The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Tuesday ~~Monday~~, August ~~20~~ 21, 1962 — 12:30 to 3:30 p.m., only

Name of pupil..... Name of school.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 If sin A is negative and cos A is positive, what is the number of the quadrant in which angle A terminates? 1.....

2 Express cos 220° as a function of a positive acute angle. 2.....

3 Express in degrees an angle of $\frac{7\pi}{6}$ radians. 3.....

4 Express $\csc^2 A$ in terms of cos A. 4.....

5 In triangle ABC, $a = 7$, $b = 3$ and $\sin A = \frac{1}{3}$. Find the value of sin B. 5.....

6 A point $P(x, y)$ is on the terminal side of an angle θ which is in standard position. Express sec θ in terms of x and y. 6.....

7 Express cot $(540° - A)$ in terms of a function of A. 7.....

8 A pendulum of length 8.4 inches swings through an arc of 18°. Find to the *nearest tenth of an inch* the length of the arc described by the end point of the pendulum. [Use the approximation $\pi = \frac{22}{7}$.] 8.....

9 Find the numerical value of sin 49° 23′. 9.....

10 Find the numerical value of log tan 74° 45′. 10.....

[OVER]

11 The ratio of the width of a rectangle to its length is 3 : 4. Determine to the *nearest minute* the angle which a diagonal makes with the longer side. 11.....

12 Find the numerical value of tan (arc tan 3 + arc tan 2). 12.....

13 Compute the area of a triangle ABC for which $a = 2$, $b = 5$ and $C = 30°$. 13.....

14 Determine the angles between 0° and 180° which are solutions of $\tan x \cos x + \sqrt{3} \cos x = 0$. 14.....

15 What is the maximum value of cos x for values of x between $-\frac{\pi}{4}$ and $\frac{\pi}{4}$ radians? 15.....

16 What is the period of the function defined by $y = \tan x$? 16.....

17 Find the *smallest* positive value of x which satisfies the equation: $2 \sin 45° \cos 45° = \sin (45° + x)$ 17.....

18 Express sin 140° — sin 60° as a product of two functions. 18.....

19 Express $\log \frac{a^3}{\sqrt{b}}$ in terms of log a and log b. 19.....

20 Express $\frac{\sin x - \cos x}{\sin x + \cos x}$ in terms of cot x. 20.....

21 In triangle ABC, $a = 4$, $b = 5$ and $c = 6$. Find the value of cos A. 21.....

22 If cos $A = \frac{2}{3}$ and A is a positive acute angle, what is the value of cos $\frac{1}{2}A$? 22.....

Directions (23–26): Indicate the correct completion for *each* of the following by writing on the line at the right the number 1, 2, 3 or 4.

23 As x increases from 0 to π radians, $\sin x$
(1) increases throughout the interval
(2) decreases throughout the interval
(3) increases, then decreases
(4) decreases, then increases
23.....

24 The following data are given for triangle ABC: $a = 10$, $b = 15$ and $B = 95°$. Using these data, it is possible to find for the length of side c
(1) one and only one value
(2) two and only two distinct values
(3) three and only three distinct values
(4) no real value
24.....

25 If $A = \arccos \frac{2}{3}$, the number of values of A between 0° and 720° is
(1) 1
(2) 2
(3) 3
(4) 4
25.....

26 If $y = \tan 6x$, then y is equal to
(1) $3 \tan 2x$
(2) $\tan (-6x)$
(3) $\dfrac{\sin 6x}{\cos (-6x)}$
(4) $\cot \dfrac{1}{6x}$
26.....

Directions (27–30): Indicate whether *each* of the following statements is true for
(1) all real values of A
(2) some but not all real values of A
(3) no real values of A
by writing on the line at the right the number 1, 2 or 3.

27 $3 \sec A = \frac{1}{2}$
27.....

28 $\sin^2 2A + \cos^2 2A = 1$
28.....

29 $\cos^2 \frac{1}{2} A - \sin^2 \frac{1}{2} A = \cos A$
29.....

30 $\sin A \neq \csc A$
30.....

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Solve the following equation for positive values of x less than $360°$: $4\cos^2 x - 5\cos x - 6 = 0$ [Determine x to the *nearest minute*.] [10]

32 *a* *Sketch* the graph of $y = 3\cos 2x$ as x varies from $-\pi$ to π radians. [6]

b On the same set of axes used in part *a*, *draw* the graph of $y = \frac{3}{2}$. [2]

c From the graphs made in answer to parts *a* and *b*, determine the number of values of x between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ radians, which satisfy the equation $\cos 2x = \frac{1}{2}$. [2]

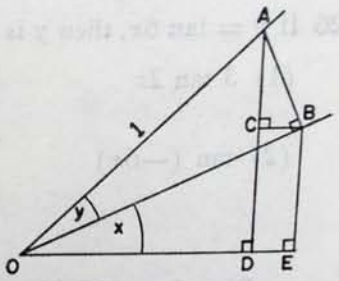
33 *a* Prove that the following equality is an identity: $\tan 2x = \dfrac{2}{\cot x - \tan x}$ [5]

b Show without the use of trigonometric tables that $\sin^2 75° = \dfrac{2+\sqrt{3}}{4}$. [5]

34 The sides of a triangle are 156, 250 and 324. Find to the *nearest degree* the largest angle of the triangle. [10]

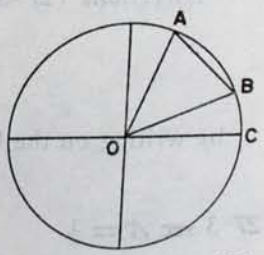
35 Answer either *a* or *b*, but *not* both:

a Using the accompanying figure in which x, y and $x + y$ are positive acute angles, *derive* the formula for $\sin(x + y)$. [10]



OR

b In the accompanying figure O is the origin and OC lies along the x-axis. The radius of circle O is 1, angle $AOC = p$, angle $BOC = q$.



(1) Show that the coordinates of A are $(\cos p, \sin p)$. [1]

(2) Find the coordinates of B. [1]

(3) Using the coordinates of A and B and the distance formula, express the square of the length AB in terms of trigonometric functions of p and q. [2]

(4) Using the law of cosines, express the square of the length AB in terms of $\cos(p - q)$. [2]

(5) Using the results obtained in (3) and (4), show that $\cos(p - q) = \cos p \cos q + \sin p \sin q$. [4]

36 A signal tower has two lights 60 feet apart, one directly above the other. From a boat the angle of elevation of the lower light is 14° and that of the upper light is 28°. Find to the *nearest foot* the distance from the boat to the tower. [10]

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING

TRIGONOMETRY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 23–26, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 4

(2) $-\cos 40°$ *or* $-\sin 50°$

(3) 210

(4) $\dfrac{1}{1-\cos^2 A}$

(5) $\frac{1}{7}$

(6) $\pm\dfrac{\sqrt{x^2+y^2}}{x}$

(7) $-\cot A$

(8) 2.6

(9) 0.7591

(10) 0.5644

(11) 36° 52′

(12) −1

(13) 2.5

(14) 90° and 120°

(15) 1

(16) π *or* 180°

(17) 45°

(18) 2 cos 100° sin 40°

(19) $3 \log a - \frac{1}{2} \log b$

(20) $\dfrac{1-\cot x}{1+\cot x}$

(21) $\frac{3}{4}$

(22) $\sqrt{\dfrac{5}{6}}$

(23) 3

(24) 1

(25) 4

(26) 3

(27) 3

(28) 1

(29) 1

(30) 2

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) 138° 35′, 221° 25′ [10]

(32) *c* 2 [2]

(34) 103 [10]

(35) *b* (2) $(\cos q, \sin q)$ [1]
(3) $(\cos p - \cos q)^2 + (\sin p - \sin q)^2$ [2]
(4) $2 - 2\cos(p - q)$ [2]

(36) 212 [10]