The University of the State of New York

319TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, August 20, 1953 — 12 m. to 3 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to Parts II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in trigonometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1953 or number and length in minutes of lessons taken in the summer of 1953 under a tutor licensed in the subject and supervised by the principal of the school you last attended.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1953 or an equivalent program of tutoring approved in advance by the Department is required.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 *a* Starting with the formulas for $\sin(x-y)$ and $\cos(x-y)$, derive the formula for $\tan(x-y)$. [4]

b Prove the identity: $\dfrac{1+\sin 2A}{\cos 2A} = \dfrac{\cos A + \sin A}{\cos A - \sin A}$ [6]

22 Find all values of x greater than 0° but less than 360° which satisfy the equation $3\cos x - 2\sin^2 x = 0$. [10]

23 *a* Sketch the graph of $y = \cos 2x$ as x varies from 0 to π radians. [4]

b On the set of axes used in *a*, sketch the graph of $y = 2\sin x$ as x varies from 0 to π radians. [4]

c From the graphs made in answer to *a* and *b*, determine the number of values of x between 0 and π radians that satisfy the equation $\cos 2x = 2\sin x$. [2]

24 The distance between two airplanes flying at the same altitude is d, and both planes are due west of an airport. From the planes the angles of depression to the airport are x and y, with x greater than y. Show that a, the altitude of the planes, is given by the following formula: [10]

$$a = \frac{d\sin x \sin y}{\sin(x-y)}$$

Part III

Answer at least two questions from part III.

25 In triangle ABC, $a = 4110$ feet, $A = 36° 50'$ and $C = 73° 30'$. Find b to the *nearest foot.* [10]

26 The distance between the two points A and B can not be measured directly but is known to be about 2 miles. From point C the distance to A is 8310 feet, and the distance from C to B is 6210 feet. Angle $CAB = 33° 50'$. Find to the *nearest ten feet* the distance from A to B. [5, 5]

27 An airplane leaves air base P on a course N 42° 20′ W and flies 42 miles to air base Q. Another airplane flies 120 miles directly west from air base P to point R. Find to the *nearest degree* the bearing of Q from R. [5, 5]

28 Two forces of 127 pounds and 244 pounds have a resultant force of 359 pounds. Find to the *nearest degree* the angle formed by the lines of action of the resultant and the smaller force. [5, 5]

Fill in the following lines:

Name of pupil..... Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 In triangle ABC, $C = 90°$ and $\tan B = \frac{\sqrt{40}}{3}$. Find $\sec A$.

[Answer may be left in radical form.] 1.....

2 If $\cos A = \frac{1}{\csc A}$, find the smallest positive value of A. 2.....

3 Find the value of $\cos(-240°)$. 3.....

4 In a circle whose radius is 5 feet, a central angle intercepts an arc of 7 feet. Find the number of radians in the angle. 4.....

5 Express in degrees an angle of $\frac{5\pi}{12}$ radians. 5.....

6 Find the logarithm of 0.003427 6.....

7 Find the number whose logarithm is 0.8316 7.....

8 Find to the *nearest minute* the positive acute angle whose cosine is 0.7630. 8.....

9 Find $\log \sin 64° 24'$ 9.....

10 If A is a positive acute angle, express $\cos A$ in terms of $\tan A$. 10.....

11 If $\tan x = 2$, find $\tan 2x$. 11.....

12 Express $\cos 40° + \cos 30°$ as the product of two functions. 12.....

13 In triangle ABC, $a = 3$, $b = 5$ and $c = 6$. Find the cosine of the largest angle. 13.....

[OVER]

14 In triangle ABC, $a = 7$, $b = 5$ and $C = 100°$. Find to the *nearest hundreth* the value of $\tan \frac{1}{2}(A - B)$. 14.....

15 A pilot in a plane at an altitude of 2000 feet observes the angle of depression of an airport to be 13°. How far, to the *nearest thousand feet,* is the airport from the point on the ground directly below the plane? 15.....

16 If $A = \tan^{-1}(-1)$, find the smallest positive value of A. 16.....

Directions (17–20): Indicate the correct completion for each of the following by writing on the line at the right the letter *a, b* or *c*.

17 Using the data $A = 33°$, $b = 10$ and $a = 5.2$, it is possible to construct (*a*) no triangle (*b*) only one triangle (*c*) two triangles 17.....

18 $\text{Sin}^2 x = 2 \sin x$ is true for (*a*) all values of x (*b*) only certain values of x (*c*) no values of x 18.....

19 The minimum value of $2 \sin 2x$ is (*a*) —4 (*b*) —2 (*c*) —1 19.....

20 As x varies from 0° to 180°, a function that always decreases is (*a*) $\sin x$ (*b*) $\cos x$ (*c*) $\tan x$ 20.....