The University of the State of New York

322ND HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Wednesday, August 25, 1954 — 12 m. to 3 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to Parts II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in trigonometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1954 or number and length in minutes of lessons taken in the summer of 1954 under a tutor licensed in the subject and supervised by the principal of the school you last attended.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1954 or an equivalent program of tutoring approved in advance by the Department is required.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II. All work should be shown in 21, 22b and 24.

21 *Derive* the formula for $\cos(x + y)$ for the case in which x, y and $(x + y)$ are positive acute angles. [10]

22 *a* Prove the following identity: [4] $\dfrac{\tan A}{\cot A} + \dfrac{\cot A}{\tan A} = \dfrac{\tan^4 A + 1}{\tan^2 A}$

b Find to the *nearest degree* the value of x between $0°$ and $90°$ that satisfies the equation $3\cos^2 x + 2\sin x - 3 = 0$. [6]

23 *a* Sketch the graph of $y = 2\cos\theta$ as θ varies from $-\dfrac{\pi}{2}$ to $+\dfrac{\pi}{2}$ radians. [4]

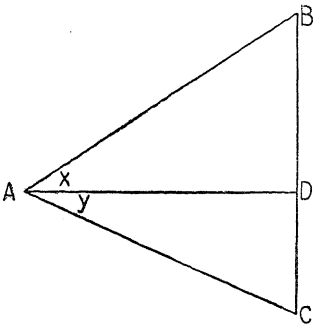
b On the same set of axes used in *a*, sketch the graph of $y = \sin 2\theta$ as θ varies from $-\dfrac{\pi}{2}$ to $+\dfrac{\pi}{2}$ radians. [5]

c From the graphs made in answer to *a* and *b*, find *one* value of θ that satisfies the following equation: [1]

$$2\cos\theta - \sin 2\theta = 2$$

24 In triangle ABC, AD is perpendicular to BC, and angles DAB and DAC are represented by x and y respectively. Prove that BC is given by the following formula: [10]

$$BC = \frac{AD \sin (x + y)}{\cos x \cos y}$$



Part III

Answer at least two questions from part III. All work, including computation, should be shown.

25 A pilot in an airplane observes that the angle of depression of a light on the ground directly below his line of flight is 30°. *One minute later* the angle of depression of the same light is 45°. If the plane is traveling horizontally and in a straight course at the rate of 150 miles per hour, find to the *nearest tenth of a mile* the altitude at which the plane is flying. [5, 5]

26 In triangle ABC, angle $A = 30° 40'$, side $AB = 150.0$, side $BC = 79.5$ and angle C is obtuse. Find angle B to the *nearest ten minutes*. [3, 7]

27 Two boats start at the same time from the same point. One sails due north at 12 knots and the other due northeast (N 45° E) at 8 knots. Find to the *nearest ten minutes* the bearing of the slower boat from the faster at the end of one hour. [4, 6]

28 A plot of ground has the form of a triangle whose sides are 50 rods, 70 rods and 80 rods in length.

a Find to the *nearest degree* the smallest angle of the triangle. [5]

b Find to the *nearest acre* the area of the plot. [One acre = 160 sq. rd.] [5]

Be sure you have answered a total of five questions from parts II and III.

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Express in degrees an angle of $\frac{\pi}{12}$ radians. 1.....

2 Express cot 265° as a function of a positive acute angle. 2.....

3 Find the smallest positive value of $\tan^{-1}\frac{1}{\sqrt{3}}$. 3.....

4 If θ is an acute angle and $\sin\theta = \frac{y}{r}$, express sec θ in terms of y and r. 4.....

5 If $\tan A = x$, express $\tan 2A$ in terms of x. 5.....

6 Find the smallest positive value of x that satisfies the equation $2\tan^2 x - \tan x - 1 = 0$. 6.....

7 In which quadrant do both the sine and the cosine decrease as the angle increases? 7.....

8 Find to the *nearest minute* the positive acute angle whose sine is 0.6634. 8.....

9 Find n if $\log n = 9.4087 - 10$ 9.....

10 Find log cos 51° 4′ 10.....

11 One angle of a parallelogram is 32° and the shorter side is 40.0. Find to the *nearest tenth* the altitude on the longer side. 11.....

12 In triangle ABC, $a = 5$, $b = 3$ and $\sin A = \frac{5}{6}$. Find $\sin B$. 12.....

13 Two angles of a triangle are A and B, and the sides opposite these angles are a and b respectively. If $A + B = 90°$, express $\tan\frac{1}{2}(A - B)$ in terms of a and b. 13.....

[OVER]

14 Is the following an identity? [Answer *yes* or *no.*]

$$2 \sin^2 \frac{A}{2} = 1 - \cos A$$ 14.....

15 Is the following an identity? [Answer *yes* or *no.*]

$$\cos (-A) \sin (-A) = \cos A \sin A$$ 15.....

16 Express $\cos 3A - \cos A$ as a product. 16.....

Directions (17–20): For *each* of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false.*

17 If a, b and c are the sides of a triangle and if $\frac{a^2 + b^2 - c^2}{2ab}$ is negative, then the angle opposite side c is obtuse. 17.....

18 If x is a positive acute angle,

$$\log \sin 2x = \log 2 + \log \sin x + \log \cos x.$$ 18.....

19 Any trigonometric function of $(180° \pm \theta)$ is numerically equal to the same function of θ. [Quantities are *numerically equal* if their absolute values are equal.] 19.....

20 If the value of the function of $a \sin \theta$ is 2 when $\theta = \frac{\pi}{2}$, then the maximum value of $a \sin \theta$ is 2. 20.....

FOR TEACHERS ONLY

INSTRUCTIONS FOR RATING
TRIGONOMETRY

Wednesday, August 25, 1954 — 12 m. to 3 p. m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit.

(1) 15
(2) tan 5° *or* cot 85°
(3) 30°
(4) $\frac{r}{\sqrt{r^2 - y^2}}$
(5) $\frac{2x}{1 - x^2}$
(6) 45°
(7) second
(8) 41° 34′
(9) .2563
(10) 9.7983 — 10
(11) 21.2
(12) $\frac{1}{2}$
(13) $\frac{a - b}{a + b}$
(14) yes
(15) no
(16) $-2 \sin 2A \sin A$
(17) true
(18) true
(19) true
(20) true