The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Monday, August 21, 1961 — 3:30 to 6:30 p.m., only

Name of pupil.....Name of school.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 If x is a positive acute angle and $\tan x = \frac{3}{\sqrt{7}}$, find the value of $\sin x$. 1.....

2 Find the number of degrees in an angle of $\frac{3\pi}{20}$ radians. 2.....

3 Express sec 130° as a function of a positive acute angle. 3.....

4 Find in degrees the smallest positive value of x that satisfies the equation $\tan \frac{1}{2}x = -\sqrt{3}$. 4.....

5 Express $\tan (180° - \theta)$ in terms of a function of θ. 5.....

6 Express $\cot^2 x$ in terms of $\sin x$. 6.....

7 The value of the sine function of a positive acute angle A is 0.5672. Find angle A to the *nearest minute*. 7.....

8 Find the number whose logarithm is 9.6583 — 10. 8.....

9 Find the value of k if $k = \log \cos 38° 17'$. 9.....

10 A central angle of 72° is drawn in a circle of radius 10 inches. Find the number of inches in the length of the arc which is intercepted by this central angle. 10.....

[OVER]

11 Write the solution of the equation $4 \sin \theta - 1 = 0$ in inverse trigonometric form. 11.....

12 If $\log N = k$, express $\log \sqrt{N^3}$ in terms of k. 12.....

13 If $\sin(-x)$ is negative, give the numbers of the two quadrants in which x may terminate. 13.....

14 Express $\cos 4x \cos 3x + \sin 4x \sin 3x$ as a single trigonometric function. 14.....

15 If A is a positive acute angle, for what value of A is the expression $\frac{1 + \tan A}{1 - \tan A}$ undefined? 15.....

16 Find the numerical value of $\sin\left(\frac{\pi}{2} + 2\pi\right)$. 16.....

17 If θ is a positive acute angle and cosine $\theta = \frac{3}{5}$, find the value of $\sin \frac{1}{2}\theta$. 17.....

18 In triangle ABC, if $a = 5$, $b = 3$ and angle $C = 60°$, find side c. 18.....

19 Find the positive value of $\sin(2 \arcsin \frac{1}{2})$. 19.....

Directions (20–30): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement.

20 In triangle ABC, $a = 2b$. Therefore,

(1) $\sin A = \sin 2B$
(2) $\sin 2A = \sin B$
(3) $\sin A = 2 \sin B$
(4) $\sin B = 2 \sin A$

20.....

21 The expression $\frac{1}{2}(\cos 5x + \cos 3x)$ is equivalent to

(1) $\cos 4x \cos x$
(2) $-\cos 4x \cos x$
(3) $\sin 4x \sin x$
(4) $-\sin 4x \sin x$

21.....

22 As x increases from 0 to 2π radians, a function which increases in all quadrants is

(1) $\sin x$
(2) $\cos x$
(3) $\sec x$
(4) $\tan x$

22.....

23 A triangle ABC is *not* uniquely determined if the following parts are given:

(1) $a = 3, b = 4, c = 5$
(2) $a = 4, b = 6, A = 30°$
(3) $a = 6, b = 4, A = 130°$
(4) $a = b = 10, C = 50°$

23.....

24 The minimum value of $3 + \sin 3x$ is

(1) 0
(2) 2
(3) 3
(4) 4

24.....

25 The statement $5^x = 25$ is equivalent to

(1) $2 = \log_x 25$
(2) $2 = \log_{25} 5$
(3) $5 = \log_{25} 2$
(4) $5 = \log_x 25$

25.....

26 The area of triangle ABC is equal to

(1) $ab \sin C$
(2) $ab \cos C$
(3) $\frac{1}{2} ab \sin C$
(4) $\frac{1}{2} ab \cos C$

26.....

27 The value of $\cos 135° + \sin 135°$ is

(1) $\sqrt{2}$
(2) 0
(3) $-\sqrt{2}$
(4) -1

27.....

28 If $\sin x = \sin 10°$, then x may equal

(1) 190°
(2) $-10°$
(3) $-350°$
(4) 80°

28.....

29 If $\cos 2\theta = 0$, then $\sin^2\theta$ is equal to

(1) 1
(2) $\frac{1}{2}$
(3) 0
(4) $\frac{1}{4}$

29.....

30 The expression $\sin (x + 30°)$ is equivalent to

(1) $\dfrac{\sqrt{3} \sin x + \cos x}{2}$
(2) $\dfrac{\sqrt{3} \sin x - \cos x}{2}$
(3) $\dfrac{\sin x + \sqrt{3} \cos x}{2}$
(4) $\dfrac{\sin x - \sqrt{3} \cos x}{2}$

30.....

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* Starting with a formula for tan $(x + y)$, *derive* a formula for tan $2x$ in terms of tan x. [4]

b Prove that $\tan 2x = \dfrac{2 \sin x \cos x}{\cos^2 x - \sin^2 x}$ is an identity. [4]

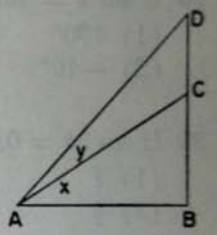
c By letting $x = 90°$ in the identity in part *b*, find tan 180°. [2]

32 Find *all* values of x between 0° and 360° which satisfy the equation $\cot^2 x \sin x - 3 \sin x - 3 = 0$. [Express approximate values of x to the *nearest degree.*] [10]

33 *a* On the same set of axes, *sketch* the graphs of $y = \cos 2x$ and $y = \frac{1}{2} \cos x$ as x varies from 0 to 2π radians. [Label each curve with its equation.] [4, 4]

b From the graphs made in answer to part *a*, determine the range of values of x between 0 and π radians for which $\cos 2x$ and $\frac{1}{2} \cos x$ are both negative. [2]

34 In the accompanying figure, DB is perpendicular to AB, angle BAC is represented by x and angle DAC is represented by y. Show that $CD = \dfrac{AB \sin y \sec x}{\cos (x + y)}$. [10]



35 In triangle ABC, $a = 25.0$ feet, $b = 16.2$ feet and $c = 34.2$ feet. Find angle C to the *nearest ten minutes.* [10]

36 A patrol boat left point A and traveled 27 miles on course 031 (N 31° E) to point B. It then traveled 18 miles on course 136 (S 44° E) to point C. Determine to the *nearest mile* the distance CA which the boat must travel to return to its starting point by the shortest route. [5, 5]

FOR TEACHERS ONLY

T

INSTRUCTIONS FOR RATING
TRIGONOMETRY

Monday, August 21, 1961 — 3:30 to 6:30 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 20–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $\frac{3}{4}$
(2) 27
(3) $-\sec 50°$ *or* $-\csc 40°$
(4) 240
(5) $-\tan\theta$
(6) $\dfrac{1-\sin^2 x}{\sin^2 x}$
(7) 34° 33′
(8) 0.4553
(9) 9.8948–10
(10) 4π *or* 12.6
(11) $\theta = \arcsin\frac{1}{4}$
(12) $\dfrac{5}{2}k$
(13) I and II
(14) $\cos x$
(15) 45°
(16) 1
(17) $\dfrac{1}{\sqrt{5}}$ *or* $\dfrac{\sqrt{5}}{5}$
(18) $\sqrt{19}$ *or* 4.4
(19) $\dfrac{\sqrt{3}}{2}$
(20) 3
(21) 1
(22) 4
(23) 2
(24) 2
(25) 1
(26) 3
(27) 2
(28) 3
(29) 2
(30) 1

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) *c* 0 [2]

(32) 14°, 166°, 270° [10]

(33) *b* $\dfrac{\pi}{2} < x < \dfrac{3\pi}{4}$ [2]

(35) 110° 20′ [10]

(36) Analysis [5]
28 [5]