The University of the State of New York

313TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Wednesday, August 22, 1951 — 12 m. to 3 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in trigonometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1951 or number and length in minutes of lessons taken in the summer of 1951 under a tutor licensed in the subject and supervised by the principal of the school you last attended.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1951 or an equivalent program of tutoring approved in advance by the Department is required.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 *a* Derive the law of sines. Consider only the case in which the triangle is acute. [6]

b Prove the identity $1 - \sin^2 x = \dfrac{\sin 2x}{2 \tan x}$ [4]

22 Find, to the *nearest* degree, all values of x greater than $0°$ and less than $360°$ that satisfy the equation $2 \cos 2x = 3 \cos x - 1$. [10]

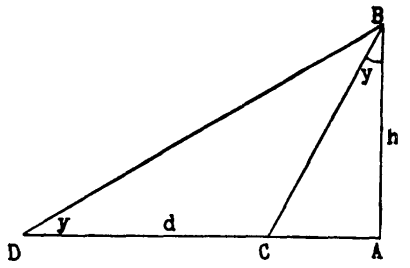
23 *a* On the same set of axes sketch the graph of $y = \tan x$ and the graph of $y = 2 \sin x$ from 0 to 2π radians inclusive. [4, 4]

b From the graphs made in answer to *a* determine the number of values of x greater than 0 and less than 2π radians for which $\tan x = 2 \sin x$. [2]

24 In the diagram at the right, $AB \perp AD$, $DC = d$, $AB = h$, $\angle ADB = y$ and $\angle CBA = y$.

a In triangle ADB express $\tan y$ in terms of sides of the triangle. [2]

b Show that $h = \dfrac{d \tan y}{1 - \tan^2 y}$ [8]



Part III

Answer at least two questions from part III.

25 In triangle ABC, $AC = 340$, $BC = 618$ and $\angle C = 108° 20'$. Find $\angle B$. [10]

26 At lookout station M, smoke from a forest fire is observed in the direction N 75° 20′ E. At lookout station R, which is 7.5 miles directly south of M, the same smoke is observed in the direction N 32° 50′ E. Find, to the *nearest tenth of a mile,* the distance of the fire from station R. [6, 4]

27 Observation posts A and B are 1750 yards apart on level ground. As an airplane flies across line AB, the angles of elevation of the airplane from A and B are 38° 40′ and 73° 10′. Find to the *nearest yard,* the height of the airplane. [5, 5]

28 A force of 47 pounds and a force of 59 pounds act on a body and produce a resultant of 76 pounds. Find, to the *nearest degree,* the angle formed by the lines of action of the resultant and the greater force. [3, 7]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Express in degrees an angle of $\frac{3\pi}{4}$ radians. 1.....

2 In a circle whose radius is 6 feet, a central angle intercepts an arc whose length is 5 feet. Find the number of radians in the angle. 2.....

3 In which quadrant does an angle lie if the sine and cosine of the angle are negative? 3.....

4 If x is a positive acute angle, express cot x in terms of sin x. 4.....

5 Find the positive value of sin $(\sec^{-1} \frac{5}{3})$ 5.....

6 Find n if $\log n = 2.3152$ 6.....

7 Find log sin 67° 32′. 7.....

8 Find, to the *nearest minute,* the positive acute angle whose cosine is .8610 8.....

9 Express tan $(x - y)$ in terms of tan x and tan y. 9.....

10 If x is a positive acute angle and $\cos x = a$, express sin $\frac{1}{2}x$ in terms of a. 10.....

11 In triangle ABC, if $a = 24$, sin $B = .8$ and sin $A = .6$, find b. 11.....

12 In triangle ABC, $\angle A = 105°$ and $\angle B = 15°$. Find the value of the ratio $\frac{a-b}{a+b}$ [Answer may be left in radical form.] 12.....

13 In triangle ABC, $a = 12$, $b = 10$, $c = 5$. Find the value of cos A. 13.....

14 Two sides of a parallelogram are 6 and 10 and the angle formed by these sides is 43°. Find, to the *nearest integer,* the area of the parallelogram. 14.....

[OVER]

15 Using the data $\angle A = 35°$, $a = 12$, $b = 10$, is it possible to construct two different triangles? [Answer *yes* or *no.*] 15.....

16 Express tan (—130°) as a function of a positive acute angle. 16.....

Directions (questions 17–20) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a, b* or *c.*

17 Cos $(180° + x)$ is equal to (*a*) —sin x (*b*) —cos x (*c*) cos x 17.....

18 Cos $6x$ + cos $2x$ is equal to (*a*) cos $8x$ (*b*) 2 cos $4x$ cos $2x$ (*c*) 2 cos $4x$ sin $2x$ 18.....

19 The maximum value of sin $2x$ is (*a*) less than (*b*) equal to (*c*) greater than the maximum value of 2 sin $\frac{1}{2}x$. 19.....

20 The statement $\sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x$ is true for (*a*) no values of x (*b*) some values of x (*c*) all values of x 20.....