The University of the State of New York
The State Education Department

EXAMINATION IN EXPERIMENTAL TWELFTH YEAR MATHEMATICS

June 1960

Name of pupil..... Name of school.....

Part I

Answer 25 questions from this part. Each correct answer will receive two credits. No partial credit will be allowed. Write your answer on the line at the right.

Directions (1-13): Indicate the correct completions by writing on the line at the right the number 1, 2, 3, 4 or 5.

1 Asserting that the conjunction of two propositions, p and q, is false (i.e., "$p \wedge q$" is false) is the same as asserting that

(1) both p and q must be false
(2) p or q is false, but both cannot be false
(3) p or q is false, or both may be false
(4) $(\sim p \wedge \sim q)$ is true
(5) $(\sim p \vee \sim q)$ is false 1.....

2 Which of the following is not a tautology?

(1) $p \vee \sim p$
(2) $[(p \vee q) \wedge \sim p] \longrightarrow q$
(3) $[(p \longrightarrow q) \wedge \sim q] \longrightarrow \sim p$
(4) $(p \wedge q) \longrightarrow (p \vee q)$
(5) $(p \longrightarrow q) \longrightarrow (q \longrightarrow \sim p)$ 2.....

3 Asserting: It is false that $\exists_x (p \longrightarrow q)$ is the same as asserting that

(1) $\exists_x \sim(p \longrightarrow q)$

(2) $\forall_x \sim(p \longrightarrow q)$

(3) $\forall_x (p \longrightarrow \sim q)$

(4) $\forall_x (\sim q \longrightarrow \sim p)$

(5) $\forall_x (p \vee \sim q)$ 3.....

4 A man promised his son, "I will give you your allowance only if you will mow the lawn." Let p stand for "I will give you your allowance" and let q stand for "You will mow the lawn." The promise may be expressed symbolically as

(1) $p \longrightarrow q$
(2) $q \longrightarrow p$
(3) $\sim q \longrightarrow p$
(4) $q \longrightarrow \sim p$
(5) $\sim p \longrightarrow q$

4.....

5 A counterexample to a universally quantified statement has the effect of proving

(1) that the statement is false
(2) that the statement may be true
(3) that the contrapositive of the statement is true
(4) that the negative of the statement is false
(5) nothing about the truth value of the statement

5.....

6 The set of non-negative integers fails to form a group under multiplication because the system

(1) is not closed under multiplication
(2) is not associative under multiplication
(3) is not commutative under multiplication
(4) does not have an identity element
(5) includes some elements that do not have inverses

6.....

7 If $f = \{(1,2), (2,3), (3,4), (5,6)\}$, which statement is true?

(1) The domain of $f = \{1,2,3,4,5\}$.
(2) The range of $f = \{2,3,4,6\}$.
(3) $f(2) = 1$.
(4) $f(f(2)) = 2$.
(5) None of these is true.

7.....

8 If f is the function defined by $f(x) = \frac{x^2 + x - 6}{x - 2}$ and the domain of x is the set of real numbers, which statement is true?

(1) $f(2) = 5$.
(2) The derivative of f at $x = 2$ is 1.
(3) The limit of f as $x \longrightarrow 2$ does not exist.
(4) The range of f is the set of real numbers.
(5) None of these is true.

8.....

THE UNIVERSITY OF THE STATE OF NEW YORK
The State Education Department
Division of Educational Testing
Albany

To: Senior High School Building Principals:

ERRATUM in Rating Key

Sample Examination in 12th Year Experimental Course in Mathematics

The answer given to number 34 in the "Instructions for Rating Examination in Experimental Twelfth Year Mathematics June 1960" is incorrect.

The correct answer is as follows:

(34) 11-12x (found by delta process)

Division of Educational Testing

9 If $f(x) = |x|$ and the domain of x is the set of real numbers, then

(1) f has no derivative at $x = 0$
(2) the derivative of f at $x = 0$ is 0
(3) f is not continuous at $x = 0$
(4) f is not defined at $x = 0$
(5) none of these is true

9.....

10 $(A \cup B)' =$

(1) $A' \cup B'$
(2) $A' \cup B$
(3) $A \cup B'$
(4) $A \cap B$
(5) $A' \cap B'$

10.....

11 The shaded area in the adjacent Venn diagram represents

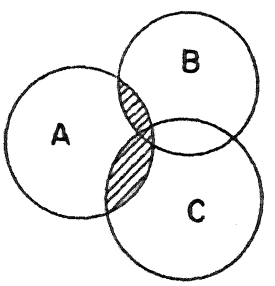
(1) $A \cup (B \cap C)$

(2) $A \cap (B \cup C)$

(3) $(A \cap B) \cap C$

(4) $A \cup (B \cup C)$

(5) $(A \cup B) \cap C$



11.....

12 Let $f = \{(1,2), (2,2), (3,0)\}$

$g = \{(1,3), (2,3), (0,1)\}$

Then it follows that

(1) $f \cup g$ is a function
(2) the domain of $(f + g) = \{1,2,3,0\}$
(3) the range of $(f + g) = \{2,3\}$
(4) the range of $(f + g) = \{5\}$
(5) none of these is true

12.....

13 Let $f = \{(1,2), (3,4), (5,6), (7,8)\}$. Then the inverse function, f^{-1}, is

(1) $\{(7,8), (5,6), (3,4), (1,2)\}$
(2) $\{(1,-2), (3,-4), (5,-6), (7,-8)\}$
(3) $\{(2,1), (4,3), (6,5), (8,7)\}$
(4) $\{(1,\frac{1}{2}), (3,\frac{1}{4}), (5,\frac{1}{6}), (7,\frac{1}{8})\}$
(5) nonexistent

13.....

Answer questions 14 and 15 on the basis of the table below, which is the multiplication table for a group of elements ESTUVW, under the operation "$\circledast$". [Note that the first factor selects the row and the second factor selects the column.]

$\circledast$	E	S	T	U	V	W
E	E	S	T	U	V	W
S	S	T	E	V	W	U
T	T	E	S	W	U	V
U	U	W	V	E	T	S
V	V	U	W	S	E	T
W	W	V	U	T	S	E

14 What element is the inverse of (W $\circledast$ V)? 14.....

15 Find x if (S $\circledast$ x) $\circledast$ T = U. 15.....

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16 Find the smallest positive integer satisfying the congruence $x^2 \equiv 2$, mod 7. 16.....

17 The integers 1, 2, 3, 4, 5, 6 form a group under ordinary multiplication modulo 7. Find the multiplicative inverse of (3 × 4). 17.....

18 Find the equation of the tangent to the graph of $y = 2 - 3x + 5x^2$ at the point where the graph cuts the y-axis. The domain of x is the set of real numbers. 18.....

19 Find the coordinates of the relative minimum of the function $f(x) = 2x^3 - 6x + 5$. The domain of x is the set of real numbers. 19.....

20 If $y = -\frac{2x^3}{3} + x^2$, find the value of $\frac{d^2y}{dx^2}$ when $x = -\frac{1}{2}$. 20.....

21 If each edge of a cube of length x inches is increasing at the rate of 3 inches per minute, how fast is the total surface area changing at the instant that $x = 2$ inches? 21.....

22 If the domain of x and y is the set of integers, find x if $x + ix + y - iy = 1 + 3i$. 22.....

23 Two roots of the equation $x^3 + px + q = 0$ are -1 and 3. Find the third root. 23.....

24 Find the constant remainder when $4x^9 - 5x^8 + 7$ is divided by $x - 1$. 24.....

25 If $(.3)^x = .2$, find x to the nearest tenth. 25.....

26 The edges of a rectangular solid are 1, 2, 3 inches. Find the sine of the smallest angle formed by a main diagonal and an edge of the solid. [The answer may be left in radical form.] 26.....

27 The larger of two similar right circular cones has its dimensions three times those of the smaller. The volume of the larger cone is how many times the volume of the smaller? 27.....

28 The line segment joining A(1, -4, 3) to B(5, 6, -1) is extended its own length through B to D. The coordinates of D are .... 28.....

29 Three distinct vertices are chosen at random from a regular hexagon. What is the probability that the vertices chosen are the vertices of an equilateral triangle? 29.....

30 If q and r are two members of the group of permutations of order six, where $q = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$ and $r = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ and "$\circ$" is the operation "then" or "followed by ", find $q \circ r$. 30.....

Part II

Answer five questions from this part.

31 Find to the nearest tenth the real root of the equation $x^3 + 2x - 15 = 0$. [10]

32 If the domain of x is the set of complex numbers, find the solution set of $2x^3 - x^2 + 2x - 1 = 0$. [10]

33 Prove by mathematical induction that $(a + 1)$ is a factor of $a^n - 1$ if n is a positive even integer and $a \neq -1$. [10]

34 Using the definition of derivative, find the derivative of the function defined by $f(x) = (3x + 2)(5 - 2x)$. [10]

35 The dimensions of a closed box are x, 2x, y feet. If the box is to have a volume of 72 cubic feet, find the dimensions that will make the total surface area a minimum. [10]

36 The vertices of a pyramid are located by the coordinate (3,0,0), (0,3,0), (0,0,4) and (0,0,0).

a Find the volume of the pyramid. [4]
b Find to the nearest integer the total area of the pyramid. [6]

37 Graph the solution set of $[y \geq x^2] \wedge [y < x + 2]$. Indicate carefully what sections of the boundary belong to the solution set. [10]

38 Find the largest subsets of the real numbers which will serve as the domain and range, respectively, for the functions defined by <u>each</u> of the following:

<u>a</u> $y = x^2$ [1,1]

<u>b</u> $y = \sqrt{x}$ [1,1]

<u>c</u> $y = \frac{1}{x(x + 1)}$ [1,1]

<u>d</u> $y = \sqrt{9 - x^2}$ [1,1]

<u>e</u> $y = |\sin x|$ [1,1]