regarding f^{-1}, the inverse of the function f?

(1) The graph of f^{-1} is always symmetric with respect to the x-axis, to the graph of f.
(2) The graph of f^{-1} is always symmetric with respect to the y-axis, to the graph of f.
(3) The graph of f^{-1} is always symmetric with respect to the line $y = x$, to the graph of f.
(4) f^{-1} is always a function.
(5) The elements of the inverse of f are $(x, \frac{1}{y})$.

7.....

8 If the function f is defined as $f(x) = \frac{2x^2 - 2}{x - 1}$, which statement concerning the the function f is correct?

(1) The function is well defined for all values of x.
(2) The derivative of the function exists when $x = 1$.
(3) The range of the function is the set of all real numbers except zero.
(4) The function approaches the limit 4 as $x \longrightarrow 1$.
(5) The function is undefined for all values of x less than 1.

8.....

9 The intersection of the sets of points described by the conditions $xy = 8$ and $x^2 + y^2 = 8$ is

(1) a set consisting of one point
(2) a set of two points
(3) a set of three points
(4) a set of four points
(5) the empty set

9.....

10 The set of points in space 4 inches from a given plane and 5 inches from a given point in the given plane is

(1) one circle
(2) two circles
(3) a set of two points
(4) a set of four points
(5) the empty set

10.....

11 At a relative minimum point on a graph of a polynomial function, which statement concerning the tangent to the curve at that point is correct?

(1) The slope of the tangent is increasing.
(2) The slope of the tangent is decreasing.
(3) The slope of the tangent is undefined.
(4) The tangent at a relative minimum point is perpendicular to the tangent at a relative maximum point.
(5) None of these

11.....

Questions 12-30: Write the answer on the line at the right.

12 Using the ordered pair (x, y) as an equivalent expression for x + yi, write as an ordered pair the product of (2, -1) and (-3, 2). 12.....

13 Write $[\sqrt{2}(\cos 30^\circ + i \sin 30^\circ)]^2$ in the form a + bi. 13.....

14 What is the remainder when $x^{14} - 2x^7 + 1$ is divided by x + 1? 14.....

15 Write an equation of lowest degree, with real coefficients, if two of its roots are -1 and 1 + i. 15.....

16 Find the solution set in the real number system if $x^2 + x - 6 \leq 0$. 16.....

17 Find $\log_2 0.25$. 17.....

18 The elements p, q, r and s form a commutative group with respect to the operation $\star$. Write an expression equivalent to $p \star r$. 18.....

19 In a throw of a pair of cubical dice, one red and one green, r is the number showing uppermost on the red die and g is the number showing uppermost on the green die. If A = $\{r \mid r \leq 4\}$ and B = $\{g \mid g > 4\}$, find the probability of (A U B). 19.....

20 What is the probability of getting exactly three "heads" in four tosses of a fair coin? 20.....

21 Write the coordinates of the point of inflection of the graph of the function $y = -x^3 - 3x^2 + x + 1$. 21.....

22 Write an equation of the tangent to the graph of the equation $y = x^2 - 2x + 1$ at the point where x = 2. 22.....

23 A frustum of a regular square pyramid has respective base edges of 3 and 9. If the distance between the bases is 4, find the area of the lateral surface of the frustum. 23.....

24 A pyramid is cut by a plane parallel to its base and bisecting its altitude. If the volume of the upper portion cut from the pyramid is 2, what is the volume of the lower portion? 24.....

25 A cylinder, whose height is 8 inches, has a radius changing at the rate of $\frac{1}{\pi}$ inches per minute. At how many cubic inches per minute is the volume changing when the radius is 2 inches? 25.....

26 The distance between point A(1,2,-1) and point B(x,-2,4) is $5\sqrt{2}$. Find a value of x. 26.....

27 Write an equation of the plane passing through the points (4,0,0) and (0,-2,0) and perpendicular to the xy-plane. 27.....

28 Express 0.15252...., where 52 repeats itself indefinitely, as a number in the form $\frac{a}{b}$ where a, b are integers. 28.....

29 Find the smallest positive integer which satisfies the congruence $3x + 4 \equiv 5$, mod 7. 29.....

30 The set of elements $\{e, p, q, r, s, t\}$ is closed under the operation $\bullet$ as shown in the following multiplication table, where the first factor selects the row and the second factor selects the column. Use this table to find y if $q \bullet (y \bullet p) = s$.

$\bullet$	e	p	q	r	s	t
e	e	p	q	r	s	t
p	p	q	e	s	t	r
q	q	e	p	t	r	s
r	r	t	s	e	q	p
s	s	r	t	p	e	q
t	t	s	r	q	p	e

30.....

Part II

Answer <u>five</u> questions from this part.

31 Find to the <u>nearest tenth</u> the smallest positive root of the equation $x^3 - 3x^2 + 3 = 0$. [10]

32 If the domain of x is the set of complex numbers, find the solution set of $3x^4 + 2x^3 + 2x^2 + 2x - 1 = 0$. [10]

33 The perimeter of a rectangular sheet of paper is to be 40 inches. The printed area must leave a margin of $1\frac{1}{2}$ inches on the bottom and 1 inch on the other three sides. Find the dimensions of the paper in inches if the printed area is to be maximum. [10]

34 The vertices of a rectangular pyramid are A(0,8,0), B(6,8,0), C(6,0,0), D(0,0,0) and P(6,8,4).

<u>a</u> Find the volume of the pyramid. [4]

<u>b</u> Find to the <u>nearest integer</u> the total area of the pyramid. [6]

35 a Graph the solution set of $(x^2 + 4y^2 < 16) \wedge (2y \geq x - 2) \wedge (x \geq 0)$. Indicate carefully what sections of the boundary belong to the solution set. [8]

b From the graph made in answer to part a, find the minimum value of y which satisfies the conditions in part a. [2]

36 Using the definition for the derivative of a function, find the derivative of the function $f(x) = ax^2 + bx + c$, where a, b and c are constants. [10]

37 a Find the largest possible real domain that will yield a real range in each of the following functions:

(1) $y = \sqrt{2 - x}$ [2]

(2) $y = \frac{x}{x^2 - 1}$ [2]

b Find the largest real range of the function $y = 1 - \frac{1}{x}$. [2]

c Write the inverse of the function f as defined by $f(x) = \sqrt{x - 1}$. [2]

d Answer yes or no to each of the following: [2]

(1) The inverse of a trigonometric function is always a function.
(2) A graph of a polynomial function is always continuous.

38 Prove by analytic geometry that the diagonals of a rhombus are perpendicular bisectors of each other. [10]

FOR TEACHERS ONLY

INSTRUCTIONS FOR RATING

EXAMINATION IN EXPERIMENTAL TWELFTH YEAR MATHEMATICS

June 1961

Part I

Allow 50 credits, 2 credits for each of 25 of the following:

(1) 1
(2) 4
(3) 4
(4) 5
(5) 1
(6) 4
(7) 3
(8) 4
(9) 5
(10) 2
(11) 1
(12) $(-4,7)$
(13) $1 + \sqrt{3}\,i$
(14) 4
(15) $x^3 - x^2 + 2 = 0$
(16) $\{x \mid -3 \leq x \leq 2\}$
(17) -2
(18) $r * p$
(19) $\frac{7}{9}$
(20) $\frac{1}{4}$
(21) $(-1,-2)$
(22) $y = 2x - 3$
(23) 120
(24) 14
(25) 32
(26) 4 or -2 (accept either)
(27) $\frac{x}{4} - \frac{y}{2} = 1$
(28) $\frac{151}{990}$
(29) 5
(30) r

Part II

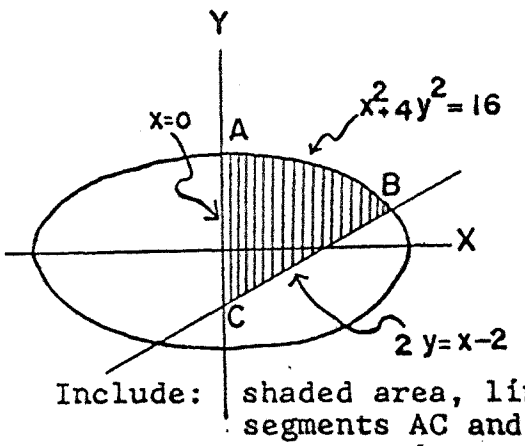
(31) 1.3 [10]

(32) $\{-1, \frac{1}{3}, i, -i\}$ [10]

(33) 10.25×9.75 [10]

(34) a 64 [4]
b 132 [6]

(35) a [8]



Include: shaded area, line segments AC and CB except points A and B.
Exclude: points on ellipse.

b -1 [2]

(37) a (1) $\{x \mid x \leq 2, x \text{ a real number}\}$ [2]
(2) D_x: all real numbers except +1 and −1. [2]

b R_y: all real numbers except +1 [2]

c $f^{-1} = \{(x,y) \mid y = x^2 + 1, x \geq 0\}$ [2]

d (1) no [1]
(2) yes [1]