

The University of the State of New York

325TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Wednesday, August 24, 1955 — 12 m. to 3 p.m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) names of schools where you have studied, (b) number of weeks and recitations a week in trigonometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1955 or number and length in minutes of lessons taken in the summer of 1955 under a tutor licensed in the subject and supervised by the principal of the school you last attended.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1955 or an equivalent program of tutoring approved in advance by the Department is required.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from this part. Show all work unless otherwise directed.

21 a Starting with the formulas for  $\sin(x - y)$  and  $\cos(x - y)$ , derive the formula for  $\tan(x - y)$  in terms of  $\tan x$  and  $\tan y$ . [5]

b Prove the identity  $\tan A = \frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A}$ . [5]

22 Find to the nearest degree all values of  $x$  greater than  $0^\circ$  but less than  $180^\circ$  that satisfy the equation  $\tan x - 3 \cot x = 2$ . [10]

23 a Sketch the graph of  $y = \frac{1}{2}x$  as  $x$  varies from 0 to  $2\pi$  radians. [4]

b On the same set of axes used in part a, sketch the graph of  $y = \frac{1}{2} \cos x$  as  $x$  varies from 0 to  $2\pi$  radians. [4]

c From the graphs made in answer to a and b, determine, as  $x$  increases from 0 to  $2\pi$  radians, the values of  $x$  between which the function  $y = \frac{1}{2} \cos x$  increases while the function  $y = \sin \frac{1}{2} x$  decreases. [2]

24 Two ships  $d$  distance apart are on opposite sides of a target. The ships and the target are on the same horizontal line. From an airplane directly above the target, the angles of depression of the ships are  $x$  and  $y$ , with  $y$  greater than  $x$ . Show that the distance  $m$  between the target and the ship nearer to it is given by the formula  $m = \frac{d \sin x \cos y}{\sin(x + y)}$ . [10]

[1]

[OVER]

## TRIGONOMETRY

### Part III

Answer at least two questions from this part. Show all work.

25 In triangle  $ABC$ ,  $a = 93.2$ ,  $b = 74.7$ ,  $B = 36^\circ 20'$  and  $A$  is an acute angle. Find  $A$  to the nearest ten minutes. [10]

26 The sides of triangle  $ABC$  are  $a = 379$ ,  $b = 287$  and  $c = 168$ . Find angle  $A$ . [Answer may be given to the nearest degree.] [10]

27 A ship sails from port  $P$  in the direction S  $45^\circ$  E for 17 miles to point  $A$ . A second ship sails from  $P$  in the direction N  $40^\circ$  E for 29 miles to point  $B$ . Find angle  $PAB$ . [Answer may be given to the nearest degree.] [3, 7]

28 A regular pentagon is inscribed in a circle whose radius is  $r$ .

*a* Show that the area  $A$  of the pentagon is equal to  $5r^2 \sin 36^\circ \cos 36^\circ$ . [5]

*b* If  $A = 3080$ , find  $r$  to the nearest integer. [5]

Be sure you have answered a total of five questions from parts II and III.

TRIGONOMETRY

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed.

- 1 Express in degrees an angle of  $\frac{4\pi}{3}$  radians. 1.....
- 2 Express  $\sin(-200^\circ)$  as a function of a positive acute angle. 2.....
- 3 Find the smallest positive value of  $\cos^{-1}(\frac{1}{2})$ . 3.....
- 4 If  $\cos A \csc A = 1$ , find the smallest positive value of  $A$ . 4.....
- 5 If  $\sec A = \frac{3}{\sqrt{5}}$  and  $A$  is acute, find  $\cot A$ . [Answer may be left in radical form.] 5.....
- 6 In triangle  $ABC$ ,  $a = 3$ ,  $b = 5$  and  $c = 7$ . Find  $\cos C$ . 6.....
- 7 In triangle  $ABC$ ,  $a = 24$ ,  $\sin A = \frac{3}{5}$  and  $\sin B = \frac{3}{5}$ . Find  $b$ . 7.....
- 8 In triangle  $ABC$ ,  $a = 12$ ,  $b = 4$  and  $C = 100^\circ$ . Find  $\tan \frac{1}{2}(A - B)$  to the *nearest hundredth*. 8.....
- 9 Two sides of a triangle are 10 and 20 and the angle between these sides is  $23^\circ$ . Find the area of the triangle to the *nearest integer*. 9.....
- 10 From a point 1000 feet from the base of a radio tower in a level plain the angle of elevation of the top of the tower is  $8^\circ 40'$ . Find the height of the tower to the *nearest foot*. 10.....
- 11 If  $\log N = 9.4688 - 10$ , find  $N$ . 11.....
- 12 Find  $\cos 19^\circ 18'$ . 12.....
- 13 Find  $\log \tan 36^\circ 43'$ . 13.....
- 14 In a circle whose radius is 4, find the length of the arc intercepted by a central angle of  $\frac{1}{3}$  radians. 14.....
- 15 Express  $\sin(A + B)$  in terms of functions of  $A$  and  $B$ . 15.....
- 16 If  $\cos x = k$ , express  $\cos^2 \frac{x}{2}$  in terms of  $k$ . 16.....

TRIGONOMETRY

*Directions (17-20):* Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

- 17  $\cos (270^\circ - A)$  is equal to (a)  $\cos A$  (b)  $\sin A$  (c)  $-\sin A$  17.....
- 18 The maximum value of  $3 \sin 2x$  is (a) 1 (b) 3 (c) 6 18.....
- 19  $\cos 80^\circ - \cos 60^\circ$  is equal to (a)  $\cos 20^\circ$  (b)  $-2 \cos 70^\circ \cos 10^\circ$   
(c)  $-2 \sin 70^\circ \sin 10^\circ$  19.....
- 20 The expression  $\frac{2 \cos A}{\sin 2A}$  can be reduced to (a)  $\cot A$  (b)  $\sec A$   
(c)  $\csc A$  20.....

# FOR TEACHERS ONLY

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## INSTRUCTIONS FOR RATING TRIGONOMETRY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

### Part I

Allow  $2\frac{1}{2}$  credits for each correct answer; allow no partial credit. For questions 17–20, allow credit if the pupil has written the correct expression instead of the letter  $a$ ,  $b$  or  $c$ .

- |  |                                      |
|--|--------------------------------------|
| (1) 240                                | (11) 0.2943                          |
| (2) $\sin 20^\circ$ or $\cos 70^\circ$ | (12) 0.9438                          |
| (3) $60^\circ$                         | (13) $9.8726 - 10$                   |
| (4) $45^\circ$                         | (14) $\frac{4}{5}$                   |
| (5) $\frac{\sqrt{5}}{2}$               | (15) $\sin A \cos B + \cos A \sin B$ |
| (6) $-\frac{1}{2}$                     | (16) $\frac{1+k}{2}$                 |
| (7) 15                                 | (17) $c$                             |
| (8) .42                                | (18) $b$                             |
| (9) 39                                 | (19) $c$                             |
| (10) 152                               | (20) $c$                             |